

# **Vehicle Dynamics Project**

**Winter- 2017**

**Modelling of vehicle using linear and non-linear models**



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## Vehicle Dynamics

To model the Dynamic characteristics of the vehicle the following Equations were Used:

$$\begin{aligned}m(\dot{V}_x - rV_y) &= F_{xf} \cos \delta_f - F_{yf} \sin \delta_f + F_{xr} \\m(\dot{V}_y + rV_x) &= F_{xf} \sin \delta_f + F_{yf} \cos \delta_f + F_{yr} \\I_{zz} \dot{r} &= a(F_{xf} \sin \delta_f + F_{yf} \cos \delta_f) - bF_{yr}\end{aligned}$$

Where m is the mass of the vehicle,  $V_x$  is the velocity in x, r is angular velocity,  $V_y$  is velocity in y,  $F_{xf}$  is force on front wheel in x direction,  $\delta_f$  is the wheel angle,  $F_{yf}$  is the force on front tyre in y direction,  $F_{xr}$  is the force on rear tyre in x direction,  $I_{zz}$  is the moment of Inertia of the vehicle about zz-axis, a is the distance of front axle from center of gravity and b is the distance of rear axle from CG.

To determine the  $F_{xf}$ ,  $F_{xr}$ ,  $F_{yf}$ ,  $F_{yr}$  two types of tire models are used which is described in later section.

## Tire Modeling

Two approaches were used to model tire behavior, namely, Linear tire model and non-linear tire(or Sine model).

### Linear tire model

Linear tire model is a simple way of modelling tire behavior, following equations depicts the behavior of linear tire model. These equations were used to model linear vehicle model.

$$\begin{aligned}F_{yf} &= C_{\alpha f} \alpha_f = C_{\alpha f} \left( \delta_f - \frac{V_y + \dot{\psi}a}{V_x} \right) \\F_{yr} &= C_{\alpha r} \alpha_r = C_{\alpha r} \left( \delta_r - \frac{V_y - \dot{\psi}b}{V_x} \right) \end{aligned}$$

$$F_{xf} = Q_f / R_{fe}$$

$$F_{xr} = Q_r / R_{re}$$

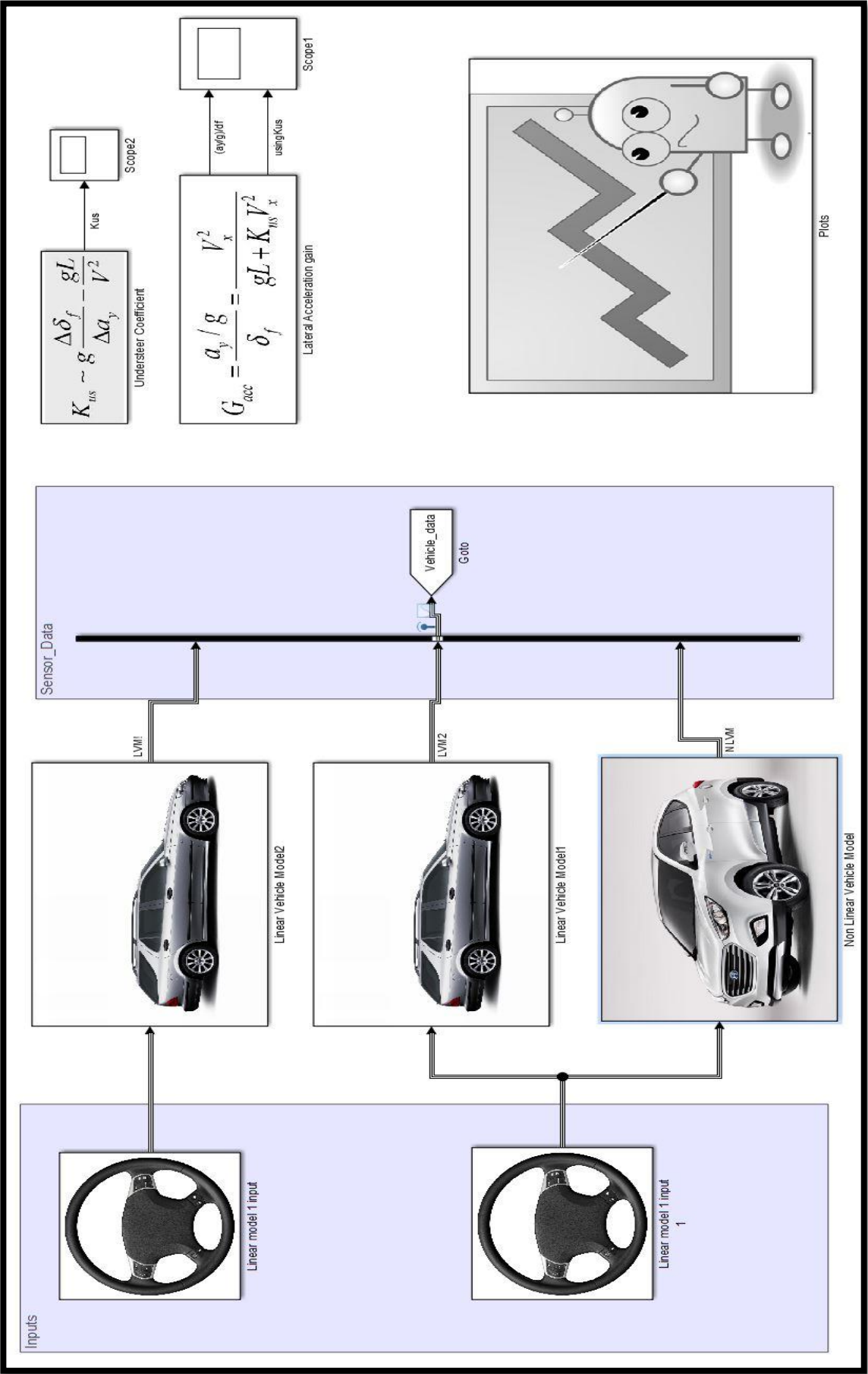
### Non-linear tire model

There are many models available for modeling nonlinear tire behavior, here we have used the sine model, here are the Equations for this model

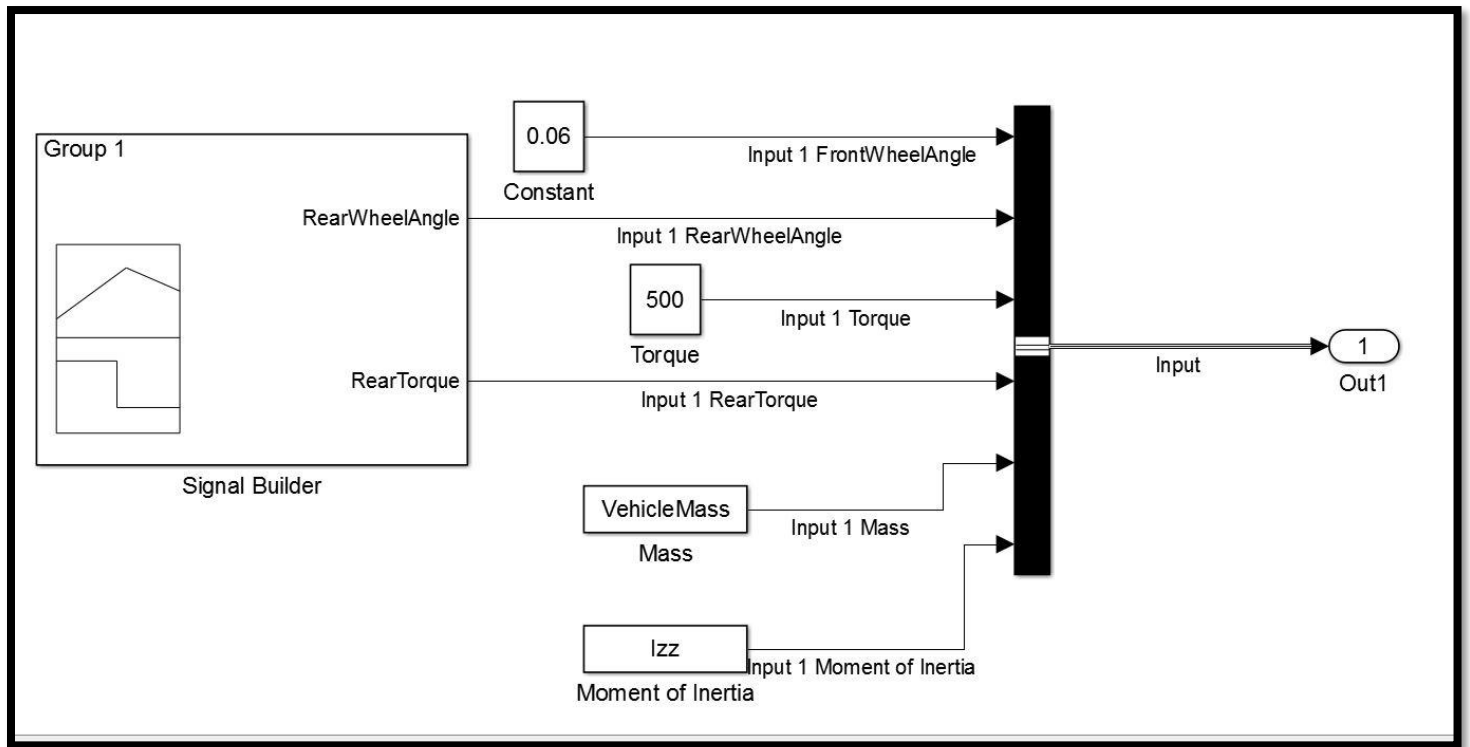
$$F_y(\alpha) = A \sin \left[ B \arctan \left( \frac{C_\alpha}{AB} \alpha \right) \right]$$

