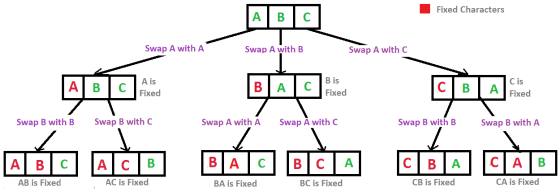
Backtracking

1. Write a C program to print all permutations of a given string

A permutation, also called an “arrangement number” or “order,” is a rearrangement of the elements of an ordered list S into a one-to-one correspondence with S itself. A string of length n has n! permutation.
Source: Mathword(http://mathworld.wolfram.com/Permutation.html)

Below are the permutations of string ABC.
ABC, ACB, BAC, BCA, CAB, CBA

Here is a solution using backtracking.



Recursion Tree for Permutations of String "ABC"

```
# include <stdio.h>

/* Function to swap values at two pointers */
void swap (char *x, char *y)
{
    char temp;
    temp = *x;
    *x = *y;
    *y = temp;
}

/* Function to print permutations of string
   This function takes three parameters:
   1. String
   2. Starting index of the string
   3. Ending index of the string. */
void permute(char *a, int i, int n)
{
   int j;
   if (i == n)
     printf("%s\n", a);
   else
   {
        for (j = i; j <= n; j++)
       {
          swap((a+i), (a+j));
          permute(a, i+1, n);
          swap((a+i), (a+j)); //backtrack
       }
   }
}

/* Driver program to test above functions */
int main()
{
   char a[] = "ABC";
   permute(a, 0, 2);
   getchar();
   return 0;
}
```

Output:

```
ABC
ACB
BAC
BCA
CBA
CAB
```

Algorithm Paradigm: Backtracking
Time Complexity: O(n*n!)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

2. Backtracking | Set 1 (The Knight's tour problem)

Backtracking is an algorithmic paradigm that tries different solutions until finds a solution that "works". Problems which are typically solved using backtracking technique have following property in common. These problems can only be solved by trying every possible configuration and each configuration is tried only once. A Naive solution for these problems is to try all configurations and output a configuration that follows given problem constraints. Backtracking works in incremental way and is an optimization over the Naive solution where all possible configurations are generated and tried.

For example, consider the following Knight's Tour problem.
The knight is placed on the first block of an empty board and, moving according to the rules of chess, must visit each square exactly once.

Let us first discuss the Naive algorithm for this problem and then the Backtracking algorithm.

Naive Algorithm for Knight's tour
The Naive Algorithm is to generate all tours one by one and check if the generated tour satisfies the constraints.

```
while there are untried tours
{
   generate the next tour
   if this tour covers all squares
   {
      print this path;
   }
}
```

Backtracking works in an incremental way to attack problems. Typically, we start from an empty solution vector and one by one add items (Meaning of item varies from problem to problem. In context of Knight's tour problem, an item is a Knight's move). When we add an item, we check if adding the current item violates the problem constraint, if it does then we remove the item and try other alternatives. If none of the alternatives work out then we go to previous stage and remove the item added in the previous stage. If we reach the initial stage back then we say that no solution exists. If adding an item doesn't violate constraints then we recursively add items one by one. If the solution vector becomes complete then we print the solution.

Backtracking Algorithm for Knight's tour

Following is the Backtracking algorithm for Knight's tour problem.

```
If all squares are visited
    print the solution
Else
   a) Add one of the next moves to solution vector and recursively
   check if this move leads to a solution. (A Knight can make maximum
   eight moves. We choose one of the 8 moves in this step).
   b) If the move chosen in the above step doesn't lead to a solution
   then remove this move from the solution vector and try other
   alternative moves.
   c) If none of the alternatives work then return false (Returning false
   will remove the previously added item in recursion and if false is
   returned by the initial call of recursion then "no solution exists" )
```

Following is C implementation for Knight's tour problem. It prints one of the possible solutions in 2D matrix form. Basically, the output is a 2D 8*8 matrix with numbers from 0 to 63 and these numbers show steps made by Knight.

```
#include<stdio.h>
#define N 8

int solveKTUtil(int x, int y, int movei, int sol[N][N], int xMove[],
                int yMove[]);

/* A utility function to check if i,j are valid indexes for N*N chessb
int isSafe(int x, int y, int sol[N][N])
{
    if ( x >= 0 && x < N && y >= 0 && y < N && sol[x][y] == -1)
        return 1;
    return 0;
}

/* A utility function to print solution matrix sol[N][N] */
void printSolution(int sol[N][N])
{
    for (int x = 0; x < N; x++)
    {
        for (int y = 0; y < N; y++)
            printf(" %2d ", sol[x][y]);
        printf("\n");
    }
}

/* This function solves the Knight Tour problem using Backtracking.  Tl
function mainly uses solveKTUtil() to solve the problem. It returns fa
no complete tour is possible, otherwise return true and prints the tou
Please note that there may be more than one solutions, this function
prints one of the feasible solutions.  */
bool solveKT()
{
    int sol[N][N];

    /* Initialization of solution matrix */
    for (int x = 0; x < N; x++)
```

```
        for (int y = 0; y < N; y++)
            sol[x][y] = -1;

    /* xMove[] and yMove[] define next move of Knight.
       xMove[] is for next value of x coordinate
       yMove[] is for next value of y coordinate */
    int xMove[8] = {  2, 1, -1, -2, -2, -1,  1,  2 };
    int yMove[8] = {  1, 2,  2,  1, -1, -2, -2, -1 };

    // Since the Knight is initially at the first block
    sol[0][0]  = 0;

    /* Start from 0,0 and explore all tours using solveKTUtil() */
    if(solveKTUtil(0, 0, 1, sol, xMove, yMove) == false)
    {
        printf("Solution does not exist");
        return false;
    }
    else
        printSolution(sol);

    return true;
}

/* A recursive utility function to solve Knight Tour problem */
int solveKTUtil(int x, int y, int movei, int sol[N][N], int xMove[N],
                int yMove[N])
{
   int k, next_x, next_y;
   if (movei == N*N)
       return true;

   /* Try all next moves from the current coordinate x, y */
   for (k = 0; k < 8; k++)
   {
       next_x = x + xMove[k];
       next_y = y + yMove[k];
       if (isSafe(next_x, next_y, sol))
       {
         sol[next_x][next_y] = movei;
         if (solveKTUtil(next_x, next_y, movei+1, sol, xMove, yMove) =
             return true;
         else
             sol[next_x][next_y] = -1;// backtracking
       }
   }

   return false;
}

/* Driver program to test above functions */
int main()
{
    solveKT();
    getchar();
    return 0;
}
```

Output:

```
 0  59  38  33  30  17   8  63
37  34  31  60   9  62  29  16
58   1  36  39  32  27  18   7
35  48  41  26  61  10  15  28
42  57   2  49  40  23   6  19
47  50  45  54  25  20  11  14
56  43  52   3  22  13  24   5
51  46  55  44  53   4  21  12
```

Note that Backtracking is not the best solution for the Knight's tour problem. See this for other better solutions. The purpose of this post is to explain Backtracking with an example.

