Odd Semester, 2020

(Held in March, 2021)

MATHEMATICS

(Elective/Honours)

(GHS–11)

(Algebra–I and Calculus–I)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that for any three sets A, B, C, $(A-C)\cap(B-C)=(A\cap B)-C$. 4

(b) Give example of a relation which is—

(i) reflexive but neither symmetric nor transitive;

(ii) reflexive, symmetric but not transitive;

(iii) symmetric and transitive but not reflexive;

(iv) reflexive and anti-symmetric. 1×4=4

(c) If $f: x \to y$ and $g: y \to z$ are one-to-one and onto mappings, then prove that $g \circ f: X \to Z$ is one-to-one and onto and $(g\circ f)^{-1} = f^{-1}\circ g^{-1}$. 5

(d) If $A=\{x, y, z\}$, then find the power set of A. 2

2. *(a)* If $f(x)=\dfrac{1+x}{1-x}$, then prove that $2\cdot f(x)\cdot f(x^2)=1+\{f(x)\}^2$. 4

(b) A survey report reveals that 59% of college students like tea whereas 72% like coffee. Find the possible range of the percentage of college students who like both tea and coffee. 4

(c) Draw the graph of the function

$$f(x)=\begin{cases}3x+2, & x<0\\ x+1, & x\geq 0\end{cases}$$

in the interval $[-2, 2]$. Is this function continuous at $x=0$? 2+2=4

(d) Use ε-δ definition to prove that

$$\lim_{x\to 0} x\sin\frac{1}{x}=0$$ 3

UNIT—II

3. *(a)* Show that the matrix

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

is nilpotent and find its index. 2+1=3

(b) Determine if the following system of equations is consistent and if so, find the solution : 4+3=7

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

(c) Reduce the following matrix to normal form and find its rank : 5

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

4. *(a)* Obtain the inverse of the matrix

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$$

by using elementary operations. 6

(b) Show that every square matrix A can be uniquely expressed as $P + iQ$ where P and Q are Hermitian matrices. 4

(c) If A and B be two matrices conformable to form the product AB, then show that $(AB)^T = B^T A^T$, where X^T represents the transpose of the matrix X. 5

UNIT—III

5. *(a)* Find $\frac{dy}{dx}$ (any *two*) : 4×2=8

(i) $(\cos x)^y = (\sin y)^x$

(ii) $x^3 + y^3 = 3axy$

(iii) $y = (\sec x)^{\tan x}$

(b) Find the derivative of $y = e^{\sqrt{x}}$ from first principle. 3

(c) Find the slope of the curve given by $x^2 + y^2 + 2x - 4y - 20 = 0$ at (2, 6). 2

(d) Evaluate the derivative of x^7 with respect to x^4. 2

6. *(a)* Let $y = \tan^{-1} x$. Show that—

(i) $(1 + x^2) y = 1$

(ii) $(1 + x^2) y_{n+1} + 2nxy_n + n(n-1) y_{n-1} = 0$ 2+4=6

(b) Using L' Hospital's rule, evaluate the following (any *two*) : 3×2=6

(i) $\underset{x \to 0}{\text{Lt}} (\sin x)^{2 \tan x}$

(ii) $\underset{x \to 0}{\text{Lt}} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

(iii) $\underset{x \to 0}{\text{Lt}}\, x \log x$

(c) The radius of a circle is increasing at the rate of 2 cm per second. At what rate is the area increasing when the radius is 10 cm? 3

UNIT—IV

7. (a) Evaluate (any *one*) : 3

(i) $\displaystyle\int \frac{x^2}{(a+bx)^3}\,dx$

(ii) $\displaystyle\int \sqrt{\frac{x-1}{x+1}}\,dx$

(b) Evaluate (any *one*) : 4

(i) $\displaystyle\int_0^1 \frac{dx}{(1+x^2)^{3/2}}$

(ii) $\displaystyle\int_0^{\pi/2} \frac{\sin x\,dx}{\sin x + \cos x}$

(c) Using the definition of definite integral as the limit of the sum, evaluate

$$\int_0^2 (x^2+1)\,dx \qquad 5$$

(d) Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$, if it converges. 3

8. (a) Show that

$$\int_0^{\pi/2} \log \sin x\,dx = \frac{\pi}{2}\log\frac{1}{2} \qquad 4$$

(b) Evaluate : 5

$$\lim_{n\to\infty}\left\{\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right\}$$

(c) If

$$I_n = \int_0^{\pi/2} \sin^n x\,dx,$$

where n is a positive integer, $n > 1$; then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