$$A = F_y^{\max} = \mu F_z$$

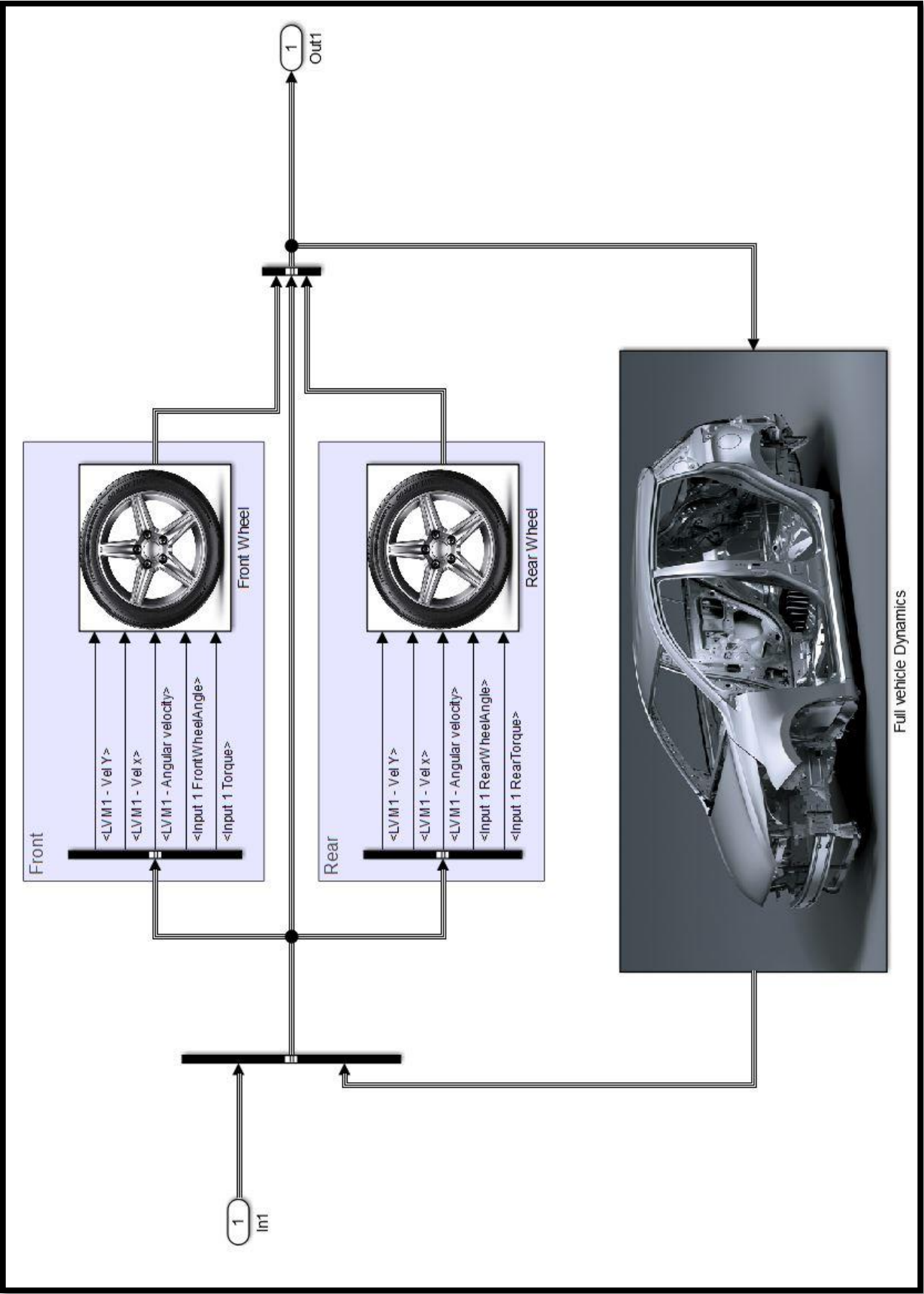
Implementation –



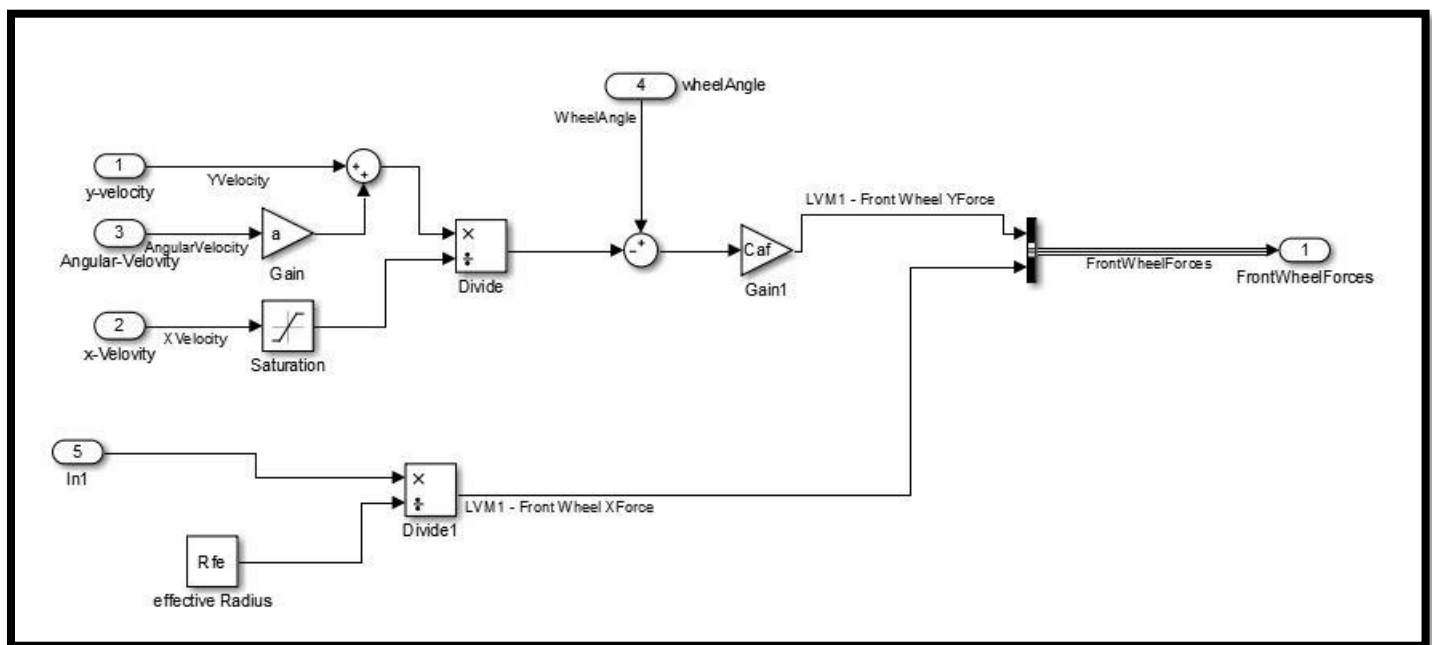
## Inputs



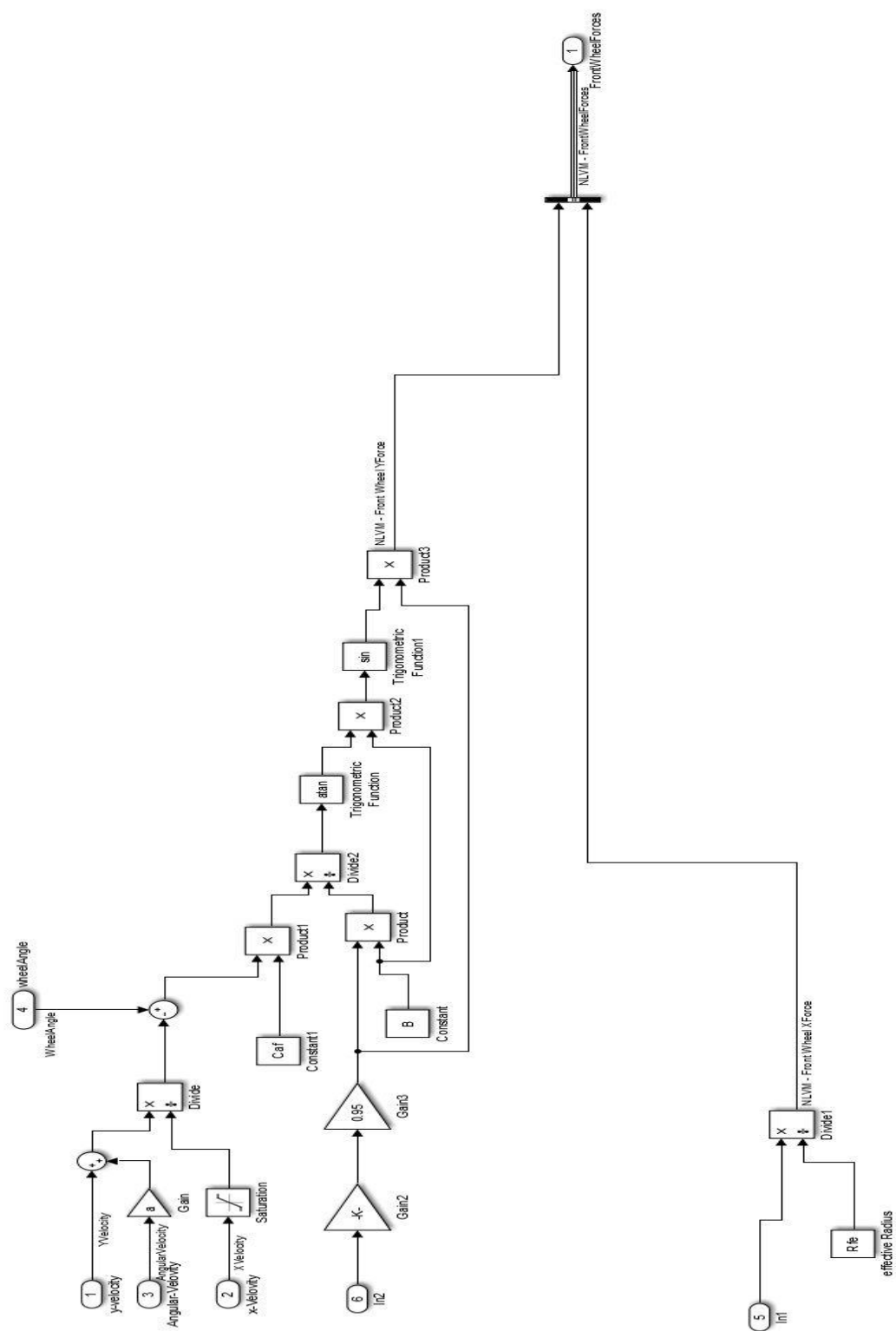
