Introduction

Simulation Model

The simulation model used to observe the movement of the vehicle and test a driver model consists of three parts: a kinematic model of the system, a driver model and a steering conversion needed to control the vehicle.

1. *Kinematic model*

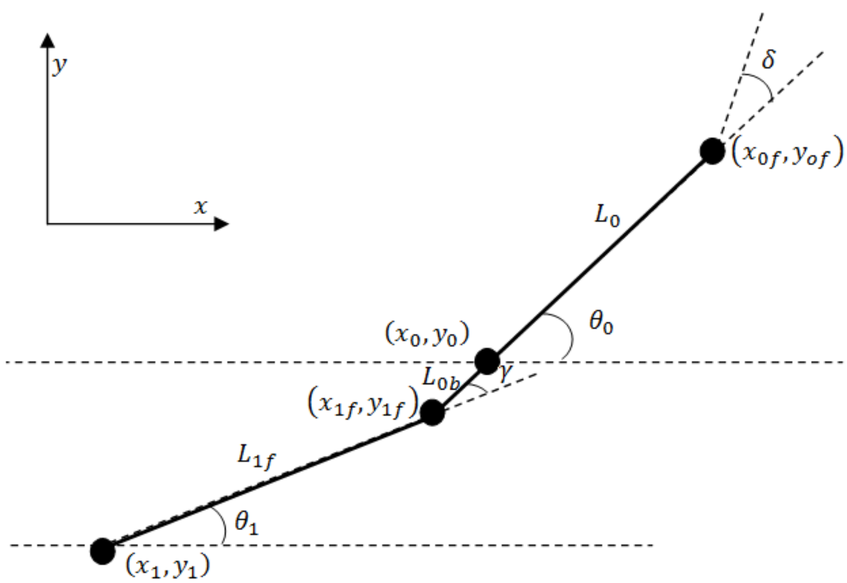


Figure 1: Single articulated vehicle model

A kinematic model is a model using mechanics describing the motion of points without considering mass of the objects or the force working on the objects in the model. Based on geometrical relationships, the movement of the system can be described [1]. The vehicle in the setup consists of a truck with one front axle and one rear axle, a trailer with one axle and a kingpin point, connecting the truck with the trailer. Figure 1 shows the single articulated vehicle setup.

The kinematic model is a single track model, assuming one fixed point at the axels of the truck and trailer. The steering angle δ at the front of the truck is an input to the model. The longitudinal velocity v_0 is the second input to the model, working on the reference point (x_0, y_0). The heading angle of the truck and trailer are indicated by θ_0 and θ_1 respectively. The articulation angle γ is described as the difference in heading angle between truck and trailer. L_0 is the wheelbase of the truck, L_{0b} is the distance between rear of the truck and the kingpin point (x_{1f}, y_{1f}) and L_{1f} is the distance between the kingpin point and the axle of the trailer (x_1, y_1). L_{0b} can be either positive or negative depending on the location of the connection between truck and trailer.

The kinematic model is based on the constraint that the wheels on the axels have no sideslip. The general form of the non-holonomic constraint is:

$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0 \tag{1.1}$$

$\dot{x}$ and $\dot{y}$ are the velocities of the axels in the given coordinate system and θ is the angle with respect to the x-axis. Based on Equation 1.1, the non-holonomic constraints for the model in Figure 1 are given by:

$$\dot{x}_{0f}\sin(\theta_0 + \delta) - \dot{y}_{0f}\cos(\theta_0 + \delta) = 0 \tag{1.2}$$
$$\dot{x}_0\sin(\theta_0) - \dot{y}_0\cos(\theta_0) = 0 \tag{1.3}$$
$$\dot{x}_1\sin(\theta_1) - \dot{y}_1\cos(\theta_1) = 0 \tag{1.4}$$

To obtain the kinematic model, the positions equations of the axel points are differentiated and substituted in Equation (1.2-1.4) to obtain the yaw rates $(\dot{\theta}_0, \dot{\theta}_1)$:

$$\dot{\theta}_0 = \frac{v_0}{L_0}\tan(\delta) \tag{1.5}$$
$$\dot{\theta}_1 = \frac{v_0}{L_{1f}}\left(\sin(\gamma)\ - \frac{L_{0b}}{L_0}\tan(\delta)\cos(\gamma)\right) \tag{1.6}$$

With the inputs of system, v_0 and δ, initial conditions for the heading angles of truck and trailer and the initial position of the vehicle, the kinematic model can be simulated. (Detailed derivations of equations are found in Appendix A)

2. Driver model

The driver model consist of a path controller which is used to follow a reference path. The reference path is an input provided externally by a path planner. It consists of two dimensional coordinates which define a path. The driver model consists of three components as seen in Figure 2. The inputs of this model consists of the variables *'positions'* and '*Vx'* that contains the position of the axle positions of the vehicle and the longitudinal speed respectively. The output of the model is the variable *'steerang'*, which is the desired steer angle for the vehicle.