References:
http://see.stanford.edu/materials/icspacs106b/H19-RecBacktrackExamples.pdf
http://www.cis.upenn.edu/~matuszek/cit594-2009/Lectures/35-backtracking.ppt
http://mathworld.wolfram.com/KnightsTour.html
http://en.wikipedia.org/wiki/Knight%27s_tour

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

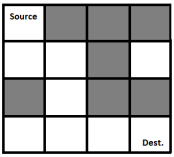
3. Backtracking | Set 2 (Rat in a Maze)

We have discussed Backtracking and Knight's tour problem in Set 1. Let us discuss Rat in a Maze as another example problem that can be solved using Backtracking.

A Maze is given as N*N binary matrix of blocks where source block is the upper left most block i.e., maze[0][0] and destination block is lower rightmost block i.e., maze[N-1][N-1]. A rat starts from source and has to reach destination. The rat can move only in two directions: forward and down.
In the maze matrix, 0 means the block is dead end and 1 means the block can be used in the path from source to destination. Note that this is a simple version of the typical Maze problem. For example, a more complex version can be that the rat can move in 4 directions and a more complex version can be with limited number of moves.

Following is an example maze.

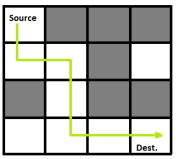
```
Gray blocks are dead ends (value = 0).
```



Following is binary matrix representation of the above maze.

```
{1, 0, 0, 0}
{1, 1, 0, 1}
{0, 1, 0, 0}
{1, 1, 1, 1}
```

Following is maze with highlighted solution path.



Following is the solution matrix (output of program) for the above input matrx.

```
{1, 0, 0, 0}
{1, 1, 0, 0}
{0, 1, 0, 0}
{0, 1, 1, 1}
All enteries in solution path are marked as 1.
```

Naive Algorithm

The Naive Algorithm is to generate all paths from source to destination and one by one check if the generated path satisfies the constraints.

```
while there are untried paths
{
   generate the next path
   if this path has all blocks as 1
   {
      print this path;
   }
}
```

Backtrackng Algorithm

```
If destination is reached
    print the solution matrix
Else
```

a) Mark current cell in solution matrix as 1.
b) Move forward in horizontal direction and recursively check if this move leads to a solution.
c) If the move chosen in the above step doesn't lead to a solution then move down and check if this move leads to a solution.
d) If none of the above solutions work then unmark this cell as 0 (BACKTRACK) and return false.

Implementation of Backtracking solution

```
#include<stdio.h>

// Maze size
#define N 4

bool solveMazeUtil(int maze[N][N], int x, int y, int sol[N][N]);

/* A utility function to print solution matrix sol[N][N] */
void printSolution(int sol[N][N])
{
    for (int i = 0; i < N; i++)
    {
        for (int j = 0; j < N; j++)
            printf(" %d ", sol[i][j]);
        printf("\n");
    }
}

/* A utility function to check if x,y is valid index for N*N maze */
bool isSafe(int maze[N][N], int x, int y)
{
    // if (x,y outside maze) return false
    if(x >= 0 && x < N && y >= 0 && y < N && maze[x][y] == 1)
        return true;

    return false;
}

/* This function solves the Maze problem using Backtracking.  It mainly
solveMazeUtil() to solve the problem. It returns false if no path is p
otherwise return true and prints the path in the form of 1s. Please no
there may be more than one solutions, this function prints one of the
solutions.*/
bool solveMaze(int maze[N][N])
{
    int sol[N][N] = { {0, 0, 0, 0},
        {0, 0, 0, 0},
        {0, 0, 0, 0},
        {0, 0, 0, 0}
    };

    if(solveMazeUtil(maze, 0, 0, sol) == false)
    {
        printf("Solution doesn't exist");
        return false;
    }

    printSolution(sol);
    return true;
```

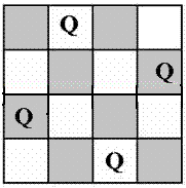
```
    return true;
}

/* A recursive utility function to solve Maze problem */
bool solveMazeUtil(int maze[N][N], int x, int y, int sol[N][N])
{
    // if (x,y is goal) return true
    if(x == N-1 && y == N-1)
    {
        sol[x][y] = 1;
        return true;
    }

    // Check if maze[x][y] is valid
    if(isSafe(maze, x, y) == true)
    {
        // mark x,y as part of solution path
        sol[x][y] = 1;

        /* Move forward in x direction */
        if (solveMazeUtil(maze, x+1, y, sol) == true)
            return true;

        /* If moving in x direction doesn't give solution then
           Move down in y direction  */
        if (solveMazeUtil(maze, x, y+1, sol) == true)
            return true;

        /* If none of the above movements work then BACKTRACK:
            unmark x,y as part of solution path */
        sol[x][y] = 0;
        return false;
    }

    return false;
}

// driver program to test above function
int main()
{
    int maze[N][N]  =  { {1, 0, 0, 0},
        {1, 1, 0, 1},
        {0, 1, 0, 0},
        {1, 1, 1, 1}
    };

    solveMaze(maze);
    getchar();
    return 0;
}
```

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

4. Backtracking | Set 3 (N Queen Problem)

We have discussed Knight's tour and Rat in a Maze problems in Set 1 and Set 2 respectively. Let us discuss N Queen as another example problem that can be solved using Backtracking.

The N Queen is the problem of placing N chess queens on an N×N chessboard so that no two queens attack each other. For example, following is a solution for 4 Queen problem.