Vehicle Model



## Inside Front Tire Model

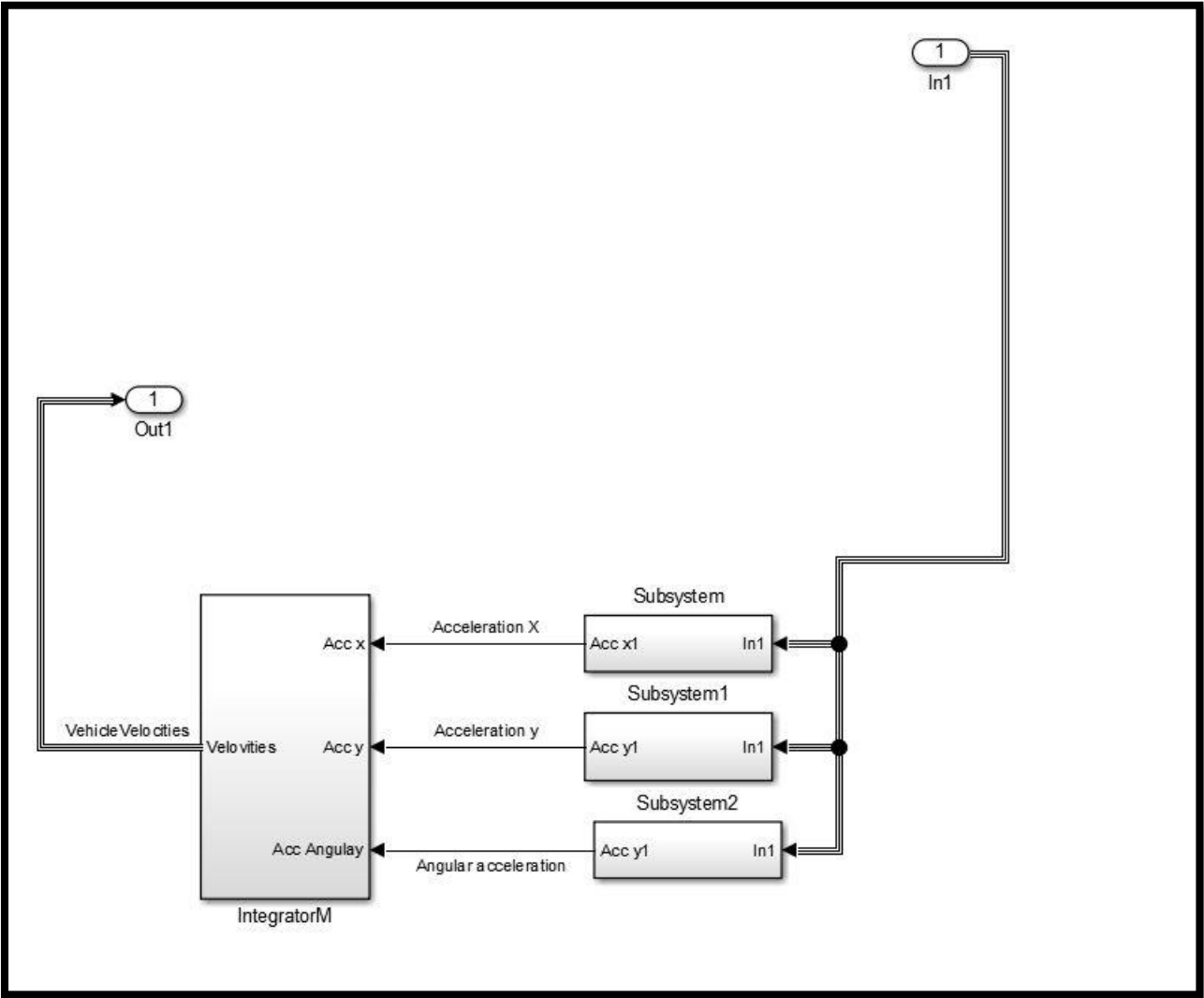


### Inside non-linear tire Model



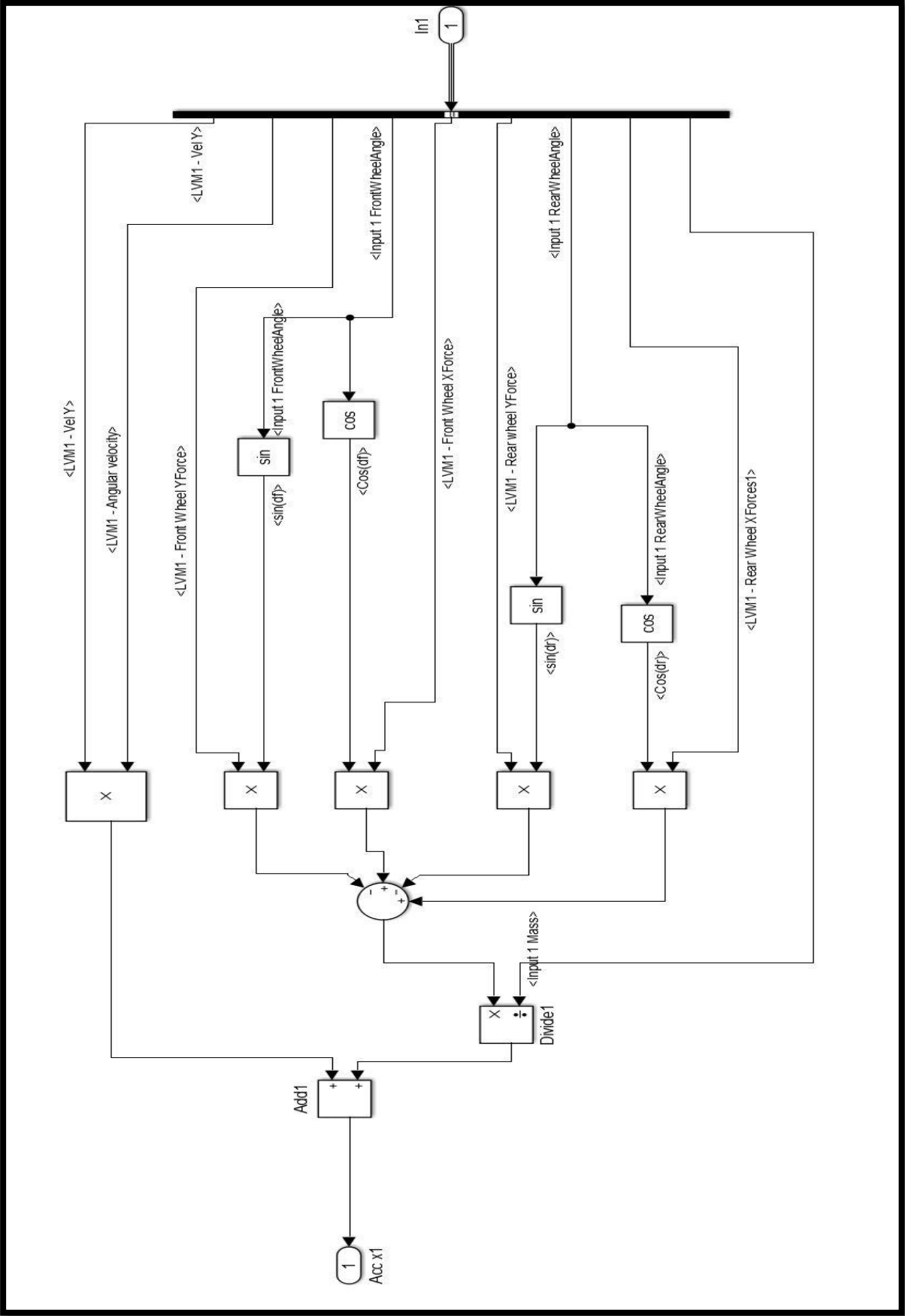


