# A Two-Point Visual Control Model of Steering


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# Abstract

When steering down a winding road, drivers have been shown to use both near and far regions of the road for guidance during steering. We propose a model of steering that explicitly embodies this idea, using both a "near point" to maintain a central lane position and a "far point" to account for the upcoming roadway. Unlike control models that integrate near and far information to compute curvature or more complex features, our model relies solely on one perceptually-plausible feature of the near and far points, namely the visual direction to each point. The resulting parsimonious model can be run in simulation within a realistic highway environment to facilitate direct comparison between model and human behavior. Using such simulations, we demonstrate that the proposed two-point model is able to account for four interesting aspects of steering behavior: curve negotiation with occluded visual regions, corrective steering after a lateral drift, lane changing, and individual differences.

# Introduction

How do drivers steer their car around the curves of a winding road? On the surface this would seem to be an effortless task that can be achieved simply by noting the car's present position in the lane and steering toward the center to correct lateral error. However, due to inherent delays between perception and action and the fact that the driver's attention can be diverted from the steering task for (possibly) extended periods of time, this simple error correction strategy is not effective at fast driving speeds. Instead, steering has been described as a two-level control strategy (e.g., Donges, 1978; Land & Horwood, 1995) that utilizes both a "near" and a "far" region of the roadway to produce successful navigation: information from the far region helps to preview the road's upcoming trajectory and (possibly) to compensate for this trajectory, and information from the near region helps to correct the car's current position within the lane boundaries.

The strongest evidence in support of the two-level model of steering comes from a driving simulator study in which only small slices of the road (1 deg height x 43 deg width) were visible to the driver when negotiating a curve (Land & Horwood, 1995). When only one segment of the road was presented to the driver, steering performance was not as accurate as when the whole road was visible regardless of the position of the segment. Adding a second visible segment greatly improved performance; in fact, when the driver was presented with one "near" segment (approximately 7-8 m from the front of the car) and one "far" segment (approximately 10-20 m from the front of the car), driving performance was as good as with the entire road visible.

Given that near and far regions are both critical for successful steering, how do drivers utilize the information in these regions to steer, and how might we model this behavior? In conjunction with the empirical work described above, Donges (1978) and Land and his colleagues (1994, 1998) describe models of steering that are consistent with the empirical evidence. Donges (1978) proposed a two-level model that combines a closed-loop control mechanism with an open-loop process that plans and executes a steering trajectory on the basis of a prediction about the desired path of travel. Donges envisioned the main control input used for this prediction to be the perceived degree of road curvature estimated from visual information a considerable distance ahead of the car's current position (10-20m). Similarly, Land and Lee (1994) posit that that drivers estimate the magnitude of road curvature using the road's tangent point i.e., the point at which the inside edge of the curve changes direction. This point has the special property that it can be used to estimate curvature without requiring a judgment of absolute distance (Land & Lee, 1994) — specifically,

$$Curvature = \frac{1}{R} = \frac{\theta^2}{2d}$$

where $R$ is the radius of the curve, $\theta$ is the visual angle between the vehicle's current direction of heading and the tangent point, and $d$ is distance between the intended path of travel and the inside lane edge. To estimate curvature from any other distant point on the road surface requires an estimate of the absolute distance to the point; a quantity which previous research has consistently demonstrated cannot be judged accurately by drivers (e.g., Groeger, 2000; Teghtsoonian & Teghtsoonian, 1970). (Although the quantity $d$ in the curvature equation is also an absolute distance, as suggested by Land, 1998, estimation of this value would be greatly simplified if the driver maintains a constant distance from the lane edge while going around the bend — that is, they do not "cut the corner" while going around the curve.) Consistent with this model, studies of eye movements during curve negotiation have shown that drivers maintain fixation a considerable distance ahead of the car's current position on the tangent point of the curve (Land & Lee, 1994). The driver's eyes appear to seek out this point roughly 1-3 seconds before entering the curve.

Despite the empirical support for these two models, one major limitation is that research has shown that human observers cannot estimate curvature accurately. In a psychophysical study, Fildes and Triggs (1985) used perspective line drawings of road curves and measured perceived curvature using a magnitude estimation technique. Observers consistently underestimated road curvature and, more importantly, judgments were not appropriately related to curve geometry.

Reductions in curve radius resulted in a paradoxical decrease in perceived curvature and curvature judgments were most strongly influenced by the curve deflection angle. This angle is not a reliable predictor of road curvature as two road curves with the same curve deflection angle can have very different curvature depending on radius. Similarly, Shinar (1977) reported a similar effect during active curve negotiation. From accident reports, he identified an "illusive curve phenomenon" where the curvature of short radius curves is dangerously underestimated (see also Virsu, 1971). Given that Donges' two-level model (and the Land & Lee, 1994, model) uses a single estimation of the curvature of the upcoming bend, these curvature judgments would often be inaccurate and would sometimes even require major corrections by the closed-loop mechanism (e.g., the planned trajectory would be much flatter than the required path of travel). This inaccurate prediction mechanism is not consistent with experimental results; for instance, visual occlusion studies have shown that drivers are able to steer successfully in the face of sustained periods of visual occlusion (Godthelp, 1986).

Without an accurate estimate of road curvature, what other aspects of the roadway — particularly aspects that might derive from near and far regions — could provide the necessary guidance for a two-level model of steering? Early research modeled steering as a control process where drivers steer so as to regulate the value of some perceptual variable around a desired value (e.g., Biggs, 1966; McRuer, 1969) and identified several sources of "near" road information. The most obvious variable that could be used for this type of control is the current perceived distance to the edge of the lane: if the driver can keep this distance constant (i.e., at the midpoint between edges of the lane) curve negotiation will be successful. Other perceptual variables that could be used for steering control are the instantaneous difference between the perceived direction of heading and the center of the lane (regulated around a value of zero) and the instantaneous visual direction (i.e., bearing) of a point on the edge of the lane (regulated around any constant value: Llewellyn, 1971). Note that all of these visual information sources can be estimated from parts of the road relatively near (<10 m) to the car. There is evidence that human observers are highly sensitive to all of these optical variables as discrimination thresholds are low for small changes in position within a boundary (e.g., bisection acuity: Gray, 1997), perceived heading (Crowell & Banks, 1992), and perceived visual direction (Ono, 1991). Therefore, these control parameters are all perceptually plausible and have proven highly effective in guiding robotic navigation (e.g., Masaki, 1995; Rushton, Wen & Allison, 2002).

Another approach to the visual guidance of steering centers on the perception of optic flow as the driver moves through the environment. Early theories in this area proposed that the direction of heading could be derived from the focus of expansion (FoE) of the optical flow pattern (Gibson, 1958) and thus steering can be achieved simply maintaining alignment of the FoE with the center of the road. As first noted by Regan & Beverley (1982), detecting the FoE is often complicated by the fact that the optical flow (produced by the observer's self-motion) is masked by the retinal flow (produced by eye and head movement), however several effective models of retinal flow decomposition have been developed (e.g., Royden, Crowell, & Banks, 1994; Warren, Li, Ehrlich, Crowell, & Banks, 1996). Models of high speed steering on the basis of optic flow that can account for many aspects of steering behavior have recently been developed (Fajen, 2001; Wilkie & Wann, 2003). However, these models cannot explicitly account for the fact that drivers appear to use information from both the near and far road during steering (Land & Horwood, 1995).

To summarize, although several models of steering behavior have been developed none of the models explicitly define perceptually plausible sources of "near" and "far" visual information that is used by the driver.

# A Two-Point Visual Control Model of Steering

We propose a two-level model of steering control that uses the perceived visual direction of two salient visual points, a "near point" in the near region and a "far point" in the far region of the roadway. This model, a generalization of an earlier prototype model (Salvucci, Boer, & Liu, 2001), was inspired by conceptually simple accounts of control and locomotion (e.g., Llewellyn, 1971; Rushton, Wen, & Allison, 2002) as well as related models of steering (e.g., Wilkie & Wann, 2003).

The key difference between our model and previous models is that our model *explicitly* includes both a near and far point, using both to guide steering in ways consistent with the empirical literature noted earlier. Rather than estimating curvature or other computable quantities, it relies solely on perceived visual direction as a directly perceivable and perceptually plausible input. This results in a straightforward computational model that, as we will demonstrate, can be validated against empirical data for several aspects of driver steering behavior — namely, curve negotiation, corrective steering, and lane changing.