The expected output is a binary matrix which has 1s for the blocks where queens are placed. For example following is the output matrix for above 4 queen solution.

```
{ 0,  1,  0,  0}
{ 0,  0,  0,  1}
{ 1,  0,  0,  0}
{ 0,  0,  1,  0}
```

Naive Algorithm
Generate all possible configurations of queens on board and print a configuration that satisfies the given constraints.

```
while there are untried conflagrations
{
   generate the next configuration
   if queens don't attack in this configuration then
   {
      print this configuration;
   }
}
```

Backtracking Algorithm
The idea is to place queens one by one in different columns, starting from the leftmost column. When we place a queen in a column, we check for clashes with already placed queens. In the current column, if we find a row for which there is no clash, we mark this row and column as part of the solution. If we do not find such a row due to clashes then we backtrack and return false.

```
1) Start in the leftmost column
```

```
2) If all queens are placed
    return true
3) Try all rows in the current column.  Do following for every tried row.
    a) If the queen can be placed safely in this row then mark this [row,
        column] as part of the solution and recursively check if placing
        queen here leads to a solution.
    b) If placing queen in [row, column] leads to a solution then return
        true.
    c) If placing queen doesn't lead to a solution then umark this [row,
        column] (Backtrack) and go to step (a) to try other rows.
3) If all rows have been tried and nothing worked, return false to trigger
    backtracking.
```

Implementation of Backtracking solution

```
#define N 4
#include<stdio.h>

/* A utility function to print solution */
void printSolution(int board[N][N])
{
    for (int i = 0; i < N; i++)
    {
        for (int j = 0; j < N; j++)
            printf(" %d ", board[i][j]);
        printf("\n");
    }
}

/* A utility function to check if a queen can be placed on board[row][
   Note that this function is called when "col" queens are already pla
   in columns from 0 to col -1. So we need to check only left side for
   attacking queens */
bool isSafe(int board[N][N], int row, int col)
{
    int i, j;

    /* Check this row on left side */
    for (i = 0; i < col; i++)
    {
        if (board[row][i])
            return false;
    }

    /* Check upper diagonal on left side */
    for (i = row, j = col; i >= 0 && j >= 0; i--, j--)
    {
        if (board[i][j])
            return false;
    }

    /* Check lower diagonal on left side */
    for (i = row, j = col; j >= 0 && i < N; i++, j--)
    {
        if (board[i][j])
            return false;
    }
```

```
    }

    return true;
}

/* A recursive utility function to solve N Queen problem */
bool solveNQUtil(int board[N][N], int col)
{
    /* base case: If all queens are placed then return true */
    if (col >= N)
        return true;

    /* Consider this column and try placing this queen in all rows
       one by one */
    for (int i = 0; i < N; i++)
    {
        /* Check if queen can be placed on board[i][col] */
        if ( isSafe(board, i, col) )
        {
            /* Place this queen in board[i][col] */
            board[i][col] = 1;

            /* recur to place rest of the queens */
            if ( solveNQUtil(board, col + 1) == true )
                return true;

            /* If placing queen in board[i][col] doesn't lead to a sol
               then remove queen from board[i][col] */
            board[i][col] = 0; // BACKTRACK
        }
    }

     /* If queen can not be place in any row in this colum col
        then return false */
    return false;
}

/* This function solves the N Queen problem using Backtracking.  It ma
solveNQUtil() to solve the problem. It returns false if queens cannot
otherwise return true and prints placement of queens in the form of 1s
note that there may be more than one solutions, this function prints o
feasible solutions.*/
bool solveNQ()
{
    int board[N][N] = { {0, 0, 0, 0},
        {0, 0, 0, 0},
        {0, 0, 0, 0},
        {0, 0, 0, 0}
    };

    if ( solveNQUtil(board, 0) == false )
    {
      printf("Solution does not exist");
      return false;
    }

    printSolution(board);
    return true;
}

// driver program to test above function
int main()
```

```
int main()
{
    solveNQ();

    getchar();
    return 0;
}
```

Sources:
http://see.stanford.edu/materials/icspacs106b/H19-RecBacktrackExamples.pdf
http://en.literateprograms.org/Eight_queens_puzzle_%28C%29
http://en.wikipedia.org/wiki/Eight_queens_puzzle

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

5. Backtracking | Set 4 (Subset Sum)

Subset sum problem is to find subset of elements that are selected from a given set whose sum adds up to a given number K. We are considering the set contains non-negative values. It is assumed that the input set is unique (no duplicates are presented).

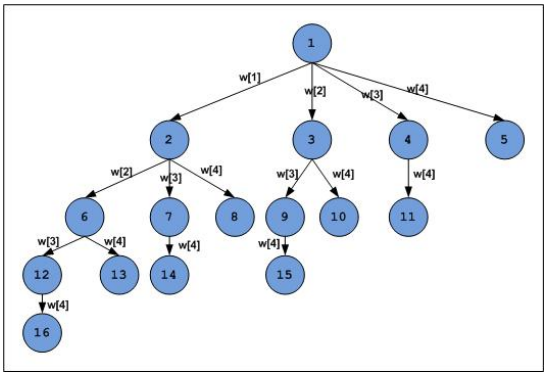
Exhaustive Search Algorithm for Subset Sum

One way to find subsets that sum to K is to consider all possible subsets. A power set contains all those subsets generated from a given set. The size of such a power set is 2^N.

Backtracking Algorithm for Subset Sum

Using exhaustive search we consider all subsets irrespective of whether they satisfy given constraints or not. Backtracking can be used to make a systematic consideration of the elements to be selected.

Assume given set of 4 elements, say **w[1] … w[4]**. Tree diagrams can be used to design backtracking algorithms. The following tree diagram depicts approach of generating variable sized tuple.



In the above tree, a node represents function call and a branch represents candidate element. The root node contains 4 children. In other words, root considers every element of the set as different branch. The next level sub-trees correspond to the subsets that includes the parent node. The branches at each level represent tuple element to be considered. For example, if we are at level 1, tuple_vector[1] can take any value of four branches generated. If we are at level 2 of left most node, tuple_vector[2] can take any value of three branches generated, and so on…

For example the left most child of root generates all those subsets that include w[1]. Similarly the second child of root generates all those subsets that includes w[2] and excludes w[1].

As we go down along depth of tree we add elements so far, and if the added sum is satisfying explicit constraints, we will continue to generate child nodes further. Whenever the constraints are not met, we stop further generation of sub-trees of that node, and backtrack to previous node to explore the nodes not yet explored. In many scenarios, it saves considerable amount of processing time.