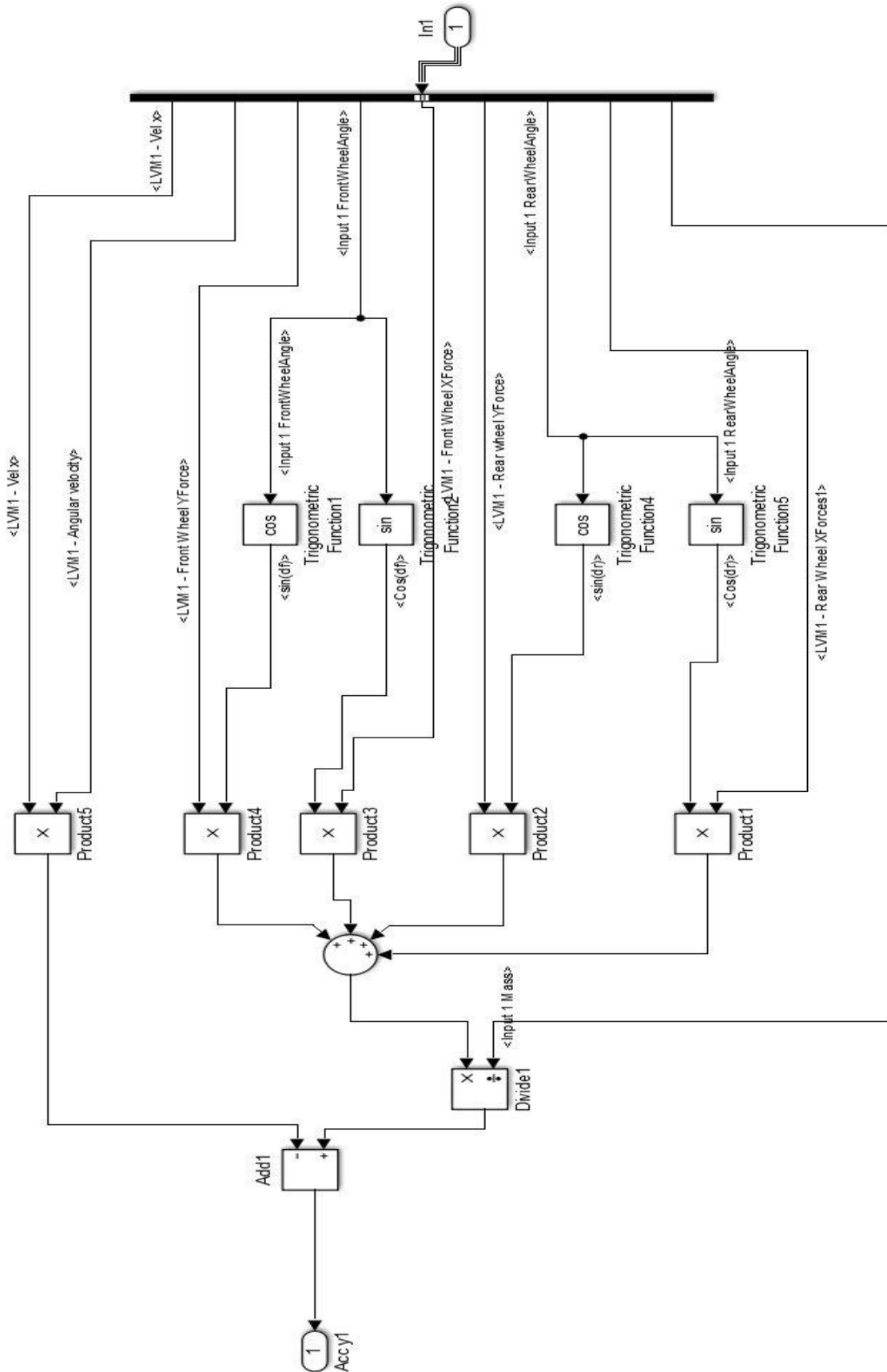
Inside Full Vehicle Dynamics Block



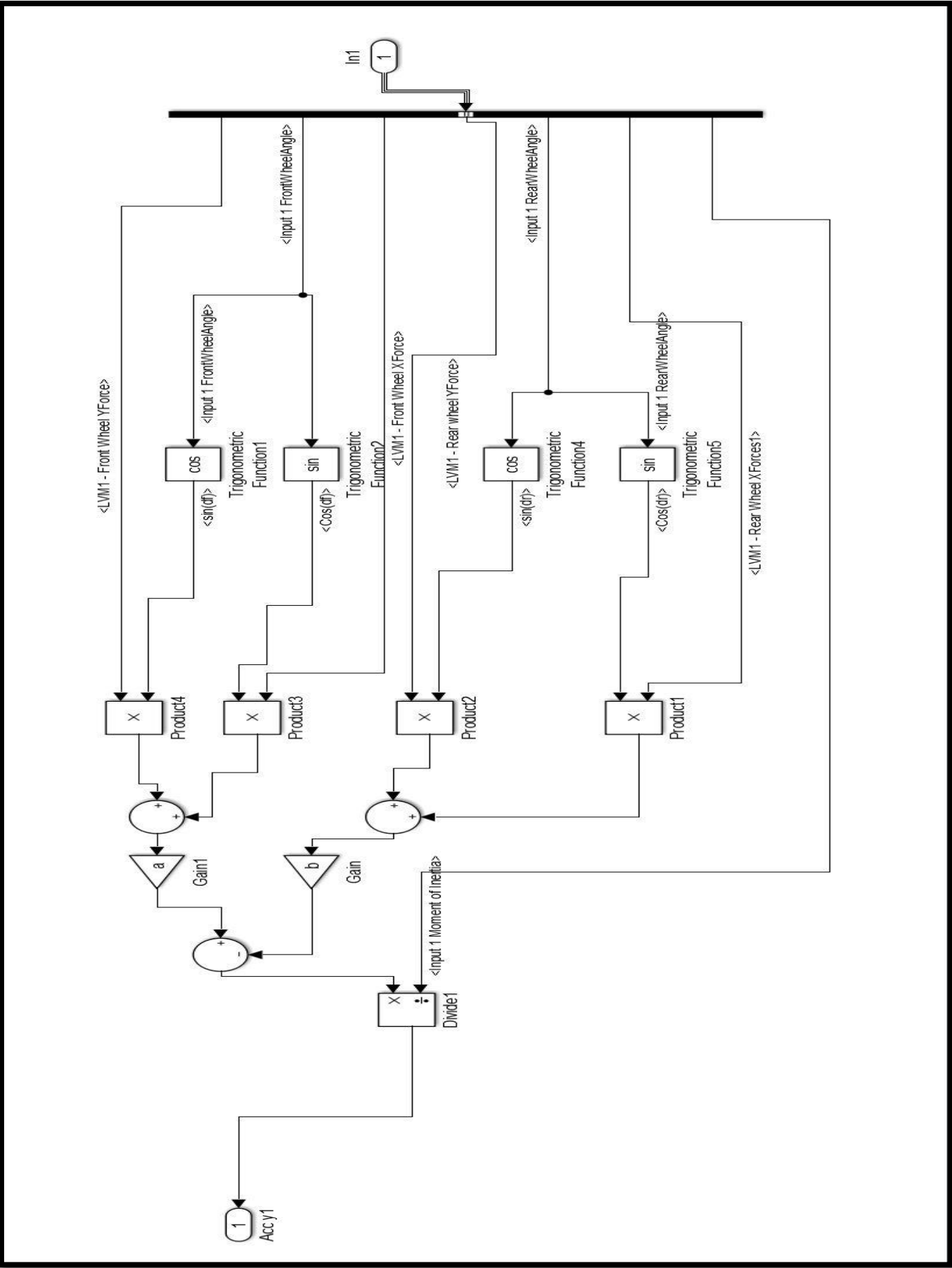
Acceleration calculation

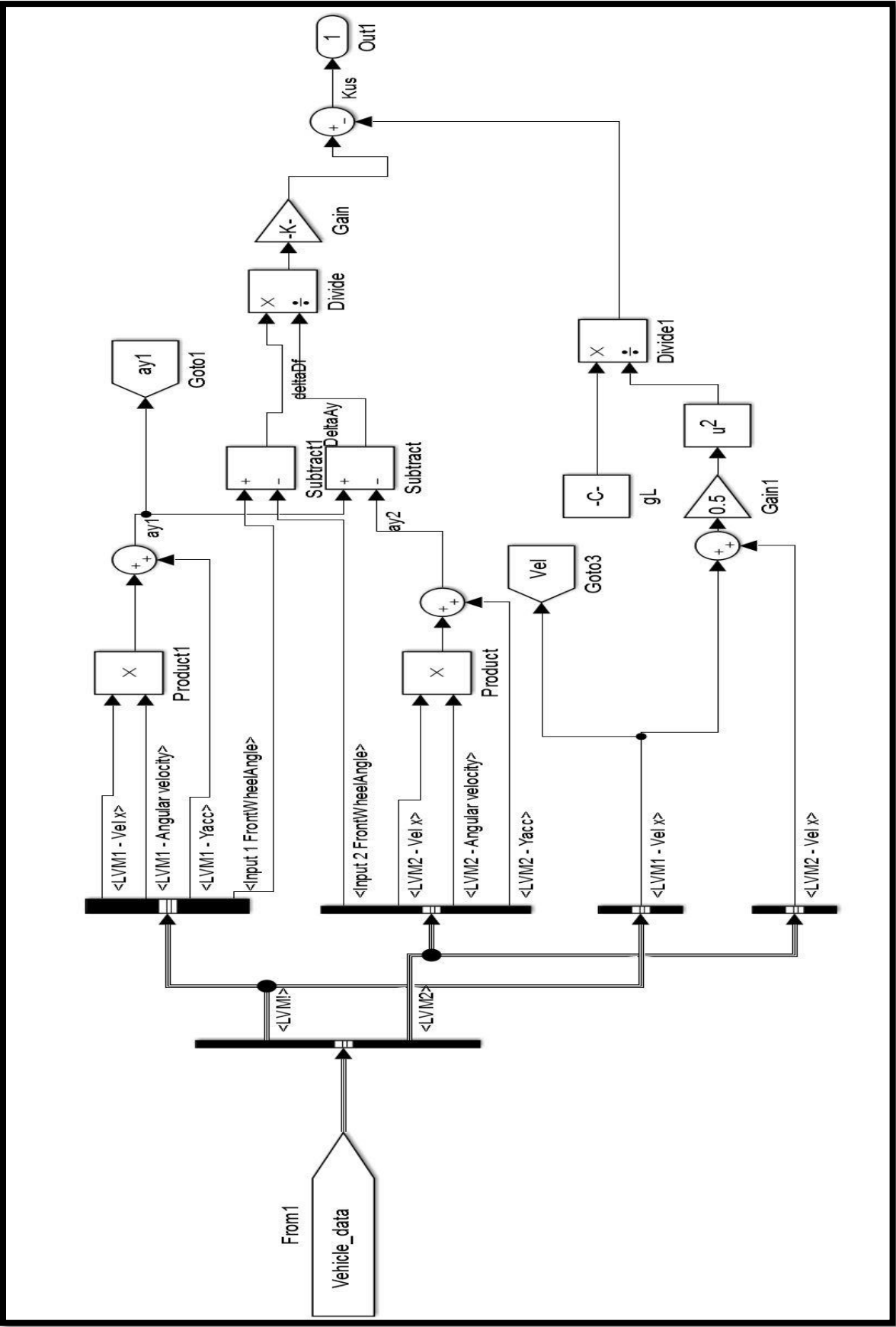
