Mathematical Algorithms

1. Write an Efficient Method to Check if a Number is Multiple of 3

The very first solution that comes to our mind is the one that we learned in school. If sum of digits in a number is multiple of 3 then number is multiple of 3 e.g., for 612 sum of digits is 9 so it's a multiple of 3. But this solution is not efficient. You have to get all decimal digits one by one, add them and then check if sum is multiple of 3.

There is a pattern in binary representation of the number that can be used to find if number is a multiple of 3. If difference between count of odd set bits (Bits set at odd positions) and even set bits is multiple of 3 then is the number.

Example: 23 (00..10111)
1) Get count of all set bits at odd positions (For 23 it's 3).
2) Get count of all set bits at even positions (For 23 it's 1).
3) If difference of above two counts is a multiple of 3 then number is also a multiple of 3.

(For 23 it's 2 so 23 is not a multiple of 3)

Take some more examples like 21, 15, etc…

```
Algorithm: isMutlipleOf3(n)
1) Make n positive if n is negative.
2) If number is 0 then return 1
3) If number is 1 then return 0
4) Initialize: odd_count = 0, even_count = 0
5) Loop while n != 0
    a) If rightmost bit is set then increment odd count.
    b) Right-shift n by 1 bit
    c) If rightmost bit is set then increment even count.
    d) Right-shift n by 1 bit
6) return isMutlipleOf3(odd_count - even_count)
```

Proof:
Above can be proved by taking the example of 11 in decimal numbers. (In this context 11 in decimal numbers is same as 3 in binary numbers)
If difference between sum of odd digits and even digits is multiple of 11 then decimal number is multiple of 11. Let's see how.

Let's take the example of 2 digit numbers in decimal
AB = 11A -A + B = 11A + (B – A)
So if (B – A) is a multiple of 11 then is AB.

Let us take 3 digit numbers.

ABC = 99A + A + 11B – B + C = (99A + 11B) + (A + C – B)
So if (A + C – B) is a multiple of 11 then is (A+C-B)

Let us take 4 digit numbers now.
ABCD = 1001A + D + 11C – C + 999B + B – A
= (1001A – 999B + 11C) + (D + B – A -C)
So, if (B + D – A – C) is a multiple of 11 then is ABCD.

This can be continued for all decimal numbers.
Above concept can be proved for 3 in binary numbers in the same way.

Time Complexity: O(logn)

Program:

```c
#include<stdio.h>

/* Fnction to check if n is a multiple of 3*/
int isMultipleOf3(int n)
{
    int odd_count = 0;
    int even_count = 0;

    /* Make no positive if +n is multiple of 3
       then is -n. We are doing this to avoid
       stack overflow in recursion*/
    if(n < 0)   n = -n;
    if(n == 0) return 1;
    if(n == 1) return 0;

    while(n)
    {
        /* If odd bit is set then
           increment odd counter */
        if(n & 1)
           odd_count++;
        n = n>>1;

        /* If even bit is set then
           increment even counter */
        if(n & 1)
            even_count++;
        n = n>>1;
    }

     return isMultipleOf3(abs(odd_count - even_count));
}

/* Program to test function isMultipleOf3 */
int main()
{
    int num = 23;
    if (isMultipleOf3(num))
        printf("num is multiple of 3");
    else
        printf("num is not a multiple of 3");
    getchar();
    return 0;
}
```

2. Efficient way to multiply with 7

We can multiply a number by 7 using bitwise operator. First left shift the number by 3 bits (you will get 8n) then subtract the original numberfrom the shifted number and return the difference (8n – n).

Program:

```
# include<stdio.h>

int multiplyBySeven(unsigned int n)
{
    /* Note the inner bracket here. This is needed
       because precedence of '-' operator is higher
       than '<<' */
    return ((n<<3) - n);
}

/* Driver program to test above function */
int main()
{
    unsigned int n = 4;
    printf("%u", multiplyBySeven(n));

    getchar();
    return 0;
}
```

Time Complexity: O(1)

Space Complexity: O(1)

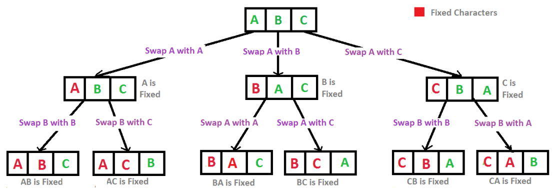
Note: Works only for positive integers.
Same concept can be used for fast multiplication by 9 or other numbers.

3. Write a C program to print all permutations of a given string

A permutation, also called an “arrangement number” or “order,” is a rearrangement of the elements of an ordered list S into a one-to-one correspondence with S itself. A string of length n has n! permutation.
Source: Mathword(http://mathworld.wolfram.com/Permutation.html)

Below are the permutations of string ABC.
ABC, ACB, BAC, BCA, CAB, CBA

Here is a solution using backtracking.



Recursion Tree for Permutations of String "ABC"

```c
# include <stdio.h>

/* Function to swap values at two pointers */
void swap (char *x, char *y)
{
    char temp;
    temp = *x;
    *x = *y;
    *y = temp;
}

/* Function to print permutations of string
   This function takes three parameters:
   1. String
   2. Starting index of the string
   3. Ending index of the string. */
void permute(char *a, int i, int n)
{
   int j;
   if (i == n)
     printf("%s\n", a);
   else
   {
        for (j = i; j <= n; j++)
       {
          swap((a+i), (a+j));
          permute(a, i+1, n);
          swap((a+i), (a+j)); //backtrack
       }
   }
}

/* Driver program to test above functions */
int main()
{
   char a[] = "ABC";
   permute(a, 0, 2);
   getchar();
   return 0;
}
```

Output:

```
ABC
ACB
BAC
BCA
CBA
CAB
```

Algorithm Paradigm: Backtracking
Time Complexity: O(n*n!)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

4. Lucky Numbers

Lucky numbers are subset of integers. Rather than going into much theory, let us see the process of arriving at lucky numbers,

Take the set of integers
1,2,3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,……

First, delete every second number, we get following reduced set.
1,3,5,7,9,11,13,15,17,19,…………

Now, delete every third number, we get
1, 3, 7, 9, 13, 15, 19,….….

Continue this process indefinitely……
Any number that does NOT get deleted due to above process is called "lucky".

Therefore, set of lucky numbers is 1, 3, 7, 13,………

Now, given an integer 'n', write a function to say whether this number is lucky or not.

```
    bool isLucky(int n)
```

Algorithm:
Before every iteration, if we calculate position of the given no, then in a given iteration, we can determine if the no will be deleted. Suppose calculated position for the given no. is P before some iteration, and each Ith no. is going to be removed in this iteration, if P < I then input no is lucky, if P is such that P%I == 0 (I is a divisor of P), then input no is not lucky.

Recursive Way:

```c
#include <stdio.h>
#define bool int

/* Returns 1 if n is a lucky no. ohterwise returns 0*/
bool isLucky(int n)
{
  static int counter = 2;

  /*variable next_position is just for readability of
     the program we can remove it and use n only */
  int next_position = n;
  if(counter > n)
    return 1;
  if(n%counter == 0)
    return 0;

 /*calculate next position of input no*/
  next_position -= next_position/counter;

  counter++;
  return isLucky(next_position);
}

/*Driver function to test above function*/
int main()
{
  int x = 5;
  if( isLucky(x) )
    printf("%d is a lucky no.", x);
  else
    printf("%d is not a lucky no.", x);
  getchar();
}
```

Example:
Let's us take an example of 19

1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,15,17,18,19,20,21,……
1,3,5,7,9,11,13,15,17,19,…..
1,3,7,9,13,15,19,……….
1,3,7,13,15,19,………
1,3,7,13,19,………

In next step every 6th no .in sequence will be deleted. 19 will not be deleted after this step because position of 19 is 5th after this step. Therefore, 19 is lucky. Let's see how above C code finds out:

Current function call	Position after this call	Counter for next call	Next Call
isLucky(19)	10	3	isLucky(10)
isLucky(10)	7	4	isLucky(7)
isLucky(7)	6	5	isLucky(6)
isLucky(6)	5	6	isLucky(5)

When isLucky(6) is called, it returns 1 (because counter > n).

Iterative Way:
Please see this comment for another simple and elegant implementation of the above algorithm.

Please write comments if you find any bug in the given programs or other ways to solve the same problem.

5. Write a program to add two numbers in base 14

Asked by Anshya.

Below are the different ways to add base 14 numbers.

Method 1
Thanks to Raj for suggesting this method.

```
  1. Convert both i/p base 14 numbers to base 10.
  2. Add numbers.
  3. Convert the result back to base 14.
```

Method 2
Just add the numbers in base 14 in same way we add in base 10. Add numerals of both numbers one by one from right to left. If there is a carry while adding two numerals, consider the carry for adding next numerals.

Let us consider the presentation of base 14 numbers same as hexadecimal numbers

```
   A --> 10
   B --> 11
   C --> 12
   D --> 13
```

```
Example:
   num1 =       1  2  A
   num2 =       C  D  3

   1. Add A and 3, we get 13(D). Since 13 is smaller than
14, carry becomes 0 and resultant numeral becomes D

  2. Add 2, D and carry(0). we get 15. Since 15 is greater
than 13, carry becomes 1 and resultant numeral is 15 - 14 = 1

  3. Add 1, C and carry(1). we get 14. Since 14 is greater
than 13, carry becomes 1 and resultant numeral is 14 - 14 = 0
```

```
Finally, there is a carry, so 1 is added as leftmost numeral and the result become
101D
```

Implementation of Method 2

```
# include <stdio.h>
# include <stdlib.h>
# define bool int

int getNumeralValue(char );
char getNumeral(int );

/* Function to add two numbers in base 14 */
char *sumBase14(char *num1,  char *num2)
{
   int l1 = strlen(num1);
   int l2 = strlen(num2);
   char *res;
   int i;
   int nml1, nml2, res_nml;
   bool carry = 0;

   if(l1 != l2)
   {
     printf("Function doesn't support numbers of different"
            " lengths. If you want to add such numbers then"
            " prefix smaller number with required no. of zeroes");
     getchar();
     assert(0);
   }

   /* Note the size of the allocated memory is one
     more than i/p lenghts for the cases where we
     have carry at the last like adding D1 and A1 */
   res = (char *)malloc(sizeof(char)*(l1 + 1));

   /* Add all numerals from right to left */
   for(i = l1-1; i >= 0; i--)
   {
     /* Get decimal values of the numerals of
       i/p numbers*/
     nml1 = getNumeralValue(num1[i]);
     nml2 = getNumeralValue(num2[i]);

     /* Add decimal values of numerals and carry */
     res_nml = carry + nml1 + nml2;

     /* Check if we have carry for next addition
        of numerals */
     if(res_nml >= 14)
     {
       carry = 1;
       res_nml -= 14;
     }
     else
     {
       carry = 0;
     }
     res[i+1] = getNumeral(res_nml);
   }

   /* if there is no carry after last iteration
   /* if there is no carry after last iteration
      then result should not include 0th character
      of the resultant string */
   if(carry == 0)
     return (res + 1);

   /* if we have carry after last iteration then
     result should include 0th character */
   res[0] = '1';
   return res;
}

/* Function to get value of a numeral
  For example it returns 10 for input 'A'
  1 for '1', etc */
int getNumeralValue(char num)
{
  if( num >= '0' && num <= '9')
    return (num - '0');
  if( num >= 'A' && num <= 'D')
    return (num - 'A' + 10);

  /* If we reach this line caller is giving
    invalid character so we assert and fail*/
  assert(0);
}

/* Function to get numeral for a value.
  For example it returns 'A' for input 10
  '1' for 1, etc */
char getNumeral(int val)
{
  if( val >= 0 && val <= 9)
    return (val + '0');
  if( val >= 10 && val <= 14)
    return (val + 'A' - 10);

  /* If we reach this line caller is giving
    invalid no. so we assert and fail*/
  assert(0);
}

/*Driver program to test above functions*/
int main()
{
    char *num1 = "DC2";
    char *num2 = "0A3";

    printf("Result is %s", sumBase14(num1, num2));
    getchar();
    return 0;
}
```

Notes:

Above approach can be used to add numbers in any base. We don't have to do string operations if base is smaller than 10.

You can try extending the above program for numbers of different lengths.

Please comment if you find any bug in the program or a better approach to do the same.

6. Babylonian method for square root

Algorithm:
This method can be derived from (but predates) Newton–Raphson method.

```
1 Start with an arbitrary positive start value x (the closer to the
   root, the better).
2 Initialize y = 1.
3. Do following until desired approximation is achieved.
  a) Get the next approximation for root using average of x and y
  b) Set y = n/x
```

Implementation:

```c
/*Returns the square root of n. Note that the function */
float squareRoot(float n)
{
  /*We are using n itself as initial approximation
   This can definitely be improved */
  float x = n;
  float y = 1;
  float e = 0.000001; /* e decides the accuracy level*/
  while(x - y > e)
  {
    x = (x + y)/2;
    y = n/x;
  }
  return x;
}

/* Driver program to test above function*/
int main()
{
  int n = 50;
  printf ("Square root of %d is %f", n, squareRoot(n));
  getchar();
}
```

Example:

```
n = 4 /*n itself is used for initial approximation*/
Initialize x = 4, y = 1
Next Approximation x = (x + y)/2 (= 2.500000),
y = n/x  (=1.600000)
Next Approximation x = 2.050000,
y = 1.951220
Next Approximation x = 2.000610,
y = 1.999390
Next Approximation x = 2.000000,
y = 2.000000
Terminate as (x - y) > e now.
```

If we are sure that n is a perfect square, then we can use following method. The method can go in infinite loop for non-perfect-square numbers. For example, for 3 the below while loop will never terminate.

```
/*Returns the square root of n. Note that the function
  will not work for numbers which are not perfect squares*/
unsigned int squareRoot(int n)
{
  int x = n;
  int y = 1;
  while(x > y)
  {
    x = (x + y)/2;
    y = n/x;
  }
  return x;
}

/* Driver program to test above function*/
int main()
{
  int n = 49;
  printf (" root of %d is %d", n, squareRoot(n));
  getchar();
}
```

References;

http://en.wikipedia.org/wiki/Square_root

http://en.wikipedia.org/wiki/Babylonian_method#Babylonian_method

Asked by Snehal

Please write comments if you find any bug in the above program/algorithm, or if you want to share more information about Babylonian method.

7. Multiply two integers without using multiplication, division and bitwise operators, and no loops

Asked by Kapil

By making use of recursion, we can multiply two integers with the given constraints.

To multiply x and y, recursively add x y times.

Thanks to geek4u for suggesting this method.

```c
#include<stdio.h>
/* function to multiply two numbers x and y*/
int multiply(int x, int y)
{
   /* 0  multiplied with anything gives 0 */
   if(y == 0)
     return 0;

   /* Add x one by one */
   if(y > 0 )
     return (x + multiply(x, y-1));

  /* the case where y is negative */
   if(y < 0 )
     return -multiply(x, -y);
}

int main()
{
  printf("\n %d", multiply(5, -11));
  getchar();
  return 0;
}
```

Time Complexity: O(y) where y is the second argument to function multiply().

Please write comments if you find any of the above code/algorithm incorrect, or find better ways to solve the same problem.

8. Print all combinations of points that can compose a given number

You can win three kinds of basketball points, 1 point, 2 points, and 3 points. Given a total score n, print out all the combination to compose n.

Examples:
For n = 1, the program should print following:
1

For n = 2, the program should print following:
1 1
2

For n = 3, the program should print following:
1 1 1
1 2
2 1

```
3
```

For n = 4, the program should print following:

```
1 1 1 1
1 1 2
1 2 1
1 3
2 1 1
2 2
3 1
```

and so on …

Algorithm:
At first position we can have three numbers 1 or 2 or 3.
First put 1 at first position and recursively call for n-1.
Then put 2 at first position and recursively call for n-2.
Then put 3 at first position and recursively call for n-3.
If n becomes 0 then we have formed a combination that compose n, so print the current combination.

Below is a generalized implementation. In the below implementation, we can change MAX_POINT if there are higher points (more than 3) in the basketball game.

```
#define MAX_POINT 3
#define ARR_SIZE 100
#include<stdio.h>

/* Utility function to print array arr[] */
void printArray(int arr[], int arr_size);

/* The function prints all combinations of numbers 1, 2, ...MAX_POINT
   that sum up to n.
   i is used in recursion keep track of index in arr[] where next
   element is to be added. Initital value of i must be passed as 0 */
void printCompositions(int n, int i)
{

  /* array must be static as we want to keep track
   of values stored in arr[] using current calls of
   printCompositions() in function call stack*/
  static int arr[ARR_SIZE];

  if (n == 0)
  {
    printArray(arr, i);
  }
  else if(n > 0)
  {
    int k;
    for (k = 1; k <= MAX_POINT; k++)
    {
      arr[i]= k;
      printCompositions(n-k, i+1);
    }
  }
}

/* UTILITY FUNCTIONS */
/* Utility function to print array arr[] */
void printArray(int arr[], int arr_size)
{
  int i;
  for (i = 0; i < arr_size; i++)
    printf("%d ", arr[i]);
  printf("\n");
}

/* Driver function to test above functions */
int main()
{
  int n = 5;
  printf("Differnt compositions formed by 1, 2 and 3 of %d are\n", n);
  printCompositions(n, 0);
  getchar();
  return 0;
}
```

Asked by Aloe

Please write comments if you find any bug in above code/algorithm, or find other ways to solve the same problem.

9. Write you own Power without using multiplication(*) and division(/) operators

Method 1 (Using Nested Loops)

We can calculate power by using repeated addition.

For example to calculate 5^6.
1) First 5 times add 5, we get 25. (5^2)
2) Then 5 times add 25, we get 125. (5^3)
3) Then 5 time add 125, we get 625 (5^4)
4) Then 5 times add 625, we get 3125 (5^5)
5) Then 5 times add 3125, we get 15625 (5^6)

```
/* Works only if a >= 0 and b >= 0  */
int pow(int a, int b)
{
  if (b == 0)
    return 1;
  int answer = a;
  int increment = a;
  int i, j;
  for(i = 1; i < b; i++)
  {
     for(j = 1; j < a; j++)
     {
        answer += increment;
     }
     increment = answer;
  }
  return answer;
}

/* driver program to test above function */
int main()
{
  printf("\n %d", pow(5, 3));
  getchar();
  return 0;
}
```

Method 2 (Using Recursion)
Recursively add a to get the multiplication of two numbers. And recursively multiply to get *a* raise to the power *b*.

```
#include<stdio.h>
/* A recursive function to get a^b
  Works only if a >= 0 and b >= 0  */
int pow(int a, int b)
{
   if(b)
     return multiply(a, pow(a, b-1));
   else
    return 1;
}

/* A recursive function to get x*y */
int multiply(int x, int y)
{
   if(y)
     return (x + multiply(x, y-1));
   else
     return 0;
}

/* driver program to test above functions */
int main()
{
  printf("\n %d", pow(5, 3));
  getchar();
  return 0;
}
```

Please write comments if you find any bug in above code/algorithm, or find other ways to solve the same problem.

10. Program for Fibonacci numbers

The Fibonacci numbers are the numbers in the following integer sequence.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 141, ……..

In mathematical terms, the sequence Fn of Fibonacci numbers is defined by the recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

with seed values

$$F_0 = 0 \quad \text{and} \quad F_1 = 1.$$

Write a function *int fib(int n)* that returns F_n. For example, if *n* = 0, then *fib()* should return 0. If n = 1, then it should return 1. For n > 1, it should return $F_{n-1} + F_{n-2}$

Following are different methods to get the nth Fibonacci number.

Method 1 (Use recursion)

A simple method that is a direct recusrive implementation mathematical recurance relation given above.

```
#include<stdio.h>
int fib(int n)
{
   if (n <= 1)
      return n;
   return fib(n-1) + fib(n-2);
}

int main ()
{
  int n = 9;
  printf("%d", fib(n));
  getchar();
  return 0;
}
```

Time Complexity: T(n) = T(n-1) + T(n-2) which is exponential.
We can observe that this implementation does a lot of repeated work (see the following recursion tree). So this is a bad implementation for nth Fibonacci number.

```
                          fib(5)
                      /              \
                fib(4)                 fib(3)
              /      \                 /      \
          fib(3)      fib(2)          fib(2)     fib(1)
         /     \        /     \       /     \
  fib(2)    fib(1)  fib(1) fib(0)  fib(1) fib(0)
  /     \
fib(1) fib(0)
```

Extra Space: O(n) if we consider the fuinction call stack size, otherwise O(1).