The tree should trigger a clue to implement the backtracking algorithm (try yourself). It prints all those subsets whose sum add up to given number. We need to explore the nodes along the breadth and depth of the tree. Generating nodes along breadth is controlled by loop and nodes along the depth are generated using recursion (post order traversal). Pseudo code given below,

```
if(subset is satisfying the constraint)
    print the subset
    exclude the current element and consider next element
else
    generate the nodes of present level along breadth of tree and
    recur for next levels
```

Following is C implementation of subset sum using variable size tuple vector. Note that the following program explores all possibilities similar to exhaustive search. It is to

demonstrate how backtracking can be used. See next code to verify, how we can optimize the backtracking solution.

```
#include <stdio.h>
#include <stdlib.h>

#define ARRAYSIZE(a) (sizeof(a))/(sizeof(a[0]))

static int total_nodes;
// prints subset found
void printSubset(int A[], int size)
{
    for(int i = 0; i < size; i++)
    {
        printf("%*d", 5, A[i]);
    }

    printf("\n");
}

// inputs
// s            - set vector
// t            - tuplet vector
// s_size       - set size
// t_size       - tuplet size so far
// sum          - sum so far
// ite          - nodes count
// target_sum   - sum to be found
void subset_sum(int s[], int t[],
                int s_size, int t_size,
                int sum, int ite,
                int const target_sum)
{
    total_nodes++;
    if( target_sum == sum )
    {
        // We found subset
        printSubset(t, t_size);
        // Exclude previously added item and consider next candidate
        subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1, targ
        return;
    }
    else
    {
        // generate nodes along the breadth
        for( int i = ite; i < s_size; i++ )
        {
            t[t_size] = s[i];
            // consider next level node (along depth)
            subset_sum(s, t, s_size, t_size + 1, sum + s[i], i + 1, ta
        }
    }
}

// Wrapper to print subsets that sum to target_sum
// input is weights vector and target_sum
void generateSubsets(int s[], int size, int target_sum)
{
    int *tuplet_vector = (int *)malloc(size * sizeof(int));

    subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);
```

```
    subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);

    free(tuplet_vector);
}

int main()
{
    int weights[] = {10, 7, 5, 18, 12, 20, 15};
    int size = ARRAYSIZE(weights);

    generateSubsets(weights, size, 35);
    printf("Nodes generated %d\n", total_nodes);
    return 0;
}
```

The power of backtracking appears when we combine explicit and implicit constraints, and we stop generating nodes when these checks fail. We can improve the above algorithm by strengthening the constraint checks and presorting the data. By sorting the initial array, we need not to consider rest of the array, once the sum so far is greater than target number. We can backtrack and check other possibilities.

Similarly, assume the array is presorted and we found one subset. We can generate next node excluding the present node only when inclusion of next node satisfies the constraints. Given below is optimized implementation (it prunes the subtree if it is not satisfying contraints).

```
#include <stdio.h>
#include <stdlib.h>

#define ARRAYSIZE(a) (sizeof(a))/(sizeof(a[0]))

static int total_nodes;

// prints subset found
void printSubset(int A[], int size)
{
    for(int i = 0; i < size; i++)
    {
        printf("%*d", 5, A[i]);
    }

    printf("\n");
}

// qsort compare function
int comparator(const void *pLhs, const void *pRhs)
{
    int *lhs = (int *)pLhs;
    int *rhs = (int *)pRhs;

    return *lhs > *rhs;
}

// inputs
// s            - set vector
// t            - tuplet vector
// s_size       - set size
// t_size       - tuplet size so far
```

```
// sum          - sum so far
// ite          - nodes count
// target_sum   - sum to be found
void subset_sum(int s[], int t[],
                int s_size, int t_size,
                int sum, int ite,
                int const target_sum)
{
    total_nodes++;

    if( target_sum == sum )
    {
        // We found sum
        printSubset(t, t_size);

        // constraint check
        if( ite + 1 < s_size && sum - s[ite] + s[ite+1] <= target_sum
        {
            // Exclude previous added item and consider next candidate
            subset_sum(s, t, s_size, t_size-1, sum - s[ite], ite + 1,
        }
        return;
    }
    else
    {
        // constraint check
        if( ite < s_size && sum + s[ite] <= target_sum )
        {
            // generate nodes along the breadth
            for( int i = ite; i < s_size; i++ )
            {
                t[t_size] = s[i];

                if( sum + s[i] <= target_sum )
                {
                    // consider next level node (along depth)
                    subset_sum(s, t, s_size, t_size + 1, sum + s[i], i
                }
            }
        }
    }
}

// Wrapper that prints subsets that sum to target_sum
void generateSubsets(int s[], int size, int target_sum)
{
    int *tuplet_vector = (int *)malloc(size * sizeof(int));

    int total = 0;

    // sort the set
    qsort(s, size, sizeof(int), &comparator);

    for( int i = 0; i < size; i++ )
    {
        total += s[i];
    }

    if( s[0] <= target_sum && total >= target_sum )
    {
```

```
        subset_sum(s, tuplet_vector, size, 0, 0, 0, target_sum);

    }

    free(tuplet_vector);
}

int main()
{
    int weights[] = {15, 22, 14, 26, 32, 9, 16, 8};
    int target = 53;

    int size = ARRAYSIZE(weights);

    generateSubsets(weights, size, target);

    printf("Nodes generated %d\n", total_nodes);

    return 0;
}
```

As another approach, we can generate the tree in fixed size tuple analogs to binary pattern. We will kill the sub-trees when the constraints are not satisfied.

– – – **Venki**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

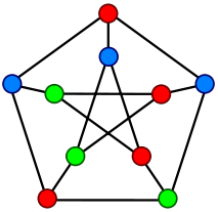
6. Backtracking | Set 5 (m Coloring Problem)

Given an undirected graph and a number m, determine if the graph can be colored with at most m colors such that no two adjacent vertices of the graph are colored with same color. Here coloring of a graph means assignment of colors to all vertices.

Input:
1) A 2D array graph[V][V] where V is the number of vertices in graph and graph[V][V] is adjacency matrix representation of the graph. A value graph[i][j] is 1 if there is a direct edge from i to j, otherwise graph[i][j] is 0.
2) An integer m which is maximum number of colors that can be used.

Output:
An array color[V] that should have numbers from 1 to m. color[i] should represent the color assigned to the ith vertex. The code should also return false if the graph cannot be colored with m colors.

Following is an example graph (from Wiki page) that can be colored with 3 colors.



Naive Algorithm

Generate all possible configurations of colors and print a configuration that satisfies the given constraints.

```
while there are untried conflagrations
{
   generate the next configuration
   if no adjacent vertices are colored with same color
   {
      print this configuration;
   }
}
```

There will be V^m configurations of colors.

Backtracking Algorithm

The idea is to assign colors one by one to different vertices, starting from the vertex 0. Before assigning a color, we check for safety by considering already assigned colors to the adjacent vertices. If we find a color assignment which is safe, we mark the color assignment as part of solution. If we do not a find color due to clashes then we backtrack and return false.