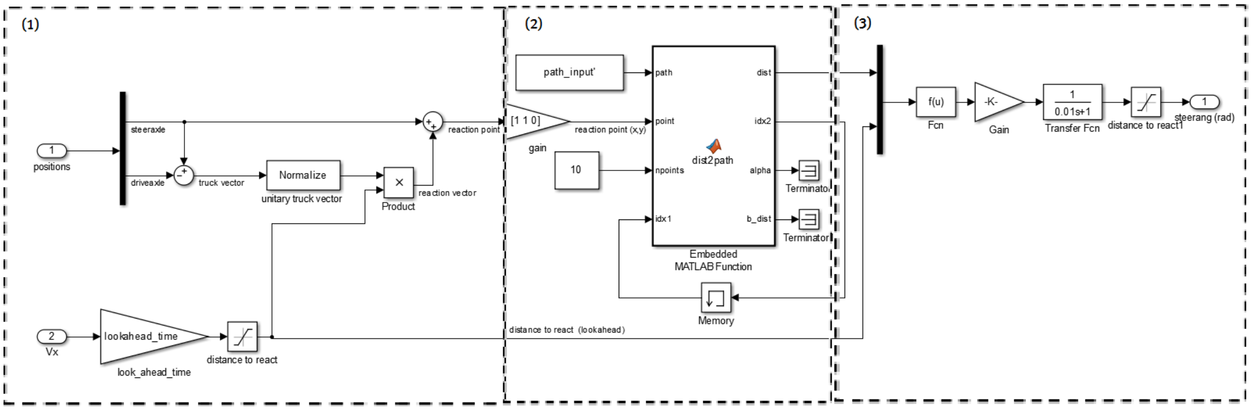


Figure 2: Driver model layout in Simulink

Firstly the positions of the vehicle axels are converted into a 'heading' vector. The heading vector consists of two parts: direction and magnitude. As seen in Figure 3, the direction of the heading vector is equal to the normalized trailer vector. Equation 2.1 calculates the magnitude of the heading vector (L_v) based on the longitudinal velocity (v_1) which has a controllable input, the look ahead time (T_l).

$$L_v = v_1 T_l \tag{2.1}$$

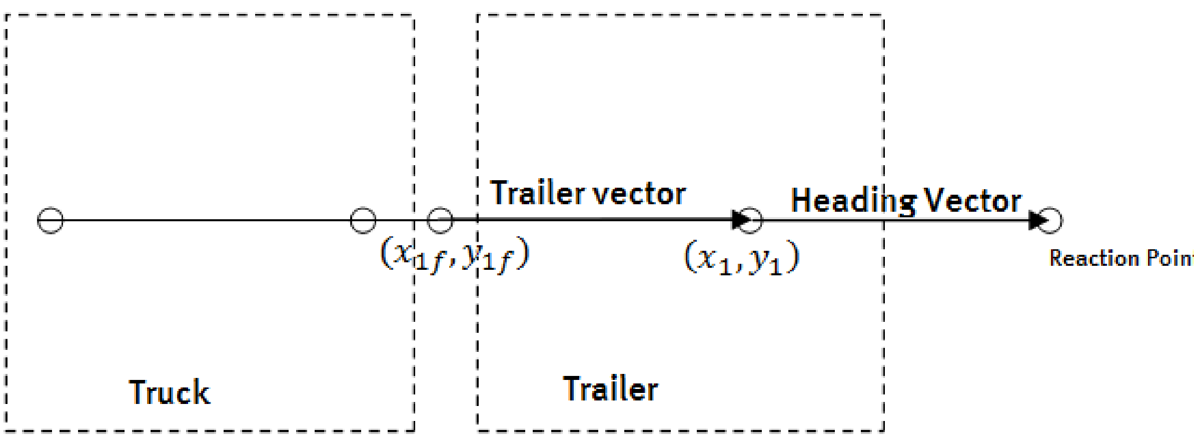


Figure 3: Schematic view of defining the heading vector

With the heading vector, a reaction point is defined from which controller operates. For every point in the reference path (Rf), the distance between the reaction point and the reference points is calculated to find the minimum distance. For every time step, the closest reference point is found. To reduce computation, a variable 'i' is introduced to present a range of reference points, depending on the previous optimal point. A schematic view of this principle is shown in Figure 4.

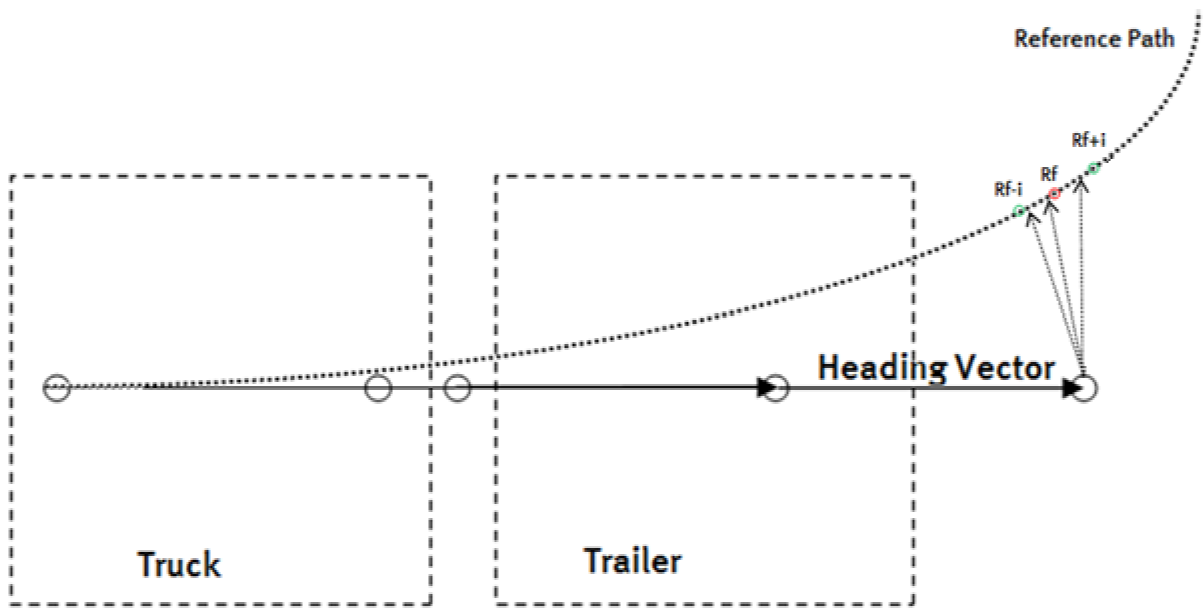


Figure 4: Schematic view of the path following calculation

Vector calculus can be used to find the orthogonal distance from the reaction point to the reference point. In this case, the reference points are fixed points with a relative distance between each other. A problem occurs when the closest reference point is not orthogonal to the reaction point, as seen in Figure 5. To solve this problem, two vectors a_1 and a_2 are created to find the orthogonal distance.