Method 2 (Use Dynamic Programming)
We can avoid the repeated work done is the method 1 by storing the Fibonacci numbers calculated so far.

```c
#include<stdio.h>

int fib(int n)
{
  /* Declare an array to store fibonacci numbers. */
  int f[n+1];
  int i;

  /* 0th and 1st number of the series are 0 and 1*/
  f[0] = 0;
  f[1] = 1;

  for (i = 2; i <= n; i++)
  {
      /* Add the previous 2 numbers in the series
         and store it */
      f[i] = f[i-1] + f[i-2];
  }

  return f[n];
}

int main ()
{
  int n = 9;
  printf("%d", fib(n));
  getchar();
  return 0;
}
```

Time Complexity: O(n)

Extra Space: O(n)

Method 3 (Space Otimized Method 2)

We can optimize the space used in method 2 by storing the previous two numbers only because that is all we need to get the next Fibannaci number in series.

```c
#include<stdio.h>
int fib(int n)
{
  int a = 0, b = 1, c, i;
  if( n == 0)
    return a;
  for (i = 2; i <= n; i++)
  {
     c = a + b;
     a = b;
     b = c;
  }
  return b;
}

int main ()
{
  int n = 9;
  printf("%d", fib(n));
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Extra Space: O(1)

Method 4 (Using power of the matrix {{1,1},{1,0}})
This another O(n) which relies on the fact that if we n times multiply the matrix M = {{1,1},{1,0}} to itself (in other words calculate power(M, n)), then we get the (n+1)th Fibonacci number as the element at row and column (0, 0) in the resultant matrix.

The matrix representation gives the following closed expression for the Fibonacci numbers:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}.$$

```
#include <stdio.h>

/* Helper function that multiplies 2 matricies F and M of size 2*2, an
  puts the multiplication result back to F[][] */
void multiply(int F[2][2], int M[2][2]);

/* Helper function that calculates F[][] raise to the power n and puts
  result in F[][]
  Note that this function is desinged only for fib() and won't work as
  power function */
void power(int F[2][2], int n);

int fib(int n)
{
  int F[2][2] = {{1,1},{1,0}};
  if (n == 0)
      return 0;
  power(F, n-1);

  return F[0][0];
}

void multiply(int F[2][2], int M[2][2])
{
  int x =  F[0][0]*M[0][0] + F[0][1]*M[1][0];
  int y =  F[0][0]*M[0][1] + F[0][1]*M[1][1];
  int z =  F[1][0]*M[0][0] + F[1][1]*M[1][0];
  int w =  F[1][0]*M[0][1] + F[1][1]*M[1][1];

  F[0][0] = x;
  F[0][1] = y;
  F[1][0] = z;
  F[1][1] = w;
}

void power(int F[2][2], int n)
{
  int i;
  int M[2][2] = {{1,1},{1,0}};

  // n - 1 times multiply the matrix to {{1,0},{0,1}}
  for (i = 2; i <= n; i++)
      multiply(F, M);
}

/* Driver program to test above function */
int main()
{
  int n = 9;
  printf("%d", fib(n));
  getchar();
  return 0;
}
```

Time Complexity: O(n)

Extra Space: O(1)

Method 5 (Optimized Method 4)

The method 4 can be optimized to work in O(Logn) time complexity. We can do recursive multiplication to get power(M, n) in the prevous method (Similar to the optimization done in this post)

```c
#include <stdio.h>

void multiply(int F[2][2], int M[2][2]);

void power(int F[2][2], int n);

/* function that returns nth Fibonacci number */
int fib(int n)
{
  int F[2][2] = {{1,1},{1,0}};
  if (n == 0)
    return 0;
  power(F, n-1);
  return F[0][0];
}

/* Optimized version of power() in method 4 */
void power(int F[2][2], int n)
{
  if( n == 0 || n == 1)
      return;
  int M[2][2] = {{1,1},{1,0}};

  power(F, n/2);
  multiply(F, F);

  if (n%2 != 0)
     multiply(F, M);
}

void multiply(int F[2][2], int M[2][2])
{
  int x =  F[0][0]*M[0][0] + F[0][1]*M[1][0];
  int y =  F[0][0]*M[0][1] + F[0][1]*M[1][1];
  int z =  F[1][0]*M[0][0] + F[1][1]*M[1][0];
  int w =  F[1][0]*M[0][1] + F[1][1]*M[1][1];

  F[0][0] = x;
  F[0][1] = y;
  F[1][0] = z;
  F[1][1] = w;
}

/* Driver program to test above function */
int main()
{
  int n = 9;
  printf("%d", fib(9));
  getchar();
  return 0;
}
```

***Time Complexity:* O(Logn)**
Extra Space: O(Logn) if we consider the function call stack size, otherwise O(1).

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

References:

http://en.wikipedia.org/wiki/Fibonacci_number
http://www.ics.uci.edu/~eppstein/161/960109.html

11. Average of a stream of numbers

Difficulty Level: Rookie
Given a stream of numbers, print average (or mean) of the stream at every point. For example, let us consider the stream as 10, 20, 30, 40, 50, 60, …

```
Average of 1 numbers is 10.00
Average of 2 numbers is 15.00
Average of 3 numbers is 20.00
Average of 4 numbers is 25.00
Average of 5 numbers is 30.00
Average of 6 numbers is 35.00
..................
```

To print mean of a stream, we need to find out how to find average when a new number is being added to the stream. To do this, all we need is count of numbers seen so far in the stream, previous average and new number. Let *n* be the count, *prev_avg* be the previous average and x be the new number being added. The average after including *x* number can be written as *(prev_avg*n + x)/(n+1)*.

```c
#include <stdio.h>

// Returns the new average after including x
float getAvg(float prev_avg, int x, int n)
{
    return (prev_avg*n + x)/(n+1);
}

// Prints average of a stream of numbers
void streamAvg(float arr[], int n)
{
   float avg = 0;
   for(int i = 0; i < n; i++)
   {
       avg  = getAvg(avg, arr[i], i);
       printf("Average of %d numbers is %f \n", i+1, avg);
   }
   return;
}

// Driver program to test above functions
int main()
{
    float arr[] = {10, 20, 30, 40, 50, 60};
    int n = sizeof(arr)/sizeof(arr[0]);
    streamAvg(arr, n);

    return 0;
}
```

The above function getAvg() can be optimized using following changes. We can avoid the use of prev_avg and number of elements by using static variables (Assuming that only this function is called for average of stream). Following is the oprimnized version.

```c
#include <stdio.h>

// Returns the new average after including x
float getAvg (int x)
{
    static int sum, n;

    sum += x;
    return (((float)sum)/++n);
}

// Prints average of a stream of numbers
void streamAvg(float arr[], int n)
{
   float avg = 0;
   for(int i = 0; i < n; i++)
   {
       avg  = getAvg(arr[i]);
       printf("Average of %d numbers is %f \n", i+1, avg);
   }
   return;
}

// Driver program to test above functions
int main()
{
    float arr[] = {10, 20, 30, 40, 50, 60};
    int n = sizeof(arr)/sizeof(arr[0]);
    streamAvg(arr, n);

    return 0;
}
```

Thanks to Abhijeet Deshpande for suggesting this optimized version.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

12. Check whether a given point lies inside a triangle or not

Given three corner points of a triangle, and one more point P. Write a function to check whether P lies within the triangle or not.

For example, consider the following program, the function should return true for P(10, 15) and false for P'(30, 15)

```
              B(10,30)
                / \
               /   \
              /     \
             /   P   \      P'
```

```
            /         \
     A(0,0) ----------- C(20,0)
```

Source: Microsoft Interview Question

Solution:
Let the coordinates of three corners be (x1, y1), (x2, y2) and (x3, y3). And coordinates of the given point P be (x, y)

1) Calculate area of the given triangle, i.e., area of the triangle ABC in the above diagram. Area A = [x1(y2 – y3) + x2(y3 – y1) + x3(y1-y2)]/2
2) Calculate area of the triangle PAB. We can use the same formula for this. Let this area be A1.
3) Calculate area of the triangle PBC. Let this area be A2.
4) Calculate area of the triangle PAC. Let this area be A3.
5) If P lies inside the triangle, then A1 + A2 + A3 must be equal to A.

```c
#include <stdio.h>
#include <stdlib.h>

/* A utility function to calculate area of triangle formed by (x1, y1)
   (x2, y2) and (x3, y3) */
float area(int x1, int y1, int x2, int y2, int x3, int y3)
{
   return abs((x1*(y2-y3) + x2*(y3-y1)+ x3*(y1-y2))/2.0);
}

/* A function to check whether point P(x, y) lies inside the triangle
   by A(x1, y1), B(x2, y2) and C(x3, y3) */
bool isInside(int x1, int y1, int x2, int y2, int x3, int y3, int x, i
{
   /* Calculate area of triangle ABC */
   float A = area (x1, y1, x2, y2, x3, y3);

   /* Calculate area of triangle PBC */
   float A1 = area (x, y, x2, y2, x3, y3);

   /* Calculate area of triangle PAC */
   float A2 = area (x1, y1, x, y, x3, y3);

   /* Calculate area of triangle PAB */
   float A3 = area (x1, y1, x2, y2, x, y);

   /* Check if sum of A1, A2 and A3 is same as A */
   return (A == A1 + A2 + A3);
}

/* Driver program to test above function */
int main()
{
   /* Let us check whether the point P(10, 15) lies inside the triangl
      formed by A(0, 0), B(20, 0) and C(10, 30) */
   if (isInside(0, 0, 20, 0, 10, 30, 10, 15))
     printf ("Inside");
   else
     printf ("Not Inside");

   return 0;
}
```

Ouptut:

```
Inside
```

Exercise: Given coordinates of four corners of a rectangle, and a point P. Write a function to check whether P lies inside the given rectangle or not.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

13. Count numbers that don’t contain 3

Given a number n, write a function that returns count of numbers from 1 to n that don’t contain digit 3 in their decimal representation.

Examples:

```
Input: n = 10
Output: 9

Input: n = 45
Output: 31
// Numbers 3, 13, 23, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43 contain digit 3.

Input: n = 578
Ouput: 385
```

Solution:
We can solve it recursively. Let count(n) be the function that counts such numbers.

```
'msd' --> the most significant digit in n
'd'   --> number of digits in n.

count(n) = n if n < 3

count(n) = n - 1 if 3 <= n < 10

count(n) = count(msd) * count(10^(d-1) - 1) +
           count(msd) +
           count(n % (10^(d-1)))
           if n > 10 and msd is not 3

count(n) = count( msd * (10^(d-1)) - 1)
           if n > 10 and msd is 3
```

```
Let us understand the solution with n = 578.
count(578) = 4*count(99) + 4 + count(78)
The middle term 4 is added to include numbers 100, 200, 400 and 500.

Let us take n = 35 as another example.
count(35) = count (3*10 - 1) = count(29)
```

```c
#include <stdio.h>

/* returns count of numbers which are in range from 1 to n and don't c
   as a digit */
int count(int n)
{
    // Base cases (Assuming n is not negative)
    if (n < 3)
        return n;
    if (n >= 3 && n < 10)
       return n-1;

    // Calculate 10^(d-1) (10 raise to the power d-1) where d is
    // number of digits in n. po will be 100 for n = 578
    int po = 1;
    while (n/po > 9)
        po = po*10;

    // find the most significant digit (msd is 5 for 578)
    int msd = n/po;

    if (msd != 3)
      // For 578, total will be 4*count(10^2 - 1) + 4 + count(78)
      return count(msd)*count(po - 1) + count(msd) + count(n%po);
    else
      // For 35, total will be equal to count(29)
      return count(msd*po - 1);
}

// Driver program to test above function
int main()
{
    printf ("%d ", count(578));
    return 0;
}
```

Output:

```
385
```

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

14. Magic Square

A magic square of order n is an arrangement of n^2 numbers, usually distinct integers, in a square, such that the n numbers in all rows, all columns, and both diagonals sum to the same constant. A magic square contains the integers from 1 to n^2.

The constant sum in every row, column and diagonal is called the magic constant or magic sum, M. The magic constant of a normal magic square depends only on n and

has the following value:
M = n(n^2+1)/2

For normal magic squares of order n = 3, 4, 5, …, the magic constants are: 15, 34, 65, 111, 175, 260, …

In this post, we will discuss how programmatically we can generate a magic square of size n. Before we go further, consider the below examples:

```
Magic Square of size 3
-----------------------
  2   7   6
  9   5   1
  4   3   8
Sum in each row & each column = 3*(3^2+1)/2 = 15


Magic Square of size 5
----------------------
  9   3  22  16  15
  2  21  20  14   8
 25  19  13   7   1
 18  12   6   5  24
 11  10   4  23  17
Sum in each row & each column = 5*(5^2+1)/2 = 65


Magic Square of size 7
----------------------
 20  12   4  45  37  29  28
 11   3  44  36  35  27  19
  2  43  42  34  26  18  10
 49  41  33  25  17   9   1
 40  32  24  16   8   7  48
 31  23  15  14   6  47  39
 22  21  13   5  46  38  30
Sum in each row & each column = 7*(7^2+1)/2 = 175
```

Did you find any pattern in which the numbers are stored?
In any magic square, the first number i.e. 1 is stored at position (n/2, n-1). Let this position be (i,j). The next number is stored at position (i-1, j+1) where we can consider each row & column as circular array i.e. they wrap around.

Three conditions hold:

1. The position of next number is calculated by decrementing row number of previous number by 1, and incrementing the column number of previous number by 1. At any time,

if the calculated row position becomes -1, it will wrap around to n-1. Similarly, if the calculated column position becomes n, it will wrap around to 0.

2. If the magic square already contains a number at the calculated position, calculated column position will be decremented by 2, and calculated row position will be incremented by 1.

3. If the calculated row position is -1 & calculated column position is n, the new position would be: (0, n-2).

```
Example:
Magic Square of size 3
----------------------
 2  7  6
 9  5  1
 4  3  8

Steps:
1. position of number 1 = (3/2, 3-1) = (1, 2)
2. position of number 2 = (1-1, 2+1) = (0, 0)
3. position of number 3 = (0-1, 0+1) = (3-1, 1) = (2, 1)
4. position of number 4 = (2-1, 1+1) = (1, 2)
   Since, at this position, 1 is there. So, apply condition 2.
   new position=(1+1,2-2)=(2,0)
5. position of number 5=(2-1,0+1)=(1,1)
6. position of number 6=(1-1,1+1)=(0,2)
7. position of number 7 = (0-1, 2+1) = (-1,3) // this is tricky, see condition 3
   new position = (0, 3-2) = (0,1)
8. position of number 8=(0-1,1+1)=(-1,2)=(2,2) //wrap around
9. position of number 9=(2-1,2+1)=(1,3)=(1,0) //wrap around
```

Based on the above approach, following is the working code:

```
#include<stdio.h>
#include<string.h>

// A function to generate odd sized magic squares
void generateSquare(int n)
{
    int magicSquare[n][n];

    // set all slots as 0
    memset(magicSquare, 0, sizeof(magicSquare));

    // Initialize position for 1
    int i = n/2;
    int j = n-1;

    // One by one put all values in magic square
    for (int num=1; num <= n*n; )
    {
        if (i==-1 && j==n) //3rd condition
        if (i==-1 && j==n) //3rd condition
        {
            j = n-2;
            i = 0;
        }
        else
        {
            //1st condition helper if next number goes to out of squar
            if (j == n)
                j = 0;
            //1st condition helper if next number is goes to out of sq
            if (i < 0)
                i=n-1;
        }
        if (magicSquare[i][j]) //2nd condition
        {
            j -= 2;
            i++;
            continue;
        }
        else
            magicSquare[i][j] = num++; //set number

        j++;  i--; //1st condition
    }


    // print magic square
    printf("The Magic Square for n=%d:\nSum of each row or column %d:\
            n, n*(n*n+1)/2);
    for(i=0; i<n; i++)
    {
        for(j=0; j<n; j++)
            printf("%3d ", magicSquare[i][j]);
        printf("\n");
    }
}

// Driver program to test above function
int main()
{
    int n = 7; // Works only when n is odd
    generateSquare (n);
    return 0;
}
```

Output:

```
The Magic Square for n=7:
Sum of each row or column 175:

 20  12   4  45  37  29  28
 11   3  44  36  35  27  19
  2  43  42  34  26  18  10
 49  41  33  25  17   9   1
 40  32  24  16   8   7  48
 31  23  15  14   6  47  39
 22  21  13   5  46  38  30
```

NOTE: This approach works only for odd values of n.

References:
http://en.wikipedia.org/wiki/Magic_square

This article is compiled by **Aashish Barnwal** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

15. Sieve of Eratosthenes

Given a number n, print all primes smaller than or equal to n. It is also given that n is a small number.
For example, if n is 10, the output should be "2, 3, 5, 7″. If n is 20, the output should be "2, 3, 5, 7, 11, 13, 17, 19″.

The sieve of Eratosthenes is one of the most efficient ways to find all primes smaller than n when n is smaller than 10 million or so (Ref Wiki).

Following is the algorithm to find all the prime numbers less than or equal to a given integer *n* by Eratosthenes' method:

1. Create a list of consecutive integers from 2 to *n*: (2, 3, 4, …, *n*).
2. Initially, let *p* equal 2, the first prime number.
3. Starting from *p*, count up in increments of *p* and mark each of these numbers greater than *p* itself in the list. These numbers will be 2*p*, 3*p*, 4*p*, etc.; note that some of them may have already been marked.
4. Find the first number greater than *p* in the list that is not marked. If there was no such number, stop. Otherwise, let *p* now equal this number (which is the next prime), and repeat from step 3.

When the algorithm terminates, all the numbers in the list that are not marked are prime.

Following is C++ implementation of the above algorithm. In the following implementation, a boolean array arr[] of size n is used to mark multiples of prime numbers.

```c
#include <stdio.h>
#include <string.h>

// marks all mutiples of 'a' ( greater than 'a' but less than equal to
void markMultiples(bool arr[], int a, int n)
{
    int i = 2, num;
    while ( (num = i*a) <= n )
    {
        arr[ num-1 ] = 1; // minus 1 because index starts from 0.
        ++i;
    }
}

// A function to print all prime numbers smaller than n
void SieveOfEratosthenes(int n)
{
    // There are no prime numbers smaller than 2
    if (n >= 2)
    {
        // Create an array of size n and initialize all elements as 0
        bool arr[n];
        memset(arr, 0, sizeof(arr));

        /* Following property is maintained in the below for loop
           arr[i] == 0 means i + 1 is prime
           arr[i] == 1 means i + 1 is not prime */
        for (int i=1; i<n; ++i)
        {
            if ( arr[i] == 0 )
            {
                //(i+1) is prime, print it and mark its multiples
                printf("%d ", i+1);
                markMultiples(arr, i+1, n);
            }
        }
    }
}

// Driver Program to test above function
int main()
{
    int n = 30;
    printf("Following are the prime numbers below %d\n", n);
    SieveOfEratosthenes(n);
    return 0;
}
```

Output:

```
Following are the prime numbers below 30
2 3 5 7 11 13 17 19 23 29
```

References:
http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes

This article is compiled by **Abhinav Priyadarshi** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

16. Find day of the week for a given date

Write a function that calculates the day of the week for any particular date in the past or future. A typical application is to calculate the day of the week on which someone was born or some other special event occurred.

Following is a simple C function suggested by Sakamoto, Lachman, Keith and Craver to calculate day. The following function returns 0 for Sunday, 1 for Monday, etc.