The central components of our control model of steering, the near point and the far point, are illustrated in Figure 1. The near point (shown with open circles) represents the center of the road at some nearby distance with which the model can monitor both lateral position and stability. The near point is assumed to be a fixed distance in front of the driver (and thus in front of the vehicle), namely a convenient distance that is near enough to monitor lateral position but far enough that the driver can comfortably see the region through the vehicle windshield. It is important to note that using the near point does not necessarily require visually fixating the near point — the driver can obtain the relevant information peripherally without explicit fixation. The far point (shown with crosses) represents some salient distant point with which the model can monitor lateral stability, and given its distance, maintain a predictive steering angle that compensates for the upcoming road profile. As such, the model has several salient visual points available as potential candidates for the far point. First, the vanishing point (Fig 1A) of a straight roadway indicates the distant convergence of the left and right road lines and thus could serve as a natural far point. Second, the tangent point (Fig 1B) of a curved roadway has been noted as a salient visual point that attracts a significant amount of drivers' gaze during curve negotiation (Land & Lee, 1994) and can guide stable steering around the curve (Land, 1998) . Third, a lead car, when present, can also serve as a stabilizing far point (Fig 1C) given that it precedes the driver's car down the roadway, and also has the added benefit of providing useful information for longitudinal control as well (i.e., acceleration and braking). Of course, much of the visual flow field would not be appropriate for use as the far point;

our model simply identifies the above three salient points as a representative sample of three very reasonable choices for use as the far point.

<< Insert Figure 1 here >>

While the near point always remains fixed, the control model can switch between far points, utilizing whatever potential far point is present and/or most convenient at the current time. For this work, we assume that the model uses the vanishing point as the far point during navigation of a straight road, but switches to using the tangent point as the far point when the tangent point appears during navigation of a curved road. Two points about the model's use of the far point warrant further explanation. First, empirical data suggests that during car following, drivers typically fixate primarily on the lead car rather than any salient road point (Salvucci, Boer, & Liu, 2001). Although we do not consider car following in this work, the model could easily do the same, using the lead car as a far point (instead of the vanishing point or tangent point) when such a lead car is present. Second, empirical data also suggests that drivers may not always utilize the tangent point on curved roads, but instead sometimes tend to fixate the center of the road (Wilkie & Wann, 2003b). Again, the model could just as easily fixate the center of road and use it as the far point to guide steering. Thus, the control model is flexible about which point is actually used as the far point, as long as the far point represents a salient stable point in the distant visual field.

We derive our steering control law from a standard proportional-integral, or PI, controller (see Ogata, 1990). Let us first assume that the driver is steering such that direction of heading remains a fixed visual angle from a single target point. Given the desired target's visual direction $\theta$, we wish to adjust the steering angle $\varphi$ to keep $\theta$ stable at 0; thus, $\theta$ represents an error term which the system continually attempts to minimize. We can describe a PI controller for this task as

$$\varphi = k_P\theta + k_I \int \theta dt \qquad \text{Equation 1}$$

or its derivative form

$$\dot{\varphi} = k_P \dot{\theta} + k_I \theta$$

Equation 2

Equation 1 keeps the steering angle $\varphi$ proportional to $\theta$ and also proportional to the cumulative error in $\theta$. Equivalently, Equation 2 keeps the change in steering proportional to the change in $\theta$ and also proportional to the actual value of $\theta$. Both formulations have constant values $k_P$ and $k_I$ to scale the proportional and integral terms, respectively. The second equation has an especially straightforward interpretation: the system simply keeps the target point stable (the first term) while keeping it as close to 0 as possible (the second term). This controller is a similar to the point-attractor approach incorporated into related models of control (e.g., Wilkie & Wann, 2003; Fajen & Warren, 2003) as will be discussed in detail below.

The key aspect of our two-point model is that it utilizes *both* a near and far point for stable control. Rather than a single variable $\theta$, we have instead a variable $\theta_n$ representing the visual direction to the near point and a variable $\theta_f$ representing the visual direction to the far point. We then reformulate Equation 2 as follows:

$$\dot{\varphi} = k_f \dot{\theta}_f + k_n \dot{\theta}_n + k_I \theta_n$$

Equation 3

The first term in Equation 2 is split into two terms, one that represents the contribution of the change in far-point visual direction $\dot{\theta}_f$ and another that represents the contribution of the change in near-point visual direction $\dot{\theta}_n$. The second term in Equation 2 is re-interpreted to represent the visual direction to the near point, $\theta_n$, since the near point is the best reflection of the vehicle's current lateral position error. Thus, the model continually adjusts steering to attempt to maintain three criteria: (1) a stable far point, $\dot{\theta}_f \approx 0$; (2) a stable near point, $\dot{\theta}_n \approx 0$; and (3) a near point centered on the roadway, $\theta_n \approx 0$. Note that the model does *not* require that the far point remain at or near the center of the roadway, thus allowing the far point to be any salient visual point in the distance that drivers can lock onto for stable navigation.

While the continuous form of the control law in Equation 3 offers a clean formulation of the underlying process, it is also convenient to express this control law in its discrete form. We derive a discrete form as follows:

$$\frac{d\varphi}{dt} = k_f \frac{d\theta_f}{dt} + k_n \frac{d\theta_n}{dt} + k_I \theta_n$$

$$\frac{\Delta\varphi}{\Delta t} = k_f \frac{\Delta\theta_f}{\Delta t} + k_n \frac{\Delta\theta_n}{\Delta t} + k_I \theta_n$$

$$\Delta\varphi = k_f \Delta\theta_f + k_n \Delta\theta_n + k_I \theta_n \Delta t$$

Equation 4

This discrete formulation allows for straightforward manipulation and use of the control law as a computational model, as we demonstrate in the next section on validation. However, the discrete form is more than a simple computational convenience; it also has important theoretical implications as a plausible human control law. First, researchers have suggested that humans operate, at least to some extent, on a discrete rhythmic "clock" (e.g., VanRullen & Koch, 2003); models of human cognition (e.g., Anderson & Lebiere, 1998; Newell, 1990; Meyer & Kieras, 1997) have estimated that the cycle time of such a "clock" is approximately 10-50 ms. The discrete control law allows the steering model to update steering output in such cycles. Second, the discrete form works both for periodic updates with a constant $\Delta t$ *and* for occasional, intermittent updates with changing values of $\Delta t$. This fact is critical when considering the fact that humans can be distracted for short or even extended periods of time before returning to the steering task. Even without distraction or inattention, it seems unreasonable that the update frequency of steering always remains perfectly constant, but rather likely varies, at least slightly, over the time course of navigation.

The final section of the paper discusses the relationship of our model to other models in the literature. First, however, we wish to demonstrate how the model accounts for various types of steering behavior, as this will elucidate some of the similarities and differences between our model and others.

# Validation Studies of the Two-Point Model

Given our two-point model of steering control, we would like to confirm that the model indeed successfully steers down a roadway and that its behavior matches that of human drivers. To this end, we performed three validation studies of the model, each focusing on a particular aspect of steering behavior. First, we tested how the model negotiates curves using the data from Land and Horwood (1995) study that explicitly tested the two-level steering model in a simulator study. Second, we tested how the model performs corrective steering using empirical results from Hildreth et al. (2000) that detail the time course of steering profiles for corrective maneuvers initiated far off-center from the roadway. Third, we tested how the model can change lanes using empirical results from Salvucci and Liu (2002) that examined drivers' gaze patterns on the start and destination lanes of the lane change. All together, these studies represent a nicely complementary set of tests with which we validated the predictions of the two-level control model.

All three studies utilize the discrete form of the steering control law (Equation 4) running in a realistic simulated environment, including realistic vehicle dynamics (taken from the simulator used by Hildreth et al., 2000; see this paper for more details on the environment). We assumed an update time for all tests of $\Delta t$ = 50 ms, which corresponds to the cycle time used in recent theories of cognitive architecture (Anderson & Lebiere, 1998, Meyer & Kieras, 1997); however, we also note that the value of $\Delta t$ could frequently be higher given the time needed to visually process the external world, and also that $\Delta t$ will in general not be the same value on every cycle — for instance, if a secondary task (e.g., cell-phone usage) interrupts steering for some time. We also set the near-point distance to the distance corresponding to 7° down from the horizon — reported by Land and Horwood (1995) as the optimum for near lane-position information — or roughly 6.2 m from vehicle center. (In general, the results reported below are fairly robust with respect to the exact distance of the near point.) The control equation parameters $k_f$, $k_n$, and $k_I$ were estimated for each study, and as we will see, these parameters can be varied to produce individual differences in steering performance. The various graphs represent the results of running one model simulation over the course of a roadway approximately 25 km in length (note that multiple runs of the model would lead to identical results for each run). Further details on the simulations are below in the descriptions of the particular studies.

