Virtual steering limitations for reversing an articulated vehicle with off-axle passive trailers

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Abstract - The paper reports on motion control for a nonholonomic vehicle pushing several passive trailers. To avoid inter-unit collisions, tractor steering limitations were successfully applied during forward motion in our previous work [1]. Here, this approach is extended to the case of backward motion for off-axle trailers. To this end, the last trailer is considered to behave as a virtual tractor. Then, virtual steering is computed by applying any path tracking method for single non-holonomic mobile robots, incorporating limitations if necessary. Finally, kinematic relationships are used to translate motion commands to the actual tractor. The method has been validated experimentally with two heterogeneous off-axle trailers attached to the tracked mobile robot Auriga-α.

Keywords: articulated vehicle; backward motion; interunit collision avoidance; kinematic relationship;mobile robot Auriga-α; motion control; nonholonomic mobile robot; nonholonomic vehicle; off-axle passive trailer; path tracking method; virtual steering; virtual tractor; collision avoidance; mobile robots; motion control; road vehicles; robot kinematics; steering systems.

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Virtual Steering Limitations for Reversing an Articulated Vehicle with Off-Axle Passive Trailers

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***Abstract*—The paper reports on motion control for a non-holonomic vehicle pushing several passive trailers. To avoid inter-unit collisions, tractor steering limitations were successfully applied during forward motion in our previous work [1]. Here, this approach is extended to the case of backward motion for off-axle trailers. To this end, the last trailer is considered to behave as a virtual tractor. Then, virtual steering is computed by applying any path tracking method for single non-holonomic mobile robots, incorporating limitations if necessary. Finally, kinematic relationships are used to translate motion commands to the actual tractor. The method has been validated experimentally with two heterogeneous off-axle trailers attached to the tracked mobile robot Auriga-α.**

I. INTRODUCTION

Articulated train-like vehicles consisting of a tractor and several passive trailers are of common use because they can increase transportation efficiency. The position of trailer hitches is relevant when driving this kind of systems. A hitching is called 'off-axle' when it lies some distance behind the preceding unit's rear axle. Off-axle kingpin hitching is widely employed in many practical applications, like airport luggage carriers, due to its simple mechanics. Backward motion control is useful in cases like navigation in narrow spaces or the parking problem.

Articulated vehicles pose specific control problems, such as the jackknife phenomenon (for backward motion), inter-unit collision avoidance, and off-tracking (i.e., trailer deviation from the towing unit's path). As a result, control strategies for single vehicles are not directly applicable for these systems.

The one-trailer problem has been treated in literature for a long time [2], including the case of reverse movement [3]. However, only few works have contemplated the complex backward motion of a tractor pushing multiple passive trailers. Most of them address nonlinear control of the tractor, with feedback linearization [4], fuzzy control [5] or switching control [6].

Controlling the last trailer as a virtual tractor has been employed as a strategy for backward motion. This allows addressing the problem much in the same way as in forward motion. Applications of this principle for one trailer systems have been founded on non-linear control [7], non-holonomic contraint [8] and simple kinematics [9]. A multi-trailer case with a holonomic vehicle of the kinematic approach has been presented for homogenous off-axle hitches that lie at the same distance between consecutive units [10]. For this setup, joint angle errors do not result in a lack of stability, but in tracking errors [11].

For path planning, mechanical steering bounds of a car-like tractor have been considered in the one trailer case [12]. The idea of imposing further steering limitations for path tracking has proven useful to avoid jackknife and collisions of a tractor with a single trailer for both forward and backward motion [9], and also for forward motion of multiple passive trailers [1].

In this paper, we propose to compute curvature limitations for the last trailer acting as a virtual vehicle during reverse motion. Virtual actuations are translated to the real tractor via kinematics of the articulated system. This is possible due to the overdamped response of joint angles. Thus, interunit collision is avoided while the virtual vehicle is controlled with a well-proven path tracking method for non-holonomic mobile robots. The method has been validated experimentally with an heterogeneous off-axle two-trailer setup attached to the tracked mobile robot Auriga-α. In this way, it has been possible to follow arbitrary paths in reverse while avoiding inter-unit collisions.