```c
/* A program to find day of a given date */
#include<stdio.h>

int dayofweek(int d, int m, int y)
{
    static int t[] = { 0, 3, 2, 5, 0, 3, 5, 1, 4, 6, 2, 4 };
    y -= m < 3;
    return ( y + y/4 - y/100 + y/400 + t[m-1] + d) % 7;
}

/* Driver function to test above function */
int main()
{
    int day = dayofweek(30, 8, 2010);
    printf ("%d", day);

    return 0;
}
```

Output: 1 (Monday)

See this for explanation of the above function.

References:
http://en.wikipedia.org/wiki/Determination_of_the_day_of_the_week

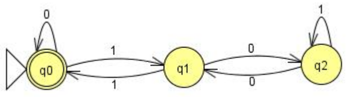
This article is compiled by **Dheeraj Jain** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

17. DFA based division

Deterministic Finite Automaton (DFA) can be used to check whether a number “num” is divisible by “k” or not. If the number is not divisible, remainder can also be obtained using DFA.

We consider the binary representation of ‘num’ and build a DFA with k states. The DFA has transition function for both 0 and 1. Once the DFA is built, we process ‘num’ over the DFA to get remainder.

Let us walk through an example. Suppose we want to check whether a given number ‘num’ is divisible by 3 or not. Any number can be written in the form: num = 3*a + b where ‘a’ is the quotient and ‘b’ is the remainder.
For 3, there can be 3 states in DFA, each corresponding to remainder 0, 1 and 2. And each state can have two transitions corresponding 0 and 1 (considering the binary representation of given ‘num’).
The transition function F(p, x) = q tells that on reading alphabet x, we move from state p to state q. Let us name the states as 0, 1 and 2. The initial state will always be 0. The final state indicates the remainder. If the final state is 0, the number is divisible.



In the above diagram, double circled state is final state.

1. When we are at state 0 and read 0, we remain at state 0.
2. When we are at state 0 and read 1, we move to state 1, why? The number so formed(1) in decimal gives remainder 1.
3. When we are at state 1 and read 0, we move to state 2, why? The number so formed(10) in decimal gives remainder 2.
4. When we are at state 1 and read 1, we move to state 0, why? The number so formed(11) in decimal gives remainder 0.
5. When we are at state 2 and read 0, we move to state 1, why? The number so formed(100) in decimal gives remainder 1.
6. When we are at state 2 and read 1, we remain at state 2, why? The number so formed(101) in decimal gves remainder 2.

The transition table looks like following:

```
state   0   1
_____________
  0     0   1
  1     2   0
  2     1   2
```

Let us check whether 6 is divisible by 3?
Binary representation of 6 is 110
state = 0

1. state=0, we read 1, new state=1
2. state=1, we read 1, new state=0
3. state=0, we read 0, new state=0

Since the final state is 0, the number is divisible by 3.

Let us take another example number as 4
state=0
1. state=0, we read 1, new state=1
2. state=1, we read 0, new state=2
3. state=2, we read 0, new state=1

Since, the final state is not 0, the number is not divisible by 3. The remainder is 1.

Note that the final state gives the remainder.

We can extend the above solution for any value of k. For a value k, the states would be 0, 1, …. , k-1. How to calculate the transition if the decimal equivalent of the binary bits seen so far, crosses the range k? If we are at state p, we have read p (in decimal). Now we read 0, new read number becomes 2*p. If we read 1, new read number becomes 2*p+1. The new state can be obtained by subtracting k from these values (2p or 2p+1) where 0 <= p < k.
Based on the above approach, following is the working code:

```c
#include <stdio.h>
#include <stdlib.h>

// Function to build DFA for divisor k
void preprocess(int k, int Table[][2])
{
    int trans0, trans1;

    // The following loop calculates the two transitions for each stat
    // starting from state 0
    for (int state=0; state<k; ++state)
    {
        // Calculate next state for bit 0
        trans0 = state<<1;
        Table[state][0] = (trans0 < k)? trans0: trans0-k;

        // Calculate next state for bit 1
        trans1 = (state<<1) + 1;
        Table[state][1] = (trans1 < k)? trans1: trans1-k;
    }
}

// A recursive utility function that takes a ‘num’ and DFA (transition
// table) as input and process ‘num’ bit by bit over DFA
void isDivisibleUtil(int num, int* state, int Table[][2])
{
    // process "num" bit by bit from MSB to LSB
    if (num != 0)
    {
        isDivisibleUtil(num>>1, state, Table);
        *state = Table[*state][num&1];
    }
}
```

```
// The main function that divides ‘num’ by k and returns the remainder
int isDivisible (int num, int k)
{
    // Allocate memory for transition table. The table will have k*2 e
    int (*Table)[2] = (int (*)[2])malloc(k*sizeof(*Table));

    // Fill the transition table
    preprocess(k, Table);

    // Process ‘num’ over DFA and get the remainder
    int state = 0;
    isDivisibleUtil(num, &state, Table);

    // Note that the final value of state is the remainder
    return state;
}

// Driver program to test above functions
int main()
{
    int num = 47; // Number to be divided
    int k = 5; // Divisor

    int remainder = isDivisible (num, k);

    if (remainder == 0)
        printf("Divisible\n");
    else
        printf("Not Divisible: Remainder is %d\n", remainder);

    return 0;
}
```

Output:

```
Not Divisible: Remainder is 2
```

DFA based division can be useful if we have a binary stream as input and we want to check for divisibility of the decimal value of stream at any time.

This article is compiled by **Aashish Barnwal** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

18. Generate integer from 1 to 7 with equal probability

Given a function foo() that returns integers from 1 to 5 with equal probability, write a function that returns integers from 1 to 7 with equal probability using foo() only. Minimize the number of calls to foo() method. Also, use of any other library function is not allowed and no floating point arithmetic allowed.

Solution:
We know foo() returns integers from 1 to 5. How we can ensure that integers from 1 to 7 occur with equal probability?
If we somehow generate integers from 1 to a-multiple-of-7 (like 7, 14, 21, …) with equal probability, we can use modulo division by 7 followed by adding 1 to get the numbers from 1 to 7 with equal probability.

We can generate from 1 to 21 with equal probability using the following expression.

```
5*foo() + foo() -5
```

Let us see how above expression can be used.
1. For each value of first foo(), there can be 5 possible combinations for values of second foo(). So, there are total 25 combinations possible.
2. The range of values returned by the above equation is 1 to 25, each integer occurring exactly once.
3. If the value of the equation comes out to be less than 22, return modulo division by 7 followed by adding 1. Else, again call the method recursively. The probability of returning each integer thus becomes 1/7.

The below program shows that the expression returns each integer from 1 to 25 exactly once.

```
#include <stdio.h>

int main()
{
    int first, second;
    for ( first=1; first<=5; ++first )
        for ( second=1; second<=5; ++second )
            printf ("%d \n", 5*first + second - 5);
    return 0;
}
```

Output:

```
1
2
.
.
24
25
```

The below program depicts how we can use foo() to return 1 to 7 with equal probability.

```
#include <stdio.h>

int foo() // given method that returns 1 to 5 with equal probability
{
    // some code here
}

int my_rand() // returns 1 to 7 with equal probability
{
    int i;
    i = 5*foo() + foo() - 5;
    if (i < 22)
        return i%7 + 1;
    return my_rand();
}

int main()
{
    printf ("%d ", my_rand());
    return 0;
}
```

This article is compiled by **Aashish Barnwal** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

19. Given a number, find the next smallest palindrome

Given a number, find the next smallest palindrome larger than this number. For example, if the input number is "2 3 5 4 5", the output should be "2 3 6 3 2". And if the input number is "9 9 9", the output should be "1 0 0 1".

The input is assumed to be an array. Every entry in array represents a digit in input number. Let the array be 'num[]' and size of array be 'n'

There can be three different types of inputs that need to be handled separately.
1) The input number is palindrome and has all 9s. For example "9 9 9". Output should be "1 0 0 1"
2) The input number is not palindrome. For example "1 2 3 4". Output should be "1 3 3 1"
3) The input number is palindrome and doesn't have all 9s. For example "1 2 2 1". Output should be "1 3 3 1".

Solution for input type 1 is easy. The output contains n + 1 digits where the corner digits are 1, and all digits between corner digits are 0.

Now let us first talk about input type 2 and 3. How to convert a given number to a greater palindrome? To understand the solution, let us first define the following two terms:
Left Side: The left half of given number. Left side of "1 2 3 4 5 6" is "1 2 3" and left side

of “1 2 3 4 5″ is “1 2″
Right Side: The right half of given number. Right side of “1 2 3 4 5 6″ is “4 5 6″ and right side of “1 2 3 4 5″ is “4 5″
To convert to palindrome, we can either take the mirror of its left side or take mirror of its right side. However, if we take the mirror of the right side, then the palindrome so formed is not guaranteed to be next larger palindrome. So, we must take the mirror of left side and copy it to right side. But there are some cases that must be handled in different ways. See the following steps.

We will start with two indices i and j. i pointing to the two middle elements (or pointing to two elements around the middle element in case of n being odd). We one by one move i and j away from each other.

Step 1. Initially, ignore the part of left side which is same as the corresponding part of right side. For example, if the number is “8 3 **4 2 2 4** 6 9″, we ignore the middle four elements. i now points to element 3 and j now points to element 6.

Step 2. After step 1, following cases arise:

Case 1: Indices i & j cross the boundary.
This case occurs when the input number is palindrome. In this case, we just add 1 to the middle digit (or digits in case n is even) propagate the carry towards MSB digit of left side and simultaneously copy mirror of the left side to the right side.
For example, if the given number is “1 2 9 2 1″, we increment 9 to 10 and propagate the carry. So the number becomes “1 3 0 3 1″

Case 2: There are digits left between left side and right side which are not same. So, we just mirror the left side to the right side & try to minimize the number formed to guarantee the next smallest palindrome.
In this case, there can be **two sub-cases**.

2.1) Copying the left side to the right side is sufficient, we don’t need to increment any digits and the result is just mirror of left side. Following are some examples of this sub-case.
Next palindrome for “7 **8** 3 3 2 2″ is “7 8 3 3 8 7″
Next palindrome for “1 2 **5** 3 2 2″ is “1 2 5 5 2 1″
Next palindrome for “1 4 **5** 8 7 6 7 8 3 2 2″ is “1 4 5 8 7 6 7 8 5 4 1″
How do we check for this sub-case? All we need to check is the digit just after the ignored part in step 1. This digit is highlighted in above examples. If this digit is greater than the corresponding digit in right side digit, then copying the left side to the right side is sufficient and we don’t need to do anything else.

2.2) Copying the left side to the right side is NOT sufficient. This happens when the above defined digit of left side is smaller. Following are some examples of this case.
Next palindrome for “7 **1** 3 3 2 2″ is “7 1 4 4 1 7″
Next palindrome for “1 2 **3** 4 6 2 8″ is “1 2 3 5 3 2 1″
Next palindrome for “9 4 **1** 8 7 9 7 8 3 2 2″ is “9 4 1 8 8 0 8 8 1 4 9″

We handle this subcase like Case 1. We just add 1 to the middle digit (or digits in ase n is even) propagate the carry towards MSB digit of left side and simultaneously copy mirror of the left side to the right side.

```
#include <stdio.h>

// A utility function to print an array
void printArray (int arr[], int n);

// A utility function to check if num has all 9s
int AreAll9s (int num[], int n );

// Returns next palindrome of a given number num[].
// This function is for input type 2 and 3
void generateNextPalindromeUtil (int num[], int n )
{
    // find the index of mid digit
    int mid = n/2;

    // A bool variable to check if copy of left side to right is suffi
    bool leftsmaller = false;

    // end of left side is always 'mid -1'
    int i = mid - 1;

    // Begining of right side depends if n is odd or even
    int j = (n % 2)? mid + 1 : mid;

   // Initially, ignore the middle same digits
    while (i >= 0 && num[i] == num[j])
        i--,j++;

    // Find if the middle digit(s) need to be incremented or not (or c
    // side is not sufficient)
    if ( i < 0 || num[i] < num[j])
        leftsmaller = true;

    // Copy the mirror of left to tight
    while (i >= 0)
    {
        num[j] = num[i];
        j++;
        i--;
    }

    // Handle the case where middle digit(s) must be incremented.
    // This part of code is for CASE 1 and CASE 2.2
    if (leftsmaller == true)
    {
        int carry = 1;
        i = mid - 1;

        // If there are odd digits, then increment
        // the middle digit and store the carry
        if (n%2 == 1)
        {
            num[mid] += carry;
            carry = num[mid] / 10;
            num[mid] %= 10;
            j = mid + 1;
        }
        else
            j = mid;

        // Add 1 to the rightmost digit of the left side, propagate the
        // towards MSB digit and simultaneously copying mirror of the 
        // to the right side.
        while (i >= 0)
        {
            num[i] += carry;
            carry = num[i] / 10;
            num[i] %= 10;
            num[j++] = num[i--]; // copy mirror to right
        }
    }
}

// The function that prints next palindrome of a given number num[]
// with n digits.
void generateNextPalindrome( int num[], int n )
{
    int i;

    printf("\nNext palindrome is:\n");

    // Input type 1: All the digits are 9, simply o/p 1
    // followed by n-1 0's followed by 1.
    if( AreAll9s( num, n ) )
    {
        printf( "1 ");
        for( i = 1; i < n; i++ )
            printf( "0 " );
        printf( "1" );
    }

    // Input type 2 and 3
    else
    {
        generateNextPalindromeUtil ( num, n );

        // print the result
        printArray (num, n);
    }
}

// A utility function to check if num has all 9s
int AreAll9s( int* num, int n )
{
    int i;
    for( i = 0; i < n; ++i )
        if( num[i] != 9 )
            return 0;
    return 1;
}

/* Utility that prints out an array on a line */
void printArray(int arr[], int n)
{
    int i;
    for (i=0; i < n; i++)
        printf("%d ", arr[i]);
    printf("\n");
}
```

```
// Driver Program to test above function
int main()
{
    int num[] = {9, 4, 1, 8, 7, 9, 7, 8, 3, 2, 2};

    int n = sizeof (num)/ sizeof(num[0]);

    generateNextPalindrome( num, n );

    return 0;
}
```

Output:

```
Next palindrome is:
9 4 1 8 8 0 8 8 1 4 9
```

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

20. Make a fair coin from a biased coin

You are given a function foo() that represents a biased coin. When foo() is called, it returns 0 with 60% probability, and 1 with 40% probability. Write a new function that returns 0 and 1 with 50% probability each. Your function should use only foo(), no other library method.

Solution:
We know foo() returns 0 with 60% probability. How can we ensure that 0 and 1 are returned with 50% probability?
The solution is similar to this post. If we can somehow get two cases with equal probability, then we are done. We call foo() two times. Both calls will return 0 with 60% probability. So the two pairs (0, 1) and (1, 0) will be generated with equal probability from two calls of foo(). Let us see how.

(0, 1): The probability to get 0 followed by 1 from two calls of foo() = 0.6 * 0.4 = 0.24
(1, 0): The probability to get 1 followed by 0 from two calls of foo() = 0.4 * 0.6 = 0.24

So the two cases appear with equal probability. The idea is to return consider only the above two cases, return 0 in one case, return 1 in other case. For other cases [(0, 0) and (1, 1)], recur until you end up in any of the above two cases.

The below program depicts how we can use foo() to return 0 and 1 with equal probability.

```
#include <stdio.h>

int foo() // given method that returns 0 with 60% probability and 1 wi
{
    // some code here
}

// returns both 0 and 1 with 50% probability
int my_fun()
{
    int val1 = foo();
    int val2 = foo();
    if (val1 == 0 && val2 == 1)
        return 0;   // Will reach here with 0.24 probability
    if (val1 == 1 && val2 == 0)
        return 1;   // // Will reach here with 0.24 probability
    return my_fun();  // will reach here with (1 - 0.24 - 0.24) probab
}

int main()
{
    printf ("%d ", my_fun());
    return 0;
}
```

References:
http://en.wikipedia.org/wiki/Fair_coin#Fair_results_from_a_biased_coin

This article is compiled by **Shashank Sinha** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.
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21. Check divisibility by 7

Given a number, check if it is divisible by 7. You are not allowed to use modulo operator, floating point arithmetic is also not allowed.

A simple method is repeated subtraction. Following is another interesting method.

Divisibility by 7 can be checked by a recursive method. A number of the form 10a + b is divisible by 7 if and only if a – 2b is divisible by 7. In other words, subtract twice the last digit from the number formed by the remaining digits. Continue to do this until a small number.

Example: the number 371: 37 – (2×1) = 37 – 2 = 35; 3 – (2 × 5) = 3 – 10 = -7; thus, since -7 is divisible by 7, 371 is divisible by 7.

Following is C implementation of the above method

```c
// A Program to check whether a number is divisible by 7
#include <stdio.h>

int isDivisibleBy7( int num )
{
    // If number is negative, make it positive
    if( num < 0 )
        return isDivisibleBy7( -num );

    // Base cases
    if( num == 0 || num == 7 )
        return 1;
    if( num < 10 )
        return 0;

    // Recur for ( num / 10 - 2 * num % 10 )
    return isDivisibleBy7( num / 10 - 2 * ( num - num / 10 * 10 ) );
}

// Driver program to test above function
int main()
{
    int num = 616;
    if( isDivisibleBy7(num ) )
        printf( "Divisible" );
    else
        printf( "Not Divisible" );
    return 0;
}
```

Output:

```
Divisible
```

How does this work? Let 'b' be the last digit of a number 'n' and let 'a' be the number we get when we split off 'b'.
The representation of the number may also be multiplied by any number relatively prime to the divisor without changing its divisibility. After observing that 7 divides 21, we can perform the following:

```
10.a + b
```

after multiplying by 2, this becomes

```
20.a + 2.b
```

and then

```
21.a - a + 2.b
```

Eliminating the multiple of 21 gives

```
-a + 2b
```

and multiplying by -1 gives

```
a - 2b
```

There are other interesting methods to check divisibility by 7 and other numbers. See following Wiki page for details.

References:
http://en.wikipedia.org/wiki/Divisibility_rule

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

22. Find the largest multiple of 3

Given an array of non-negative integers. Find the largest multiple of 3 that can be formed from array elements.

For example, if the input array is {8, 1, 9}, the output should be “9 8 1″, and if the input array is {8, 1, 7, 6, 0}, output should be “8 7 6 0″.

Method 1 (Brute Force)
The simple & straight forward approach is to generate all the combinations of the elements and keep track of the largest number formed which is divisible by 3.

Time Complexity: O(n x 2^n). There will be 2^n combinations of array elements. To compare each combination with the largest number so far may take O(n) time.
Auxiliary Space: O(n) // to avoid integer overflow, the largest number is assumed to be stored in the form of array.

Method 2 (Tricky)
This problem can be solved efficiently with the help of O(n) extra space. This method is based on the following facts about numbers which are multiple of 3.

1) A number is multiple of 3 if and only if the sum of digits of number is multiple of 3. For example, let us consider 8760, it is a multiple of 3 because sum of digits is 8 + 7+ 6+ 0 = 21, which is a multiple of 3.

2) If a number is multiple of 3, then all permutations of it are also multiple of 3. For example, since 6078 is a multiple of 3, the numbers 8760, 7608, 7068, ….. are also multiples of 3.

3) We get the same remainder when we divide the number and sum of digits of the number. For example, if divide number 151 and sum of it digits 7, by 3, we get the same remainder 1.

What is the idea behind above facts?
The value of 10%3 and 100%3 is 1. The same is true for all the higher powers of 10, because 3 divides 9, 99, 999, … etc.
Let us consider a 3 digit number n to prove above facts. Let the first, second and third digits of n be 'a', 'b' and 'c' respectively. n can be written as

```
n = 100.a + 10.b + c
```

Since (10^x)%3 is 1 for any x, the above expression gives the same remainder as following expression

```
 1.a + 1.b + c
```

So the remainder obtained by sum of digits and 'n' is same.

Following is a solution based on the above observation.

1. Sort the array in non-decreasing order.

2. Take three queues. One for storing elements which on dividing by 3 gives remainder as 0.The second queue stores digits which on dividing by 3 gives remainder as 1. The third queue stores digits which on dividing by 3 gives remainder as 2. Call them as queue0, queue1 and queue2

3. Find the sum of all the digits.

4. Three cases arise:
……**4.1** The sum of digits is divisible by 3. Dequeue all the digits from the three queues. Sort them in non-increasing order. Output the array.

……**4.2** The sum of digits produces remainder 1 when divided by 3.
Remove one item from queue1. If queue1 is empty, remove two items from queue2. If queue2 contains less than two items, the number is not possible.

……**4.3** The sum of digits produces remainder 2 when divided by 3.
Remove one item from queue2. If queue2 is empty, remove two items from queue1. If queue1 contains less than two items, the number is not possible.

5. Finally empty all the queues into an auxiliary array. Sort the auxiliary array in non-increasing order. Output the auxiliary array.

Based on the above, below is the implementation:

```
/* A program to find the largest multiple of 3 from an array of elemen
#include <stdio.h>
#include <stdlib.h>

// A queue node
typedef struct Queue
{
```

```
{
    int front;
    int rear;
    int capacity;
    int* array;
} Queue;

// A utility function to create a queue with given capacity
Queue* createQueue( int capacity )
{
    Queue* queue = (Queue *) malloc (sizeof(Queue));
    queue->capacity = capacity;
    queue->front = queue->rear = -1;
    queue->array = (int *) malloc (queue->capacity * sizeof(int));
    return queue;
}

// A utility function to check if queue is empty
int isEmpty (Queue* queue)
{
    return queue->front == -1;
}

// A function to add an item to queue
void Enqueue (Queue* queue, int item)
{
    queue->array[ ++queue->rear ] = item;
    if ( isEmpty(queue) )
        ++queue->front;
}