Hence evaluate $\displaystyle\int_0^{\pi/2} \sin^5 x\,dx$. 3+3=6

UNIT—V

9. (a) Show that $v = \dfrac{A}{r} + B$ satisfies the differential equation

$$\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0 \qquad 2$$

(b) Solve : 5

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

(c) Show that the equation

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

is exact. 2

(d) Solve any *two* of the following : 3×2=6

(i) $xy^2dy - y^3dx + y^2dy = dx$

(ii) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

(iii) $\frac{dy}{dx} = \frac{6x - 2y - 7}{3x - y + 4}$

10. *(a)* Find the differential equation of the family of curves

$$y = e^x (a \cos x + b \sin x)$$

where a and b are arbitrary constants. 2

(b) Solve completely $y = px + \frac{a}{p}$ where $p = \frac{dy}{dx}$. 4

(c) Find the orthogonal trajectories of the series of hyperbolas $xy = a^2$. 3

(d) Solve any *two* of the following : 3×2=6

(i) $(D^2 - D - 2)y = e^{2x}$

(ii) $(D^2 - 8D + 15)y = 0$

(iii) $(D^3 - 1)y = 0$

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(February)

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(Algebra—I and Calculus—I)

(GHS–11)

Marks : 75

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Answer **five** questions, taking one from each unit

UNIT—I

1. *(a)* Find the domain of the function

$$f(x) \quad \frac{1}{\sqrt{|x| \quad x}}$$ 3

(b) If $f(x) \quad \frac{x \quad 1}{x \quad 1}$, show that

$$\frac{f(x) \quad f(y)}{1 \quad f(x)f(y)} \quad \frac{x \quad y}{1 \quad xy}$$ 3

(c) Draw the graph of the function $f(x) \quad [x]$, where $[x]$ denotes the greatest integer not greater than x. 3

(d) Let $f(x) \quad |x|$, show that

$$\underset{h \quad 0}{\text{Lt}} \frac{f(h) \quad f(0)}{h}$$

does not exist. 3

(e) Examine the continuity of the function f given as in Q. No. **1***(d)* on $\mathbb{R}$. 3

2. *(a)* Which of the following statements are true? 2

For a set A

(i) $A \quad P(A)$

(ii) $A \quad P(A)$

(iii) $\{A\} \quad P(A)$

(iv) $\{A\} \quad P(A)$

(b) Prove that for any two sets A and B $A \quad B \quad A \quad B^c$. 2

(c) Show that

$$\lim_{x \quad 0} \frac{1 \quad \cos x}{x^2} \quad 1$$ 3

(d) If R is a relation on $\mathbb{Z} \quad \{0\}$ and xRy if and only if $xy \quad 0$, prove that R is an equivalence relation. 3

(e) If A and B are two sets and $A \quad A \quad B \quad B$, then prove that $A \quad B$. 2

(f) $f : Q \quad Q$ is defined by $f(x) \quad 3x \quad 4$. Show that f is invertible and find $f^{\quad 1}$. 3

UNIT—II

3. *(a)* Define, with example, the following terms : 2×3=6

(i) Symmetric matrix

(ii) Skew-symmetric matrix

(iii) Diagonal matrix

(b) Reduce the matrix A to the normal form and find its rank, where

$$A \quad \begin{array}{cccc} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{array}$$

6

(c) Let A and B are Hermitian matrices, show that $AB \quad BA$ is Hermitian. 3

4. *(a)* Examine if the following system of equations are consistent :

$$\begin{array}{c} x \quad y \quad 2z \quad 4 \\ 3x \quad y \quad 4z \quad 6 \\ x \quad y \quad z \quad 1 \end{array}$$

If consistent, solve the system. 3+4=7

(b) Using elementary operation find the inverse of the matrix A, where

$$A \quad \begin{array}{ccc} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{array}$$

6

(c) Show that the matrix

$$A \quad \begin{array}{ccc} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{array}$$

is idempotent. 2

UNIT—III

5. *(a)* State the intermediate value theorem. Prove that the equation $x^4 \quad x^3 \quad 3 \quad 0$ has a real root between 1 and 2. 2+1=3