## Study 1: Curve Negotiation

Our two-level model of steering, like the original Donges (1978) model, posits that areas far from the vehicle are used for stability while areas near the vehicle help to keep the vehicle centered. This implies that moving or removing the near or far regions of the visual field would affect the vehicle's lateral position or stability, respectively. Land and Horwood (1995) directly tested this prediction in a study in which the driver could see only small segments of the roadway in a simulated environment. Specifically, they created two segments, a near segment and a far segment, each only one degree of visual angle in height. They then varied the positions of these segments from 1 to 9 degrees below the horizon, leading to three sets of conditions (using our own terminology): a *move-near* condition, where the far segment remained at the top of the view and the near segment moved up and down; a *move-far* condition, where the near segment remained at the bottom of the view and the far segment moved up and down; and a *move-both* condition, where both segments overlapped and moved up and down (i.e., the drivers saw only one region). These three conditions are illustrated in Figure 2. In the experiment, drivers navigated a winding roadway at a constant speed of 16.9 m/s, with each driver completing all three conditions as well as a *clear-view* condition in which the entire roadway was visible. They then measured "steering accuracy" as the ratio of the standard deviation of angle between the vehicle heading and road center line with the whole road visible divided by the standard deviation with one or two segments visible. Figure 3(a) shows the empirical results for each of the experimental conditions, with the full road visible baseline accuracy plotted as a dotted line across the graph (note that by definition this condition has an accuracy ratio of 1). The results are described in detail below.

<< Insert Figure 2 here >>

<< Insert Figure 3 here >>

To validate the model with Land and Horwood data, we provided the model with the same viewing conditions and analyzed its steering behavior with respect to the same accuracy metric. The driving environment was a realistic roadway approximately 25 km long taken from a previous study (Salvucci & Liu, 2002) with curved segments of a variety road curvatures. The model's near and far points were locked at the center of the near and far road segments (respectively); in the move-both condition, both the near and far points were locked at the center of the single road segment. We also estimated the value of the model parameters to achieve the best fit to the empirical results: $k_f = 30$, $k_n = 13.5$, and $k_I = 36$. Because the near and far point visual angles scale according to visual angle down from the horizon, we scaled their associated parameters to compensate — specifically, we scaled the parameters linearly down or up depending on the vertical angular distance up or down from the horizon. The model's speed along the roadway was maintained at a constant 16.9 m/s, just as in the original study. With this setup, the model, like human drivers, was forced to maintain stability and centeredness using only the small visual segments given.

Figure 3(b) shows the results of the model's steering accuracy when driving the entire length of the roadway in each experimental condition. As in Figure 3(a), the dotted line notes the baseline performance with the full road visible, and data points from two simulations where the model was not able to successfully maintain position on the road (i.e., went off the road) are omitted. (Recall that with the full road visible, the model utilizes the vanishing point and tangent points rather than being restricted to a particular region.) The model's behavior in all conditions matches that of the human drivers reasonably well ($R^2$ = .59, point-by-point correlation over all conditions) particularly in the qualitative shapes of the curves and the relative differences among the conditions. In the move-near condition, the model performed best when the near region was relatively far from the horizon (5-9°). In the move-far condition, the model performed best when the far region was relatively close to the horizon (2-4°). In the move-both condition, the model performed best when the single region was located roughly in the middle of the roadway (4-6°).

Thus, the model's predictions in all three conditions nicely reflected the human drivers' overall performance as well as their relative performance between the various conditions and the baseline (full-view) condition. The largest discrepancies between model and human drivers arises in the extreme cases — that is, when the near and far regions were in the regions closest to and farther from the horizon (1° and 9° down, respectively). We believe that in these extreme cases, it is very likely that the human drivers were utilizing additional compensatory mechanisms to counter the lack or instability of information in the extreme regions; the model, not having such additional compensatory mechanisms, did its best with the single proposed mechanism but could not attain the same performance as human drivers. Nevertheless, the model clearly replicates the most important trend in these data, namely that drivers perform best when the near and far regions are separated rather than close together.

Another critical result of the Land and Horwood study showed how the near and far point serve different and complementary roles, with the near point guiding lane centeredness and the far point facilitating stability. In particular, they demonstrated that driving using only a far region resulted in smooth but less inaccurate lane keeping, while driving using only a near region resulted in more accurate but "jerky" control — in their terms, more of a "bang-bang" steering approach. Our two-level model, as discussed, incorporates complementary roles for the near and far points, and thus also produces different effects when given only a near region versus a far region for control. To demonstrate these effects, we recorded lateral deviation data (from lane center) for the model in the move-both condition where the single visible region occurred in the far area (4° down from horizon) and in the near area (6° down from horizon); these two conditions were chosen because the model produces almost identical overall accuracies for in the two conditions as can be seen in Figure 3(b) (.70 and .67, respectively). Figure 4 shows the lateral deviation of the model's vehicle as it proceeds down the road for a period of approximately 85 seconds. In Figure 4(a), with the visible region in the far area, the model exhibits fairly smooth lateral control but also exhibits large inaccuracies around the curves (a curve to the right in the first half of the time period, a curve to the left in the second half). This result arises because the far region nicely guides the model's stability but does not necessarily keep the vehicle in the center of the lane. In Figure 4(b), with the visible region in the near area, the model exhibits the same "bang-bang" steering observed by Land and Horwood. This result comes about because the near region consistently tries to keep the vehicle in the lane center, but there is no far region to help maintain stability and thus the vehicle wavers more side-to-side. Thus, although the overall accuracy in the two conditions (in Figure 3(b)) is approximately equal, the model clearly exhibits how the complementary roles of the near and far points have differential effects on steering behavior.

<< Insert Figure 4 here >>

## Study 2: Corrective Steering

The validation of curve negotiation demonstrates how the model's steering closely resembles that of human drivers during normal control — that is, when the model or driver is in command of the vehicle at all times and the vehicle never (or rarely) strays too far from the road center. However, we would also like to know if the model exhibits reasonable performance during corrective maneuvers — that is, those in which the vehicle has strayed far from the center (perhaps because of inattention) and the driver must quickly steer the vehicle back to a stable central lateral position. Hildreth et al. (2000) examined driver behavior in these types of scenarios. In one study, drivers drove down a straight road at a constant speed of 20 m/s, but occasionally the system took control and turned the car to follow a straight path at an angle off the road centerline. When the lateral position (i.e., distance to the centerline) reached a predetermined threshold, the driver saw a flash and then took control of the vehicle, performing a corrective maneuver to adjust the car back to, and aligned, with the centerline. We will address two studies in Hildreth et al. (2000), the first analyzing the effect of vehicle heading on the resulting steering profile, and the second analyzing the effect of speed on these profiles.

For our validation, we integrated the model with the same simulated environment as for Study 1, except that it used a straight roadway without curves. The model simulations were set up just as the original studies: the system took control until a particular lateral position was reached, and then the model resumed control and maneuvered the vehicle back to center. For each simulation, the model drove approximately 20 km down the straight road, continually performing trials (i.e., corrective maneuvers) but with 10 s delays between the end of one trial and the start of the next. The results below represent the aggregate profile averaged over all trials.

In Experiment 1, Hildreth et al. examined the effect of vehicle heading on steering profiles. Given a constant speed of 25 m/s, the heading angle was varied over five values: 1.0º, 1.5º, 2.0º, 2.5º, and 3.0º. Figure 5(a) shows the steering profiles for two individual subjects, Driver 1 and Driver 3, for each heading angle averaged over all trials. Overall, the main effect found by Hildreth et al. was that larger angles led to steering profiles of larger magnitude, but did not lead to significantly longer overall maneuver times. The results for the two extreme cases bear this out: for Driver 1, the steering angles for the 3.0º heading reach a minimum of approximately 40º, while the steering angles for the 1.0º heading reach a minimum of approximately 30º; nonetheless, the maneuvers for both cases take roughly four seconds to complete. In addition, the data for the two drivers indicate the type of individual variability exhibited by the human drivers: some steered more aggressively than others to arrive at the centerline more quickly, and these drivers (e.g., Driver 1) exhibited steering of overall greater magnitude compared to drivers who accepted longer times to reach the centerline (e.g., Driver 3).

<< Insert Figure 5 here >>

We compared the model to these data by running the model in these same conditions. To model the two individual drivers highlighted by Hildreth et al., we estimated model parameters for each human driver to capture their particular tendencies to steer more or less aggressively during the maneuver; specifically, $k_n$ and $k_I$ were varied to represent aggressiveness to center, while $k_f$ was left constant between drivers. Figure 5(b) shows the model's predicted steering profiles at each initial heading angle and for each simulated driver. The model nicely reproduced the main effect, $R^2 = .84$ (across all conditions and both drivers), with larger heading angles leading to profiles of larger magnitude. In addition, as for human subjects, the overall maneuver times were roughly equal across heading angles. In essence, the larger heading angles resulted in larger values of $\theta_n$ and $\Delta\theta_n$, in turn producing larger steering increments and total values in the profiles. Also, by varying the $k_n$ and $k_I$ parameters, the model is able to capture basic individual differences between drivers with respect to corrective steering. Parameter values for the model are shown in Table 1. In comparison to Study 1, drivers appear to place less weight on the direction of the near point during corrective steering maneuvers as compared to steering around a bend, perhaps to avoid extremely large changes in steering angle outside the bounds of what the vehicle can comfortably handle. It can also be seen that the more aggressive driver (Driver 1) placed more weight on the change in direction of the near point and the direction of the near point than the less aggressive driver.