// A function to remove an item from queue
int Dequeue (Queue* queue)
{
    int item = queue->array[ queue->front ];
    if( queue->front == queue->rear )
        queue->front = queue->rear = -1;
    else
        queue->front++;

    return item;
}

// A utility function to print array contents
void printArr (int* arr, int size)
{
    int i;
    for (i = 0; i< size; ++i)
        printf ("%d ", arr[i]);
}

/* Following two functions are needed for library function qsort().
   Refer following link for help of qsort()
   http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */
int compareAsc( const void* a, const void* b )
{
    return *(int*)a > *(int*)b;
}
int compareDesc( const void* a, const void* b )
{
    return *(int*)a < *(int*)b;
}
```

```c
// This function puts all elements of 3 queues in the auxiliary array
void populateAux (int* aux, Queue* queue0, Queue* queue1,
                            Queue* queue2, int* top )
{
    // Put all items of first queue in aux[]
    while ( !isEmpty(queue0) )
        aux[ (*top)++ ] = Dequeue( queue0 );

    // Put all items of second queue in aux[]
    while ( !isEmpty(queue1) )
        aux[ (*top)++ ] = Dequeue( queue1 );

    // Put all items of third queue in aux[]
    while ( !isEmpty(queue2) )
        aux[ (*top)++ ] = Dequeue( queue2 );
}
```

```c
// The main function that finds the largest possible multiple of
// 3 that can be formed by arr[] elements
int findMaxMultupleOf3( int* arr, int size )
{
    // Step 1: sort the array in non-decreasing order
    qsort( arr, size, sizeof( int ), compareAsc );

    // Create 3 queues to store numbers with remainder 0, 1
    // and 2 respectively
    Queue* queue0 = createQueue( size );
    Queue* queue1 = createQueue( size );
    Queue* queue2 = createQueue( size );

    // Step 2 and 3 get the sum of numbers and place them in
    // corresponding queues
    int i, sum;
    for ( i = 0, sum = 0; i < size; ++i )
    {
        sum += arr[i];
        if ( (arr[i] % 3) == 0 )
            Enqueue( queue0, arr[i] );
        else if ( (arr[i] % 3) == 1 )
            Enqueue( queue1, arr[i] );
        else
            Enqueue( queue2, arr[i] );
    }

    // Step 4.2: The sum produces remainder 1
    if ( (sum % 3) == 1 )
    {
        // either remove one item from queue1
        if ( !isEmpty( queue1 ) )
            Dequeue( queue1 );

        // or remove two items from queue2
        else
        {
            if ( !isEmpty( queue2 ) )
                Dequeue( queue2 );
            else
                return 0;

            if ( !isEmpty( queue2 ) )
                Dequeue( queue2 );
            else
                return 0;
                return 0;
        }
    }

    // Step 4.3: The sum produces remainder 2
    else if ((sum % 3) == 2)
    {
        // either remove one item from queue2
        if ( !isEmpty( queue2 ) )
            Dequeue( queue2 );

        // or remove two items from queue1
        else
        {
            if ( !isEmpty( queue1 ) )
                Dequeue( queue1 );
            else
                return 0;

            if ( !isEmpty( queue1 ) )
                Dequeue( queue1 );
            else
                return 0;
        }
    }

    int aux[size], top = 0;

    // Empty all the queues into an auxiliary array.
    populateAux (aux, queue0, queue1, queue2, &top);

    // sort the array in non-increasing order
    qsort (aux, top, sizeof( int ), compareDesc);

    // print the result
    printArr (aux, top);

    return top;
}

// Driver program to test above functions
int main()
{
    int arr[] = {8, 1, 7, 6, 0};
    int size = sizeof(arr)/sizeof(arr[0]);

    if (findMaxMultupleOf3( arr, size ) == 0)
        printf( "Not Possible" );

    return 0;
}
```

The above method can be optimized in following ways.
1) We can use Heap Sort or Merge Sort to make the time complexity O(nLogn).

2) We can avoid extra space for queues. We know at most two items will be removed from the input array. So we can keep track of two items in two variables.

3) At the end, instead of sorting the array again in descending order, we can print the ascending sorted array in reverse order. While printing in reverse order, we can skip the

two elements to be removed.

The above code works only if the input arrays has numbers from 0 to 9. It can be easily extended for any positive integer array. We just have to modify the part where we sort the array in decreasing order, at the end of code.

Time Complexity: O(nLogn), assuming a O(nLogn) algorithm is used for sorting.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

23. Lexicographic rank of a string

Given a string, find its rank among all its permutations sorted lexicographically. For example, rank of “abc” is 1, rank of “acb” is 2, and rank of “cba” is 6.

For simplicity, let us assume that the string does not contain any duplicated characters.

One simple solution is to initialize rank as 1, generate all permutations in lexicographic order. After generating a permutation, check if the generated permutation is same as given string, if same, then return rank, if not, then increment the rank by 1. The time complexity of this solution will be exponential in worst case. Following is an efficient solution.

Let the given string be “STRING”. In the input string, ‘S’ is the first character. There are total 6 characters and 4 of them are smaller than ‘S’. So there can be 4 * 5! smaller strings where first character is smaller than ‘S’, like following

R X X X X X
I X X X X X
N X X X X X
G X X X X X

Now let us Fix S’ and find the smaller strings staring with ‘S’.

Repeat the same process for T, rank is 4*5! + 4*4! +…

Now fix T and repeat the same process for R, rank is 4*5! + 4*4! + 3*3! +…

Now fix R and repeat the same process for I, rank is 4*5! + 4*4! + 3*3! + 1*2! +…

Now fix I and repeat the same process for N, rank is 4*5! + 4*4! + 3*3! + 1*2! + 1*1! +…

Now fix N and repeat the same process for G, rank is 4*5! + 4*4 + 3*3! + 1*2! + 1*1! + 0*0!

Rank = 4*5! + 4*4! + 3*3! + 1*2! + 1*1! + 0*0! = 597

Since the value of rank starts from 1, the final rank = 1 + 597 = 598

```
#include <stdio.h>
#include <string.h>

// A utility function to find factorial of n
int fact(int n)
{
    return (n <= 1)? 1 :n * fact(n-1);
}

// A utility function to count smaller characters on right
// of arr[low]
int findSmallerInRight(char* str, int low, int high)
{
    int countRight = 0, i;

    for (i = low+1; i <= high; ++i)
        if (str[i] < str[low])
            ++countRight;

    return countRight;
}

// A function to find rank of a string in all permutations
// of characters
int findRank (char* str)
{
    int len = strlen(str);
    int mul = fact(len);
    int rank = 1;
    int countRight;

    int i;
    for (i = 0; i < len; ++i)
    {
        mul /= len - i;

        // count number of chars smaller than str[i]
        // fron str[i+1] to str[len-1]
        countRight = findSmallerInRight(str, i, len-1);

        rank += countRight * mul ;
    }

    return rank;
}

// Driver program to test above function
int main()
{
    char str[] = "string";
    printf ("%d", findRank(str));
    return 0;
}
```

Output

```
598
```

The time complexity of the above solution is O(n^2). We can reduce the time complexity to O(n) by creating an auxiliary array of size 256. See following code.

```
// A O(n) solution for finding rank of string
#include <stdio.h>
#include <string.h>
#define MAX_CHAR 256

// A utility function to find factorial of n
int fact(int n)
{
    return (n <= 1)? 1 :n * fact(n-1);
}

// Construct a count array where value at every index
// contains count of smaller characters in whole string
void populateAndIncreaseCount (int* count, char* str)
{
    int i;

    for( i = 0; str[i]; ++i )
        ++count[ str[i] ];

    for( i = 1; i < 256; ++i )
        count[i] += count[i-1];
}

// Removes a character ch from count[] array
// constructed by populateAndIncreaseCount()
void updatecount (int* count, char ch)
{
    int i;
    for( i = ch; i < MAX_CHAR; ++i )
        --count[i];
}

// A function to find rank of a string in all permutations
// of characters
int findRank (char* str)
{
    int len = strlen(str);
    int mul = fact(len);
    int rank = 1, i;
    int count[MAX_CHAR] = {0};  // all elements of count[] are initiali

    // Populate the count array such that count[i] contains count of
    // characters which are present in str and are smaller than i
    populateAndIncreaseCount( count, str );

    for (i = 0; i < len; ++i)
    {
        mul /= len - i;

        // count number of chars smaller than str[i]
        // fron str[i+1] to str[len-1]
        rank += count[ str[i] - 1] * mul;

        // Reduce count of characters greater than str[i]
        updatecount (count, str[i]);
    }

    return rank;
    return rank;
}

// Driver program to test above function
int main()
{
    char str[] = "string";
    printf ("%d", findRank(str));
    return 0;
}
```

The above programs don't work for duplicate characters. To make them work for duplicate characters, find all the characters that are smaller (include equal this time also), do the same as above but, this time divide the rank so formed by p! where p is the count of occurrences of the repeating character.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

24. Print all permutations in sorted (lexicographic) order

Given a string, print all permutations of it in sorted order. For example, if the input string is "ABC", then output should be "ABC, ACB, BAC, BCA, CAB, CBA".

We have discussed a program to print all permutations in this post, but here we must print the permutations in increasing order.

Following are the steps to print the permutations lexicographic-ally

1. Sort the given string in non-decreasing order and print it. The first permutation is always the string sorted in non-decreasing order.

2. Start generating next higher permutation. Do it until next higher permutation is not possible. If we reach a permutation where all characters are sorted in non-increasing order, then that permutation is the last permutation.

Steps to generate the next higher permutation:
1. Take the previously printed permutation and find the rightmost character in it, which is smaller than its next character. Let us call this character as 'first character'.

2. Now find the ceiling of the 'first character'. Ceiling is the smallest character on right of 'first character', which is greater than 'first character'. Let us call the ceil character as 'second character'.

3. Swap the two characters found in above 2 steps.

4. Sort the substring (in non-decreasing order) after the original index of 'first character'.

Let us consider the string “ABCDEF”. Let previously printed permutation be “DCFEBA”. The next permutation in sorted order should be “DEABCF”. Let us understand above steps to find next permutation. The ‘first character’ will be ‘C’. The ‘second character’ will be ‘E’. After swapping these two, we get “DEFCBA”. The final step is to sort the substring after the character original index of ‘first character’. Finally, we get “DEABCF”.

Following is C++ implementation of the algorithm.

```
// Program to print all permutations of a string in sorted order.
#include <stdio.h>
#include <stdlib.h>
#include <string.h>

/* Following function is needed for library function qsort(). Refer
   http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */
int compare (const void *a, const void * b)
{  return ( *(char *)a - *(char *)b ); }

// A utility function two swap two characters a and b
void swap (char* a, char* b)
{
    char t = *a;
    *a = *b;
    *b = t;
}

// This function finds the index of the smallest character
// which is greater than 'first' and is present in str[l..h]
int findCeil (char str[], char first, int l, int h)
{
    // initialize index of ceiling element
    int ceilIndex = l;

    // Now iterate through rest of the elements and find
    // the smallest character greater than 'first'
    for (int i = l+1; i <= h; i++)
      if (str[i] > first && str[i] < str[ceilIndex])
            ceilIndex = i;

    return ceilIndex;
}

// Print all permutations of str in sorted order
void sortedPermutations ( char str[] )
{
    // Get size of string
    int size = strlen(str);

    // Sort the string in increasing order
    qsort( str, size, sizeof( str[0] ), compare );

    // Print permutations one by one
    bool isFinished = false;
    while ( ! isFinished )
    {
        // print this permutation
        printf ("%s \n", str);

        // Find the rightmost character which is smaller than its next
        // character. Let us call it 'first char'
        int i;
        for ( i = size - 2; i >= 0; --i )
           if (str[i] < str[i+1])
              break;

        // If there is no such chracter, all are sorted in decreasing 
        // means we just printed the last permutation and we are done.
        if ( i == -1 )
            isFinished = true;
        else
        {
            // Find the ceil of 'first char' in right of first charact
            // Ceil of a character is the smallest character greater t
            int ceilIndex = findCeil( str, str[i], i + 1, size - 1 );

            // Swap first and second characters
            swap( &str[i], &str[ceilIndex] );

            // Sort the string on right of 'first char'
            qsort( str + i + 1, size - i - 1, sizeof(str[0]), compare
        }
    }
}

// Driver program to test above function
int main()
{
    char str[] = "ABCD";
    sortedPermutations( str );
    return 0;
}
```

Output:

```
ABCD
ABDC
....
....
DCAB
DCBA
```

The upper bound on time complexity of the above program is O(n^2 x n!). We can optimize step 4 of the above algorithm for finding next permutation. Instead of sorting the subarray after the ‘first character’, we can reverse the subarray, because the subarray we get after swapping is always sorted in non-increasing order. This optimization makes the time complexity as O(n x n!). See following optimized code.

```
// An optimized version that uses reverse instead of sort for
// finding the next permutation

// A utility function to reverse a string str[l..h]
void reverse(char str[], int l, int h)
{
   while (l < h)
   {
       swap(&str[l], &str[h]);
       l++;
       h--;
   }
}
```

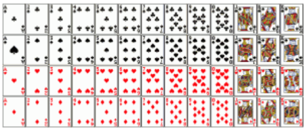
```
// Print all permutations of str in sorted order
void sortedPermutations ( char str[] )
{
    // Get size of string
    int size = strlen(str);

    // Sort the string in increasing order
    qsort( str, size, sizeof( str[0] ), compare );

    // Print permutations one by one
    bool isFinished = false;
    while ( ! isFinished )
    {
        // print this permutation
        printf ("%s \n", str);

        // Find the rightmost character which is smaller than its next
        // character. Let us call it 'first char'
        int i;
        for ( i = size - 2; i >= 0; --i )
           if (str[i] < str[i+1])
              break;

        // If there is no such chracter, all are sorted in decreasing
        // means we just printed the last permutation and we are done.
        if ( i == -1 )
            isFinished = true;
        else
        {
            // Find the ceil of 'first char' in right of first characte
            // Ceil of a character is the smallest character greater th
            int ceilIndex = findCeil( str, str[i], i + 1, size - 1 );

            // Swap first and second characters
            swap( &str[i], &str[ceilIndex] );

            // reverse the string on right of 'first char'
            reverse( str, i + 1, size - 1 );
        }
    }
}
```

The above programs print duplicate permutation when characters are repeated. We can avoid it by keeping track of the previous permutation. While printing, if the current permutation is same as previous permutation, we won’t print it.

This article is compiled by **Aashish Barnwal** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

25. Shuffle a given array

Given an array, write a program to generate a random permutation of array elements. This question is also asked as “shuffle a deck of cards” or “randomize a given array”.



Let the given array be *arr[]*. A simple solution is to create an auxiliary array *temp[]* which is initially a copy of *arr[]*. Randomly select an element from *temp[]*, copy the randomly selected element to *arr[0]* and remove the selected element from *temp[]*. Repeat the same process n times and keep copying elements to *arr[1], arr[2], … .* The time complexity of this solution will be O(n^2).

Fisher–Yates shuffle Algorithm works in O(n) time complexity. The assumption here is, we are given a function rand() that generates random number in O(1) time.
The idea is to start from the last element, swap it with a randomly selected element from the whole array (including last). Now consider the array from 0 to n-2 (size reduced by 1), and repeat the process till we hit the first element.

Following is the detailed algorithm

```
To shuffle an array a of n elements (indices 0..n-1):
  for i from n - 1 downto 1 do
       j = random integer with 0 <= j <= i
       exchange a[j] and a[i]
```

Following is C++ implementation of this algorithm.

```c
// C Program to shuffle a given array

#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// A utility function to swap to integers
void swap (int *a, int *b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// A utility function to print an array
void printArray (int arr[], int n)
{
    for (int i = 0; i < n; i++)
        printf("%d ", arr[i]);
    printf("\n");
}

// A function to generate a random permutation of arr[]
void randomize ( int arr[], int n )
{
    // Use a different seed value so that we don't get same
    // result each time we run this program
    srand ( time(NULL) );

    // Start from the last element and swap one by one. We don't
    // need to run for the first element that's why i > 0
    for (int i = n-1; i > 0; i--)
    {
        // Pick a random index from 0 to i
        int j = rand() % (i+1);

        // Swap arr[i] with the element at random index
        swap(&arr[i], &arr[j]);
    }
}

// Driver program to test above function.
int main()
{
    int arr[] = {1, 2, 3, 4, 5, 6, 7, 8};
    int n = sizeof(arr)/ sizeof(arr[0]);
    randomize (arr, n);
    printArray(arr, n);

    return 0;
}
```

Output:

```
7 8 4 6 3 1 2 5
```

The above function assumes that rand() generates a random number.

Time Complexity: O(n), assuming that the function rand() takes O(1) time.

How does this work?
The probability that ith element (including the last one) goes to last position is 1/n, because we randomly pick an element in first iteration.

The probability that ith element goes to second last position can be proved to be 1/n by dividing it in two cases.
Case 1: i = n-1 (index of last element):
The probability of last element going to second last position is = (probability that last element doesn't stay at its original position) x (probability that the index picked in previous step is picked again so that the last element is swapped)
So the probability = ((n-1)/n) x (1/(n-1)) = 1/n
Case 2: 0 < i < n-1 (index of non-last):
The probability of ith element going to second position = (probability that ith element is not picked in previous iteration) x (probability that ith element is picked in this iteration)
So the probability = ((n-1)/n) x (1/(n-1)) = 1/n

We can easily generalize above proof for any other position.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

26. Space and time efficient Binomial Coefficient

Write a function that takes two parameters n and k and returns the value of Binomial Coefficient C(n, k). For example, your function should return 6 for n = 4 and k = 2, and it should return 10 for n = 5 and k = 2.

We have discussed a O(n*k) time and O(k) extra space algorithm in this post. The value of C(n, k) can be calculated in O(k) time and O(1) extra space.

```
C(n, k) = n! / (n-k)! * k!
        = [n * (n-1) *....* 1]  / [ ( (n-k) * (n-k-1) * .... * 1) *
                                    ( k * (k-1) * .... * 1 ) ]
After simplifying, we get
C(n, k) = [n * (n-1) * .... * (n-k+1)] / [k * (k-1) * .... * 1]

Also, C(n, k) = C(n, n-k)  // we can change r to n-r if r > n-r
```

Following implementation uses above formula to calculate C(n, k)

```c
// Program to calculate C(n ,k)
#include <stdio.h>

// Returns value of Binomial Coefficient C(n, k)
int binomialCoeff(int n, int k)
{
    int res = 1;

    // Since C(n, k) = C(n, n-k)
    if ( k > n - k )
        k = n - k;

    // Calculate value of [n * (n-1) *---* (n-k+1)] / [k * (k-1) *----
    for (int i = 0; i < k; ++i)
    {
        res *= (n - i);
        res /= (i + 1);
    }

    return res;
}

/* Drier program to test above function*/
int main()
{
    int n = 8, k = 2;
    printf ("Value of C(%d, %d) is %d ", n, k, binomialCoeff(n, k) );
    return 0;
}
```

```
Value of C(8, 2) is 28
```

Time Complexity: O(k)
Auxiliary Space: O(1)

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

27. Reservoir Sampling

Reservoir sampling is a family of randomized algorithms for randomly choosing *k* samples from a list of *n* items, where *n* is either a very large or unknown number. Typically *n* is large enough that the list doesn't fit into main memory. For example, a list of search queries in Google and Facebook.

So we are given a big array (or stream) of numbers (to simplify), and we need to write an efficient function to randomly select *k* numbers where *1* <= *k* <= *n*. Let the input array be *stream[]*.

A **simple solution** is to create an array *reservoir[]* of maximum size *k*. One by one randomly select an item from *stream[0..n-1]*. If the selected item is not previously selected, then put it in *reservoir[]*. To check if an item is previously selected or not, we need to search the item in *reservoir[]*. The time complexity of this algorithm will be *O(k^2)*. This can be costly if *k* is big. Also, this is not efficient if the input is in the form of a stream.

It **can be solved in *O(n)* time**. The solution also suits well for input in the form of stream. The idea is similar to this post. Following are the steps.

1) Create an array *reservoir[0..k-1]* and copy first *k* items of *stream[]* to it.
2) Now one by one consider all items from *(k+1)*th item to *n*th item.
…**a)** Generate a random number from 0 to *i* where *i* is index of current item in *stream[]*. Let the generated random number is *j*.
…**b)** If *j* is in range 0 to *k-1*, replace *reservoir[j]* with *arr[i]*

Following is C implementation of the above algorithm.