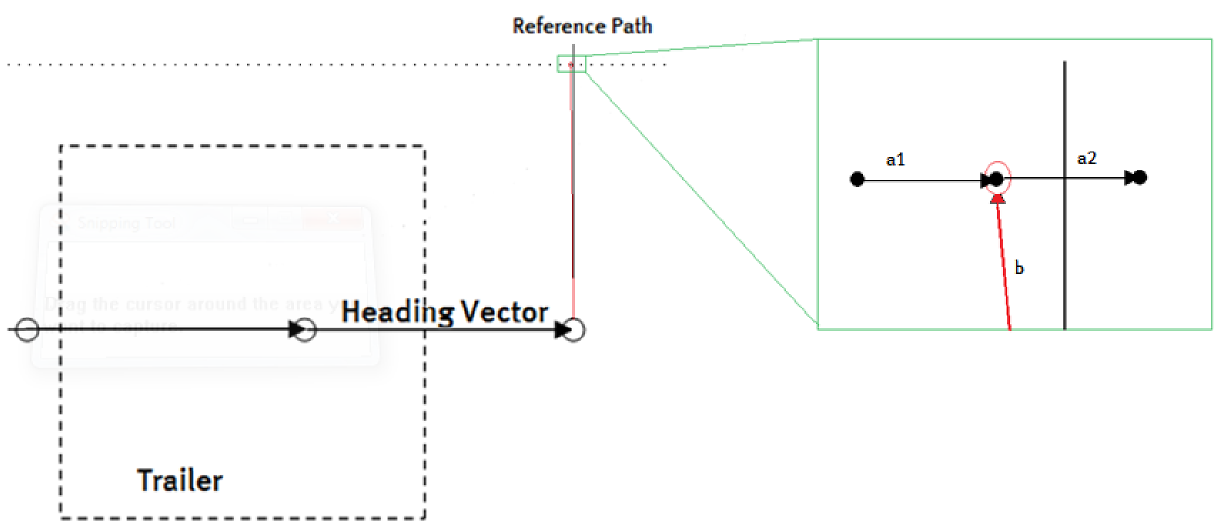


Figure 5: Reference point problem

To find the orthogonal vector, vector b is projected on vector a_1 and a_2 :

$$\alpha_i = \frac{a_i \cdot b}{a_i \cdot a_i} \tag{2.2}$$

If the angle between the vector a_i and b is within 0 and 90 degrees, an orthogonal vector with length dx can be found by:

$$dx = \|b - \alpha_i a_i\| \tag{2.3}$$

To calculate the angle in radians between the heading vector and the orthogonal vector, the following trigonometric identity is used:

$$\varphi = \arctan\left(\frac{dx}{L_v}\right) \tag{2.4}$$

The output of the path controller is given by Equation 2.5. The steering sensitivity gain K_s is the second controller input.

$$steerang = K_s \varphi \tag{2.5}$$

The path controller is adjustable by changing two control parameters. The *'lookahead'* parameter influences the magnitude of the heading vector. With increasing value for the look ahead time, the magnitude of the heading vector is increased and the path controller is more focused on future path references. Deviations of a straight path reference, for example a curve, are earlier observed. The controller can anticipate the change in curvature, and provide a smoother response. The upper limit for the value of the look ahead time is reached when deviations of a straight path are ignored by the controller or the reference is slowly followed, which can be seen in Figure 6. The path controller will be very sensitive at the lower limit, the controller is not able to properly create a heading vector. It will not be able to create a realistic input for the kinematic model, which can turn it into an instable vehicle as shown in Figure 6.

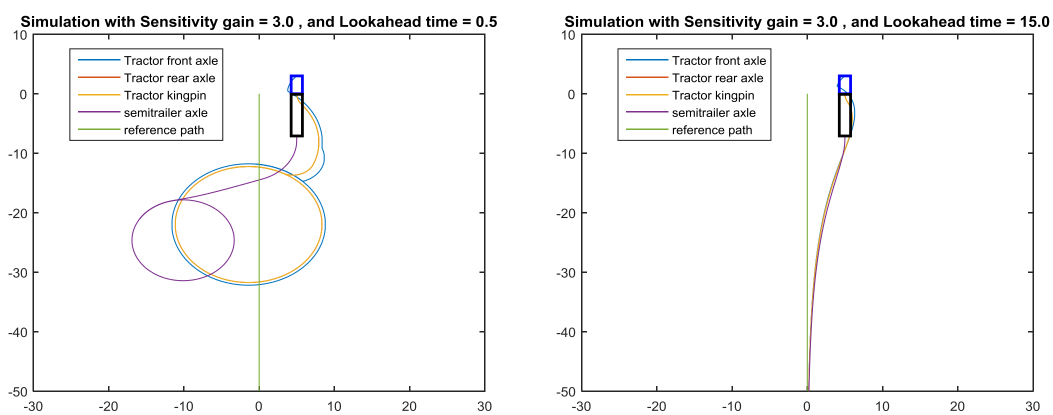


Figure 6: Upper (left) and lower (right) limit on the look ahead time

The steer sensitivity is the second parameter to influence the controller. This parameter directly influences the output of the controller, φ. Increasing the value for the steer sensitivity results into a more direct response on the desired input of the kinematic model. An example of the upper limit is shown in Figure 7. The controller overcompensates the steering angle and the movement of the truck moves into an oscillation motion. At the lower limit, the truck barely responds to the output of the controller, due to a low steering sensitivity.