The paper is organized as follows. Backward motion control of the articulated system via the virtual tractor is described in the next section. The steering limitation idea for driving articulated vehicles in reverse is presented in section III. Section IV deals with implementation details and experimental results with Auriga-α. Conclusions and future work are discussed in section V. Finally, acknowledgments and references complete the paper.

II. BACKWARD MOTION CONTROL BASED ON KINEMATICS

A. Kinematic model for trailers with off-axle hitching

An n-trailer system can be considered as a kinematic chain consisting of $n+1$ units. Unit 0 is the tractor vehicle, and trailers are numbered from 1 to n. Local coordinate frames are attached to each unit i, with the Y_i axis coincident with the longitudinal axis and the X_i axis lying on the rear axle. The state of this system can be defined by the position and orientation of the tractor, plus the angle of each joint.

The parameters of the kinematic chain are shown in Fig. 1. Distances L_{if} and L_{ib} are always positive constants for

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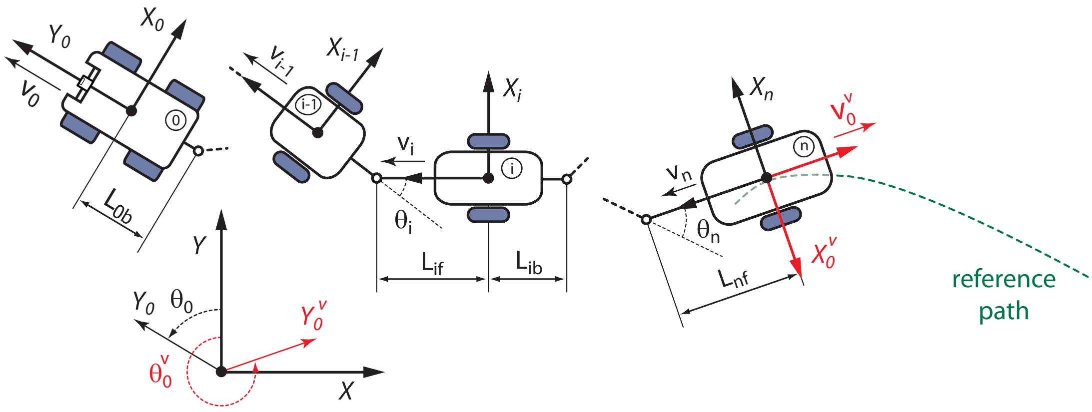


Fig. 1. Parameters of the kinematic chain.

trailers with off-axle hitching. The relative angle of the i^{th} trailer with respect to the $(i-1)^{th}$ unit is represented by θ_i. θ_0 is the tractor's heading with respect to the global coordinate system XY. All these angles are considered counterclockwise positive.

Heading ϕ_i of the i^{th} trailer with respect to the global frame can be computed as:

$$\phi_i = \sum_{j=0}^{i} \theta_j . \tag{1}$$

Besides, the relative angular velocity ω_i of the i^{th} unit with respect to the preceding one is defined by:

$$\omega_i = \frac{d\theta_i}{dt}. \tag{2}$$

Consequently, the absolute angular velocity Ω_i can be calculated as:

$$\Omega_i = \sum_{j=0}^{i} \omega_j . \tag{3}$$

Since $v_i = ds_i/dt$, where s_i is the distance traveled by the i^{th} unit, curvature γ_i is related to Ω_i and v_i in the following way:

$$\gamma_i = \frac{d\phi_i}{ds_i} = \frac{\Omega_i}{v_i}. \tag{4}$$

The only control inputs to the articulated system are the longitudinal v_0 and angular speed Ω_0 of the tractor. The angular Ω_i and linear v_i speeds of trailer i depend on how v_0 and Ω_0 are propagated through the kinematic chain:

$$v_i = -v_{i-1}\, \cos(\theta_i) + \Omega_{i-1}\, L_{i-1\,b}\, \sin(\theta_i), \tag{5}$$

$$\Omega_i = -\frac{v_{i-1}\, \sin(\theta_i) + \Omega_{i-1}\, L_{i-1\,b}\, \cos(\theta_i)}{L_{i\,f}}, \tag{6}$$

where dynamic effects are neglected under the assumption of not overloaded trailers moving with moderate velocities and accelerations [1].