X



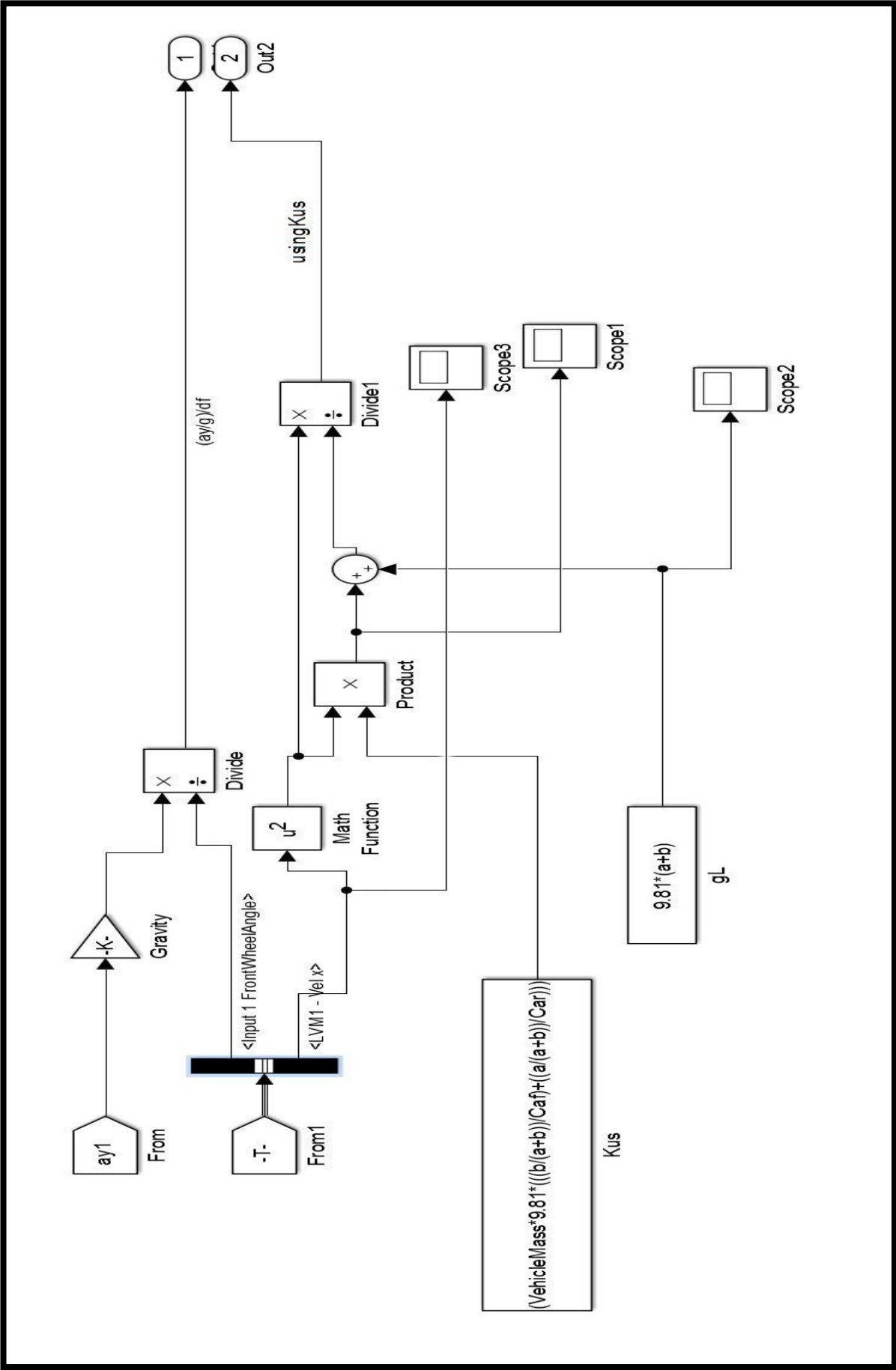


Angular Acceleration





Lateral Acceleration gain

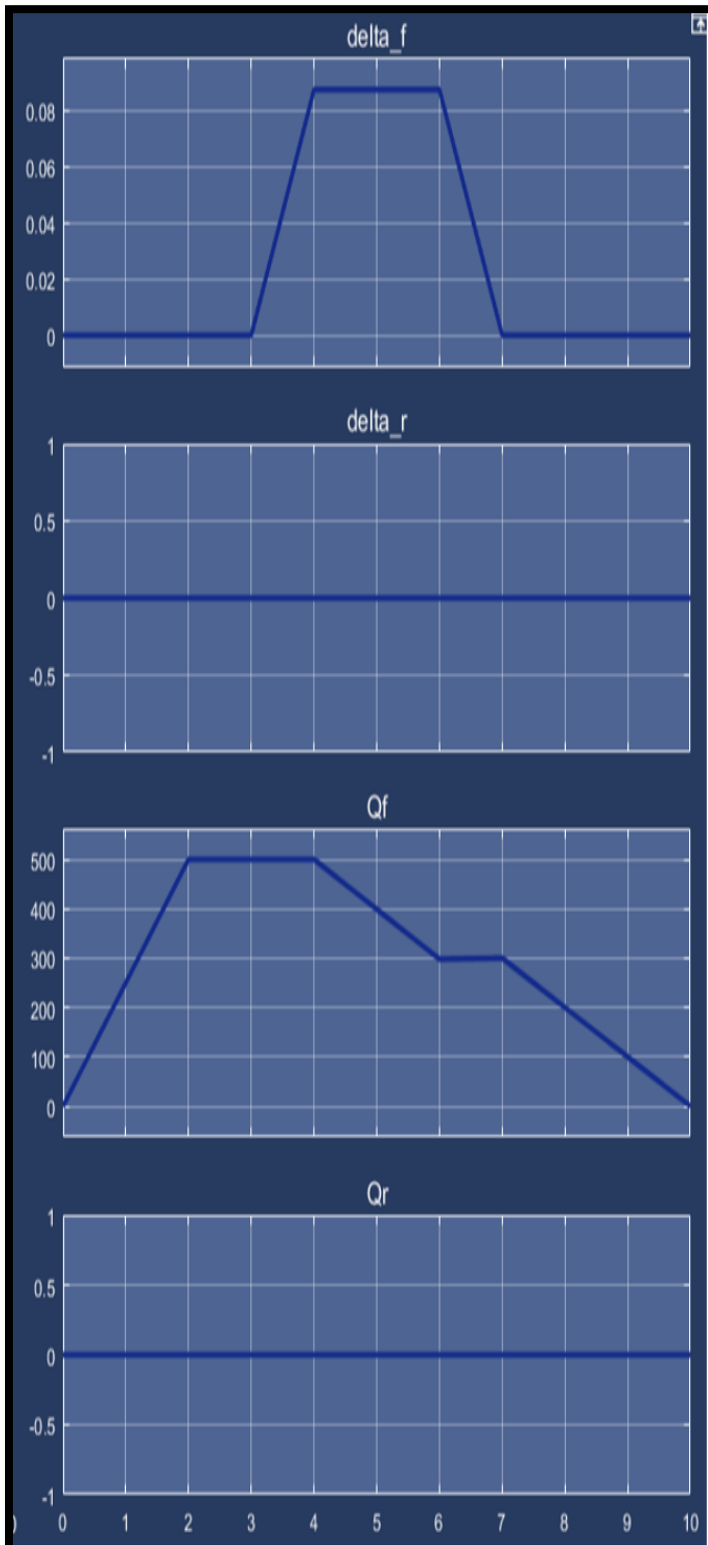


## Result and Analysis