```
// An efficient program to randomly select k items from a stream of it

#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// A utility function to print an array
void printArray(int stream[], int n)
{
    for (int i = 0; i < n; i++)
        printf("%d ", stream[i]);
    printf("\n");
}

// A function to randomly select k items from stream[0..n-1].
void selectKItems(int stream[], int n, int k)
{
    int i;  // index for elements in stream[]

    // reservoir[] is the output array. Initialize it with
    // first k elements from stream[]
    int reservoir[k];
    for (i = 0; i < k; i++)
        reservoir[i] = stream[i];

    // Use a different seed value so that we don't get
    // same result each time we run this program
    srand(time(NULL));

    // Iterate from the (k+1)th element to nth element
    for (; i < n; i++)
    {
        // Pick a random index from 0 to i.
        int j = rand() % (i+1);

        // If the randomly  picked index is smaller than k, then repla
        // the element present at the index with new element from stre
        if (j < k)
          reservoir[j] = stream[i];
    }

    printf("Following are k randomly selected items \n");
    printArray(reservoir, k);
}

// Driver program to test above function.
int main()
{
    int stream[] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12};
    int n = sizeof(stream)/sizeof(stream[0]);
    int k = 5;
    selectKItems(stream, n, k);
    return 0;
}
```

Output:

```
 Following are k randomly selected items
6 2 11 8 12
```

Time Complexity: O(n)

How does this work?
To prove that this solution works perfectly, we must prove that the probability that any item *stream[i]* where *0 <= i < n* will be in final *reservoir[]* is *k/n*. Let us divide the proof in two cases as first *k* items are treated differently.

Case 1: For last *n-k* stream items, i.e., for *stream[i]* where *k <= i < n*
For every such stream item *stream[i]*, we pick a random index from 0 to *i* and if the picked index is one of the first *k* indexes, we replace the element at picked index with *stream[i]*

To simplify the proof, let us first consider the *last item*. The probability that the last item is in final reservoir = The probability that one of the first *k* indexes is picked for last item = *k/n* (the probability of picking one of the *k* items from a list of size *n*)

Let us now consider the *second last item*. The probability that the second last item is in final *reservoir[]* = [Probability that one of the first *k* indexes is picked in iteration for *stream[n-2]*] X [Probability that the index picked in iteration for *stream[n-1]* is not same as index picked for *stream[n-2]*] = $[k/(n-1)]*[(n-1)/n] = k/n$.

Similarly, we can consider other items for all stream items from *stream[n-1]* to *stream[k]* and generalize the proof.

Case 2: For first *k* stream items, i.e., for *stream[i]* where *0 <= i < k*
The first *k* items are initially copied to *reservoir[]* and may be removed later in iterations for *stream[k]* to *stream[n]*.
The probability that an item from *stream[0..k-1]* is in final array = Probability that the item is not picked when items *stream[k], stream[k+1], …. stream[n-1]* are considered = $[k/(k+1)] \times [(k+1)/(k+2)] \times [(k+2)/(k+3)] \times \ldots \times [(n-1)/n] = k/n$

References:
http://en.wikipedia.org/wiki/Reservoir_sampling

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

28. Pascal’s Triangle

Pascal’s triangle is a triangular array of the binomial coefficients. Write a function that takes an integer value n as input and prints first n lines of the Pascal’s triangle. Following are the first 6 rows of Pascal’s Triangle.

```
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
```

Method 1 (O(n^3) time complexity)
Number of entries in every line is equal to line number. For example, the first line has “1”, the second line has “1 1″, the third line has “1 2 1″,.. and so on. Every entry in a line is value of a Binomial Coefficient. The value of ***i***th entry in line number *line* is *C(line, i)*. The value can be calculated using following formula.

```
C(line, i)   = line! / ( (line-i)! * i! )
```

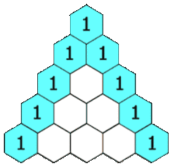
A simple method is to run two loops and calculate the value of Binomial Coefficient in inner loop.

```c
// A simple O(n^3) program for Pascal's Triangle
#include <stdio.h>

// See http://www.geeksforgeeks.org/archives/25621 for details of this
int binomialCoeff(int n, int k);

// Function to print first n lines of Pascal's Triangle
void printPascal(int n)
{
  // Iterate through every line and print entries in it
  for (int line = 0; line < n; line++)
  {
    // Every line has number of integers equal to line number
    for (int i = 0; i <= line; i++)
      printf("%d ", binomialCoeff(line, i));
    printf("\n");
  }
}

// See http://www.geeksforgeeks.org/archives/25621 for details of this
int binomialCoeff(int n, int k)
{
    int res = 1;
    if (k > n - k)
       k = n - k;
    for (int i = 0; i < k; ++i)
    {
        res *= (n - i);
        res /= (i + 1);
    }
    return res;
}

// Driver program to test above function
int main()
{
  int n = 7;
  printPascal(n);
  return 0;
}
```

Time complexity of this method is O(n^3). Following are optimized methods.

Method 2(O(n^2) time and O(n^2) extra space)

If we take a closer at the triangle, we observe that every entry is sum of the two values above it. So we can create a 2D array that stores previously generated values. To generate a value in a line, we can use the previously stored values from array.



```
// A O(n^2) time and O(n^2) extra space method for Pascal's Triangle
void printPascal(int n)
{
  int arr[n][n]; // An auxiliary array to store generated pscal triang

  // Iterate through every line and print integer(s) in it
  for (int line = 0; line < n; line++)
  {
    // Every line has number of integers equal to line number
    for (int i = 0; i <= line; i++)
    {
      // First and last values in every row are 1
      if (line == i || i == 0)
           arr[line][i] = 1;
      else // Other values are sum of values just above and left of ab
           arr[line][i] = arr[line-1][i-1] + arr[line-1][i];
      printf("%d ", arr[line][i]);
    }
    printf("\n");
  }
}
```

This method can be optimized to use O(n) extra space as we need values only from previous row. So we can create an auxiliary array of size n and overwrite values. Following is another method uses only O(1) extra space.

Method 3 (O(n^2) time and O(1) extra space)

This method is based on method 1. We know that *i*th entry in a line number *line* is Binomial Coefficient *C(line, i)* and all lines start with value 1. The idea is to calculate *C(line, i)* using *C(line, i-1)*. It can be calculated in O(1) time using the following.

```
C(line, i)   = line! / ( (line-i)! * i! )
C(line, i-1) = line! / ( (line - i + 1)! * (i-1)! )
We can derive following expression from above two expressions.
C(line, i) = C(line, i-1) * (line - i + 1) / i


So C(line, i) can be calculated from C(line, i-1) in O(1) time
```

```
// A O(n^2) time and O(1) extra space function for Pascal's Triangle
void printPascal(int n)
{
  for (int line = 1; line <= n; line++)
  {
    int C = 1;  // used to represent C(line, i)
    for (int i = 1; i <= line; i++)
    {
      printf("%d ", C);  // The first value in a line is always 1
      C = C * (line - i) / i;
    }
    printf("\n");
  }
}
```

So method 3 is the best method among all, but it may cause integer overflow for large values of n as it multiplies two integers to obtain values.

This article is compiled by **Rahul** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

29. Select a random number from stream, with O(1) space

Given a stream of numbers, generate a random number from the stream. You are allowed to use only O(1) space and the input is in the form of stream, so can't store the previously seen numbers.

So how do we generate a random number from the whole stream such that the probability of picking any number is 1/n. with O(1) extra space? This problem is a variation of Reservoir Sampling. Here the value of k is 1.

1) Initialize 'count' as 0, 'count' is used to store count of numbers seen so far in stream.
2) For each number 'x' from stream, do following
…..**a)** Increment 'count' by 1.
…..**b)** If count is 1, set result as x, and return result.
…..**c)** Generate a random number from 0 to 'count-1′. Let the generated random number be i.
…..**d)** If i is equal to 'count – 1′, update the result as x.

```
// An efficient program to randomly select a number from stream of numl
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// A function to randomly select a item from stream[0], stream[1], .. 
int selectRandom(int x)
{
    static int res;    // The resultant random number
    static int count = 0;  //Count of numbers visited so far in stream

    count++;  // increment count of numbers seen so far

    // If this is the first element from stream, return it
    if (count == 1)
        res = x;
    else
    {
        // Generate a random number from 0 to count - 1
        int i = rand() % count;

        // Replace the prev random number with new number with 1/count
        if (i == count - 1)
            res  = x;
    }
    return res;
}

// Driver program to test above function.
int main()
{
    int stream[] = {1, 2, 3, 4};
    int n = sizeof(stream)/sizeof(stream[0]);

    // Use a different seed value for every run.
    srand(time(NULL));
    for (int i = 0; i < n; ++i)
        printf("Random number from first %d numbers is %d \n",
                                i+1, selectRandom(stream[i]));
    return 0;
}
```

Output:

```
Random number from first 1 numbers is 1
Random number from first 2 numbers is 1
Random number from first 3 numbers is 3
Random number from first 4 numbers is 4
```

Auxiliary Space: O(1)

How does this work

We need to prove that every element is picked with 1/n probability where n is the number of items seen so far. For every new stream item x, we pick a random number from 0 to ‘count -1’, if the picked number is ‘count-1’, we replace the previous result with x.

To simplify proof, let us first consider the last element, the last element replaces the previously stored result with 1/n probability. So probability of getting last element as result is 1/n.

Let us now talk about second last element. When second last element processed first time, the probability that it replaced the previous result is 1/(n-1). The probability that previous result stays when nth item is considered is (n-1)/n. So probability that the second last element is picked in last iteration is [1/(n-1)] * [(n-1)/n] which is 1/n.

Similarly, we can prove for third last element and others.

References:
Reservoir Sampling

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

30. Find the largest multiple of 2, 3 and 5

An array of size n is given. The array contains digits from 0 to 9. Generate the largest number using the digits in the array such that the number is divisible by 2, 3 and 5.
For example, if the arrays is {1, 8, 7, 6, 0}, output must be: 8760. And if the arrays is {7, 7, 7, 6}, output must be: “no number can be formed”.

Source: Amazon Interview | Set 7

This problem is a variation of “Find the largest multiple of 3“.

Since the number has to be divisible by 2 and 5, it has to have last digit as 0. So if the given array doesn’t contain any zero, then no solution exists.

Once a 0 is available, extract 0 from the given array. Only thing left is, the number should be is divisible by 3 and the largest of all. Which has been discussed here.

Thanks to shashank for suggesting this solution. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

31. Efficient program to calculate e^x

The value of Exponential Function e^x can be expressed using following Taylor Series.

```
e^x = 1 + x/1! + x^2/2! + x^3/3! + ......
```

How to efficiently calculate the sum of above series?
The series can be re-written as

```
e^x = 1 + (x/1) (1 + (x/2) (1 + (x/3) (........) ) )
```

Let the sum needs to be calculated for n terms, we can calculate sum using following loop.

```
for (i = n - 1, sum = 1; i > 0; --i )
    sum = 1 + x * sum / i;
```

Following is implementation of the above idea.

```
// Efficient program to calculate e raise to the power x
#include <stdio.h>

//Returns approximate value of e^x using sum of first n terms of Taylo
float exponential(int n, float x)
{
    float sum = 1.0f; // initialize sum of series

    for (int i = n - 1; i > 0; --i )
        sum = 1 + x * sum / i;

    return sum;
}

// Driver program to test above function
int main()
{
    int n = 10;
    float x = 1.0f;
    printf("e^x = %f", exponential(n, x));
    return 0;
}
```

Output:

```
e^x = 2.718282
```

This article is compiled by **Rahul** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

32. Measure one litre using two vessels and infinite water supply

There are two vessels of capacities ‘a’ and ‘b’ respectively. We have infinite water supply. Give an efficient algorithm to make exactly 1 litre of water in one of the vessels. You can throw all the water from any vessel any point of time. Assume that ‘a’ and ‘b’ are Coprimes.

Following are the steps:
Let V1 be the vessel of capacity ‘a’ and V2 be the vessel of capacity ‘b’ and ‘a’ is smaller than ‘b’.
1) Do following while the amount of water in V1 is not 1.
….**a)** If V1 is empty, then completely fill V1
….**b)** Transfer water from V1 to V2. If V2 becomes full, then keep the remaining water in V1 and empty V2
2) V1 will have 1 litre after termination of loop in step 1. Return.

Following is C++ implementation of the above algorithm.

```
/* Sample run of the Algo for V1 with capacity 3 and V2 with capacity
1. Fill V1:                               V1 = 3, V2 = 0
2. Transfer from V1 to V2, and fill V1:   V1 = 3, V2 = 3
2. Transfer from V1 to V2, and fill V1:   V1 = 3, V2 = 6
3. Transfer from V1 to V2, and empty V2:  V1 = 2, V2 = 0
4. Transfer from V1 to V2, and fill V1:   V1 = 3, V2 = 2
5. Transfer from V1 to V2, and fill V1:   V1 = 3, V2 = 5
6. Transfer from V1 to V2, and empty V2:  V1 = 1, V2 = 0
7. Stop as V1 now contains 1 litre.

Note that V2 was made empty in steps 3 and 6 because it became full */

#include <iostream>
using namespace std;

// A utility function to get GCD of two numbers
int gcd(int a, int b) { return b? gcd(b, a % b) : a; }

// Class to represent a Vessel
class Vessel
{
    // A vessel has capacity, and current amount of water in it
    int capacity, current;
public:
    // Constructor: initializes capacity as given, and current as 0
    Vessel(int capacity) { this->capacity = capacity; current = 0; }

    // The main function to fill one litre in this vessel. Capacity of
    // must be greater than this vessel and two capacities must be co-
    void makeOneLitre(Vessel &V2);

    // Fills vessel with given amount and returns the amount of water
    // transferred to it. If the vessel becomes full, then the vessel
    // is made empty.
    int transfer(int amount);
};

// The main function to fill one litre in this vessel. Capacity
// of V2 must be greater than this vessel and two capacities
// must be coprime
```

```
void Vessel:: makeOneLitre(Vessel &V2)
{
    // solution exists iff a and b are co-prime
    if (gcd(capacity, V2.capacity) != 1)
        return;

    while (current != 1)
    {
        // fill A (smaller vessel)
        if (current == 0)
            current = capacity;

        cout << "Vessel 1: " << current << "   Vessel 2: "
             << V2.current << endl;

        // Transfer water from V1 to V2 and reduce current of V1 by
        //  the amount equal to transferred water
        current = current - V2.transfer(current);
    }

    // Finally, there will be 1 litre in vessel 1
    cout << "Vessel 1: " << current << "   Vessel 2: "
         << V2.current << endl;
}

// Fills vessel with given amount and returns the amount of water
// transferred to it. If the vessel becomes full, then the vessel
// is made empty
int Vessel::transfer(int amount)
{
    // If the vessel can accommodate the given amount
    if (current + amount < capacity)
    {
        current += amount;
        return amount;
    }

    // If the vessel cannot accommodate the given amount, then
    // store the amount of water transferred
    int transferred = capacity - current;

    // Since the vessel becomes full, make the vessel
    // empty so that it can be filled again
    current = 0;

    return transferred;
}

// Driver program to test above function
int main()
{
    int a = 3, b = 7;  // a must be smaller than b

    // Create two vessels of capacities a and b
    Vessel V1(a), V2(b);

    // Get 1 litre in first vessel
    V1.makeOneLitre(V2);

    return 0;
}
```

Output:

```
Vessel 1: 3   Vessel 2: 0
Vessel 1: 3   Vessel 2: 3
Vessel 1: 3   Vessel 2: 6
Vessel 1: 2   Vessel 2: 0
Vessel 1: 3   Vessel 2: 2
Vessel 1: 3   Vessel 2: 5
Vessel 1: 1   Vessel 2: 0
```

How does this work?

To prove that the algorithm works, we need to proof that after certain number of iterations in the while loop, we will get 1 litre in V1.
Let ‘a’ be the capacity of vessel V1 and ‘b’ be the capacity of V2. Since we repeatedly transfer water from V1 to V2 until V2 becomes full, we will have ‘a – b (mod a)’ water in V1 when V2 becomes full first time . Once V2 becomes full, it is emptied. We will have ‘a – 2b (mod a)’ water in V1 when V2 is full second time. We repeat the above steps, and get ‘a – nb (mod a)’ water in V1 after the vessel V2 is filled and emptied ‘n’ times. We need to prove that the value of ‘a – nb (mod a)’ will be 1 for a finite integer ‘n’. To prove this, let us consider the following property of coprime numbers.
For any two coprime integers ‘a’ and ‘b’, the integer ‘b’ has a multiplicative inverse modulo ‘a’. In other words, there exists an integer ‘y’ such that $by \equiv 1 (mod\, a)$ (See 3rd point here). After ‘(a – 1)*y’ iterations, we will have ‘a – [(a-1)*y*b (mod a)]’ water in V1, the value of this expression is ‘a – [(a – 1) * 1] mod a’ which is 1. So the algorithm converges and we get 1 litre in V1.

This article is compiled by Aashish Barnwal. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

33. Efficient program to print all prime factors of a given number

Given a number n, write an efficient function to print all prime factors of n. For example, if the input number is 12, then output should be “2 2 3″. And if the input number is 315, then output should be “3 3 5 7″.

Following are the steps to find all prime factors.
1) While n is divisible by 2, print 2 and divide n by 2.
2) After step 1, n must be odd. Now start a loop from i = 3 to square root of n. While i divides n, print i and divide n by i, increment i by 2 and continue.
3) If n is a prime number and is greater than 2, then n will not become 1 by above two steps. So print n if it is greater than 2.

```c
// Program to print all prime factors
# include <stdio.h>
# include <math.h>

// A function to print all prime factors of a given number n
void primeFactors(int n)
{
    // Print the number of 2s that divide n
    while (n%2 == 0)
    {
        printf("%d ", 2);
        n = n/2;
    }

    // n must be odd at this point.  So we can skip one element (Note
    for (int i = 3; i <= sqrt(n); i = i+2)
    {
        // While i divides n, print i and divide n
        while (n%i == 0)
        {
            printf("%d ", i);
            n = n/i;
        }
    }

    // This condition is to handle the case whien n is a prime number
    // greater than 2
    if (n > 2)
        printf ("%d ", n);
}

/* Driver program to test above function */
int main()
{
    int n = 315;
    primeFactors(n);
    return 0;
}
```

Output:

```
3 3 5 7
```

How does this work?
The steps 1 and 2 take care of composite numbers and step 3 takes care of prime numbers. To prove that the complete algorithm works, we need to prove that steps 1 and 2 actually take care of composite numbers. This is clear that step 1 takes care of even numbers. And after step 1, all remaining prime factor must be odd (difference of two prime factors must be at least 2), this explains why i is incremented by 2.
Now the main part is, the loop runs till square root of n not till. To prove that this optimization works, let us consider the following property of composite numbers.
Every composite number has at least one prime factor less than or equal to square root of itself.
This property can be proved using counter statement. Let a and b be two factors of n such that a*b = n. If both are greater than $\sqrt{n}$, then a.b > $\sqrt{n} * \sqrt{n}$ which contradicts the expression “a * b = n”.

In step 2 of the above algorithm, we run a loop and do following in loop
a) Find the least prime factor i (must be less than $\sqrt{n}$)
b) Remove all occurrences i from n by repeatedly dividing n by i.
c) Repeat steps a and b for divided n and i = i + 2. The steps a and b are repeated till n becomes either 1 or a prime number.

Thanks to **Vishwas Garg** for suggesting the above algorithm. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

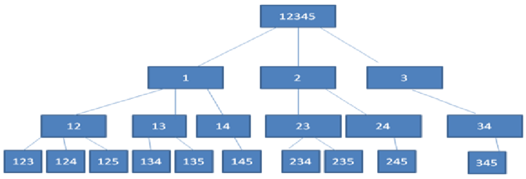
34. Print all possible combinations of r elements in a given array of size n

Given an array of size n, generate and print all possible combinations of r elements in array. For example, if input array is {1, 2, 3, 4} and r is 2, then output should be {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4} and {3, 4}.

Following are two methods to do this.

Method 1 (Fix Elements and Recur)
We create a temporary array 'data[]' which stores all outputs one by one. The idea is to start from first index (index = 0) in data[], one by one fix elements at this index and recur for remaining indexes. Let the input array be {1, 2, 3, 4, 5} and r be 3. We first fix 1 at index 0 in data[], then recur for remaining indexes, then we fix 2 at index 0 and recur. Finally, we fix 3 and recur for remaining indexes. When number of elements in data[] becomes equal to r (size of a combination), we print data[].

Following diagram shows recursion tree for same input.



Following is C++ implementation of above approach.