B. Virtual tractor for backward motion control

Backward motion control of an articulated vehicle can be implemented by considering the last trailer n as a virtual tractor that moves forward (see Fig. 1). In this way, the actual tractor is commanded to act as the last trailer so that the articulated system moves forward virtually. The virtual tractor local axis XY_0^v is the same as the last trailer local frame XY_n but rotated 180°. Thus, the position and heading of the virtual tractor with respect to the global coordinate system XY are:

$$x_0^v = x_n, \quad y_0^v = y_n, \quad \phi_0^v = \phi_n + \pi. \tag{7}$$

Each control interval for the trailer system in backward motion requires three steps:

1) Compute position (x_0^v, y_0^v) and orientation ϕ_0^v of the virtual tractor with respect to the global coordinate system XY. This can be accomplished by mounting specific sensors in the last trailer [10], or by applying kinematic relationships using the tractor sensors [9].
2) Apply a forward motion controller to the virtual tractor to obtain its input set-points $(v_{0s}^v, \gamma_{0s}^v)$. This can be done with controllers for single non-holonomic vehicles [9] [10], but inter unit collisions should be avoided [9].
3) Transform virtual set-points $(v_{0s}^v, \gamma_{0s}^v)$ into set-points (v_{0s}, Ω_{0s}) for the actual tractor. To implement this, virtual tractor inputs are firstly related to v_{ns} and Ω_{ns} as follows:

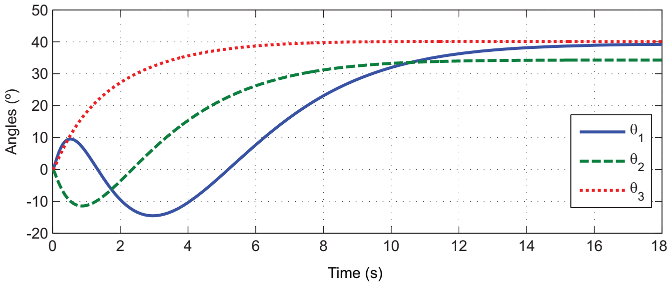
$$v_{ns} = -v_{0s}^v, \quad \Omega_{ns} = \gamma_{0s}^v v_{0s}^v . \tag{8}$$

Then, v_{ns} and Ω_{ns} are backward-propagated until the actual tractor is reached, using the inverse of equations (5) and (6):

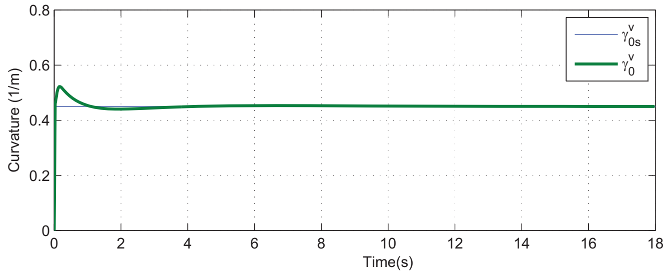
$$v_{i-1s} = -\Omega_{is}\, L_{i\,f}\, \sin(\theta_i) + v_{is}\, \cos(\theta_i), \tag{9}$$

$$\Omega_{i-1s} = -\frac{\Omega_{is}\, L_{i\,f}\, \cos(\theta_i) + v_{is}\, \sin(\theta_i)}{L_{i-1\,b}}, \tag{10}$$

which depends on the current values of the joint angles θ_i. Note that this transformation cannot be applied when any of the trailers are on-axle because $L_{i-1\,b}$ is null.



(a)



(b)

Fig. 2. Simulated transitory responses for a step curvature virtual set-point for a three trailer setup. (a) Hitch angles. (b) Curvatures.

C. Response of the articulated vehicle to virtual set-points

If the response of the actual tractor were instantaneous, so it would be the response of the virtual one. In this case, (5) and (6) would be transformed into a model of the kinematic chain where the last trailer becomes the tractor. Consequently, the articulated vehicle would behave as a forward motion n-trailer system, where the response of all joint angles to tractor steering is overdamped [1].