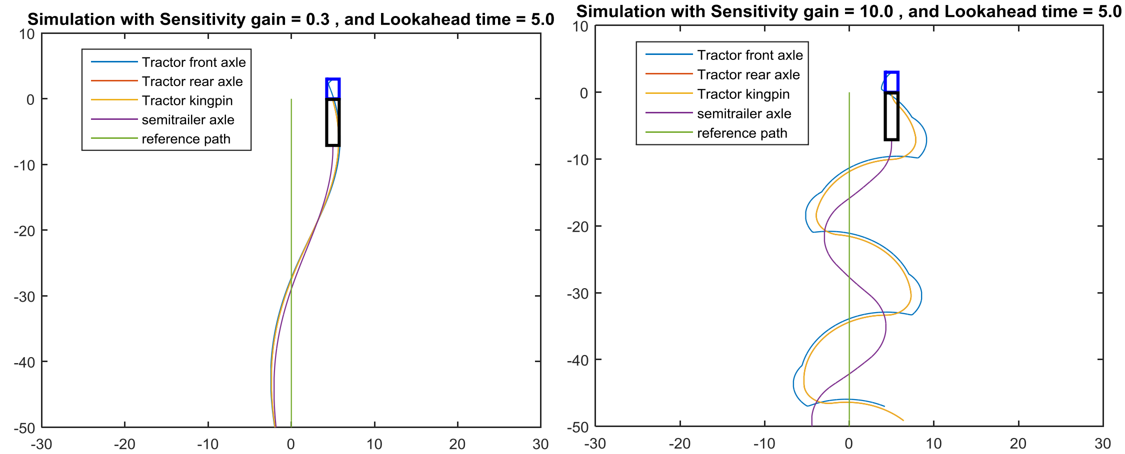


Figure 7: Upper (right) and lower (left) limit on the steering sensitivity

Originally there were no limitations for the actuator in the steering wheel implemented. An actuator has a limited operating speed. The output of the path controller is changed to:

$$\varphi = \int \dot{\varphi}\, dt \qquad (2.6)$$

$\dot{\varphi}$ is the desired angular velocity of the trailer and it is limited to 1 rad/s (+/- 57 deg/s).

3. Steering conversion

The kinematic model has one steering input δ. However the path controller gives the desired angle φ, and the axel at that point is not a steering axle. Therefore, through a kinematic relation, the desired δ can be determined to achieve the angular displacement φ.

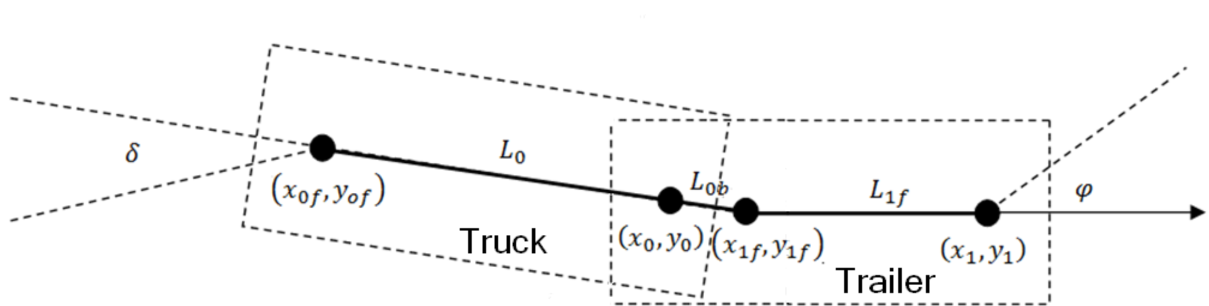


Figure 8: Kinematic Chain

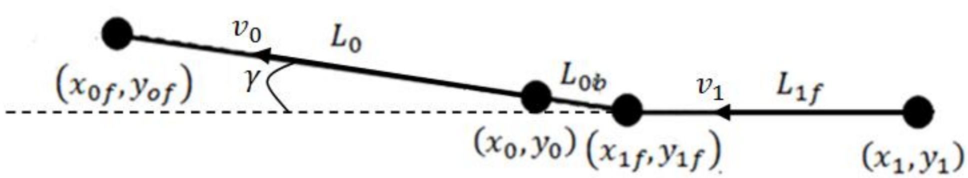
Equation 1.5 can be rewritten as:

$$\delta = \arctan\left(\frac{L_0\dot{\theta}_0}{v_0}\right) \qquad (3.1)$$

Using the trailer as virtual truck, φ can be written as:

$$\varphi = \arctan\left(\frac{L_{1f}\dot{\theta}_1}{-v_1}\right)$$
$$\dot{\theta}_1 = \tan(\varphi)\left(\frac{-v_1}{L_{1f}}\right) \qquad (3.2)$$

Through kinematic relationship seen in f9, v_0 can be determined from v_1



$$v_0 = -\dot{\gamma}\, L_{1f}\sin(\gamma) + v_1\cos(\gamma)$$
$$v_0 = \tan(\varphi)\left(\frac{-v_1}{L_{1f}}\right)\sin(\gamma) + v_1\cos(\gamma) \qquad (3.3)$$

Combining Equation 3.1 and 3.3, a direct relationship is found between φ and δ.

4. Simulation Tests

To test the controller capabilities, certain scenarios are tested to validate the controller and models performance. A direct problem with the model was found when testing the first scenario. The reference path is created by a forward driving model turning 180 degrees with maximum steering for this scenario of 42 degrees. Driving the truck in reverse results into an truck following the reference path as seen in Figure 9. However, a problem is seen at the end of the simulation. Figure 10 shows the angular positions of the vehicle. The articulation angle converges to zero which means the axel positions should be on a vertical line. The heading angle of the truck converges to 180 degrees which corresponds with the vertical reference line. However, as seen Figure 11, The axel positions do not verify this data.