<< Insert Table 1 here >>

In Experiment 2, Hildreth et al. examined the effect of speed on driver steering profiles. The vehicle was deflected at a heading angle of 2.0º while moving at five possible constant speeds: 17.5, 20, 22.5, 25, and 27.5 m/s. Figure 6(a) shows the aggregate steering profiles for Driver 1 and Driver 3 at each of these speeds. Contrary to the effect of heading angle, Hildreth et al. found a main effect of speed on overall maneuver time, with higher speeds leading to shorter times to complete the full maneuver. However, again contrary to the heading study, speed did not affect the overall magnitude of the steering profiles. Not surprisingly, the two drivers exhibit similar differences as in the heading study, namely that Driver 1 steered generally more aggressively toward the centerline than Driver 3.

<< Insert Figure 6 here >>

We ran the model in these speed conditions, maintaining the same model parameters for both drivers as estimated in the heading study. The results are shown in Figure 6(b). The model again captured the major trends in the human driver data, $R^2$ = .84: higher speeds led to shorter maneuvers, but not to significantly different magnitudes in steering. Thus, maintaining a constant heading angle keeps $\theta_n$ and $\Delta\theta_n$ roughly equal across conditions; however, the resulting steering angle needs to be maintained for a shorter period of time given the vehicle's higher speeds. And again, the parameter settings for each driver — and importantly, the same settings as in the heading study — nicely captured each driver's performance.

## Study 3: Lane Changing

In addition to curve negotiation and corrective steering, one of the most common aspects of driving in the real world is that of lane changing — smoothly steering from one lane to an adjacent lane. While lane changing has not been studied as rigorously as other aspects of steering such as curve negotiation, there are nevertheless several studies that have elucidated important aspects of how drivers accomplish this maneuver (e.g., van Winsum, de Waard, & Brookhuis, 1999). One study that is particularly relevant to our two-point model examined the time course of driver behavior before and during a lane change (Salvucci & Liu, 2002). In the study, drivers navigated a straight two-lane highway in a naturalistic way (i.e., with no special instructions) while their eye movements were collected and analyzed. The study found an interesting effect of lane changing on driver gaze: as soon as drivers made the decision to change lanes (indicated with a verbal response), they would shift the bulk of their gaze time (60-70%) from the start/current lane to the destination lane, well before the vehicle had entered the destination lane or even moved significantly in the start lane. This indicated that drivers used salient visual features of the destination lane to smoothly guide their control in maneuvering over to this lane. To illustrate driver behavior during control of a lane change, Figure 7(a) plots the aggregate steering profiles for the leftward lane changes of three subjects in the study. The profiles, similar to those for corrective steering, include an initial larger steering movement to turn the vehicle toward the destination lane and a shallower movement to straighten the vehicle in this lane (mixed with slight back-and-forth adjustments for some drivers).

<< Insert Figure 7 here >>

This study has an interesting and straightforward implication for how to change lanes with our two-point control model: when the model "decides" to change lanes, it simply switches from using the near and far points of the start/current lane to the near and far points of the destination lane. Thus, we tested this idea by running the model in a two-lane environment identical to Study 2 and having the model change lanes back and forth, with 10 seconds of normal driving in between lane changes. In addition, to mimic different individual drivers performing lane changes, we varied parameter values as done in Study 2 to attempt to produce a set of profiles roughly similar to those of the three drivers in Figure 7(a).

Figure 7(b) shows the steering profiles for three model-simulated "drivers" with different parameter values, averaged over all instances of leftward lane changes. (Lane changes in the other direction produced identical but mirrored results.) Parameter values for the individual drivers are shown in Table 2. The steering profiles correspond well qualitatively to typical lane-change profiles seen in Salvucci and Liu (2002) and other work (e.g., van Winsum, de Waard, & Brookhuis, 1999): we see the initial sharp steering movement to the left, producing the high first peak in the profile, followed by a shallower corrective movement to the right, producing the lower second peak in the profile. The key to the steering model's ability to predict these profiles lies in the interaction among the three terms of the control equation. At the start of the lane change, the model shifts to using the near and far points of the destination lane. The new near and far points are likely fairly stable, but the location of the new near point creates a large steering movement toward the destination lane. At the same time, the equation terms specifying the rate of change of near and far point angles ensure that this movement is done in a controlled way, thus resulting in the smooth curves in the steering profile. The second corrective maneuver (the second peak in the profile) is smaller in amplitude because of the decreasing effect of the new near-point distance (due to the vehicle moving closer to this point).

Figure 7(b) also illustrates how different model parameters can produce different steering profiles. Our estimated parameter values for the three simulated "drivers," shown in Table 2, nicely reproduce the qualitative shape of the three human driver profiles in Figure 7(a). However, we should note that the graphs in the two figures are shown in different scales to emphasize the similarity of the qualitative shape; the model does not match the human data as well with respect to the absolute magnitude of the steering profiles. We believe that the model’s inability to capture the profile magnitude lies in the nature of lane changes and a major difference from corrective maneuvers: while a corrective maneuver embodies some urgency to return quickly to lane center, a lane change typically has leeway for less urgency at the discretion of the driver. The model, not aware of any distinctions of urgency, treats the lane change as a large corrective maneuver and thus executes it more sharply than a human driver’s typical lane change. Nevertheless, we believe that a future version of this model could incorporate such distinctions and allow the model to adapt its control during lane changes to take urgency and/or necessity of maneuvering into account.

<< Insert Table 2 here >>

# Discussion

We have shown that our two-point model of steering corresponds closely to human steering in four significant ways:

1. The model successfully accounts for curve negotiation on winding roads, even when given only small segments of the road. Its steering accuracy and instability for various combinations of near, far, and single segments is similar to that of human drivers.
2. The model successfully predicts steering profiles for corrective steering maneuvers, matching human steering for different initial vehicle speeds and headings.
3. The model successfully accounts for human-like steering profiles when making a lane change, with no additional terms or mechanisms other than switching from the current lane’s visual points to those of the destination lane.
4. The model demonstrates the ability to capture some aspects of individual differences among drivers, in particular with respect to the execution of steering maneuvers.

While the model is clearly not yet a fully comprehensive model of steering or driving behavior, we believe that its ability to capture these four issues demonstrates significant promise as a foundation for a more comprehensive model.

Typically driving behaviors such as steering, lane changing and car following have been modeled as completely separate perceptual-motor control tasks that rely on very different control inputs. Here we demonstrate that seemingly different driving tasks can be controlled by the same small set of perceptual inputs. In order to switch the maneuver that is being performed (e.g., from steering around a bend to changing lanes) the driver need only switch far points (e.g., from the current lane to the desired lane) and/or change the relative weighting given to information provided by the near and far points. Our model could also potentially account for changes in driving behavior produced by restricted visibility (e.g., fog or snow) by varying the relative weighting of the far point based on the reliability of the visual inputs.

Of the numerous models of steering and control in the literature, the model of Wann and Wilkie (in press) is perhaps the most closely related to our own. As mentioned, they utilize a point-attractor approach in which steering emulates (conceptually) the idea of a spring returning to its equilibrium state. The two most significant differences between their model and ours is that theirs utilizes a single visual direction angle rather than angles for two separate (near and far) points, and at the same time, incorporates retinal flow into the steering control law. Given these two differences, it is not clear that the Wann and Wilkie model would be able to account for the four issues above, particularly the empirical results of Land and Horwood (1995); the retinal-flow component of the model could potentially provide some of the necessary information to model these results, but how exactly this could be realized is not clear. Nevertheless, the incorporation of retinal flow into the visual-direction model is intriguing; in fact, we believe that our two-point model is quite amenable to a similar treatment of retinal flow, but have not attempted this as yet.