However, there is a transient from (v_0, Ω_0) to $(v_{0\,s}, \Omega_{0\,s})$ which depends on the dynamics of the tractor actuators. The response is propagated through the kinematic chain with (5) and (6). This provokes a transient on the curvature and speed of the virtual tractor, from $(v_0^v,\, \gamma_0^v)$ to $(v_{0s}^v,\, \gamma_{0s}^v)$.

Nevertheless, when moving at moderate speeds, the transient response of the tractor actuators is faster than the transitory of the articulated system. In practice, this implies that all joint angle transients are overdamped, as in forward motion. This is illustrated in Fig. 2, where a virtual step curvature for a simulated three off-axle trailer setup is presented. Note that the transitory of virtual curvature is much faster than the setting time of joint angles. As in forward motion, a response similar to that of nonminimal phase linear systems appears in the angles associated to off-axle joints that follow the first virtual trailer (i.e., θ_1 and θ_2) [1].

In forward motion, the tractor is always moving forward, but the trailers can experience negative linear speeds. Thus, actual motion commands for the tractor when acting as the last virtual trailer, may include both positive and negative speeds.

III. VIRTUAL STEERING SET-POINT LIMITATIONS

The overdamped response of every θ_i allows that the same strategy for calculating the steering limitations of an articulated system moving forward [1] can be employed for backward motion control.

Three types of steady operational limits for the curvature of the i^{th} trailer can be considered for the $(i-1)^{th}$ unit:

- *Equilibrium limit* $\gamma_{i\,m_1}$ ensures that equilibrium points exist.
- *Mechanical limit* $\gamma_{i\,m_2}$ avoids inter-unit collision.
- *Forward-propagated limit* $\gamma_{i\,m_3}$ guarantees equilibrium and mechanical limits for all the previous units.

The formulas for steering limitations proposed for forward motion in [1] have to be adapted to the backwards case. Thus, the role of lengths $L_{i-1\,b}$ and $L_{i\,f}$ is swapped for each unit and the propagation of limitations is forward instead of backward. For the sake of clarity, it has been assumed that mechanical limits are symmetrical, which provides symmetrical curvature limits represented by their absolute values.

First, it is necessary that every unit tends to an equilibrium point. The equilibrium angle θ_{is} is given by:

$$\theta_{i\,s} = -\arctan(\gamma_{i-1s} L_{i-1\,b}) - \arctan(\gamma_{is} L_{i\,f}). \tag{11}$$

During virtual forward motion, this implies that the absolute value of curvature for the i^{th} unit should be maintained below the following limit:

$$\gamma_{i\,m_1} = \begin{cases} \frac{1}{\sqrt{L_{i-1\,b}^2 - L_{i\,f}^2}}, & L_{i\,f} < L_{i-1\,b}, \\ \infty, & L_{i\,f} \geq L_{i-1\,b}. \end{cases} \tag{12}$$

Second, the joint between every pair of consecutive units i and $(i-1)$ can have an inherent mechanical bound $\theta_{i\,m}$. If this angular bound is surpassed, then inter-unit collision or link breakage occurs. Hence, a further limit should be imposed:

$$\gamma_{i\,m_2} = \left| \frac{\sin(\theta_{i\,m})}{L_{i-1\,b} + L_{i\,f}\cos(\theta_{i\,m})} \right|. \tag{13}$$

Third, a new curvature constraint $\gamma_{i\,m_3}$ is inherited from the propagation of the most restrictive limitation from the previous unit $\gamma_{i-1\,m}$, if it exists. This is accomplished by means of:

$$\gamma_{im_3} = \begin{cases} \frac{\gamma_{i-1m}}{\sqrt{1+\gamma_{i-1m}^2 (L_{i-1\,b}^2 - L_{i\,f}^2)}}, & \begin{matrix} (1<i<n) \text{ and} \\ (\gamma_{i-1m}^2 (L_{i\,f}^2 - L_{i-1\,b}^2) < 1), \end{matrix} \\ \infty, & \text{otherwise.} \end{cases} \tag{14}$$

Among these three limits for the following unit's curvature, the most restrictive one is chosen:

$$\gamma_{i\,m} = \min(\gamma_{i\,m_1}, \gamma_{i\,m_2}, \gamma_{i\,m_3}). \tag{15}$$

By recursively applying (15) starting from the first trailer, this procedure eventually results on the safest steady curvature limitation for the last trailer $\gamma_{n\,m}$, and hence, the virtual tractor $\gamma_{0\,m}^v = |-\gamma_{n\,m}| = \gamma_{n\,m}$.