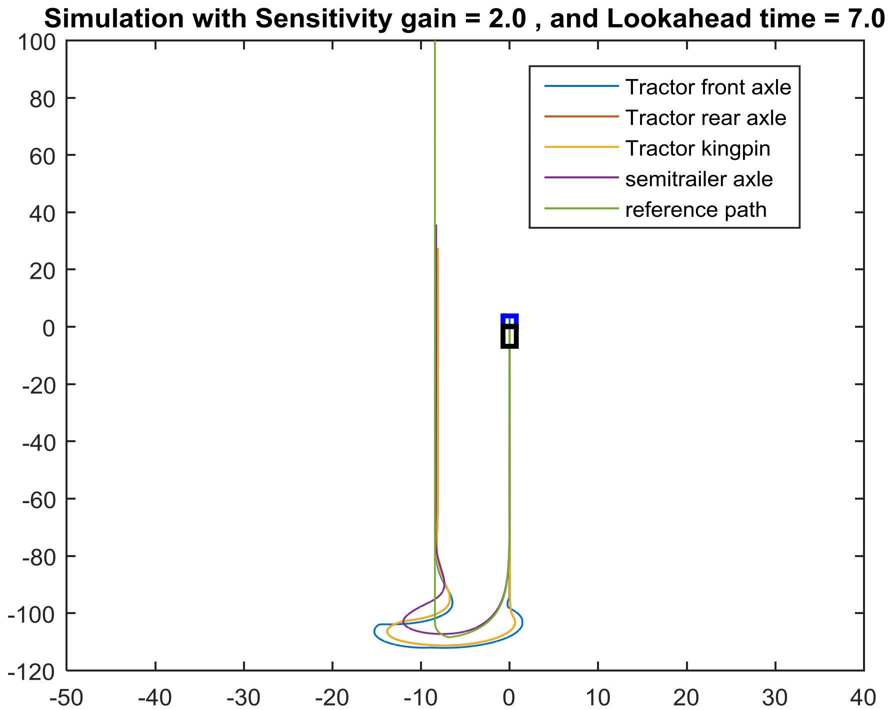


Figure 9: Simulation result with extended straight path

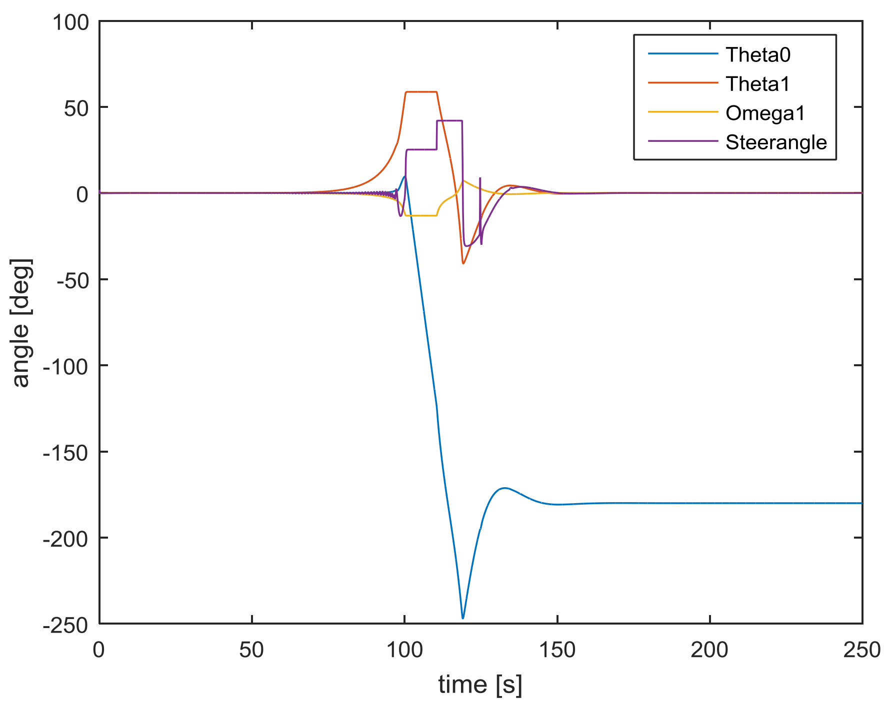


Figure 10: Heading, steering and articulation angle of the vehicle during simulation

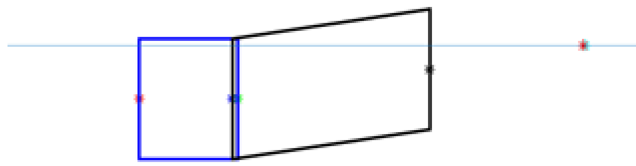


Figure 11: Truck-trailer position at t=250 [s]

The reason for this error is found at the calculation of the positions of the trailer, which is based on integrating the velocities of the trailer. The position of the axle of the trailer is calculated by:

$$x_1 = \int -v_1 \sin(\theta_1)\, dt \tag{4.1}$$
$$y_1 = \int v_1 \cos(\theta_1)\ dt \tag{4.2}$$

Integrating the velocity in the simulation results into a position change per time step. If the velocity changes in the time step, an error on the position is found. When the position is directly calculated through the position of the truck and the articulation angle, the deviation is removed. The position of trailer axel of the kinematic model is given by:

$$x_1 = x_0 - L_{0b} \cos(\theta_0) - L_{1f} \cos(\theta_1) \tag{4.3}$$
$$y_1 = y_0 - L_{0b} \sin(\theta_0) - L_{1f} \sin(\theta_1) \tag{4.4}$$