Implementation of Backtracking solution

```
#include<stdio.h>

// Number of vertices in the graph
#define V 4

void printSolution(int color[]);

/* A utility function to check if the current color assignment
   is safe for vertex v */
bool isSafe (int v, bool graph[V][V], int color[], int c)
{
    for (int i = 0; i < V; i++)
        if (graph[v][i] && c == color[i])
            return false;
    return true;
}

/* A recursive utility function to solve m coloring problem */
bool graphColoringUtil(bool graph[V][V], int m, int color[], int v)
{
```

```
    /* base case: If all vertices are assigned a color then
       return true */
    if (v == V)
        return true;

    /* Consider this vertex v and try different colors */
    for (int c = 1; c <= m; c++)
    {
        /* Check if assignment of color c to v is fine*/
        if (isSafe(v, graph, color, c))
        {
           color[v] = c;

           /* recur to assign colors to rest of the vertices */
           if (graphColoringUtil (graph, m, color, v+1) == true)
             return true;

            /* If assigning color c doesn't lead to a solution
               then remove it */
           color[v] = 0;
        }
    }

    /* If no color can be assigned to this vertex then return false */
    return false;
}

/* This function solves the m Coloring problem using Backtracking.
  It mainly uses graphColoringUtil() to solve the problem. It returns
  false if the m colors cannot be assigned, otherwise return true and
  prints assignments of colors to all vertices. Please note that there
  may be more than one solutions, this function prints one of the
  feasible solutions.*/
bool graphColoring(bool graph[V][V], int m)
{
    // Initialize all color values as 0. This initialization is needed
    // correct functioning of isSafe()
    int *color = new int[V];
    for (int i = 0; i < V; i++)
       color[i] = 0;

    // Call graphColoringUtil() for vertex 0
    if (graphColoringUtil(graph, m, color, 0) == false)
    {
      printf("Solution does not exist");
      return false;
    }

    // Print the solution
    printSolution(color);
    return true;
}

/* A utility function to print solution */
void printSolution(int color[])
{
    printf("Solution Exists:"
            " Following are the assigned colors \n");
    for (int i = 0; i < V; i++)
      printf(" %d ", color[i]);
    printf("\n");
}
```

```
}

// driver program to test above function
int main()
{
    /* Create following graph and test whether it is 3 colorable
      (3)---(2)
       |   / |
       |  /  |
       | /   |
      (0)---(1)
    */
    bool graph[V][V] = {{0, 1, 1, 1},
        {1, 0, 1, 0},
        {1, 1, 0, 1},
        {1, 0, 1, 0},
    };
    int m = 3; // Number of colors
    graphColoring (graph, m);
    return 0;
}
```

Output:

```
Solution Exists: Following are the assigned colors
 1  2  3  2
```

References:
http://en.wikipedia.org/wiki/Graph_coloring

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

7. Backtracking | Set 6 (Hamiltonian Cycle)

Hamiltonian Path in an undirected graph is a path that visits each vertex exactly once. A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian Path such that there is an edge (in graph) from the last vertex to the first vertex of the Hamiltonian Path. Determine whether a given graph contains Hamiltonian Cycle or not. If it contains, then print the path. Following are the input and output of the required function.

Input:
A 2D array graph[V][V] where V is the number of vertices in graph and graph[V][V] is adjacency matrix representation of the graph. A value graph[i][j] is 1 if there is a direct edge from i to j, otherwise graph[i][j] is 0.

Output:

An array path[V] that should contain the Hamiltonian Path. path[i] should represent the ith vertex in the Hamiltonian Path. The code should also return false if there is no Hamiltonian Cycle in the graph.

For example, a Hamiltonian Cycle in the following graph is {0, 1, 2, 4, 3, 0}. There are more Hamiltonian Cycles in the graph like {0, 3, 4, 2, 1, 0}

```
(0)--(1)--(2)
 |   / \   |
 |  /   \  |
 | /     \ |
(3)-------(4)
```

And the following graph doesn't contain any Hamiltonian Cycle.

```
(0)--(1)--(2)
 |   / \   |
 |  /   \  |
 | /     \ |
(3)       (4)
```

Naive Algorithm
Generate all possible configurations of vertices and print a configuration that satisfies the given constraints. There will be n! (n factorial) configurations.

```
while there are untried conflagrations
{
   generate the next configuration
   if ( there are edges between two consecutive vertices of this
      configuration and there is an edge from the last vertex to
      the first ).
   {
      print this configuration;
      break;
   }
}
```

Backtracking Algorithm
Create an empty path array and add vertex 0 to it. Add other vertices, starting from the vertex 1. Before adding a vertex, check for whether it is adjacent to the previously added vertex and not already added. If we find such a vertex, we add the vertex as part of the solution. If we do not find a vertex then we return false.