```c
// Program to print all combination of size r in an array of size n
#include <stdio.h>
void combinationUtil(int arr[], int data[], int start, int end, int in

// The main function that prints all combinations of size r
// in arr[] of size n. This function mainly uses combinationUtil()
void printCombination(int arr[], int n, int r)
{
    // A temporary array to store all combination one by one
    int data[r];

    // Print all combination using temprary array 'data[]'
    combinationUtil(arr, data, 0, n-1, 0, r);
}

/* arr[]  ---> Input Array
   data[] ---> Temporary array to store current combination
   start & end ---> Staring and Ending indexes in arr[]
   index  ---> Current index in data[]
   r ---> Size of a combination to be printed */
void combinationUtil(int arr[], int data[], int start, int end, int in
{
    // Current combination is ready to be printed, print it
    if (index == r)
    {
        for (int j=0; j<r; j++)
            printf("%d ", data[j]);
        printf("\n");
        return;
    }

    // replace index with all possible elements. The condition
    // "end-i+1 >= r-index" makes sure that including one element
    // at index will make a combination with remaining elements
    // at remaining positions
    for (int i=start; i<=end && end-i+1 >= r-index; i++)
    {
        data[index] = arr[i];
        combinationUtil(arr, data, i+1, end, index+1, r);
    }
}

// Driver program to test above functions
int main()
{
    int arr[] = {1, 2, 3, 4, 5};
    int r = 3;
    int n = sizeof(arr)/sizeof(arr[0]);
    printCombination(arr, n, r);
}
```

Output:

```
1 2 3
1 2 4
1 2 5
1 3 4
1 3 5
1 4 5
```

```
2 3 4
2 3 5
2 4 5
3 4 5
```

How to handle duplicates?
Note that the above method doesn't handle duplicates. For example, if input array is {1, 2, 1} and r is 2, then the program prints {1, 2} and {2, 1} as two different combinations. We can avoid duplicates by adding following two additional things to above code.
1) Add code to sort the array before calling combinationUtil() in printCombination()
2) Add following lines at the end of for loop in combinationUtil()

```
        // Since the elements are sorted, all occurrences of an element
        // must be together
        while (arr[i] == arr[i+1])
             i++;
```

See **this** for an implementation that handles duplicates.

Method 2 (Include and Exclude every element)
Like the above method, We create a temporary array data[]. The idea here is similar to Subset Sum Problem. We one by one consider every element of input array, and recur for two cases:

1) The element is included in current combination (We put the element in data[] and increment next available index in data[])
2) The element is excluded in current combination (We do not put the element and do not change index)

When number of elements in data[] become equal to r (size of a combination), we print it.

This method is mainly based on Pascal's Identity, i.e. $\mathbf{n_{c_r} = n\text{-}1_{c_r} + n\text{-}1_{c_{r-1}}}$

Following is C++ implementation of method 2.

```c
// Program to print all combination of size r in an array of size n
#include<stdio.h>
void combinationUtil(int arr[],int n,int r,int index,int data[],int i)

// The main function that prints all combinations of size r
// in arr[] of size n. This function mainly uses combinationUtil()
void printCombination(int arr[], int n, int r)
{
    // A temporary array to store all combination one by one
    int data[r];

    // Print all combination using temprary array 'data[]'
    combinationUtil(arr, n, r, 0, data, 0);
}

/* arr[]  ---> Input Array
   n      ---> Size of input array
   r      ---> Size of a combination to be printed
   index  ---> Current index in data[]
   data[] ---> Temporary array to store current combination
   i      ---> index of current element in arr[]     */
void combinationUtil(int arr[], int n, int r, int index, int data[], i
{
    // Current cobination is ready, print it
    if (index == r)
    {
        for (int j=0; j<r; j++)
            printf("%d ",data[j]);
        printf("\n");
        return;
    }

    // When no more elements are there to put in data[]
    if (i >= n)
        return;

    // current is included, put next at next location
    data[index] = arr[i];
    combinationUtil(arr, n, r, index+1, data, i+1);

    // current is excluded, replace it with next (Note that
    // i+1 is passed, but index is not changed)
    combinationUtil(arr, n, r, index, data, i+1);
}

// Driver program to test above functions
int main()
{
    int arr[] = {1, 2, 3, 4, 5};
    int r = 3;
    int n = sizeof(arr)/sizeof(arr[0]);
    printCombination(arr, n, r);
    return 0;
}
```

Output:

```
1 2 3
1 2 4
1 2 5
```

```
1 3 4
1 3 5
1 4 5
2 3 4
2 3 5
2 4 5
3 4 5
```

How to handle duplicates in method 2?
Like method 1, we can following two things to handle duplicates.
1) Add code to sort the array before calling combinationUtil() in printCombination()
2) Add following lines between two recursive calls of combinationUtil() in combinationUtil()

```
        // Since the elements are sorted, all occurrences of an element
        // must be together
        while (arr[i] == arr[i+1])
             i++;
```

See **this** for an implementation that handles duplicates.

This article is contributed by **Bateesh**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

35. Random number generator in arbitrary probability distribution fashion

Given n numbers, each with some frequency of occurrence. Return a random number with probability proportional to its frequency of occurrence.

Example:

```
Let following be the given numbers.
  arr[] = {10, 30, 20, 40}

Let following be the frequencies of given numbers.
  freq[] = {1, 6, 2, 1}

The output should be
  10 with probability 1/10
  30 with probability 6/10
  20 with probability 2/10
  40 with probability 1/10
```

It is quite clear that the simple random number generator won't work here as it doesn't keep track of the frequency of occurrence.

We need to somehow transform the problem into a problem whose solution is known to us.

One simple method is to take an auxiliary array (say aux[]) and duplicate the numbers according to their frequency of occurrence. Generate a random number(say r) between 0 to Sum-1(including both), where Sum represents summation of frequency array (freq[] in above example). Return the random number aux[r] (Implementation of this method is left as an exercise to the readers).

The limitation of the above method discussed above is huge memory consumption when frequency of occurrence is high. If the input is 997, 8761 and 1, this method is clearly not efficient.

How can we reduce the memory consumption? Following is detailed algorithm that uses O(n) extra space where n is number of elements in input arrays.

1. Take an auxiliary array (say prefix[]) of size n.
2. Populate it with prefix sum, such that prefix[i] represents sum of numbers from 0 to i.
3. Generate a random number(say r) between 1 to Sum(including both), where Sum represents summation of input frequency array.
4. Find index of Ceil of random number generated in step #3 in the prefix array. Let the index be index**c**.
5. Return the random number arr[indexc], where arr[] contains the input n numbers.

Before we go to the implementation part, let us have quick look at the algorithm with an example:

    arr[]: {10, 20, 30}
    freq[]: {2, 3, 1}
    Prefix[]: {2, 5, 6}

Since last entry in prefix is 6, all possible values of r are [1, 2, 3, 4, 5, 6]

      1: Ceil is 2. Random number generated is 10.
      2: Ceil is 2. Random number generated is 10.
      3: Ceil is 5. Random number generated is 20.
      4: Ceil is 5. Random number generated is 20.
      5: Ceil is 5. Random number generated is 20.
      6. Ceil is 6. Random number generated is 30.

In the above example

    10 is generated with probability 2/6.
    20 is generated with probability 3/6.
    30 is generated with probability 1/6.

How does this work?
Any number input[i] is generated as many times as its frequency of occurrence because there exists count of integers in range(prefix[i – 1], prefix[i]] is input[i]. Like in the above

example 3 is generated thrice, as there exists 3 integers 3, 4 and 5 whose ceil is 5.

```
//C program to generate random numbers according to given frequency di
#include <stdio.h>
#include <stdlib.h>

// Utility function to find ceiling of r in arr[l..h]
int findCeil(int arr[], int r, int l, int h)
{
    int mid;
    while (l < h)
    {
         mid = l + ((h - l) >> 1);  // Same as mid = (l+h)/2
        (r > arr[mid]) ? (l = mid + 1) : (h = mid);
    }
    return (arr[l] >= r) ? l : -1;
}

// The main function that returns a random number from arr[] according
// distribution array defined by freq[]. n is size of arrays.
int myRand(int arr[], int freq[], int n)
{
    // Create and fill prefix array
    int prefix[n], i;
    prefix[0] = freq[0];
    for (i = 1; i < n; ++i)
        prefix[i] = prefix[i - 1] + freq[i];

    // prefix[n-1] is sum of all frequencies. Generate a random number
    // with value from 1 to this sum
    int r = (rand() % prefix[n - 1]) + 1;

    // Find index of ceiling of r in prefix arrat
    int indexc = findCeil(prefix, r, 0, n - 1);
    return arr[indexc];
}

// Driver program to test above functions
int main()
{
    int arr[]  = {1, 2, 3, 4};
    int freq[] = {10, 5, 20, 100};
    int i, n = sizeof(arr) / sizeof(arr[0]);

    // Use a different seed value for every run.
    srand(time(NULL));

    // Let us generate 10 random numbers accroding to
    // given distribution
    for (i = 0; i < 5; i++)
      printf("%d\n", myRand(arr, freq, n));

    return 0;
}
```

Output: May be different for different runs

```
4
3
4
```

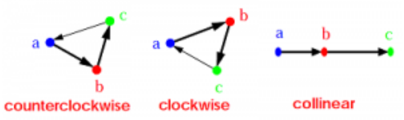
```
4
4
```

This article is compiled by Aashish Barnwal. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

36. How to check if two given line segments intersect?

Given two line segments (p1, q1) and (p2, q2), find if the given line segments intersect with each other.

Before we discuss solution, let us define notion of **orientation**. Orientation of an ordered triplet of points in the plane can be
–counterclockwise
–clockwise
–colinear
The following diagram shows different possible orientations of (a, b, c)



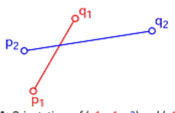
Note the word 'ordered' here. Orientation of (a, b, c) may be different from orientation of (c, b, a).

How is Orientation useful here?
Two segments (p1,q1) and (p2,q2) intersect if and only if one of the following two conditions is verified

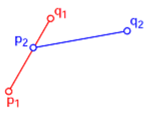
1. ***General Case:***
– (p1, q1, p2) and (p1, q1, q2) have different orientations and
– (p2, q2, p1) and (p2, q2, q1) have different orientations

2. ***Special Case***
– (p1, q1, p2), (p1, q1, q2), (p2, q2, p1), and (p2, q2, q1) are all collinear and
– the x-projections of (p1, q1) and (p2, q2) intersect
– the y-projections of (p1, q1) and (p2, q2) intersect

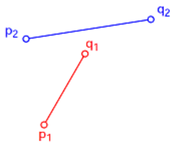
Examples of General Case:



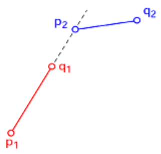
Example 1: Orientations of (p1, q1, p2) and (p1, q1, q2) are different. Orientations of (p2, q2, p1) and (p2, q2, q1) are also different



Example 2: Orientations of (p1, q1, p2) and (p1, q1, q2) are different. Orientations of (p2, q2, p1) and (p2, q2, q1) are also different

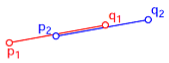


Example 3: Orientations of (p1, q1, p2) and (p1, q1, q2) are different. Orientations of (p2, q2, p1) and (p2, q2, q1) are **same**

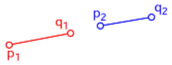


Example 4: Orientations of (p1, q1, p2) and (p1, q1, q2) are different. Orientations of (p2, q2, p1) and (p2, q2, q1) are **same**

Examples of Special Case:



Example 1: All points are colinear. The x-projections of (p1, q1) and (p2, q2) intersect. The y-projections of (p1, q1) and (p2, q2) intersect



Example 2: All points are colinear. The x-projections of (p1, q1) and (p2, q2) do not intersect. The y-projections of (p1, q1) and do not (p2, q2) intersect

Following is C++ implementation based on above idea.

```cpp
// A C++ program to check if two given line segments intersect
#include <iostream>
using namespace std;

struct Point
{
    int x;
    int y;
};

// Given three colinear points p, q, r, the function checks if
// point q lies on line segment 'pr'
bool onSegment(Point p, Point q, Point r)
{
    if (q.x <= max(p.x, r.x) && q.x >= min(p.x, r.x) &&
        q.y <= max(p.y, r.y) && q.y >= min(p.y, r.y))
       return true;

    return false;
}

// To find orientation of ordered triplet (p, q, r).
// The function returns following values
// 0 --> p, q and r are colinear
// 1 --> Clockwise
// 2 --> Counterclockwise
int orientation(Point p, Point q, Point r)
{
    // See 10th slides from following link for derivation of the formu
    // http://www.dcs.gla.ac.uk/~pat/52233/slides/Geometry1x1.pdf
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);

    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2; // clock or counterclock wise
}

// The main function that returns true if line segment 'p1q1'
// and 'p2q2' intersect.
bool doIntersect(Point p1, Point q1, Point p2, Point q2)
{
    // Find the four orientations needed for general and
    // special cases
    int o1 = orientation(p1, q1, p2);
    int o2 = orientation(p1, q1, q2);
    int o3 = orientation(p2, q2, p1);
    int o4 = orientation(p2, q2, q1);

    // General case
    if (o1 != o2 && o3 != o4)
        return true;

    // Special Cases
    // p1, q1 and p2 are colinear and p2 lies on segment p1q1
    if (o1 == 0 && onSegment(p1, p2, q1)) return true;

    // p1, q1 and p2 are colinear and q2 lies on segment p1q1
    if (o2 == 0 && onSegment(p1, q2, q1)) return true;

    // p2, q2 and p1 are colinear and p1 lies on segment p2q2
    if (o3 == 0 && onSegment(p2, p1, q2)) return true;

     // p2, q2 and q1 are colinear and q1 lies on segment p2q2
    if (o4 == 0 && onSegment(p2, q1, q2)) return true;

    return false; // Doesn't fall in any of the above cases
}

// Driver program to test above functions
int main()
{
    struct Point p1 = {1, 1}, q1 = {10, 1};
    struct Point p2 = {1, 2}, q2 = {10, 2};

    doIntersect(p1, q1, p2, q2)? cout << "Yes\n": cout << "No\n";

    p1 = {10, 0}, q1 = {0, 10};
    p2 = {0, 0}, q2 = {10, 10};
    doIntersect(p1, q1, p2, q2)? cout << "Yes\n": cout << "No\n";

    p1 = {-5, -5}, q1 = {0, 0};
    p2 = {1, 1}, q2 = {10, 10};
    doIntersect(p1, q1, p2, q2)? cout << "Yes\n": cout << "No\n";

    return 0;
}
```

Output:

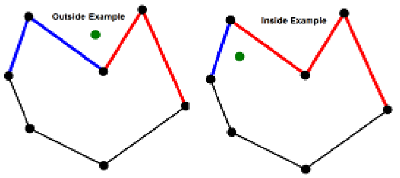
```
No
Yes
No
```

Sources:
http://www.dcs.gla.ac.uk/~pat/52233/slides/Geometry1x1.pdf
Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

37. How to check if a given point lies inside or outside a polygon?

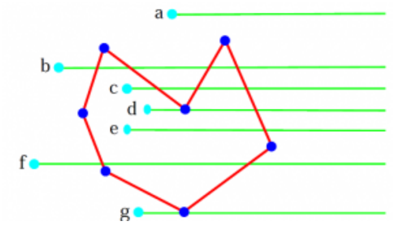
Given a polygon and a point 'p', find if 'p' lies inside the polygon or not. The points lying on the border are considered inside.



We strongly recommend to see the following post first.
How to check if two given line segments intersect?

Following is a simple idea to check whether a point is inside or outside.

```
1) Draw a horizontal line to the right of each point and extend it to infinity

1) Count the number of times the line intersects with polygon edges.

2) A point is inside the polygon if either count of intersections is odd or
   point lies on an edge of polygon.  If none of the conditions is true, then
   point lies outside.
```



How to handle point 'g' in the above figure?
Note that we should returns true if the point lies on the line or same as one of the vertices of the given polygon. To handle this, after checking if the line from 'p' to

extreme intersects, we check whether ‘p’ is colinear with vertices of current line of polygon. If it is coliear, then we check if the point ‘p’ lies on current side of polygon, if it lies, we return true, else false.

Following is C++ implementation of the above idea.

```
// A C++ program to check if a given point lies inside a given polygon
// Refer http://www.geeksforgeeks.org/check-if-two-given-line-segments
// for explanation of functions onSegment(), orientation() and doInter
#include <iostream>
using namespace std;

// Define Infinite (Using INT_MAX caused overflow problems)
#define INF 10000

struct Point
{
    int x;
    int y;
};

// Given three colinear points p, q, r, the function checks if
// point q lies on line segment 'pr'
bool onSegment(Point p, Point q, Point r)
{
    if (q.x <= max(p.x, r.x) && q.x >= min(p.x, r.x) &&
            q.y <= max(p.y, r.y) && q.y >= min(p.y, r.y))
        return true;
    return false;
}

// To find orientation of ordered triplet (p, q, r).
// The function returns following values
// 0 --> p, q and r are colinear
// 1 --> Clockwise
// 2 --> Counterclockwise
int orientation(Point p, Point q, Point r)
{
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);

    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2; // clock or counterclock wise
}

// The function that returns true if line segment 'p1q1'
// and 'p2q2' intersect.
bool doIntersect(Point p1, Point q1, Point p2, Point q2)
{
    // Find the four orientations needed for general and
    // special cases
    int o1 = orientation(p1, q1, p2);
    int o2 = orientation(p1, q1, q2);
    int o3 = orientation(p2, q2, p1);
    int o4 = orientation(p2, q2, q1);

    // General case
    if (o1 != o2 && o3 != o4)
        return true;

    // Special Cases
    // p1, q1 and p2 are colinear and p2 lies on segment p1q1
    if (o1 == 0 && onSegment(p1, p2, q1)) return true;

    // p1, q1 and p2 are colinear and q2 lies on segment p1q1
    if (o2 == 0 && onSegment(p1, q2, q1)) return true;

    // p2, q2 and p1 are colinear and p1 lies on segment p2q2
    if (o3 == 0 && onSegment(p2, p1, q2)) return true;

     // p2, q2 and q1 are colinear and q1 lies on segment p2q2
    if (o4 == 0 && onSegment(p2, q1, q2)) return true;

    return false; // Doesn't fall in any of the above cases
}

// Returns true if the point p lies inside the polygon[] with n vertic
bool isInside(Point polygon[], int n, Point p)
{
    // There must be at least 3 vertices in polygon[]
    if (n < 3)  return false;

    // Create a point for line segment from p to infinite
    Point extreme = {INF, p.y};

    // Count intersections of the above line with sides of polygon
    int count = 0, i = 0;
    do
    {
        int next = (i+1)%n;

        // Check if the line segment from 'p' to 'extreme' intersects
        // with the line segment from 'polygon[i]' to 'polygon[next]'
        if (doIntersect(polygon[i], polygon[next], p, extreme))
        {
            // If the point 'p' is colinear with line segment 'i-next'
            // then check if it lies on segment. If it lies, return tr
            // otherwise false
            if (orientation(polygon[i], p, polygon[next]) == 0)
               return onSegment(polygon[i], p, polygon[next]);

            count++;
        }
        i = next;
    } while (i != 0);

    // Return true if count is odd, false otherwise
    return count&1;  // Same as (count%2 == 1)
}

// Driver program to test above functions
int main()
{
    Point polygon1[] = {{0, 0}, {10, 0}, {10, 10}, {0, 10}};
    int n = sizeof(polygon1)/sizeof(polygon1[0]);
    Point p = {20, 20};
    isInside(polygon1, n, p)? cout << "Yes \n": cout << "No \n";

    p = {5, 5};
    isInside(polygon1, n, p)? cout << "Yes \n": cout << "No \n";

    Point polygon2[] = {{0, 0}, {5, 5}, {5, 0}};
    p = {3, 3};
    n = sizeof(polygon2)/sizeof(polygon2[0]);
    isInside(polygon2, n, p)? cout << "Yes \n": cout << "No \n";

    p = {5, 1};
    isInside(polygon2, n, p)? cout << "Yes \n": cout << "No \n";

    p = {8, 1};
    isInside(polygon2, n, p)? cout << "Yes \n": cout << "No \n";

    Point polygon3[] =  {{0, 0}, {10, 0}, {10, 10}, {0, 10}};
    p = {-1,10};
    n = sizeof(polygon3)/sizeof(polygon3[0]);
    isInside(polygon3, n, p)? cout << "Yes \n": cout << "No \n";

    return 0;
}
```

Output:

```
No
Yes
Yes
Yes
No
No
```

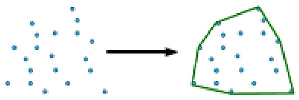
Time Complexity: O(n) where n is the number of vertices in the given polygon.

Source:
http://www.dcs.gla.ac.uk/~pat/52233/slides/Geometry1x1.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

38. Convex Hull | Set 1 (Jarvis's Algorithm or Wrapping)

Given a set of points in the plane. the convex hull of the set is the smallest convex polygon that contains all the points of it.



We strongly recommend to see the following post first.
How to check if two given line segments intersect?

The idea of Jarvis's Algorithm is simple, we start from the leftmost point (or point with minimum x coordinate value) and we keep wrapping points in counterclockwise direction. The big question is, given a point p as current point, how to find the next point in output? The idea is to use orientation() here. Next point is selected as the point that beats all other points at counterclockwise orientation, i.e., next point is q if for any other point r, we have “orientation(p, r, q) = counterclockwise”. Following is the detailed algorithm.

1) Initialize p as leftmost point.
2) Do following while we don’t come back to the first (or leftmost) point.
…..**a)** The next point q is the point such that the triplet (p, q, r) is counterclockwise for any other point r.
…..**b)** next[p] = q (Store q as next of p in the output convex hull).
…..**c)** p = q (Set p as q for next iteration).