In practice, $\gamma_{0\,m}^v$ requires further reduction to guarantee that equilibrium and mechanical limits are also observed despite non-minimal phase-like response in trailers with off-axle hitching. This can be accomplished by a transient analysis via simulation in order to obtain a tentative value that should be refined experimentally. The worst case transition between two steady-states is obtained with a set-point change from $\gamma_{0\,m}^v$

to $-\gamma^v_{0\,m}$ and viceversa. If the transient is unsatisfactory, the limit $\gamma^v_{0\,m}$ is gradually decremented by a certain $\Delta\gamma$ until the transient response becomes acceptable (see Algorithm 1).

It must be noted that the steady-state for the $\gamma^v_{0\,m}$ set-point implies limit equilibrium hitch angles $\theta_{i\,e}$, as stated by (11). If the initial configuration of the n-trailer system includes some angles that exceed these limits, then no constant curvature set-points for the virtual tractor, not even straight-line motion, can guarantee a safe transition. On the contrary, if these angular limits are initially satisfied, the application of arbitrary curvature set-points below the steering limits will never result on exceeding the equilibrium hitch angles.

Algorithm 1: Virtual Tractor Steering Limitation

for $i = 1$ *to* n **do** *(Steady analysis)*
  Compute *equilibrium limit* $\gamma_{i\,m_1}$ (12)
  Compute *mechanical limit* $\gamma_{i\,m_2}$ (13)
  Compute *forward-propagation limit* $\gamma_{i\,m_3}$ (14)
  $\gamma_{i\,m} = \min(\gamma_{i\,m_1}, \gamma_{i\,m_2}, \gamma_{i\,m_3})$
$\gamma^v_{0\,m} = \gamma_{nm}$
repeat *(Transient analysis)*
  $\gamma^v_{0\,m} \leftarrow (\gamma^v_{0\,m} - \Delta\gamma)$
  Evaluate step response from $\mp\gamma^v_{0\,m}$ to $\pm\gamma^v_{0\,m}$
until *equilibrium is reached without inter-unit collision*
Result: $\gamma^v_{0\,m}$

IV. Implementation

A. Auriga-α and its trailers

The backward motion control with steering limitations presented in the previous section has been tested on the Auriga-α mobile robot (see Fig. 3). Its dimensions are $1.24\,\mathrm{m}$ (l), $0.75\,\mathrm{m}$ (w) and $0.84\,\mathrm{m}$ (h), and it weights $258\,\mathrm{kg}$.

Auriga-α is a tracked vehicle driven by two DC motors with gear-reduction and incremental shaft encoders for dead-reckoning. The maximum speed of each track is $1\,\mathrm{m/s}$. The vehicle top speed coincides with this limit in straight-line motion, but it decreases to zero according to the increase in the demanded curvature. The track speed controller runs in an on-board DSP every $10\,\mathrm{ms}$, which also provides odometric data every $30\,\mathrm{ms}$.

The tractor can be controlled as a differential drive vehicle using the approximated kinematic model presented in [13]. This also allows to define the virtual rear axle of this vehicle, which is relevant to determine hitch parameter L_{0b} (see Fig. 1).

A Sick LMS 200 time-of-flight laser scanner is mounted on the forward part of the vehicle at a distance of $0.5\,\mathrm{m}$ ahead of its coordinate center. To correct odometric estimations of the actual tractor, an accurate laser scan matching technique has been employed every $270\,\mathrm{ms}$ [14].

The dimensions of the two wheeled trailers are similar to the tractor. The first trailer is employed for carrying loads, while the second one is for spraying (see Fig. 3). Each angle θ_i is indirectly obtained with a draw-wire displacement sensor.

Kinematic parameters and mechanical angle limits for this particular setup are the following: $L_{0b} = 0.71\,\mathrm{m}$, $L_{1f} = 0.99\,\mathrm{m}$, $L_{1b} = 0.61\,\mathrm{m}$, $L_{2f} = 0.81\,\mathrm{m}$, $\theta_{1\,m} = 68°$ and $\theta_{2\,m} = 43.6°$.