Implementation of Backtracking solution
Following is C/C++ implementation of the Backtracking solution.

```
// Program to print Hamiltonian cycle
#include<stdio.h>
```

```
#include<stdio.h>

// Number of vertices in the graph
#define V 5

void printSolution(int path[]);

/* A utility function to check if the vertex v can be added at index '
   in the Hamiltonian Cycle constructed so far (stored in 'path[]') */
bool isSafe(int v, bool graph[V][V], int path[], int pos)
{
    /* Check if this vertex is an adjacent vertex of the previously
       added vertex. */
    if (graph [ path[pos-1] ][ v ] == 0)
        return false;

    /* Check if the vertex has already been included.
      This step can be optimized by creating an array of size V */
    for (int i = 0; i < pos; i++)
        if (path[i] == v)
            return false;

    return true;
}

/* A recursive utility function to solve hamiltonian cycle problem */
bool hamCycleUtil(bool graph[V][V], int path[], int pos)
{
    /* base case: If all vertices are included in Hamiltonian Cycle */
    if (pos == V)
    {
        // And if there is an edge from the last included vertex to th
        // first vertex
        if ( graph[ path[pos-1] ][ path[0] ] == 1 )
           return true;
        else
          return false;
    }

    // Try different vertices as a next candidate in Hamiltonian Cycle
    // We don't try for 0 as we included 0 as starting point in in ham
    for (int v = 1; v < V; v++)
    {
        /* Check if this vertex can be added to Hamiltonian Cycle */
        if (isSafe(v, graph, path, pos))
        {
            path[pos] = v;

            /* recur to construct rest of the path */
            if (hamCycleUtil (graph, path, pos+1) == true)
                return true;

            /* If adding vertex v doesn't lead to a solution,
               then remove it */
            path[pos] = -1;
        }
    }

    /* If no vertex can be added to Hamiltonian Cycle constructed so f
       then return false */
    return false;
}
```

```
}

/* This function solves the Hamiltonian Cycle problem using Backtracki
  It mainly uses hamCycleUtil() to solve the problem. It returns false
  if there is no Hamiltonian Cycle possible, otherwise return true and
  prints the path. Please note that there may be more than one solutio
  this function prints one of the feasible solutions. */
bool hamCycle(bool graph[V][V])
{
    int *path = new int[V];
    for (int i = 0; i < V; i++)
        path[i] = -1;

    /* Let us put vertex 0 as the first vertex in the path. If there i
       a Hamiltonian Cycle, then the path can be started from any poin
       of the cycle as the graph is undirected */
    path[0] = 0;
    if ( hamCycleUtil(graph, path, 1) == false )
    {
        printf("\nSolution does not exist");
        return false;
    }

    printSolution(path);
    return true;
}

/* A utility function to print solution */
void printSolution(int path[])
{
    printf ("Solution Exists:"
            " Following is one Hamiltonian Cycle \n");
    for (int i = 0; i < V; i++)
        printf(" %d ", path[i]);

    // Let us print the first vertex again to show the complete cycle
    printf(" %d ", path[0]);
    printf("\n");
}

// driver program to test above function
int main()
{
   /* Let us create the following graph
      (0)--(1)--(2)
       |   / \   |
       |  /   \  |
       | /     \ |
      (3)-------(4)    */
   bool graph1[V][V] = {{0, 1, 0, 1, 0},
                        {1, 0, 1, 1, 1},
                        {0, 1, 0, 0, 1},
                        {1, 1, 0, 0, 1},
                        {0, 1, 1, 1, 0},
                       };

    // Print the solution
    hamCycle(graph1);

   /* Let us create the following graph
      (0)--(1)--(2)
       |   / \   |
```

```
        | /   \ |
        | /     \ |
      (3)       (4)    */
    bool graph2[V][V] = {{0, 1, 0, 1, 0},
                         {1, 0, 1, 1, 1},
                         {0, 1, 0, 0, 1},
                         {1, 1, 0, 0, 0},
                         {0, 1, 1, 0, 0},
                        };

    // Print the solution
    hamCycle(graph2);

    return 0;
}
```

Output:

```
Solution Exists: Following is one Hamiltonian Cycle
 0  1  2  4  3  0


Solution does not exist
```

Note that the above code always prints cycle starting from 0. Starting point should not matter as cycle can be started from any point. If you want to change the starting point, you should make two changes to above code.
Change “path[0] = 0;” to “path[0] = s;” where s is your new starting point. Also change loop “for (int v = 1; v < V; v++)” in hamCycleUtil() to “for (int v = 0; v < V; v++)”.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

8. Backtracking | Set 7 (Sudoku)

Given a partially filled 9×9 2D array ‘grid[9][9]’, the goal is to assign digits (from 1 to 9) to the empty cells so that every row, column, and subgrid of size 3×3 contains exactly one instance of the digits from 1 to 9.

3		6	5		8	4		
5	2							
	8	7					3	1
		3		1			8	
9			8	6	3			5
	5			9		6		
1	3					2	5	
							7	4
		5	2		6	3		

Naive Algorithm
The Naive Algorithm is to generate all possible configurations of numbers from 1 to 9 to fill the empty cells. Try every configuration one by one until the correct configuration is found.

Backtracking Algorithm
Like all other Backtracking problems, we can solve Sudoku by one by one assigning numbers to empty cells. Before assigning a number, we check whether it is safe to assign. We basically check that the same number is not present in current row, current column and current 3X3 subgrid. After checking for safety, we assign the number, and recursively check whether this assignment leads to a solution or not. If the assignment doesn't lead to a solution, then we try next number for current empty cell. And if none of number (1 to 9) lead to solution, we return false.

```
Find row, col of an unassigned cell
If there is none, return true
For digits from 1 to 9
  a) If there is no conflict for digit at row,col
      assign digit to row,col and recursively try fill in rest of grid
  b) If recursion successful, return true
  c) Else, remove digit and try another
If all digits have been tried and nothing worked, return false
```

Following is C++ implementation for Sudoku problem. It prints the completely filled grid as output.

```
// A Backtracking program  in C++ to solve Sudoku problem
#include <stdio.h>

// UNASSIGNED is used for empty cells in sudoku grid
#define UNASSIGNED 0

// N is used for size of Sudoku grid. Size will be NxN
#define N 9

// This function finds an entry in grid that is still unassigned
bool FindUnassignedLocation(int grid[N][N], int &row, int &col);

// Checks whether it will be legal to assign num to the given row,col
bool isSafe(int grid[N][N], int row, int col, int num);

/* Takes a partially filled-in grid and attempts to assign values to
  all unassigned locations in such a way to meet the requirements
  for Sudoku solution (non-duplication across rows, columns, and boxes
bool SolveSudoku(int grid[N][N])
{
    int row, col;

    // If there is no unassigned location, we are done
    if (!FindUnassignedLocation(grid, row, col))
       return true; // success!

    // consider digits 1 to 9
```

```
    for (int num = 1; num <= 9; num++)
    {
        // if looks promising
        if (isSafe(grid, row, col, num))
        {
            // make tentative assignment
            grid[row][col] = num;

            // return, if success, yay!
            if (SolveSudoku(grid))
                return true;

            // failure, unmake & try again
            grid[row][col] = UNASSIGNED;
        }
    }
    return false; // this triggers backtracking
}

/* Searches the grid to find an entry that is still unassigned. If
   found, the reference parameters row, col will be set the location
   that is unassigned, and true is returned. If no unassigned entries
   remain, false is returned. */
bool FindUnassignedLocation(int grid[N][N], int &row, int &col)
{
    for (row = 0; row < N; row++)
        for (col = 0; col < N; col++)
            if (grid[row][col] == UNASSIGNED)
                return true;
    return false;
}

/* Returns a boolean which indicates whether any assigned entry
   in the specified row matches the given number. */
bool UsedInRow(int grid[N][N], int row, int num)
{
    for (int col = 0; col < N; col++)
        if (grid[row][col] == num)
            return true;
    return false;
}

/* Returns a boolean which indicates whether any assigned entry
   in the specified column matches the given number. */
bool UsedInCol(int grid[N][N], int col, int num)
{
    for (int row = 0; row < N; row++)
        if (grid[row][col] == num)
            return true;
    return false;
}

/* Returns a boolean which indicates whether any assigned entry
   within the specified 3x3 box matches the given number. */
bool UsedInBox(int grid[N][N], int boxStartRow, int boxStartCol, int n
{
    for (int row = 0; row < 3; row++)
        for (int col = 0; col < 3; col++)
            if (grid[row+boxStartRow][col+boxStartCol] == num)
                return true;
    return false;
```

```
}

/* Returns a boolean which indicates whether it will be legal to assig
   num to the given row,col location. */
bool isSafe(int grid[N][N], int row, int col, int num)
{
    /* Check if 'num' is not already placed in current row,
       current column and current 3x3 box */
    return !UsedInRow(grid, row, num) &&
           !UsedInCol(grid, col, num) &&
           !UsedInBox(grid, row - row%3 , col - col%3, num);
}

/* A utility function to print grid  */
void printGrid(int grid[N][N])
{
    for (int row = 0; row < N; row++)
    {
       for (int col = 0; col < N; col++)
            printf("%2d", grid[row][col]);
        printf("\n");
    }
}

/* Driver Program to test above functions */
int main()
{
    // 0 means unassigned cells
    int grid[N][N] = {{3, 0, 6, 5, 0, 8, 4, 0, 0},
                      {5, 2, 0, 0, 0, 0, 0, 0, 0},
                      {0, 8, 7, 0, 0, 0, 0, 3, 1},
                      {0, 0, 3, 0, 1, 0, 0, 8, 0},
                      {9, 0, 0, 8, 6, 3, 0, 0, 5},
                      {0, 5, 0, 0, 9, 0, 6, 0, 0},
                      {1, 3, 0, 0, 0, 0, 2, 5, 0},
                      {0, 0, 0, 0, 0, 0, 0, 7, 4},
                      {0, 0, 5, 2, 0, 6, 3, 0, 0}};
    if (SolveSudoku(grid) == true)
          printGrid(grid);
    else
         printf("No solution exists");

    return 0;
}
```