```
// A C++ program to find convex hull of a set of points
// Refer http://www.geeksforgeeks.org/check-if-two-given-line-segments
// for explanation of orientation()
#include <iostream>
using namespace std;

// Define Infinite (Using INT_MAX caused overflow problems)
#define INF 10000

struct Point
{
    int x;
    int y;
};

// To find orientation of ordered triplet (p, q, r).
// The function returns following values
// 0 --> p, q and r are colinear
// 1 --> Clockwise
// 2 --> Counterclockwise
int orientation(Point p, Point q, Point r)
{
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);

    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2; // clock or counterclock wise
}

// Prints convex hull of a set of n points.
void convexHull(Point points[], int n)
{
    // There must be at least 3 points
    if (n < 3) return;

    // Initialize Result
    int next[n];
    for (int i = 0; i < n; i++)
        next[i] = -1;

    // Find the leftmost point
    int l = 0;
    for (int i = 1; i < n; i++)
        if (points[i].x < points[l].x)
            l = i;
    // Start from leftmost point, keep moving counterclockwise
    // until reach the start point again
    int p = l, q;
    do
    {
        // Search for a point 'q' such that orientation(p, i, q) is
        // counterclockwise for all points 'i'
        q = (p+1)%n;
        for (int i = 0; i < n; i++)
          if (orientation(points[p], points[i], points[q]) == 2)
             q = i;

        next[p] = q;  // Add q to result as a next point of p
        p = q; // Set p as q for next iteration
    } while (p != l);

    // Print Result
    for (int i = 0; i < n; i++)
    {
        if (next[i] != -1)
           cout << "(" << points[i].x << ", " << points[i].y << ")\n";
    }
}

// Driver program to test above functions
int main()
{
    Point points[] = {{0, 3}, {2, 2}, {1, 1}, {2, 1},
                      {3, 0}, {0, 0}, {3, 3}};
    int n = sizeof(points)/sizeof(points[0]);
    convexHull(points, n);
    return 0;
}
```

Output: The output is points of the convex hull.

```
(0, 3)
(3, 0)
(0, 0)
(3, 3)
```

Time Complexity: For every point on the hull we examine all the other points to determine the next point. Time complexity is $\Theta(m * n)$ where n is number of input points and m is number of output or hull points (m <= n). In worst case, time complexity is O(n ^{2}). The worst case occurs when all the points are on the hull (m = n)

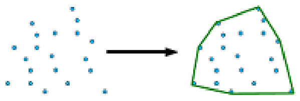
We will soon be discussing other algorithms for finding convex hulls.

Sources:
http://www.cs.uiuc.edu/~jeffe/teaching/373/notes/x05-convexhull.pdf
http://www.dcs.gla.ac.uk/~pat/52233/slides/Hull1x1.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

39. Convex Hull | Set 2 (Graham Scan)

Given a set of points in the plane. the convex hull of the set is the smallest convex polygon that contains all the points of it.



We strongly recommend to see the following post first.
How to check if two given line segments intersect?

We have discussed Jarvis's Algorithm for Convex Hull. Worst case time complexity of Jarvis's Algorithm is O(n^2). Using Graham's scan algorithm, we can find Convex Hull in O(nLogn) time. Following is Graham's algorithm

Let points[0..n-1] be the input array.

1) Find the bottom-most point by comparing y coordinate of all points. If there are two points with same y value, then the point with smaller x coordinate value is considered. Put the bottom-most point at first position.

2) Consider the remaining n-1 points and sort them by polor angle in counterclockwise order around points[0]. If polor angle of two points is same, then put the nearest point first.

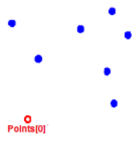
3) Create an empty stack 'S' and push points[0], points[1] and points[2] to S.

4) Process remaining n-3 points one by one. Do following for every point 'points[i]'
    4.1) Keep removing points from stack while orientation of following 3 points is not counterclockwise (or they don't make a left turn).
        a) Point next to top in stack
        b) Point at the top of stack
        c) points[i]
    4.2) Push points[i] to S

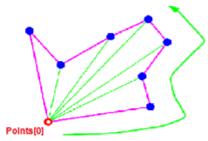
5) Print contents of S

The above algorithm can be divided in two phases.

Phase 1 (Sort points): We first find the bottom-most point. The idea is to pre-process points be sorting them with respect to the bottom-most point. Once the points are sorted, they form a simple closed path (See following diagram).



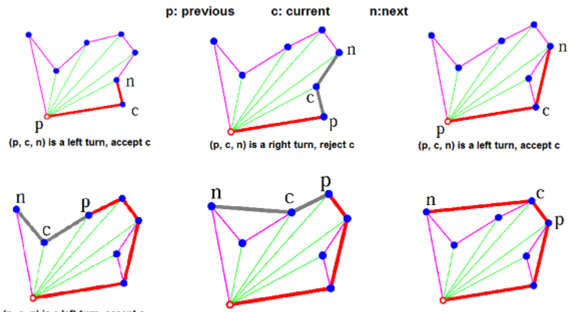
Given Points



Points considered according to increasing angle
with respect to points[0] form a simple closed path

What should be the sorting criteria? computation of actual angles would be inefficient since trigonometric functions are not simple to evaluate. The idea is to use the orientation to compare angles without actually computing them (See the compare() function below)

Phase 2 (Accept or Reject Points): Once we have the closed path, the next step is to traverse the path and remove concave points on this path. How to decide which point to remove and which to keep? Again, orientation helps here. The first two points in sorted array are always part of Convex Hull. For remaining points, we keep track of recent three points, and find the angle formed by them. Let the three points be prev(p), curr(c) and next(n). If orientation of these points (considering them in same order) is not counterclockwise, we discard c, otherwise we keep it. Following diagram shows step by step process of this phase (Source of these diagrams is Ref 2).



(p, c, n) is a left turn, accept c — (p, c, n) is a right turn, reject c — (p, c, n) is a left turn, accept c

(p, c, n) is a left turn, accept c — (p, c, n) is a right turn, reject c — (p, c, n) is a left turn, accept c

In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Following is C++ implementation of the above algorithm.

```
// A C++ program to find convex hull of a set of points
// Refer http://www.geeksforgeeks.org/check-if-two-given-line-segments
// for explanation of orientation()
#include <iostream>
#include <stack>
#include <stdlib.h>
using namespace std;

struct Point
{
    int x;
    int y;
};

// A globle point needed for  sorting points with reference to the fir
// Used in compare function of qsort()
Point p0;
```

```
// A utility function to find next to top in a stack
Point nextToTop(stack<Point> &S)
{
    Point p = S.top();
    S.pop();
    Point res = S.top();
    S.push(p);
    return res;
}

// A utility function to swap two points
int swap(Point &p1, Point &p2)
{
    Point temp = p1;
    p1 = p2;
    p2 = temp;
}

// A utility function to return square of distance between p1 and p2
int dist(Point p1, Point p2)
{
    return (p1.x - p2.x)*(p1.x - p2.x) + (p1.y - p2.y)*(p1.y - p2.y);
}

// To find orientation of ordered triplet (p, q, r).
// The function returns following values
// 0 --> p, q and r are colinear
// 1 --> Clockwise
// 2 --> Counterclockwise
int orientation(Point p, Point q, Point r)
{
    int val = (q.y - p.y) * (r.x - q.x) -
              (q.x - p.x) * (r.y - q.y);

    if (val == 0) return 0;  // colinear
    return (val > 0)? 1: 2; // clock or counterclock wise
}

// A function used by library function qsort() to sort an array of
// points with respect to the first point
int compare(const void *vp1, const void *vp2)
{
   Point *p1 = (Point *)vp1;
   Point *p2 = (Point *)vp2;

   // Find orientation
   int o = orientation(p0, *p1, *p2);
   if (o == 0)
     return (dist(p0, *p2) >= dist(p0, *p1))? -1 : 1;

   return (o == 2)? -1: 1;
}

// Prints convex hull of a set of n points.
void convexHull(Point points[], int n)
{
   // Find the bottommost point
   int ymin = points[0].y, min = 0;
   for (int i = 1; i < n; i++)
   {
     int y = points[i].y;
```

```
      // Pick the bottom-most or chose the left most point in case of t
      if ((y < ymin) || (ymin == y && points[i].x < points[min].x))
         ymin = points[i].y, min = i;
   }

   // Place the bottom-most point at first position
   swap(points[0], points[min]);

   // Sort n-1 points with respect to the first point.  A point p1 com
   // before p2 in sorted ouput if p2 has larger polar angle (in
   // counterclockwise direction) than p1
   p0 = points[0];
   qsort(&points[1], n-1, sizeof(Point), compare);

   // Create an empty stack and push first three points to it.
   stack<Point> S;
   S.push(points[0]);
   S.push(points[1]);
   S.push(points[2]);

   // Process remaining n-3 points
   for (int i = 3; i < n; i++)
   {
      // Keep removing top while the angle formed by points next-to-to
      // top, and points[i] makes a non-left turn
      while (orientation(nextToTop(S), S.top(), points[i]) != 2)
         S.pop();
      S.push(points[i]);
   }

   // Now stack has the output points, print contents of stack
   while (!S.empty())
   {
       Point p = S.top();
       cout << "(" << p.x << ", " << p.y <<")" << endl;
       S.pop();
   }
}

// Driver program to test above functions
int main()
{
    Point points[] = {{0, 3}, {1, 1}, {2, 2}, {4, 4},
                       {0, 0}, {1, 2}, {3, 1}, {3, 3}};
    int n = sizeof(points)/sizeof(points[0]);
    convexHull(points, n);
    return 0;
}
```

Output:

```
(0, 3)
(4, 4)
(3, 1)
(0, 0)
```

Time Complexity: Let n be the number of input points. The algorithm takes O(nLogn) time if we use a O(nLogn) sorting algorithm.

The first step (finding the bottom-most point) takes O(n) time. The second step (sorting points) takes O(nLogn) time. In third step, every element is pushed and popped at most one time. So the third step to process points one by one takes O(n) time, assuming that the stack operations take O(1) time. Overall complexity is O(n) + O(nLogn) + O(n) which is O(nLogn)

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest

http://www.dcs.gla.ac.uk/~pat/52233/slides/Hull1x1.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

40. How to check if a given number is Fibonacci number?

Given a number ‘n’, how to check if n is a Fibonacci number.

A simple way is to generate Fibonacci numbers until the generated number is greater than or equal to ‘n’. Following is an interesting property about Fibonacci numbers that can also be used to check if a given number is Fibonacci or not.

A number is Fibonacci if and only if one or both of ($5*n^2 + 4$) or ($5*n^2 – 4$) is a perfect square (Source: Wiki). Following is a simple program based on this concept.

```cpp
// C++ program to check if x is a perfect square
#include <iostream>
#include <math.h>
using namespace std;

// A utility function that returns true if x is perfect square
bool isPerfectSquare(int x)
{
    int s = sqrt(x);
    return (s*s == x);
}

// Returns true if n is a Fibinacci Number, else false
bool isFibonacci(int n)
{
    // n is Fibinacci if one of 5*n*n + 4 or 5*n*n - 4 or both
    // is a perferct square
    return isPerfectSquare(5*n*n + 4) ||
           isPerfectSquare(5*n*n - 4);
}

// A utility function to test above functions
int main()
{
  for (int i = 1; i <= 10; i++)
     isFibonacci(i)? cout << i << " is a Fibonacci Number \n":
                     cout << i << " is a not Fibonacci Number \n" ;
  return 0;
}
```

Output:

```
1 is a Fibonacci Number
2 is a Fibonacci Number
3 is a Fibonacci Number
4 is a not Fibonacci Number
5 is a Fibonacci Number
6 is a not Fibonacci Number
7 is a not Fibonacci Number
8 is a Fibonacci Number
9 is a not Fibonacci Number
10 is a not Fibonacci Number
```

This article is contributed by **Abhay Rathi**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

41. Russian Peasant Multiplication

Given two integers, write a function to multiply them without using multiplication operator.

There are many other ways to multiply two numbers (For example, see this). One interesting method is the Russian peasant algorithm. The idea is to double the first number and halve the second number repeatedly till the second number doesn't become 1. In the process, whenever the second number become odd, we add the first number to result (result is initialized as 0)
The following is simple algorithm.

```
Let the two given numbers be 'a' and 'b'
1) Initialize result 'res' as 0.
2) Do following while 'b' is greater than 0
   a) If 'b' is odd, add 'a' to 'res'
   b) Double 'a' and halve 'b'
3) Return 'res'.
```

```
#include <iostream>
using namespace std;

// A method to multiply two numbers using Russian Peasant method
unsigned int russianPeasant(unsigned int a, unsigned int b)
{
    int res = 0;  // initialize result

    // While second number doesn't become 1
    while (b > 0)
    {
         // If second number becomes odd, add the first number to resu
         if (b & 1)
             res = res + a;

         // Double the first number and halve the second number
         a = a << 1;
         b = b >> 1;
     }
     return res;
}

// Driver program to test above function
int main()
{
    cout << russianPeasant(18, 1) << endl;
    cout << russianPeasant(20, 12) << endl;
    return 0;
}
```

Output:

```
18
240
```

How does this work?
The value of a*b is same as (a*2)*(b/2) if b is even, otherwise the value is same as ((a*2)*(b/2) + a). In the while loop, we keep multiplying 'a' with 2 and keep dividing 'b' by 2. If 'b' becomes odd in loop, we add 'a' to 'res'. When value of 'b' becomes 1, the value of 'res' + 'a', gives us the result.

Note that when ‘b’ is a power of 2, the ‘res’ would remain 0 and ‘a’ would have the multiplication. See the reference for more information.

Reference:
http://mathforum.org/dr.math/faq/faq.peasant.html

This article is compiled by **Shalki Agarwal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

42. Program for nth Catalan Number

Catalan numbers are a sequence of natural numbers that occurs in many interesting counting problems like following.

1) Count the number of expressions containing n pairs of parentheses which are correctly matched. For n = 3, possible expressions are ((())), ()(()), ()()(), (())(), (()()).

2) Count the number of possible Binary Search Trees with n keys (See this)

3) Count the number of full binary trees (A rooted binary tree is full if every vertex has either two children or no children) with n+1 leaves.

See this for more applications.

The first few Catalan numbers for n = 0, 1, 2, 3, … are **1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, …**

Recursive Solution
Catalan numbers satisfy the following recursive formula.
$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i} \quad \text{for } n \geq 0;$

Following is C++ implementation of above recursive formula.

```
#include<iostream>
using namespace std;

// A recursive function to find nth catalan number
unsigned long int catalan(unsigned int n)
{
    // Base case
    if (n <= 1) return 1;

    // catalan(n) is sum of catalan(i)*catalan(n-i-1)
    unsigned long int res = 0;
    for (int i=0; i<n; i++)
        res += catalan(i)*catalan(n-i-1);

    return res;
}

// Driver program to test above function
int main()
{
    for (int i=0; i<10; i++)
        cout << catalan(i) << " ";
    return 0;
}
```

Output :

```
1 1 2 5 14 42 132 429 1430 4862
```

Time complexity of above implementation is equivalent to nth catalan number.

$$T(n) = \sum_{i=0}^{n-1} T(i) * T(n-i) \quad \text{for } n \geq 0;$$

The value of nth catalan number is exponential that makes the time complexity exponential.

Dynamic Programming Solution

We can observe that the above recursive implementation does a lot of repeated work (we can the same by drawing recursion tree). Since there are overlapping subproblems, we can use dynamic programming for this. Following is a Dynamic programming based implementation in C++.

```
#include<iostream>
using namespace std;

// A dynamic programming based function to find nth
// Catalan number
unsigned long int catalanDP(unsigned int n)
{
    // Table to store results of subproblems
    unsigned long int catalan[n+1];

    // Initialize first two values in table
    catalan[0] = catalan[1] = 1;

    // Fill entries in catalan[] using recursive formula
    for (int i=2; i<=n; i++)
    {
        catalan[i] = 0;
        for (int j=0; j<i; j++)
            catalan[i] += catalan[j] * catalan[i-j-1];
    }

    // Return last entry
    return catalan[n];
}

// Driver program to test above function
int main()
{
    for (int i = 0; i < 10; i++)
        cout << catalanDP(i) << " ";
    return 0;
}
```

Output:

```
1 1 2 5 14 42 132 429 1430 4862
```

Time Complexity: Time complexity of above implementation is O(n^2)

Using Binomial Coefficient

We can also use the below formula to find nth catalan number in O(n) time.

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

We have discussed a O(n) approach to find binomial coefficient nCr.

```
#include<iostream>
using namespace std;

// Returns value of Binomial Coefficient C(n, k)
unsigned long int binomialCoeff(unsigned int n, unsigned int k)
{
    unsigned long int res = 1;

    // Since C(n, k) = C(n, n-k)
    if (k > n - k)
        k = n - k;

    // Calculate value of [n*(n-1)*---*(n-k+1)] / [k*(k-1)*---*1]
    for (int i = 0; i < k; ++i)
    {
        res *= (n - i);
        res /= (i + 1);
    }

    return res;
}

// A Binomial coefficient based function to find nth catalan
// number in O(n) time
unsigned long int catalan(unsigned int n)
{
    // Calculate value of 2nCn
    unsigned long int c = binomialCoeff(2*n, n);

    // return 2nCn/(n+1)
    return c/(n+1);
}

// Driver program to test above functions
int main()
{
    for (int i = 0; i < 10; i++)
        cout << catalan(i) << " ";
    return 0;
}
```

Output:

```
1 1 2 5 14 42 132 429 1430 4862
```

Time Complexity: Time complexity of above implementation is O(n).

We can also use below formula to find nth catalan number in O(n) time.

$$C_n = \frac{(2n)!}{(n+1)!\,n!} = \prod_{k=2}^{n} \frac{n+k}{k} \qquad \text{for } n \geq 0.$$

References:
http://en.wikipedia.org/wiki/Catalan_number

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

43. Count trailing zeroes in factorial of a number

Given an integer n, write a function that returns count of trailing zeroes in n!.

```
Examples:
Input: n = 5
Output: 1
Factorial of 5 is 20 which has one trailing 0.

Input: n = 20
Output: 4
Factorial of 20 is 2432902008176640000 which has
4 trailing zeroes.

Input: n = 100
Output: 24
```

A simple method is to first calculate factorial of n, then count trailing 0s in the result (We can count trailing 0s by repeatedly dividing the factorial by 10 till the remainder is 0).

The above method can cause overflow for a slightly bigger numbers as factorial of a number is a big number (See factorial of 20 given in above examples). The idea is to consider prime factors of a factorial n. A trailing zero is always produced by prime factors 2 and 5. If we can count the number of 5s and 2s, our task is done. Consider the following examples.

n = 5: There is one 5 and 3 2s in prime factors of 5! (2 * 2 * 2 * 3 * 5). So count of trailing 0s is 1.

n = 11: There are two 5s and three 2s in prime factors of 11! (2^{8} * 3^{4} * 5^{2} * 7). So count of trailing 0s is 2.

We can easily observe that the number of 2s in prime factors is always more than or equal to the number of 5s. So if we count 5s in prime factors, we are done. *How to count total number of 5s in prime factors of n!?* A simple way is to calculate floor(n/5). For example, 7! has one 5, 10! has two 5s. It is done yet, there is one more thing to consider. Numbers like 25, 125, etc have more than one 5. For example if we consider 28!, we get one extra 5 and number of 0s become 6. Handling this is simple, first divide n by 5 and remove all single 5s, then divide by 25 to remove extra 5s and so on. Following is the summarized formula for counting trailing 0s.

```
Trailing 0s in n! = Count of 5s in prime factors of n!
                  = floor(n/5) + floor(n/25) + floor(n/125) + ....
```

Following is C++ program based on above formula.