B. Backward motion controller

To estimate the virtual tractor pose with respect to the global frame (step 1 of section II-B), θ_i measurements and geometric relations between units are employed:

$$x_i = x_{i-1} + L_{i-1\,b}\sin(\phi_{i-1}) + L_{i\,f}\sin(\phi_i), \qquad (16)$$

$$y_i = y_{i-1} - L_{i-1\,b}\cos(\phi_{i-1}) - L_{i\,f}\cos(\phi_i), \qquad (17)$$

including (1) to propagate heading.

Curvature for the virtual tractor γ^v_{0s} is obtained (step 2 of section II-B) using the Pure-Pursuit path tracker [15] with positive constant speed set-point v^v_{0s}. The path tracking controller has been implemented in an on-board Pentium-IV industrial PC at $2.2\,\mathrm{GHz}$ under a real-time operating system providing virtual set-points every $30\,\mathrm{ms}$.

To avoid inter-unit collision, virtual tractor curvature is limited before actual tractor inputs are obtained with step 3 of section II-B. By applying Algorithm 1, $\gamma_{1\,m1} = \infty$, $\gamma_{1\,m2} = 0.85\,\mathrm{m}^{-1}$ and $\gamma_{1\,m3} = \infty$. Then, $\gamma_{1m} < 0.85\,\mathrm{m}^{-1}$, which is propagated to the second trailer as $\gamma_{2\,m3} = 0.95\,\mathrm{m}^{-1}$. The steady analysis renders $\gamma_{2m} = \gamma^v_{0\,m} = 0.57\,\mathrm{m}^{-1}$ since $\gamma_{2\,m1} = \infty$ and $\gamma_{2\,m2} = 0.57\,\mathrm{m}^{-1}$.

Simulated transient analysis provides $\gamma^v_{0\,m} = 0.45\,\mathrm{m}^{-1}$ after a few of simulated iterations with $\Delta\gamma = 0.01\,\mathrm{m}^{-1}$ decrements. Experimentally, this limit is too conservative and can be increased to $\gamma^v_{0\,m} = 0.5\,\mathrm{m}^{-1}$. The equilibrium hitch angles associated to this virtual steering limit are $\theta_{1\,e} = 43.58°$ and $\theta_{2\,e} = 38.48°$.

C. Experimental results

Fig. 4 shows the U-shaped reference path in the indoor environment where the experiment took place. Starting from the position depicted as a circle, the path turns around a column with $0.44\,\mathrm{m}^{-1}$ constant curvature, which is close to γ^v_{0m}. In the experiment, the lookahead distance of the Pure Pursuit path tracker has been set to $1.1\,\mathrm{m}$ and the constant virtual tractor forward speed to $v^v_{0s} = 0.3\,\mathrm{m/s}$.

The paths followed by Auriga-α and the last trailer are shown in Fig. 5, as well as the reference path for the virtual tractor. Respective initial points are marked with small circles. Initially, tractor and trailers are far from the path but with the virtual tractor almost aligned with it. The controller successfully takes the virtual tractor to the reference path. Auriga-α, i.e. the last virtual trailer, exhibits a small off-tracking error which is natural for this type of heterogeneous articulated system. Off-tracking tends to disappear in the following straight line segment.

Fig. 6(a) shows that trailer angles are maintained under their corresponding limit values. Fig. 6(b) presents the curvature set-points for the virtual tractor. Since the position of the virtual tractor is computed from the hitch angle readings, these set-points are affected by noise. This noise is eventually filtered by

Fig. 3. Top view of the mobile robot Auriga-α (left) with a utility trailer (center) and a spraying trailer (right).

Fig. 4. Photograph of the indoor environment and the reference path.