Output:

```
3 1 6 5 7 8 4 9 2
5 2 9 1 3 4 7 6 8
4 8 7 6 2 9 5 3 1
2 6 3 4 1 5 9 8 7
9 7 4 8 6 3 1 2 5
8 5 1 7 9 2 6 4 3
1 3 8 9 4 7 2 5 6
6 9 2 3 5 1 8 7 4
7 4 5 2 8 6 3 1 9
```

References:
http://see.stanford.edu/materials/icspacs106b/H19-RecBacktrackExamples.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

9. Tug of War

Given a set of n integers, divide the set in two subsets of n/2 sizes each such that the difference of the sum of two subsets is as minimum as possible. If n is even, then sizes of two subsets must be strictly n/2 and if n is odd, then size of one subset must be (n-1)/2 and size of other subset must be (n+1)/2.

For example, let given set be {3, 4, 5, -3, 100, 1, 89, 54, 23, 20}, the size of set is 10. Output for this set should be {4, 100, 1, 23, 20} and {3, 5, -3, 89, 54}. Both output subsets are of size 5 and sum of elements in both subsets is same (148 and 148).
Let us consider another example where n is odd. Let given set be {23, 45, -34, 12, 0, 98, -99, 4, 189, -1, 4}. The output subsets should be {45, -34, 12, 98, -1} and {23, 0, -99, 4, 189, 4}. The sums of elements in two subsets are 120 and 121 respectively.

The following solution tries every possible subset of half size. If one subset of half size is formed, the remaining elements form the other subset. We initialize current set as empty and one by one build it. There are two possibilities for every element, either it is part of current set, or it is part of the remaining elements (other subset). We consider both possibilities for every element. When the size of current set becomes n/2, we check whether this solutions is better than the best solution available so far. If it is, then we update the best solution.

Following is C++ implementation for Tug of War problem. It prints the required arrays.

```
#include <iostream>
#include <stdlib.h>
#include <limits.h>
using namespace std;

// function that tries every possible solution by calling itself recur
void TOWUtil(int* arr, int n, bool* curr_elements, int no_of_selected_
             bool* soln, int* min_diff, int sum, int curr_sum, int cur
{
    // checks whether the it is going out of bound
    if (curr_position == n)
        return;

    // checks that the numbers of elements left are not less than the
    // number of elements required to form the solution
    if ((n/2 - no_of_selected_elements) > (n - curr_position))
```

```
        return;

    // consider the cases when current element is not included in the
    TOWUtil(arr, n, curr_elements, no_of_selected_elements,
              soln, min_diff, sum, curr_sum, curr_position+1);

    // add the current element to the solution
    no_of_selected_elements++;
    curr_sum = curr_sum + arr[curr_position];
    curr_elements[curr_position] = true;

    // checks if a solution is formed
    if (no_of_selected_elements == n/2)
    {
        // checks if the solution formed is better than the best solut
        if (abs(sum/2 - curr_sum) < *min_diff)
        {
            *min_diff = abs(sum/2 - curr_sum);
            for (int i = 0; i<n; i++)
                soln[i] = curr_elements[i];
        }
    }
    else
    {
        // consider the cases where current element is included in the
        TOWUtil(arr, n, curr_elements, no_of_selected_elements, soln,
                  min_diff, sum, curr_sum, curr_position+1);
    }

    // removes current element before returning to the caller of this
    curr_elements[curr_position] = false;
}

// main function that generate an arr
void tugOfWar(int *arr, int n)
{
    // the boolen array that contains the inclusion and exclusion of a
    // in current set. The number excluded automatically form the othe
    bool* curr_elements = new bool[n];

    // The inclusion/exclusion array for final solution
    bool* soln = new bool[n];

    int min_diff = INT_MAX;

    int sum = 0;
    for (int i=0; i<n; i++)
    {
        sum += arr[i];
        curr_elements[i] =  soln[i] = false;
    }

    // Find the solution using recursive function TOWUtil()
    TOWUtil(arr, n, curr_elements, 0, soln, &min_diff, sum, 0, 0);

    // Print the solution
    cout << "The first subset is: ";
    for (int i=0; i<n; i++)
    {
        if (soln[i] == true)
            cout << arr[i] << " ";
    }
```

```
    cout << "\nThe second subset is: ";
    for (int i=0; i<n; i++)
    {
        if (soln[i] == false)
            cout << arr[i] << " ";
    }
}

// Driver program to test above functions
int main()
{
    int arr[] = {23, 45, -34, 12, 0, 98, -99, 4, 189, -1, 4};
    int n = sizeof(arr)/sizeof(arr[0]);
    tugOfWar(arr, n);
    return 0;
}
```

Output:

```
The first subset is: 45 -34 12 98 -1
The second subset is: 23 0 -99 4 189 4
```

This article is compiled by Ashish Anand and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

10. Backtracking | Set 8 (Solving Cryptarithmetic Puzzles)

Newspapers and magazines often have crypt-arithmetic puzzles of the form:

```
  SEND
+ MORE
--------
 MONEY
--------
```

The goal here is to assign each letter a digit from 0 to 9 so that the arithmetic works out correctly. The rules are that all occurrences of a letter must be assigned the same digit, and no digit can be assigned to more than one letter.