As mentioned, there are several models of control derived from a two-level approach that require estimation of road curvature (Donges, 1978, Land, 1998). Also, there are models that do not compute curvature per se but rather compute the curvature profile of the vehicle's optimal trajectory through a curve (Boer, 1996; Hildreth et al., 2000). We believe that the two-point model offers a simpler account for steering performance based on directly perceivable features (i.e., visual direction of the near and far points), and is very much in line with the literature on the use of perceived direction to guide control (e.g., Rushton et al., 1998; Harris & Rogers, 1999; Wood et al., 2000; Rushton & Salvucci, 2001; Wilkie & Wann, 2003). Nevertheless, one interesting aspect of some of these models (e.g., Donges, 1978; Hildreth et al., 2000) is the integration of an open-loop steering process to complement the closed-loop process. Such a model's open-loop component provides the model with the ability to generate truly predictive steering behavior, thus compensating for the roughly 0.8 s delay between visual access and steering correction (Land, 1998). Also, open-loop steering can steer through periods of visual occlusion or inattention (e.g., when looking over one's shoulder before changing lanes, reading a road sign or dialing a cellular phone) resulting in intermittent rather than continuous visual feedback. It has been demonstrated that these shifts of attention can lead to degraded judgments of heading (Wann, Swapp, & Rushton, 2000) and motion in depth (Gray, 2000). Some studies have shown that humans are reasonably proficient at generating predictive steering for longer periods of occlusion — for instance, for up to 1.5 s (Godthelp, 1986). However, a recent study on lane changing has also raised questions about human open-loop steering ability, noting a common failure to initiate the return phase of a lane-change maneuver when performing these maneuvers with no visual feedback (Wallis, Chatziastros & Bulthoff, 2002). The two-point model presented here is not, at present, able to capture open-loop steering during visual occlusions. However, we could imagine an open-loop component to the model that predicts the movement of the near and far points given the current (and future) steering trajectories and generates predictive steering using this information.

Another important aspect of our two-point control model, and indeed of any such control model, is the potential for integration into fuller cognitive models of driver behavior. In fact, an earlier version of our two-point model has been incorporated into a model of driving (Salvucci, Boer, & Liu, 2001) developed in a production-system "cognitive architecture," which in turn incorporates theories and model implementations of both basic cognitive processes (e.g., memory learning and recall) and various perceptual and motor processes. The cognitive architecture thus embeds the control model in a computational framework that enforces certain constraints of cognition and performance. For example, the process of visually encoding the outside environment and subsequently using this information for steering control requires some amount of time (as mentioned earlier), and thus the driver model must abide by this delay and somehow manage the delay to steer successfully; Salvucci, Boer, and Liu (2001) demonstrate that the driver model captures basic profiles for curve negotiation and lane changing even with such a delay. As another example, the architecture allows for inclusion of secondary tasks to perform while driving (e.g., dialing a cellular phone); Salvucci (2001) has explored how integration of such secondary-task models with the main driver model can successfully predict adverse effects of “driver distraction” on driver performance. The two-point control model described here serves as the foundation for such work, providing a sound and rigorous computational model that can be integrated into larger-scale models to account for a wider range of driver behavior.

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Table 1

Model Parameter Values for Study #2 (Corrective Steering)

| Driver | $k_f$ | $k_n$ | $k_I$ |
|---|---|---|---|
| 1 | 20 | 6 | 6 |
| 3 | 20 | 1.8 | 1.8 |

Table 2

Model Parameter Values for Study #3 (Lane Changing)

| Driver | $k_f$ | $k_n$ | $k_I$ |
|---|---|---|---|
| 1 | 20 | 12.6 | 8.4 |
| 2 | 20 | 9.0 | 6.0 |
| 3 | 20 | 5.4 | 3.6 |

# Figure Captions

Figure 1: Near and far points for three scenarios: (a) straight roadway with vanishing point, (b) curved roadway with tangent point, and (c) presence of lead car.

Figure 2: Conditions for the Land and Horwood (1995) empirical study.

Figure 3: Curve negotiation accuracy ratios for (a) human drivers, adapted from Land and Horwood (1995), and (b) model simulations.

Figure 4: Lateral deviation from lane center over a selected period of time in the move-both condition, (a) with the visible region in the far area (4° down from horizon) and (b) with the visible region in the near area (6° down from horizon).

Figure 5: Corrective steering profiles varying initial heading for (a) human drivers, adapted from Hildreth et al. (2000), and (b) model simulations.

Figure 6: Corrective steering profiles varying initial speed for (a) human drivers, adapted from Hildreth et al. (2000), and (b) model simulations.

Figure 7: Lane-change steering profiles for (a) three human drivers taken from Salvucci & Liu (2002), and (b) model simulations.

# Figures

Figure 1.

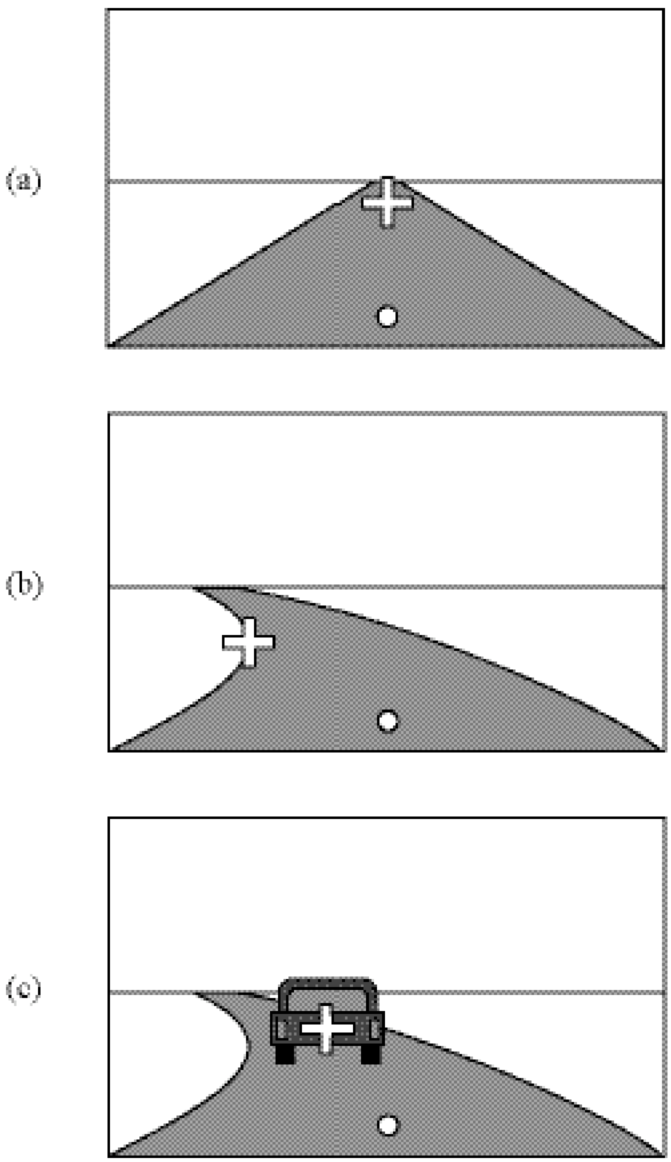
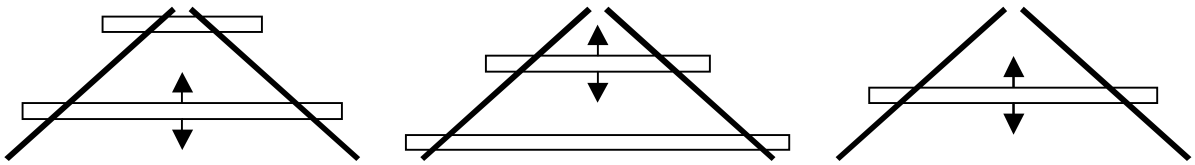
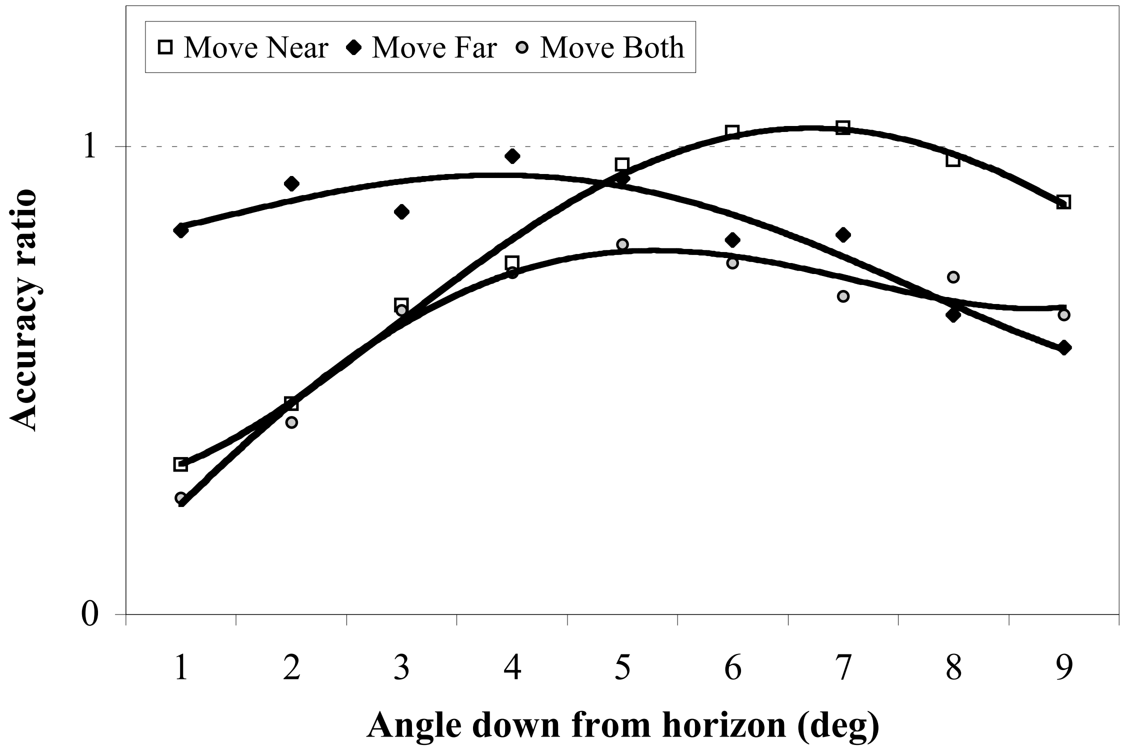


Figure 2.