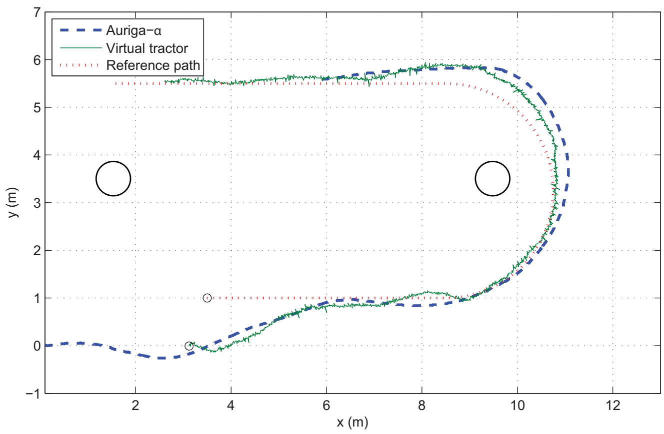


Fig. 5. Path tracking of Auriga-α pushing 2 trailers with virtual steering limitations.

the actual tractor actuators. The effect of steering limitations is visible at the beginning of the approximation path and, also, while tracking the arc (see Fig. 6(b)). This limitation avoids that θ_2 reaches its limit. If no steering limitation for the virtual tractor were applied, then a collision between both trailers would occur.

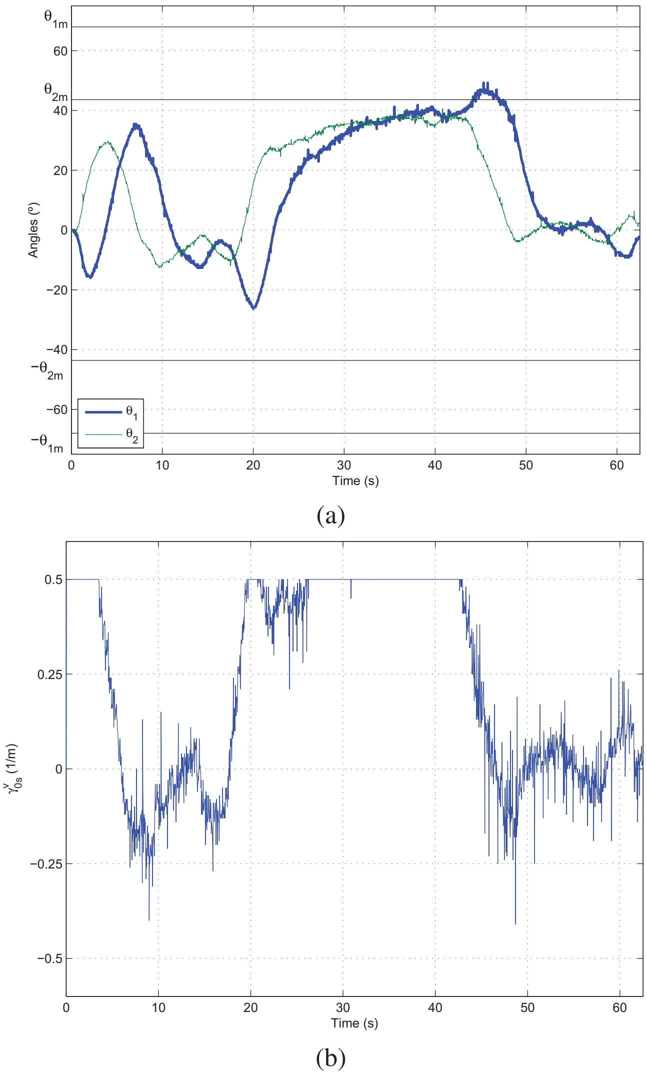


Fig. 6. Experimental data for the tracked path. (a) Hitch angles of the first and second trailers. (b) Curvature set-points for the virtual tractor.

V. Conclusions

This paper proposes a pragmatic approach for backward driving an articulated vehicle with several off-axle passive trailers. To this end, the last trailer is defined as a virtual tractor. The method introduces a backward motion controller with steering limitations for the virtual tractor based on the kinematic analysis of inter-unit collisions and the existence of equilibrium of the trailers. Thus, path-tracking and path-planning methods for single non-holonomic vehicles can be easily adapted to push passive trailers.

Experimental results have shown the successful application of this approach for the tracked mobile robot Auriga-α pushing two heterogeneous off-axle trailers.

Future work includes the relaxation of the virtual steering limits when instantaneous hitch angles are far from their corresponding limits. It is also of interest the application of steering limitations to articulated vehicles with on-axle trailers during backward motion.

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