- First, create a list of all the characters that need assigning to pass to Solve
- If all characters are assigned, return true if puzzle is solved, false otherwise
- Otherwise, consider the first unassigned character
- for (every possible choice among the digits not in use)

make that choice and then recursively try to assign the rest of the characters
if recursion sucessful, return true
if !successful, unmake assignment and try another digit

If all digits have been tried and nothing worked, return false to trigger backtracking

```
/* ExhaustiveSolve
* --------------
* This is the "not-very-smart" version of cryptarithmetic solver. It t
* the puzzle itself (with the 3 strings for the two addends and sum) a
* string of letters as yet unassigned. If no more letters to assign
* then we've hit a base-case, if the current letter-to-digit mapping s
* the puzzle, we're done, otherwise we return false to trigger backtra
* If we have letters to assign, we take the first letter from that lis
* try assigning it the digits from 0 to 9 and then recursively working
* through solving puzzle from here. If we manage to make a good assign
* that works, we've succeeded, else we need to unassign that choice an
* another digit. This version is easy to write, since it uses a simple
* approach (quite similar to permutations if you think about it) but i
* not so smart because it doesn't take into account the structure of t
* puzzle constraints (for example, once the two digits for the addends
* been assigned, there is no reason to try anything other than the cor
* digit for the sum) yet it tries a lot of useless combos regardless
*/
bool ExhaustiveSolve(puzzleT puzzle, string lettersToAssign)
{
    if (lettersToAssign.empty()) // no more choices to make
        return PuzzleSolved(puzzle); // checks arithmetic to see if wo
    for (int digit = 0; digit <= 9; digit++)   // try all digits
    {
        if (AssignLetterToDigit(lettersToAssign[0], digit))
        {
            if (ExhaustiveSolve(puzzle, lettersToAssign.substr(1)))
                return true;
            UnassignLetterFromDigit(lettersToAssign[0], digit);
        }
    }
    return false;  // nothing worked, need to backtrack
}
```

The algorithm above actually has a lot in common with the permutations algorithm, it pretty much just creates all arrangements of the mapping from characters to digits and tries each until one works or all have been successfully tried. For a large puzzle, this could take a while.
A smarter algorithm could take into account the structure of the puzzle and avoid going down dead-end paths. For example, if we assign the characters starting from the ones place and moving to the left, at each stage, we can verify the correctness of what we have so far before we continue onwards. This definitely complicates the code but leads to a tremendous improvement in efficiency, making it much more feasible to solve large puzzles.

Below pseudocode in this case has more special cases, but the same general design

- Start by examining the rightmost digit of the topmost row, with a carry of 0

- If we are beyond the leftmost digit of the puzzle, return true if no carry, false otherwise
- If we are currently trying to assign a char in one of the addends
  If char already assigned, just recur on row beneath this one, adding value into sum
  If not assigned, then
    - for (every possible choice among the digits not in use)
      make that choice and then on row beneath this one, if successful, return true
      if !successful, unmake assignment and try another digit
    - return false if no assignment worked to trigger backtracking
- Else if trying to assign a char in the sum
- If char assigned & matches correct,
  recur on next column to the left with carry, if success return true,
- If char assigned & doesn't match, return false
- If char unassigned & correct digit already used, return false
- If char unassigned & correct digit unused,
  assign it and recur on next column to left with carry, if success return true
- return false to trigger backtracking

Source:
http://see.stanford.edu/materials/icspacs106b/H19-RecBacktrackExamples.pdf

11. Fill two instances of all numbers from 1 to n in a specific way

Given a number n, create an array of size 2n such that the array contains 2 instances of every number from 1 to n, and the number of elements between two instances of a number i is equal to i. If such a configuration is not possible, then print the same.

Examples:

```
Input: n = 3
Output: res[] = {3, 1, 2, 1, 3, 2}

Input: n = 2
Output: Not Possible

Input: n = 4
Output: res[] = {4, 1, 3, 1, 2, 4, 3, 2}
```

We strongly recommend to minimize the browser and try this yourself first.

One solution is to Backtracking. The idea is simple, we place two instances of n at a

place, then recur for n-1. If recurrence is successful, we return true, else we backtrack and try placing n at different location. Following is C implementation of the idea.

```
// A backtracking based C Program to fill two instances of all numbers
// from 1 to n in a specific way
#include <stdio.h>
#include <stdbool.h>

// A recursive utility function to fill two instances of numbers from
// 1 to n in res[0..2n-1].  'curr' is current value of n.
bool fillUtil(int res[], int curr, int n)
{
     // If current number becomes 0, then all numbers are filled
     if (curr == 0) return true;

     // Try placing two instances of 'curr' at all possible locations
     // till solution is found
     int i;
     for (i=0; i<2*n-curr-1; i++)
     {
        // Two 'curr' should be placed at 'curr+1' distance
        if (res[i] == 0 && res[i + curr + 1] == 0)
        {
           // Plave two instances of 'curr'
           res[i] = res[i + curr + 1] = curr;

           // Recur to check if the above placement leads to a solutio
           if (fillUtil(res, curr-1, n))
               return true;

           // If solution is not possible, then backtrack
           res[i] = res[i + curr + 1] = 0;
        }
     }
     return false;
}

// This function prints the result for input number 'n' using fillUtil
void fill(int n)
{
    // Create an array of size 2n and initialize all elements in it as
    int res[2*n], i;
    for (i=0; i<2*n; i++)
       res[i] = 0;

    // If solution is possible, then print it.
    if (fillUtil(res, n, n))
    {
        for (i=0; i<2*n; i++)
           printf("%d ", res[i]);
    }
    else
        puts("Not Possible");
}

// Driver program
int main()
{
  fill(7);
  return 0;
}
```

Output:

```
7 3 6 2 5 3 2 4 7 6 5 1 4 1
```

The above solution may not be the best possible solution. There seems to be a pattern in the output. I an Looking for a better solution from other geeks.

This article is contributed by **Asif**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above