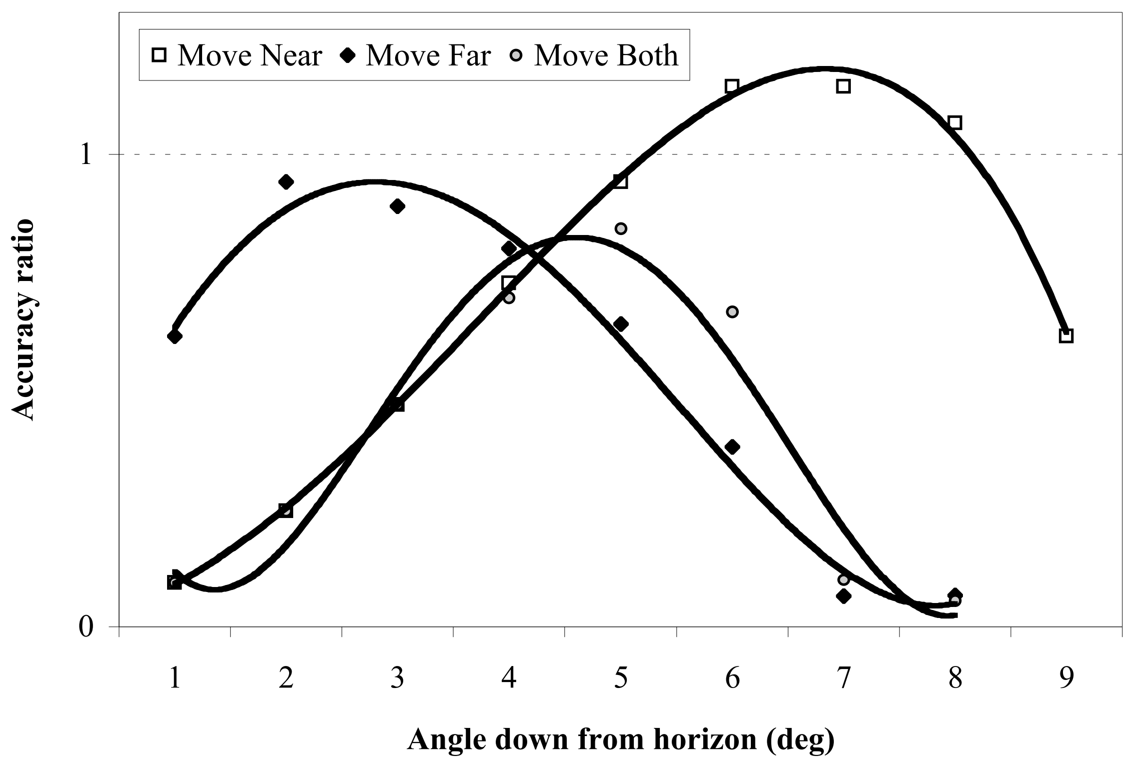


move-near move-far move-both

Figure 3.


Move Near
Move Far
Move Both
Accuracy ratio
1
0
1
2
3
4
5
6
7
8
9
Angle down from horizon (deg)

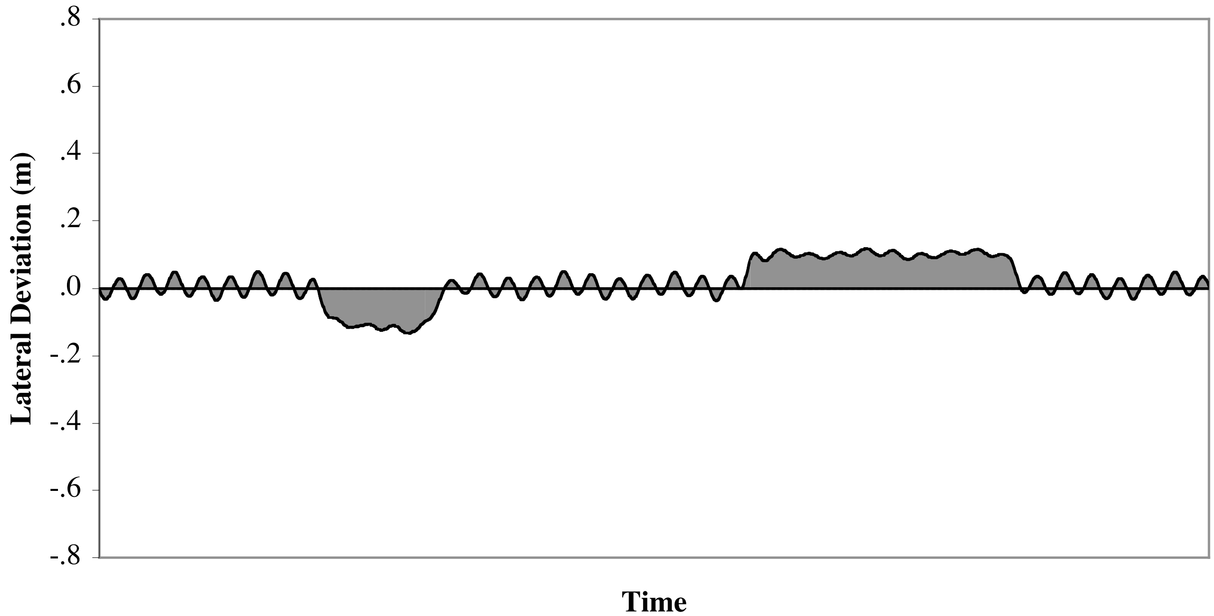
(a)


Move Near
Move Far
Move Both
Accuracy ratio
1
0
1
2
3
4
5
6
7
8
9
Angle down from horizon (deg)

(b)

Figure 4.

(a)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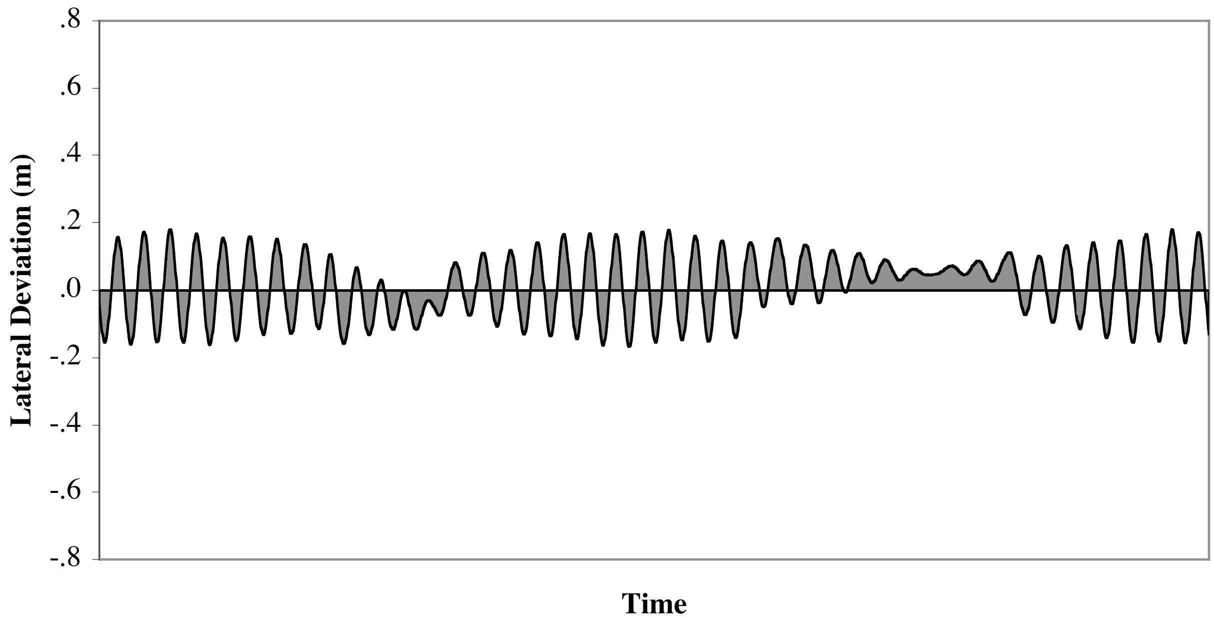
.6
.4
.2
.0
-.2
-.4
-.6
-.8
Lateral Deviation (m)
Time