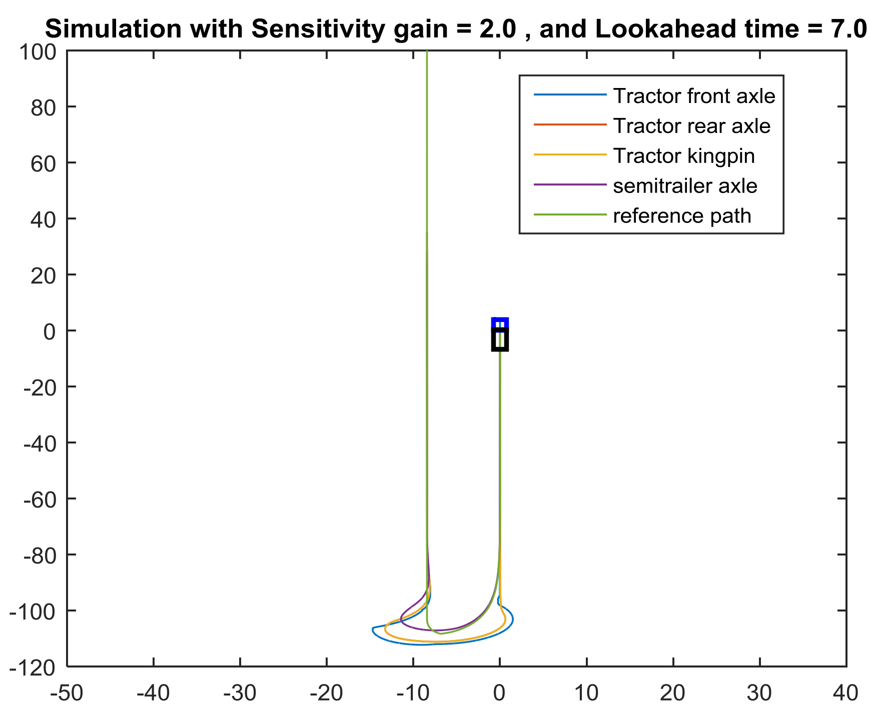


Figure 12: Simulation using improved trailer model

The result is shown in Figure 12. As seen, the position lines up with the rest of the vehicle, meaning the deviation on the position is solved. It also shows that the controller cannot cope with 180 degrees without overshooting the reference path. Therefore, the turn is lowered to a 90 degrees turn. A new forward simulation reference path was made using a 90 degrees turn. The maximum steering angle was 50 degrees. The reference path and the result of the simulation is shown in Figure 13. To test the performance of the controller, the the reference path is followed in reverse. The steering angle is limited to 48 degrees. As seen the vehicle is capable to follow the trajectory, even with limited steering angle. The deviation on the reference path in the curvature is at maximum 1 meter.

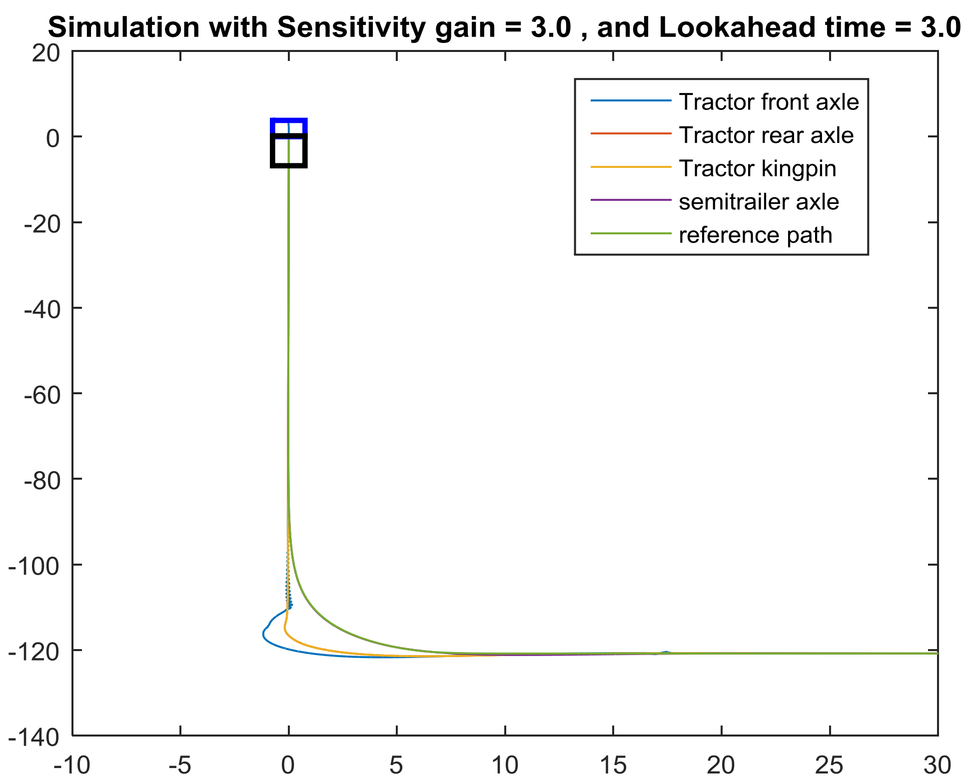


Figure 13: Simulation result of vehicle reversing the path starting at the endpoint

To find a limit for the controller, a reference path was made using the maximum steering of 60 degrees. The reference path and the simulation result of the reverse movement is shown in Figure 14. The control parameters are adjusted to give decent response and there is no limit on the steering angle in this simulation.