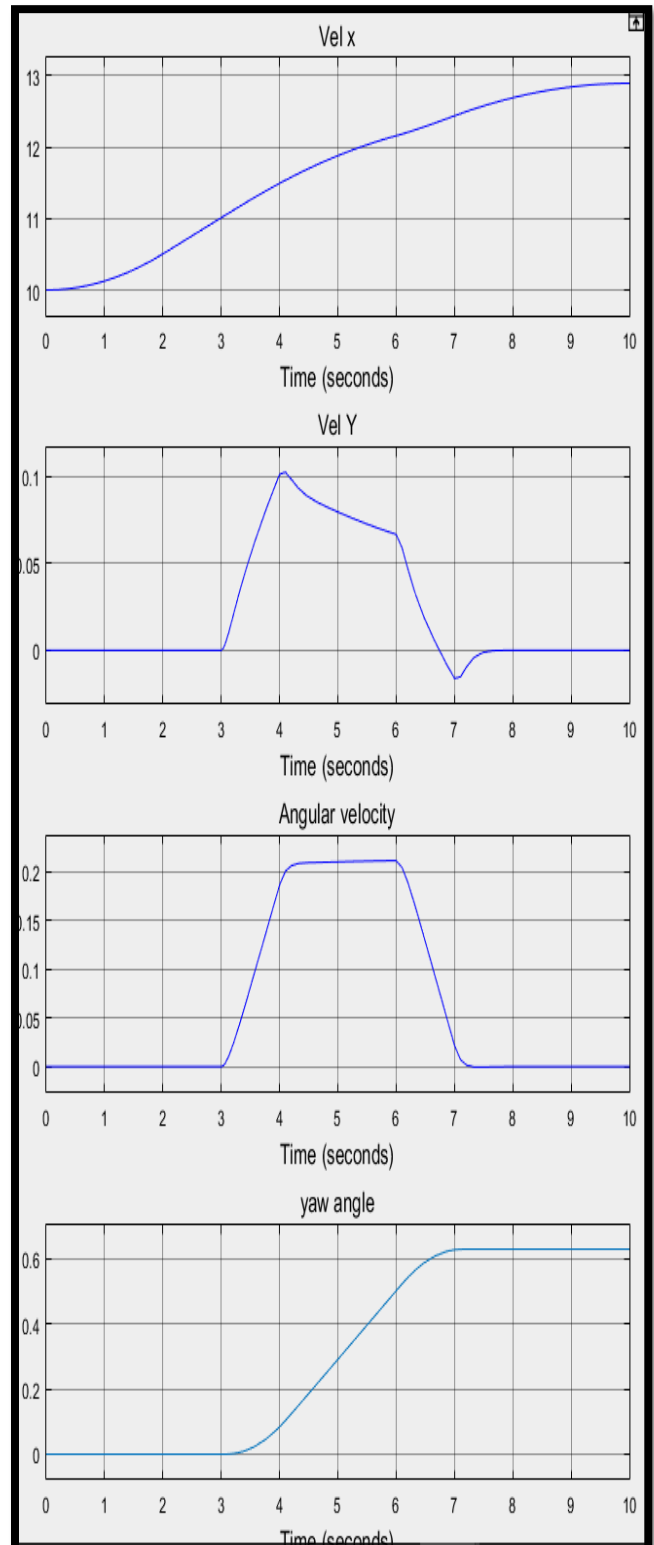
### Baseline Validation

For Baseline validation, the output acquired is same as expected hence the validation was approved

Inputs:



Outputs:

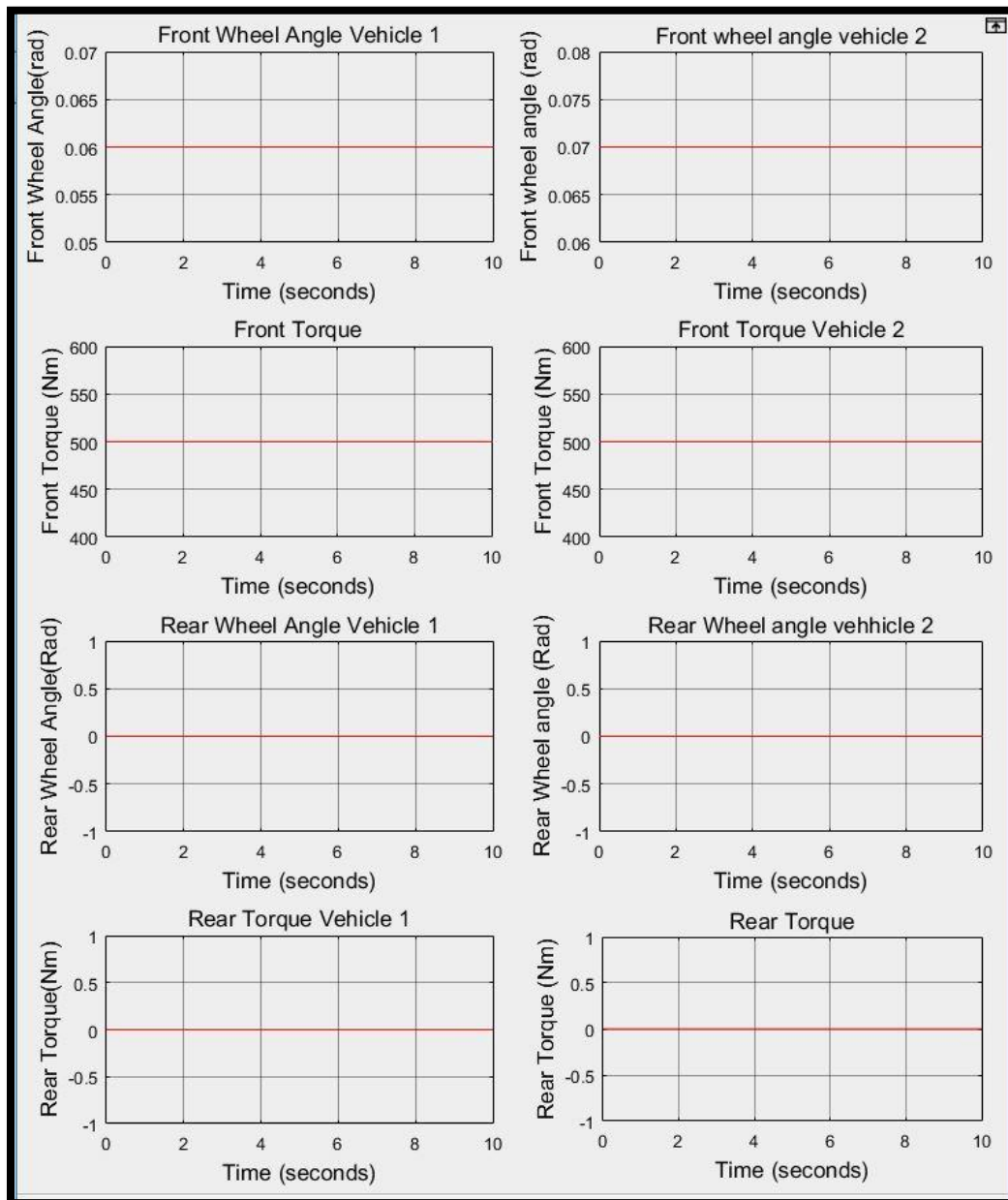


## Understeer Coefficient:

To find the understeer coefficient constant velocity test was used under which the understeer coefficient can be determined using following formula

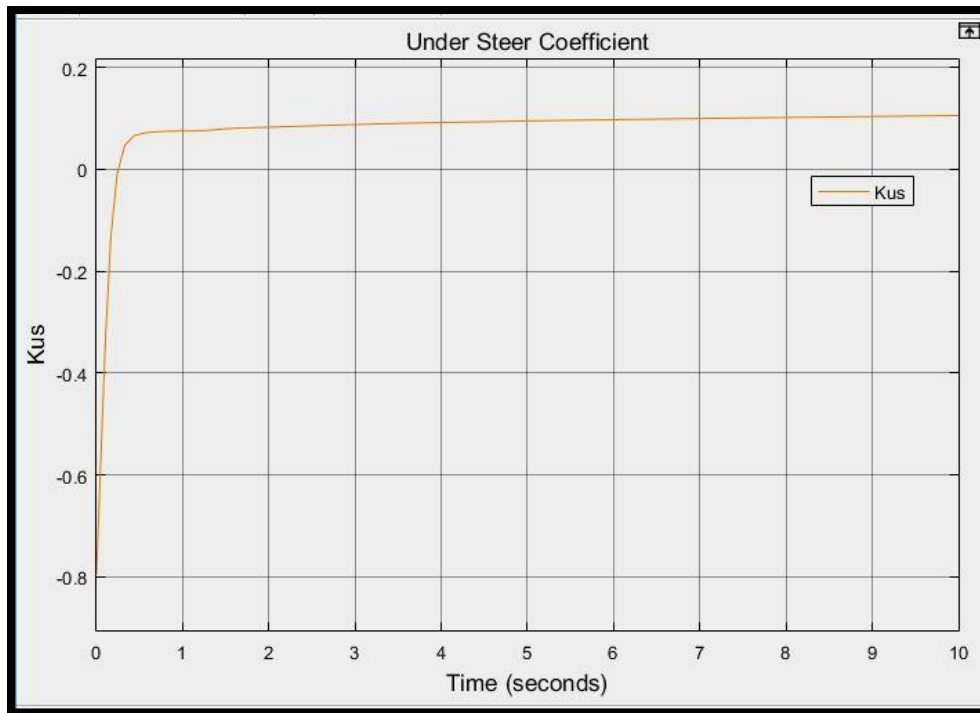
$$K_{us} \sim g \frac{\Delta\delta_f}{\Delta a_y} - \frac{gL}{V^2}$$

Inputs





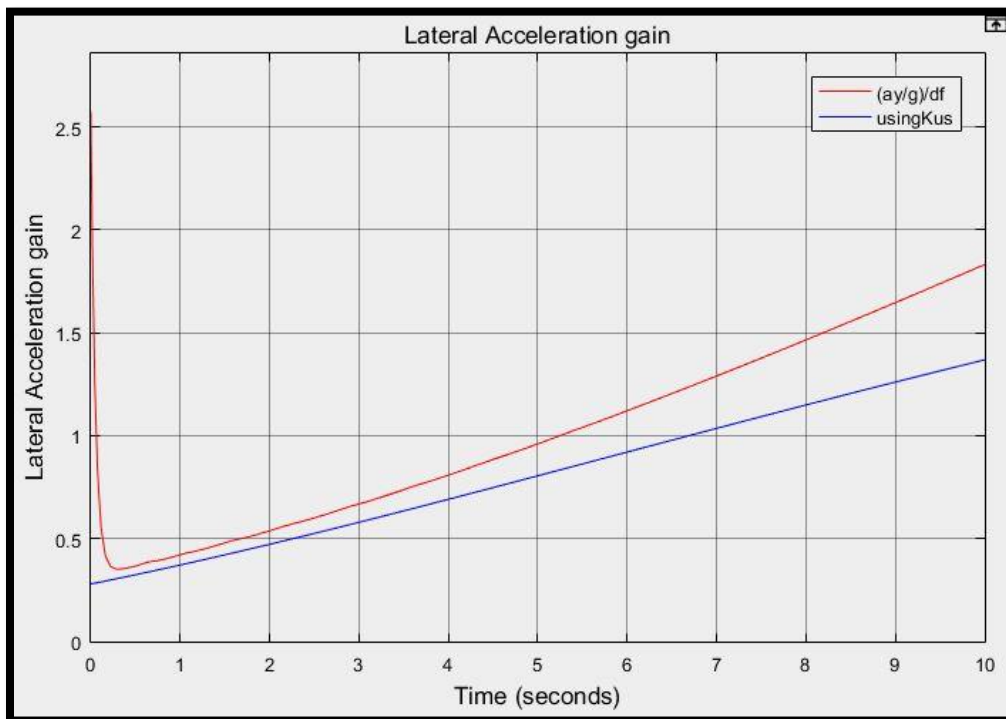
And the following plot was obtained:



### Lateral Acceleration gain

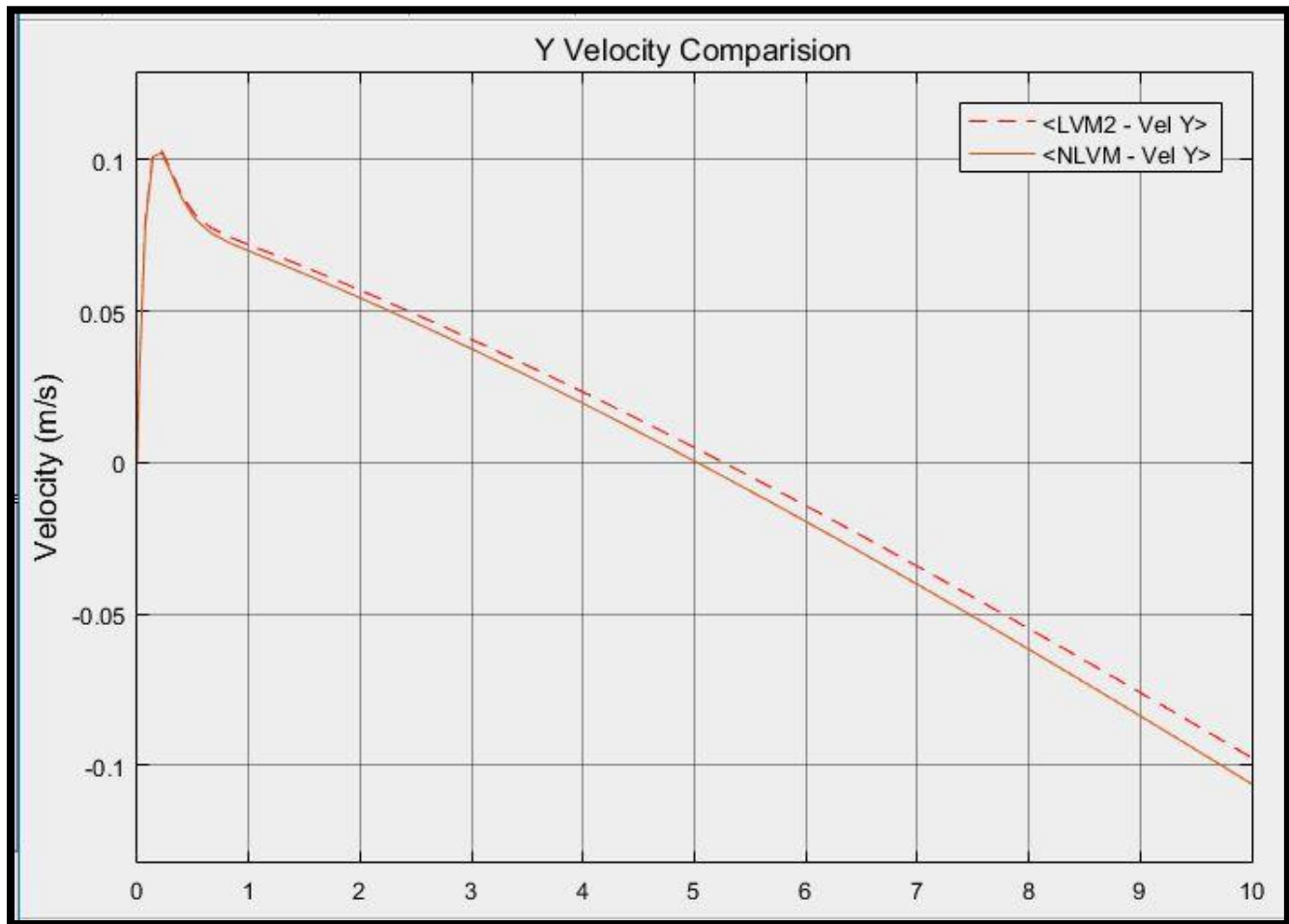
Lateral Acceleration gain was calculated using the following two formulas the values obtained were similar at lower values of  $V_y$  and  $\alpha$ .