(b)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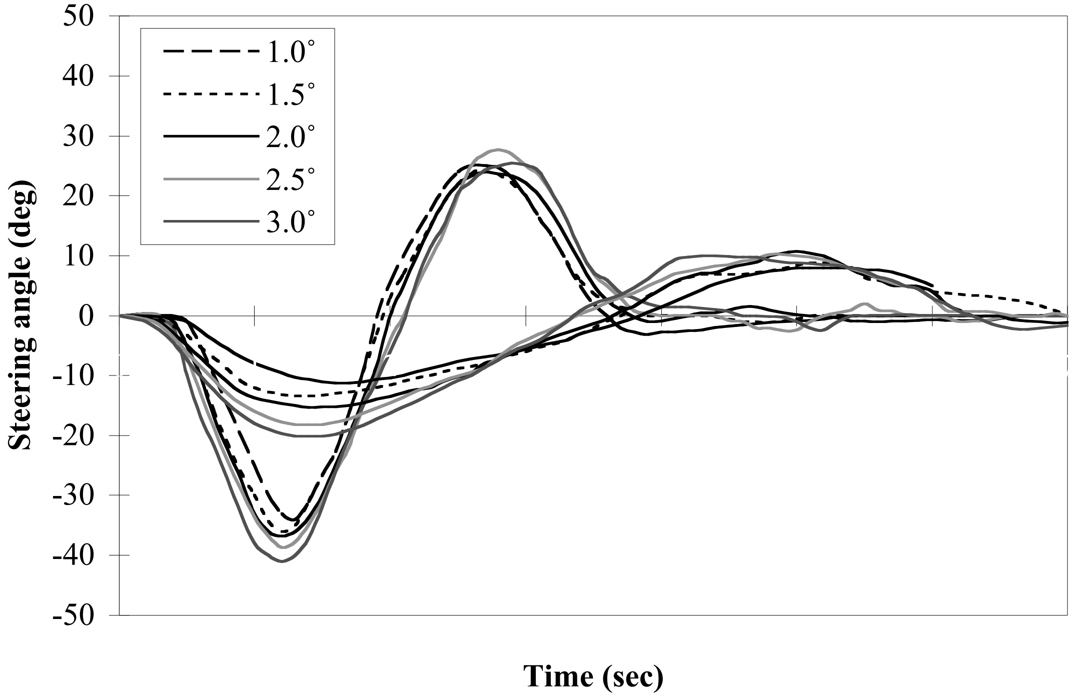
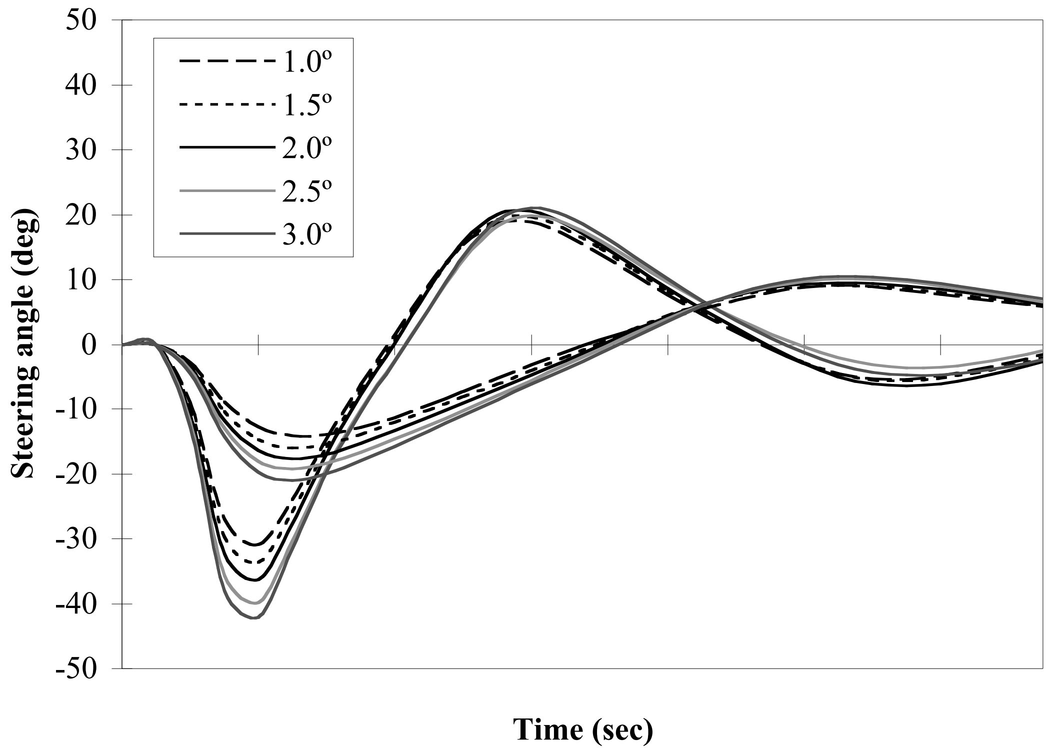
.6
.4
.2
.0
-.2
-.4
-.6
-.8
Lateral Deviation (m)
Time

Figure 5.



50
40
30
20
10
0
-10
-20
-30
-40
-50
1.0˚
1.5˚
2.0˚
2.5˚
3.0˚
Steering angle (deg)
Time (sec)

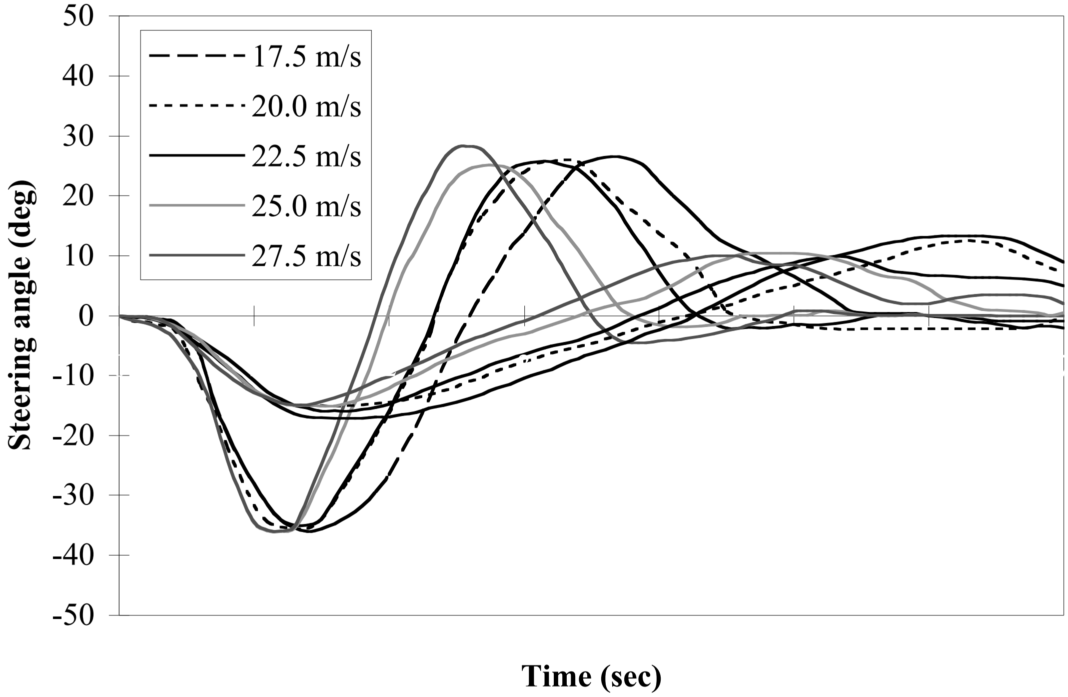

(a)