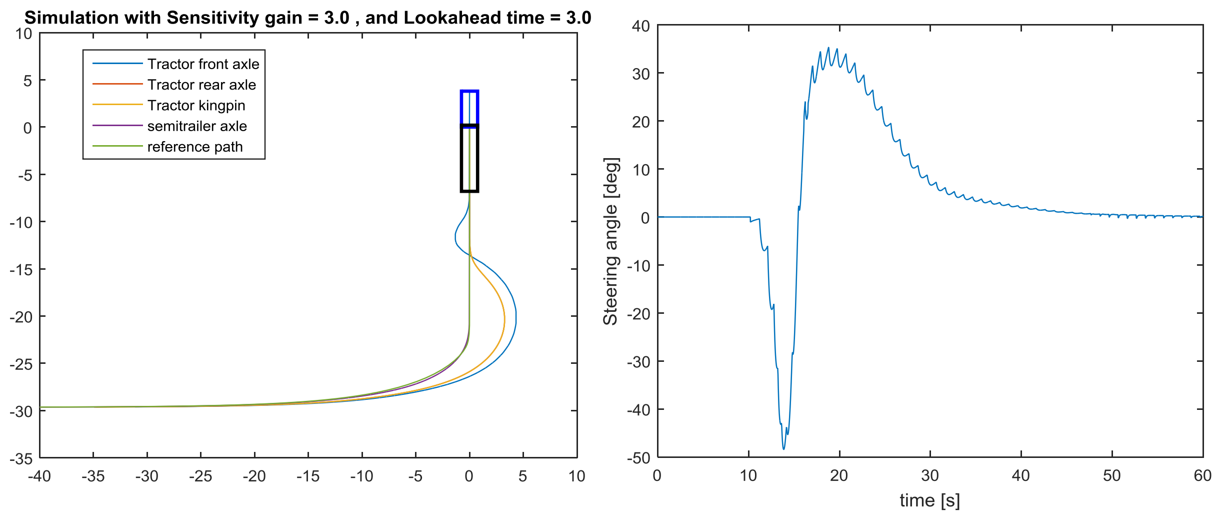


Figure 14: Reverse simulation with 60 degree corner

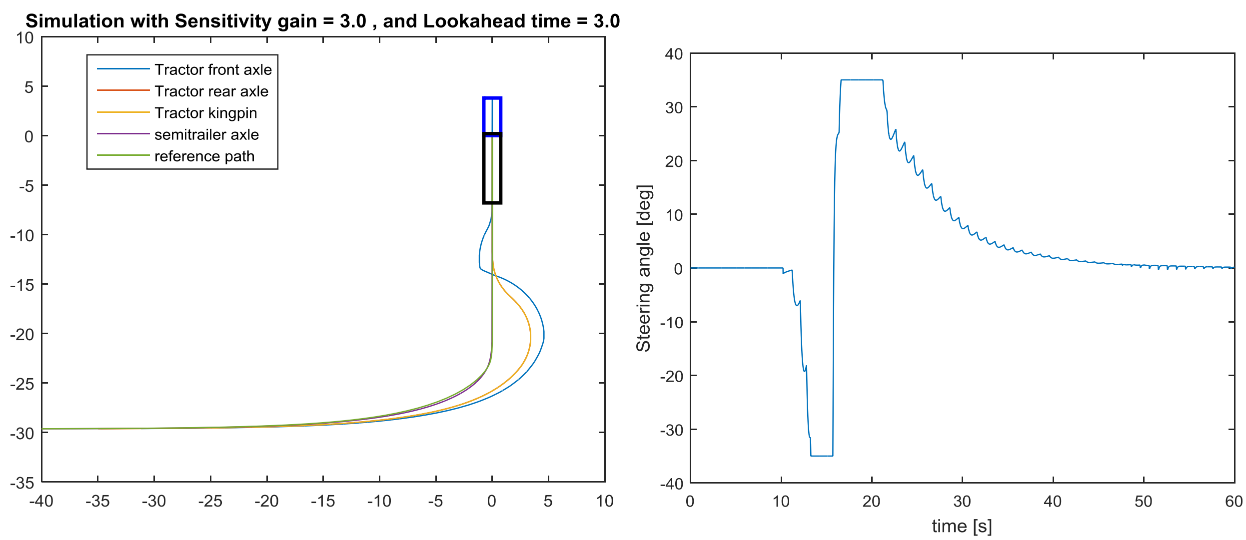


Figure 15: Reverse simulation with 35 degree limit

Figure 15 shows the same simulation but with limit on the steering angle. The limit of the steering angle to control the vehicle in a 90 degree corner is 35 degrees. The steering angle can be reduced to nearly half the value needed for the vehicle to perform the maneuver in forwards motion.

%% Further test on S-shape turns and test controller on response with noise on the signals.

References

[1] K. Patel (Oktober 2014). Driver assistance during rearward maneuvering of a Longer Heavier Vehicle, HAN Univiversity of Applied Sciences

[2] Morales, Jesus; Mandow, Anthony; Martinez, Jorge L.; Martínez, Jorge L.; Garcia-Cerezo, Alfonso J.; ,"Driver assistance system for backward maneuvers in passive multi-trailer vehicles,"2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp.4853-4858, 7-12 Oct. 2012 doi: 10.1109/IROS.2012.6385799

Appendix A

The positions and velocities of the truck-trailer combination is given in a (x, y)coordinate system. The reference point for the vehicle is given by (x_0, y_0). A longitudinal velocity v_0 acts on the reference point as input of the system.

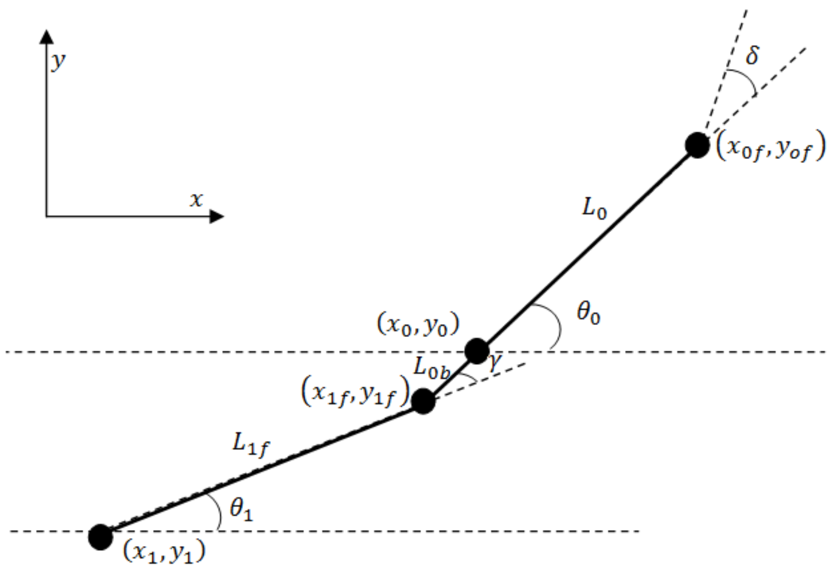


Figure 16: Single articulated vehicle model

The positions of the axels can be defined based on the reference position:

$$x_{0f} = x_0 + L_0 \cos(\theta_0) \tag{A.1}$$
$$y_{0f} = y_0 + L_0 \sin(\theta_0) \tag{A.2}$$
$$x_{1f} = x_0 - L_{0b} \cos(\theta_0) \tag{A.3}$$
$$y_{1f} = y_0 - L_{0b} \sin(\theta_0) \tag{A.4}$$
$$x_1 = x_1 f - L_{1f} \cos(\theta_1) = x_0 - L_{0b} \cos(\theta_0) - L_{1f} \cos(\theta_1) \tag{A.5}$$
$$y_1 = y_{1f} - L_{1f} \sin(\theta_1) = y_0 - L_{0b} \sin(\theta_0) - L_{1f} \sin(\theta_1) \tag{A.6}$$