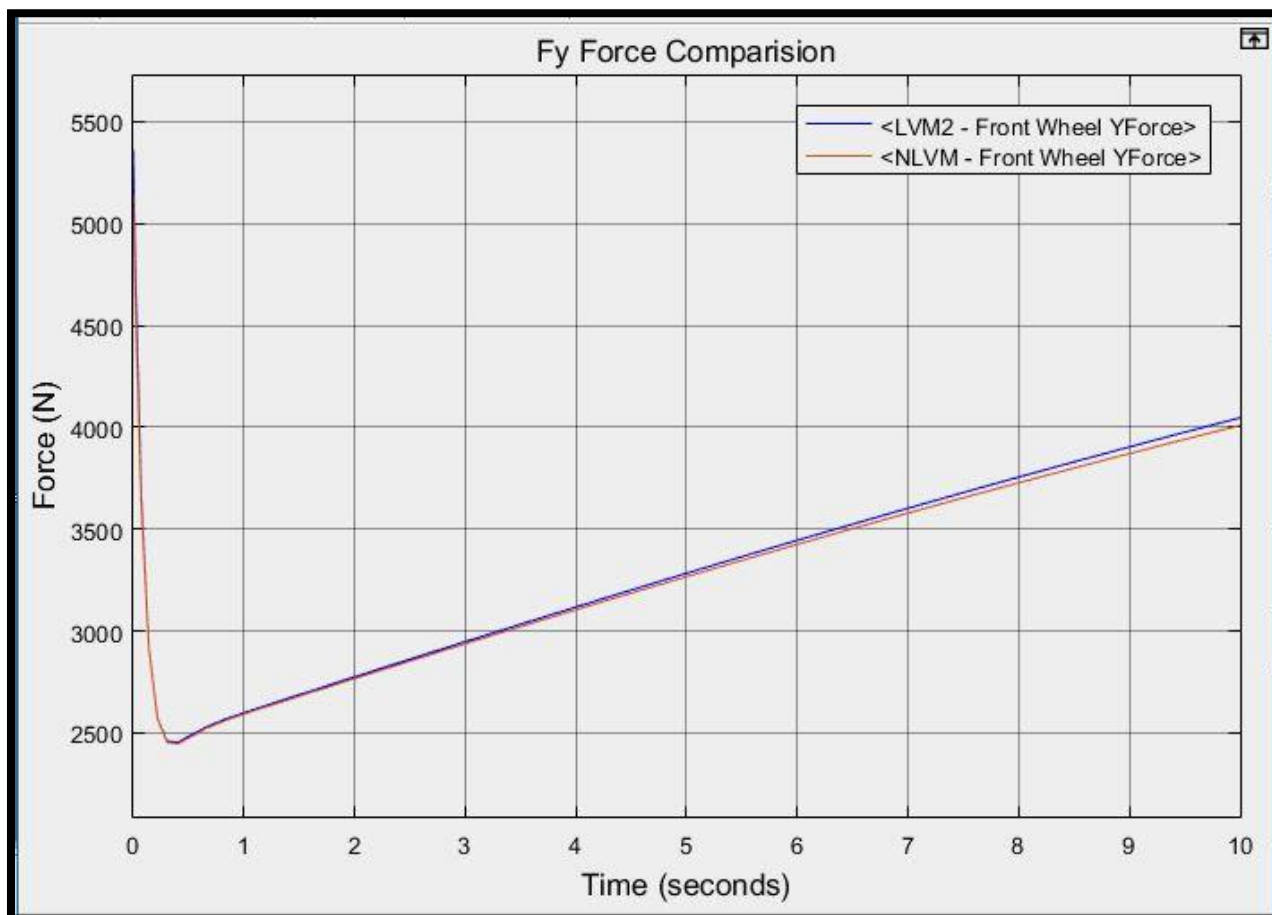
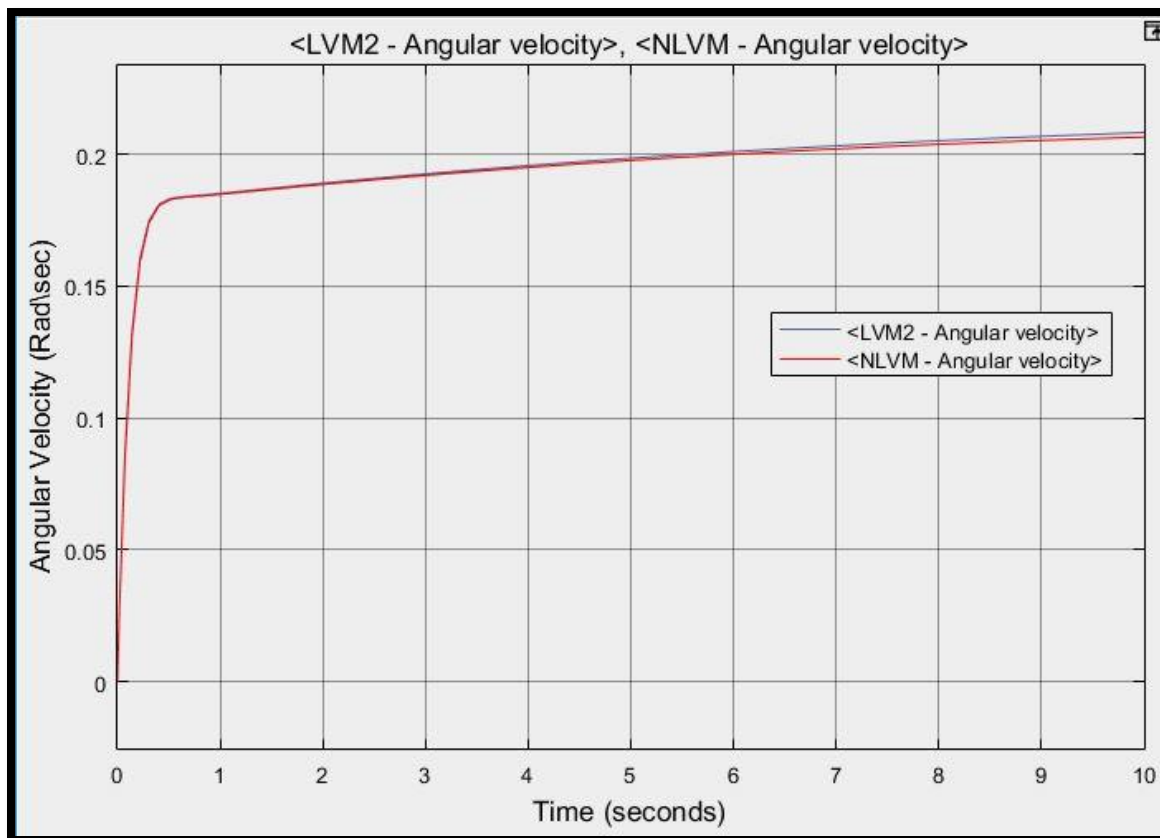
$$G_{acc} = \frac{a_y / g}{\delta_f} = \frac{V_x^2}{gL + K_{us} V_x^2}$$



## Comparison of Linear and nonlinear model

After the above modelling and testing the model was found to be satisfactory and then the comparison of results obtained by linear and nonlinear model was done although the initial values obtained by both the model are similar but with increasing values of velocity in  $y$  and  $\alpha$  the linear model drifts away from the nonlinear model hence showing the lack of accuracy in linear model at higher slip angles





**References:**

- Gillespie, T.D., Fundamentals of Vehicle Dynamics, Society of Automotive Engineers, 1992.
- Wong, J.Y., Theory of Ground Vehicles, Wiley-Interscience, 2001.
- Pacejka, H.B. and Besselink, I., Tire and Vehicle Dynamics, Butterworth-Heinemann, 2012.