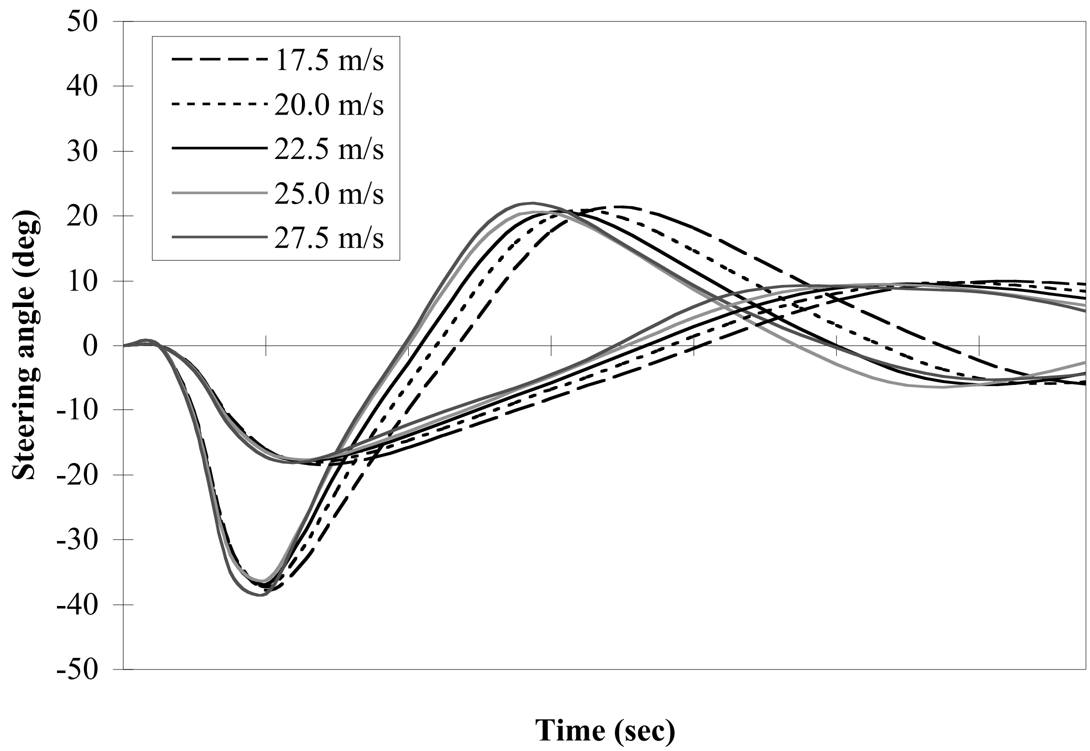
50
40
30
20
10
0
-10
-20
-30
-40
-50
1.0º
1.5º
2.0º
2.5º
3.0º
Steering angle (deg)
Time (sec)


(b)

Figure 6.

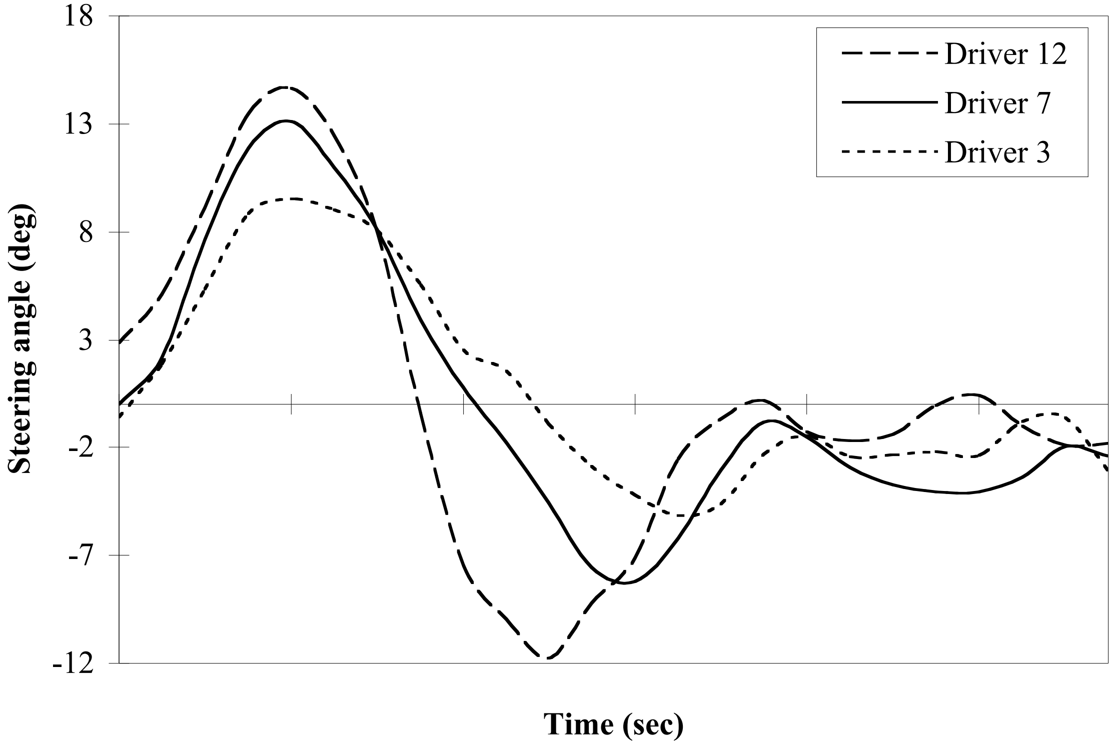

50
40
30
20
10
0
-10
-20
-30
-40
-50
Steering angle (deg)
Time (sec)
17.5 m/s
20.0 m/s
22.5 m/s
25.0 m/s
27.5 m/s

(a)

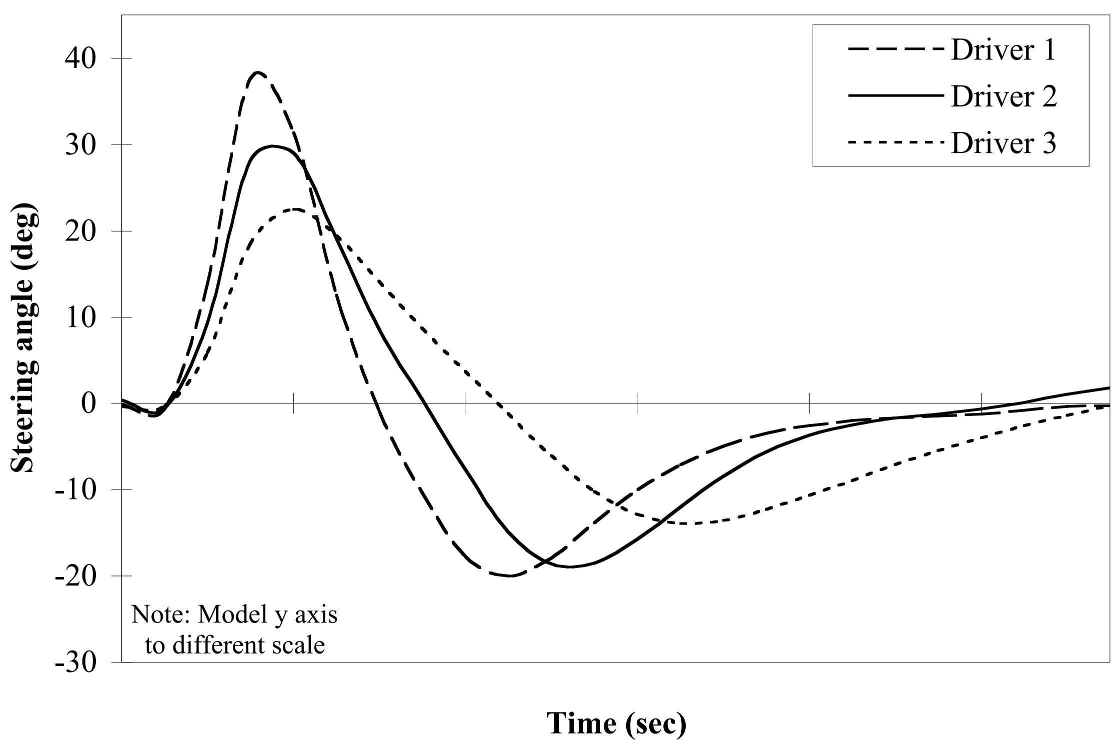

50
40
30
20
10
0
-10
-20
-30
-40
-50
Steering angle (deg)
Time (sec)
17.5 m/s
20.0 m/s
22.5 m/s
25.0 m/s
27.5 m/s

(b)

Figure 7.


18
13
8
3
-2
-7
-12
Steering angle (deg)
Time (sec)
Driver 12
Driver 7
Driver 3

(a)


40
30
20
10
0
-10
-20
-30
Steering angle (deg)
Time (sec)
Driver 1
Driver 2
Driver 3
Note: Model y axis
to different scale

(b)