Differentiating the reference point and Equation (A.1-A.6) results into:

$$\dot{x}_0 = v_0 \cos(\theta_0) \tag{A.7}$$
$$\dot{y}_0 = v_0 sin(\theta_0) \tag{A.8}$$
$$\dot{x_{0f}} = \dot{x_0} - L_0 \sin(\theta_0)\dot{\theta_0} \tag{A.9}$$
$$\dot{y_{0f}} = \dot{y_0} + L_0 \cos(\theta_0)\dot{\theta_0} \tag{A.10}$$
$$\dot{x_{1f}} = \dot{x_0} + L_{0b} \sin(\theta_0)\dot{\theta_0} \tag{A.11}$$
$$\dot{y_{1f}} = \dot{y_0} - L_{0b} \cos(\theta_0)\, \dot{\theta_0} \tag{A.12}$$

$$\dot{x_1} = \dot{x_0} + L_{0b}\sin(\theta_0)\dot{\theta_0} + L_{1f}\sin(\theta_1)\dot{\theta_1} \tag{A.13}$$
$$\dot{y_1} = \dot{y_0} - L_{0b}\cos(\theta_0)\dot{\theta_0} - L_{1f}\cos(\theta_1)\dot{\theta_1} \tag{A.14}$$

The articulation angle and rate is given by:

$$\gamma = \theta_0 - \theta_1 \tag{A.15}$$
$$\dot{\gamma} = \dot{\theta_0} - \dot{\theta_1} \tag{A.16}$$

The general form of the non-holonomic constraint is given by Equation A.17.

$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0 \tag{A.17}$$

$\dot{x}$ and $\dot{y}$ are the velocities of the axels in the given coordinate system and θ is the angle with respect to the x-axis. Based on Equation A.17, the non-holonomic constraints for model in Figure 16 are given by:

$$\dot{x}_{0f}\sin(\theta_0 + \delta) - \dot{y}_{0f}\cos(\theta_0 + \delta) = 0 \tag{A.18}$$
$$\dot{x}_0\sin(\theta_0) - \dot{y}_0\cos(\theta_0) = 0 \tag{A.19}$$
$$\dot{x}_1\sin(\theta_1) - \dot{y}_1\cos(\theta_1) = 0 \tag{A.20}$$

The yaw rate of the truck can be defined by substituting Equation A.9 and A.10 into Equation A.18.

$$\left(\dot{x_0} - L_0\sin(\theta_0)\,\dot{\theta_0}\right)\sin(\theta_0 + \delta) - \left(\dot{y_0} + L_0\cos(\theta_0)\dot{\theta_0}\right)\cos(\theta_0 + \delta) = 0 \tag{A.21}$$

Substituting Equations A.7 and 8 into Equation A.21 results into:

$$\left(v_0\cos(\theta_0) - L_0\sin(\theta_0)\,\dot{\theta_0}\right)\sin(\theta_0 + \delta)\,...$$
$$-\left(v_0 sin(\theta_0) + L_0\cos(\theta_0)\dot{\theta_0}\right)\cos(\theta_0 + \delta) = 0$$

$$v_0(\cos(\theta_0)\sin(\theta_0 + \delta) - \sin(\theta_0)\cos(\theta_0 + \delta))\,...$$
$$-L_0\dot{\theta}_0(\sin(\theta_0)\sin(\theta_0 + \delta) + \cos(\theta_0)\cos(\theta_0 + \delta))=0 \tag{A.22}$$

Known are the trigonometric identities:

$$2\cos(a)\cos(b) = \cos(a - b) + \cos(a + b) \tag{A.23}$$
$$2\sin(a)\sin(b) = \cos(a - b) - \cos(a + b) \tag{A.24}$$
$$2\sin(a)\cos(b) = \sin(a + b) + \sin(a - b) \tag{A.25}$$
$$2\cos(a)\sin(b) = \sin(a + b) - \sin(a - b) \tag{A.26}$$

With Equation (A.23-A.26), Equation A.22 can be simplified to:

$$v_0\sin(\delta) - L_0\dot{\theta}_0\cos(\delta) = 0$$

$$\dot{\theta}_0 = \frac{v_0}{L_0}\tan(\delta) \tag{A.27}$$

Using the constraint in Equation A.20, the yaw rate of the trailer can be defined. Substituting Equation A.7, A.8, A.13 and A.14 into Equation A.20:

$$\left(v_0\cos(\theta_0) + L_{0b}\sin(\theta_0)\dot{\theta}_0 + L_{1f}\sin(\theta_1)\,\dot{\theta}_1\right)\sin(\theta_1)\ldots$$
$$-\left(v_0 sin(\theta_0) - L_{0b}\cos(\theta_0)\dot{\theta}_0 - L_{1f}\cos(\theta_1)\dot{\theta}_1\right)\cos(\theta_1) = 0$$

$$v_0\,(\,\cos(\theta_0)\sin(\theta_1) - \sin(\theta_0)\cos(\theta_1)\,)\; + L_{0b}\dot{\theta}_0(\sin(\theta_0)\sin(\theta_1)\; + \cos(\theta_0)\cos(\theta_1))\ldots$$
$$+L_{1f}\dot{\theta}_1(\sin(\theta_1)\sin(\theta_1) + \cos(\theta_1)\cos(\theta_1)) = 0 \tag{A.28}$$

Using the trigonometric identities given in Equation (A.23-A.26), Equation A.28 can be simplified:

$$-v_0\sin(\theta_0 - \theta_1) + L_{0b}\dot{\theta}_0\cos(\theta_0 - \theta_1) + L_{1f}\dot{\theta}_1$$
$$\dot{\theta}_1 = \frac{v_0\sin(\theta_0-\theta_1) - L_{0b}\dot{\theta}_0\cos(\theta_0-\theta_1)}{L_{1f}} \tag{A.29}$$

Substituting Equations A.15 and A.27 into A.29, results into:

$$\dot{\theta}_1 = \frac{v_0}{L_{1f}}\left(\sin(\gamma)\; - \frac{L_{0b}}{L_0}\tan(\delta)\cos(\gamma)\right) \tag{A.30}$$