(b) Evaluate $\dfrac{dy}{dx}$ of any *two* of the following : 3×2=6

where

(i) $x^y y^x \quad 1$

(ii) $x \quad \sqrt{1 \quad u},\ y \quad \sqrt{1 \quad u}$

(iii) $y \quad \tan^{\quad 1} \sqrt{\dfrac{1 \quad \cos}{1 \quad \cos}}$

(c) Find the points on the curve

$$y \quad 2x^3 \quad 15x^2 \quad 34x \quad 20$$

where the tangents are parallel to $y \quad 2x \quad 0$. 3

(d) From the first principle obtain the derivative of $\frac{1}{\sqrt{x}}$. 3

6. *(a)* If the area of a circle increases at a uniform rate, prove that the rate of increase of the perimeter varies inversely as the radius. 3

(b) Evaluate : 2½×2=5

(i) $\underset{x \quad 0}{\mathrm{Lt}} \; \frac{1}{x^2} \quad \frac{1}{\sin^2 x}$

(ii) $\underset{x \quad 0}{\mathrm{Lt}} \; \frac{e^x \quad \sin x \quad 1}{\log(1 \quad x)}$

(c) If $y \quad \tan^{\quad 1} x$, show that

(i) $(1 \quad x^2)y_1 \quad 1$

(ii) $(1 \quad x^2)y_{n \quad 1} \quad 2nxy_n \quad n(n \quad 1)y_{n \quad 1} \quad 0$

Find also $(y_n)_0$. 1½+3½+2=7

UNIT—IV

7. *(a)* Evaluate any *two* of the following : 3×2=6

(i) $\frac{d}{5 \quad 4\cos}$

(ii) $\frac{dx}{x(x \quad 1)^2}$

(iii) $\sqrt{2ax \quad x^2}\, dx$

(b) Show that

$$\int_0^{\frac{}{2}} \sin^6 \quad \cos^3 \quad d \quad \frac{2}{63}$$ 3

(c) Find by method of summation

$$\int_0^1 x^3 dx$$ 3

(d) Evaluate, if possible,

$$\int_0 \frac{dx}{1 \quad \cos x}$$ 3

8. *(a)* Show that

$$\int_0 x \log \sin x \, dx \quad \frac{^2}{2} \log \frac{1}{2}$$ 4

(b) Evaluate : 4

$$\underset{n}{\mathrm{Lt}} \; \frac{1^2}{n^3 \quad 1^3} \quad \frac{2^2}{n^3 \quad 2^3} \quad \cdots \quad \frac{n^2}{2n^3}$$

(c) If $u_n \quad {}_0^{\frac{}{2}} x^n \sin x\, dx,\ n \quad 0$, prove that

$$u_n \quad n(n \quad 1)u_{n \quad 2} \quad n \left(\frac{}{2}\right)^{n \quad 1}$$ 5

(d) Evaluate : 2

$$_0^1 x^3 \sqrt{1 \quad 3x^4}\, dx$$

UNIT—V

9. *(a)* All the circles which touch the Y-axis at the origin is given by the equation $x^2 \quad y^2 \quad 2cx$. Obtain the differential equation of the family. 3

(b) Solve any *four* of the following : 3×4=12

(i) $(3y \quad 2x \quad 4)\, dx \quad (4x \quad 6y \quad 5)\, dy$

(ii) $x^2 dy \quad (xy \quad 2y^2) dx \quad 0$

(iii) $3e^x \tan y\, dx \quad (1 \quad e^x) \sec^2 y\, dy \quad 0$

(iv) $x\, dy \quad y\, dx \quad \cos \frac{1}{x}\, dx$

(v) $x(x^2 \quad y^2 \quad 4)\, dx \quad y(x^2 \quad y^2 \quad 9)\, dy \quad 0$

(vi) $(1 \quad y^2)\, dx \quad (\tan^{\ 1} y \quad x)\, dy$

10. *(a)* Solve any *two* of the following : 2½×2=5

(i) $p(p^2 \quad xy) \quad p^2(x \quad y)$

(ii) $y \quad 2px \quad p^2$

(iii) $p^2 \quad py \quad x \quad 0$

(p stands for $\frac{dy}{dx}$

(b) Find the orthogonal trajectories of the family of confocal conics

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$$

is a parameter. 4

(c) Reduce the equation $(px \quad y)(x \quad py) \quad 2p$ to Clairaut's form by the substitution $x^2 \quad u,\ y^2 \quad v$. 2

(d) Solve any *two* of the following : 2×2=4

(i) $\frac{d^2y}{dx^2} \quad 8\frac{dy}{dx} \quad 25y \quad 0$

(ii) $\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad y \quad 0$

(iii) $\frac{d^2y}{dx^2} \quad 6\frac{dy}{dx} \quad 25y \quad 0$

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