```
// C++ program to count trailing 0s in n!
#include <iostream>
using namespace std;

// Function to return trailing 0s in factorial of n
int findTrailingZeros(int  n)
{
    // Initialize result
    int count = 0;

    // Keep dividing n by powers of 5 and update count
    for (int i=5; n/i>=1; i *= 5)
          count += n/i;

    return count;
}

// Driver program to test above function
int main()
{
    int n = 100;
    cout << "Count of trailing 0s in " << 100
         << "! is " << findTrailingZeros(n);
    return 0;
}
```

Output:

```
Count of trailing 0s in 100! is 24
```

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

44. Horner's Method for Polynomial Evaluation

Given a polynomial of the form $c_nx^n + c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \ldots + c_1x + c_0$ and a value of x, find the value of polynomial for a given value of x. Here c_n, c_{n-1}, .. are integers (may be negative) and n is a positive integer.

Input is in the form of an array say *poly[]* where poly[0] represents coefficient for x^n and poly[1] represents coefficient for x^{n-1} and so on.

Examples:

```
// Evaluate value of 2x^3 - 6x^2 + 2x - 1 for x = 3
Input: poly[] = {2, -6, 2, -1}, x = 3
Output: 5
```

```
// Evaluate value of 2x3 + 3x + 1 for x = 2
Input: poly[] = {2, 0, 3, 1}, x = 2
Output: 23
```

A naive way to evaluate a polynomial is to one by one evaluate all terms. First calculate x^n, multiply the value with c_n, repeat the same steps for other terms and return the sum. Time complexity of this approach is $O(n^2)$ if we use a simple loop for evaluation of x^n. Time complexity can be improved to O(nLogn) if we use O(Logn) approach for evaluation of x^n.

Horner's method can be used to evaluate polynomial in O(n) time. To understand the method, let us consider the example of $2x^3 – 6x^2 + 2x – 1$. The polynomial can be evaluated as $((2x – 6)x + 2)x – 1$. The idea is to initialize result as coefficient of x^n which is 2 in this case, repeatedly multiply result with x and add next coefficient to result. Finally return result.

Following is C++ implementation of Horner's Method.

```cpp
#include <iostream>
using namespace std;

// returns value of poly[0]x(n-1) + poly[1]x(n-2) + .. + poly[n-1]
int horner(int poly[], int n, int x)
{
    int result = poly[0];  // Initialize result

    // Evaluate value of polynomial using Horner's method
    for (int i=1; i<n; i++)
        result = result*x + poly[i];

    return result;
}

// Driver program to test above function.
int main()
{
    // Let us evaluate value of 2x3 - 6x2 + 2x - 1 for x = 3
    int poly[] = {2, -6, 2, -1};
    int x = 3;
    int n = sizeof(poly)/sizeof(poly[0]);
    cout << "Value of polynomial is " << horner(poly, n, x);
    return 0;
}
```

Output:

```
Value of polynomial is 5
```

Time Complexity: O(n)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

45. Write a function that generates one of 3 numbers according to given probabilities

You are given a function rand(a, b) which generates equiprobable random numbers between [a, b] inclusive. Generate 3 numbers x, y, z with probability P(x), P(y), P(z) such that P(x) + P(y) + P(z) = 1 using the given rand(a,b) function.

The idea is to utilize the equiprobable feature of the rand(a,b) provided. ***Let the given probabilities be in percentage form, for example P(x)=40%, P(y)=25%, P(z)=35%.*.**

Following are the detailed steps.
1) Generate a random number between 1 and 100. Since they are equiprobable, the probability of each number appearing is 1/100.
2) Following are some important points to note about generated random number 'r'.
a) 'r' is smaller than or equal to P(x) with probability P(x)/100.
b) 'r' is greater than P(x) and smaller than or equal P(x) + P(y) with P(y)/100.
c) 'r' is greater than P(x) + P(y) and smaller than or equal 100 (or P(x) + P(y) + P(z)) with probability P(z)/100.

```
// This function generates 'x' with probability px/100, 'y' with
// probability py/100  and 'z' with probability pz/100:
// Assumption: px + py + pz = 100 where px, py and pz lie
// between 0 to 100
int random(int x, int y, int z, int px, int py, int pz)
{
        // Generate a number from 1 to 100
        int r = rand(1, 100);

        // r is smaller than px with probability px/100
        if (r <= px)
            return x;

         // r is greater than px and smaller than or equal to px+py
         // with probability py/100
        if (r <= (px+py))
            return y;

         // r is greater than px+py and smaller than or equal to 100
         // with probability pz/100
        else
            return z;
}
```

This function will solve the purpose of generating 3 numbers with given three probabilities.

This article is contributed by **Harsh Agarwal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

46. Find the smallest number whose digits multiply to a given number n

Given a number ‘n’, find the smallest number ‘p’ such that if we multiply all digits of ‘p’, we get ‘n’. The result ‘p’ should have minimum two digits.

Examples:

```
Input:  n = 36
Output: p = 49
// Note that 4*9 = 36 and 49 is the smallest such number

Input:  n = 100
Output: p = 455
// Note that 4*5*5 = 100 and 455 is the smallest such number

Input: n = 1
Output:p = 11
// Note that 1*1 = 1

Input: n = 13
Output: Not Possible
```

For a given n, following are the two cases to be considered.
Case 1: n < 10 When n is smaller than n, the output is always n+10. For example for n = 7, output is 17. For n = 9, output is 19.

Case 2: n >= 10 Find all factors of n which are between 2 and 9 (both inclusive). The idea is to start searching from 9 so that the number of digits in result are minimized. For example 9 is preferred over 33 and 8 is preferred over 24.
Store all found factors in an array. The array would contain digits in non-increasing order, so finally print the array in reverse order.

Following is C implementation of above concept.

```c
#include<stdio.h>

// Maximum number of digits in output
#define MAX 50

// prints the smallest number whose digits multiply to n
void findSmallest(int n)
{
    int i, j=0;
    int res[MAX]; // To sore digits of result in reverse order

    // Case 1: If number is smaller than 10
    if (n < 10)
    {
        printf("%d", n+10);
        return;
    }

    // Case 2: Start with 9 and try every possible digit
    for (i=9; i>1; i--)
    {
        // If current digit divides n, then store all
        // occurrences of current digit in res
        while (n%i == 0)
        {
            n = n/i;
            res[j] = i;
            j++;
        }
    }

    // If n could not be broken in form of digits (prime factors of n
    // are greater than 9)
    if (n > 10)
    {
        printf("Not possible");
        return;
    }

    // Print the result array in reverse order
    for (i=j-1; i>=0; i--)
        printf("%d", res[i]);
}

// Driver program to test above function
int main()
{
    findSmallest(7);
    printf("\n");

    findSmallest(36);
    printf("\n");

    findSmallest(13);
    printf("\n");

    findSmallest(100);
    return 0;
}
```

Output:

```
17
49
Not possible
455
```

This article is contributed by **Ashish Bansal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

47. Calculate the angle between hour hand and minute hand

This problem is know as Clock angle problem where we need to find angle between hands of an analog clock at .

Examples:

```
Input:  h = 12:00, m = 30.00
Output: 165 degree

Input:  h = 3.00, m = 30.00
Output: 75 degree
```

The idea is to take 12:00 (h = 12, m = 0) as a reference. Following are detailed steps.

1) Calculate the angle made by hour hand with respect to 12:00 in h hours and m minutes.
2) Calculate the angle made by minute hand with respect to 12:00 in h hours and m minutes.
3) The difference between two angles is the angle between two hands.

How to calculate the two angles with respect to 12:00?
The minute hand moves 360 degree in 60 minute(or 6 degree in one minute) and hour hand moves 360 degree in 12 hours(or 0.5 degree in 1 minute). In h hours and m minutes, the minute hand would move (h*60 + m)*6 and hour hand would move (h*60 + m)*0.5.

```c
// C program to find angle between hour and minute hands
#include <stdio.h>
#include <stdlib.h>

// Utility function to find minimum of two integers
int min(int x, int y) { return (x < y)? x: y; }

int calcAngle(double h, double m)
{
    // validate the input
    if (h <0 || m < 0 || h >12 || m > 60)
        printf("Wrong input");

    if (h == 12) h = 0;
    if (m == 60) m = 0;

    // Calculate the angles moved by hour and minute hands
    // with reference to 12:00
    int hour_angle = 0.5 * (h*60 + m);
    int minute_angle = 6*m;

    // Find the difference between two angles
    int angle = abs(hour_angle - minute_angle);

    // Return the smaller angle of two possible angles
    angle = min(360-angle, angle);

    return angle;
}

// Driver program to test above function
int main()
{
    printf("%d \n", calcAngle(9, 60));
    printf("%d \n", calcAngle(3, 30));
    return 0;
}
```

Output:

```
90
75
```

Exercise: Find all times when hour and minute hands get superimposed.

This article is contributed by **Ashish Bansal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

48. Count Possible Decodings of a given Digit Sequence

Let 1 represent ‘A’, 2 represents ‘B’, etc. Given a digit sequence, count the number of possible decodings of the given digit sequence.

Examples:

```
Input:  digits[] = "121"
Output: 3
// The possible decodings are "ABA", "AU", "LA"

Input: digits[] = "1234"
Output: 3
// The possible decodings are "ABCD", "LCD", "AWD"
```

An empty digit sequence is considered to have one decoding. It may be assumed that the input contains valid digits from 0 to 9 and there are no leading 0’s, no extra trailing 0’s and no two or more consecutive 0’s.

We strongly recommend to minimize the browser and try this yourself first.

This problem is recursive and can be broken in sub-problems. We start from end of the given digit sequence. We initialize the total count of decodings as 0. We recur for two subproblems.
1) If the last digit is non-zero, recur for remaining (n-1) digits and add the result to total count.
2) If the last two digits form a valid character (or smaller than 27), recur for remaining (n-2) digits and add the result to total count.

Following is C++ implementation of the above approach.

```cpp
// A naive recursive C++ implementation to count number of decodings
// that can be formed from a given digit sequence
#include <iostream>
#include <cstring>
using namespace std;

// Given a digit sequence of length n, returns count of possible
// decodings by replacing 1 with A, 2 woth B, ... 26 with Z
int countDecoding(char *digits, int n)
{
    // base cases
    if (n == 0 || n == 1)
        return 1;

    int count = 0;  // Initialize count

    // If the last digit is not 0, then last digit must add to
    // the number of words
    if (digits[n-1] > '0')
        count =  countDecoding(digits, n-1);

    // If the last two digits form a number smaller than or equal to 2
    // then consider last two digits and recur
    if (digits[n-2] < '2' || (digits[n-2] == '2' && digits[n-1] < '7'))
        count +=  countDecoding(digits, n-2);

    return count;
}

// Driver program to test above function
int main()
{
    char digits[] = "1234";
    int n = strlen(digits);
    cout << "Count is " << countDecoding(digits, n);
    return 0;
}
```

Output:

```
Count is 3
```

The time complexity of above the code is exponential. If we take a closer look at the above program, we can observe that the recursive solution is similar to Fibonacci Numbers. Therefore, we can optimize the above solution to work in O(n) time using Dynamic Programming. Following is C++ implementation for the same.

```
// A Dynamic Programming based C++ implementation to count decodings
#include <iostream>
#include <cstring>
using namespace std;

// A Dynamic Programming based function to count decodings
int countDecodingDP(char *digits, int n)
{
    int count[n+1]; // A table to store results of subproblems
    count[0] = 1;
    count[1] = 1;

    for (int i = 2; i <= n; i++)
    {
        count[i] = 0;

        // If the last digit is not 0, then last digit must add to
        // the number of words
        if (digits[i-1] > '0')
            count[i] = count[i-1];

        // If second last digit is smaller than 2 and last digit is
        // smaller than 7, then last two digits form a valid character
        if (digits[i-2] < '2' || (digits[i-2] == '2' && digits[i-1] <
            count[i] += count[i-2];
    }
    return count[n];
}

// Driver program to test above function
int main()
{
    char digits[] = "1234";
    int n = strlen(digits);
    cout << "Count is " << countDecodingDP(digits, n);
    return 0;
}
```

Output:

```
Count is 3
```

Time Complexity of the above solution is O(n) and it requires O(n) auxiliary space. We can reduce auxiliary space to O(1) by using space optimized version discussed in the Fibonacci Number Post.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

49. Find next greater number with same set of digits

Given a number n, find the smallest number that has same set of digits as n and is greater than n. If x is the greatest possible number with its set of digits, then print “not possible”.

Examples:
For simplicity of implementation, we have considered input number as a string.

```
Input:  n = "218765"
Output: "251678"

Input:  n = "1234"
Output: "1243"

Input: n = "4321"
Output: "Not Possible"

Input: n = "534976"
Output: "536479"
```

We strongly recommend to minimize the browser and try this yourself first.

Following are few observations about the next greater number.
1) If all digits sorted in descending order, then output is always “Not Possible”. For example, 4321.
2) If all digits are sorted in ascending order, then we need to swap last two digits. For example, 1234.
3) For other cases, we need to process the number from rightmost side (why? because we need to find the smallest of all greater numbers)

You can now try developing an algorithm yourself.

Following is the algorithm for finding the next greater number.
I) Traverse the given number from rightmost digit, keep traversing till you find a digit which is smaller than the previously traversed digit. For example, if the input number is “534976”, we stop at **4** because 4 is smaller than next digit 9. If we do not find such a digit, then output is “Not Possible”.

II) Now search the right side of above found digit ‘d’ for the smallest digit greater than ‘d’. For “53**4**976″, the right side of 4 contains “976”. The smallest digit greater than 4 is **6**.

III) Swap the above found two digits, we get 53**6**97**4** in above example.

IV) Now sort all digits from position next to ‘d’ to the end of number. The number that we get after sorting is the output. For above example, we sort digits in bold 536**974**. We get “536**479**” which is the next greater number for input 534976.

Following is C++ implementation of above approach.

```
// C++ program to find the smallest number which greater than a given
// and has same set of digits as given number
#include <iostream>
#include <cstring>
#include <algorithm>
using namespace std;

// Utility function to swap two digits
void swap(char *a, char *b)
{
    char temp = *a;
    *a = *b;
    *b = temp;
}

// Given a number as a char array number[], this function finds the
// next greater number.  It modifies the same array to store the resul
void findNext(char number[], int n)
{
    int i, j;

    // I) Start from the right most digit and find the first digit tha
    // smaller than the digit next to it.
    for (i = n-1; i > 0; i--)
        if (number[i] > number[i-1])
           break;

    // If no such digit is found, then all digits are in descending or
    // means there cannot be a greater number with same set of digits
    if (i==0)
    {
        cout << "Next number is not possible";
        return;
    }

    // II) Find the smallest digit on right side of (i-1)'th digit tha
    // greater than number[i-1]
    int x = number[i-1], smallest = i;
    for (j = i+1; j < n; j++)
        if (number[j] > x && number[j] < number[smallest])
            smallest = j;

    // III) Swap the above found smallest digit with number[i-1]
    swap(&number[smallest], &number[i-1]);

    // IV) Sort the digits after (i-1) in ascending order
    sort(number + i, number + n);

    cout << "Next number with same set of digits is " << number;

    return;
}

// Driver program to test above function
int main()
{
    char digits[] = "534976";
    int n = strlen(digits);
    findNext(digits, n);
    return 0;
}
```

Output:

```
Next number with same set of digits is 536479
```

The above implementation can be optimized in following ways.
1) We can use binary search in step II instead of linear search.
2) In step IV, instead of doing simple sort, we can apply some clever technique to do it in linear time. Hint: We know that all digits are linearly sorted in reverse order except one digit which was swapped.

With above optimizations, we can say that the time complexity of this method is O(n).

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

50. Print squares of first n natural numbers without using *, / and -

Given a natural number ‘n’, print squares of first n natural numbers without using *, / and -
.

```
Input:   n = 5
Output: 0 1 4 9 16

Input:   n = 6
Output: 0 1 4 9 16 25
```

We strongly recommend to minimize the browser and try this yourself first.

Method 1: The idea is to calculate next square using previous square value. Consider the following relation between square of x and (x-1). We know square of (x-1) is $(x-1)^2$ – 2*x + 1. We can write x^2 as

```
x^2 = (x-1)^2 + 2*x - 1
x^2 = (x-1)^2 + x + (x - 1)
```

When writing an iterative program, we can keep track of previous value of x and add the current and previous values of x to current value of square. This way we don’t even use the ‘-‘ operator.

```
// C++ program to print squares of first 'n' natural numbers
// wothout using *, / and -
#include<iostream>
using namespace std;

void printSquares(int n)
{
    // Initialize 'square' and previous value of 'x'
    int square = 0, prev_x = 0;

    // Calculate and print squares
    for (int x = 0; x < n; x++)
    {
        // Update value of square using previous value
        square = (square + x + prev_x);

        // Print square and update prev for next iteration
        cout << square << " ";
        prev_x = x;
    }
}

// Driver program to test above function
int main()
{
   int n = 5;
   printSquares(n);
}
```

Output:

```
0 1 4 9 16
```

Method 2: Sum of first n odd numbers are squares of natural numbers from 1 to n. For example 1, 1+3, 1+3+5, 1+3+5+7, 1+3+5+7+9, ….

Following is C++ program based on above concept. Thanks to Aadithya Umashanker and raviteja for suggesting this method.

```
// C++ program to print squares of first 'n' natural numbers
// wothout using *, / and -
#include<iostream>
using namespace std;

void printSquares(int n)
{
    // Initialize 'square' and first odd number
    int square = 0, odd = 1;

    // Calculate and print squares
    for (int x = 0; x < n; x++)
    {
        // Print square
        cout << square << " ";

        // Update 'square' and 'odd'
        square = square + odd;
        odd = odd + 2;
    }
}

// Driver program to test above function
int main()
{
   int n = 5;
   printSquares(n);
}
```

Output:

```
0 1 4 9 16
```

This article is contributed by **Sachin**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

51. Find n’th number in a number system with only 3 and 4

Given a number system with only 3 and 4. Find the nth number in the number system. First few numbers in the number system are: 3, 4, 33, 34, 43, 44, 333, 334, 343, 344, 433, 434, 443, 444, 3333, 3334, 3343, 3344, 3433, 3434, 3443, 3444, …

Source: Zoho Interview

We strongly recommend to minimize the browser and try this yourself first.

We can generate all numbers with i digits using the numbers with (i-1) digits. The idea is to first add a ‘3’ as prefix in all numbers with (i-1) digit, then add a ‘4’. For example, the numbers with 2 digits are 33, 34, 43 and 44. The numbers with 3 digits are 333, 334, 343, 344, 433, 434, 443 and 444 which can be generated by first adding a 3 as prefix, then 4.

Following are detailed steps.

```
1) Create an array 'arr[]' of strings size n+1.
2) Initialize arr[0] as empty string. (Number with 0 digits)
3) Do following while array size is smaller than or equal to n
.....a) Generate numbers by adding a 3 as prefix to the numbers generated
        in previous iteration.  Add these numbers to arr[]
.....a) Generate numbers by adding a 4 as prefix to the numbers generated
        in previous iteration. Add these numbers to arr[]
```

Thanks to kaushik Lele for suggesting this idea in a comment here. Following is C++ implementation for the same.

```cpp
// C++ program to find n'th number in a number system with only 3 and 
#include <iostream>
using namespace std;

// Function to find n'th number in a number system with only 3 and 4
void find(int n)
{
    // An array of strings to store first n numbers. arr[i] stores i't
    string arr[n+1];
    arr[0] = ""; // arr[0] stores the empty string (String with 0 digi

    // size indicates number of current elements in arr[]. m indicates
    // number of elements added to arr[] in previous iteration.
    int size = 1, m = 1;

    // Every iteration of following loop generates and adds 2*m number
    // arr[] using the m numbers generated in previous iteration.
    while (size <= n)
    {
        // Consider all numbers added in previous iteration, add a pre
        // "3" to them and add new numbers to arr[]
        for (int i=0; i<m && (size+i)<=n; i++)
            arr[size + i] = "3" +  arr[size - m + i];

        // Add prefix "4" to numbers of previous iteration and add new
        // numbers to arr[]
        for (int i=0; i<m && (size + m + i)<=n; i++)
            arr[size + m + i] = "4" +  arr[size - m + i];

        // Update no. of elements added in previous iteration
        m = m<<1; // Or m = m*2;

        // Update size
        size = size + m;
    }
    cout << arr[n] << endl;
}

// Driver program to test above functions
int main()
{
    for (int i = 1; i < 16; i++)
        find(i);
    return 0;
}
```

Output:

```
3
4
33
34
43
44
333
334
343
```

```
344
433
434
443
444
3333
```

This article is contributed by **Raman**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

52. Count Distinct Non-Negative Integer Pairs (x, y) that Satisfy the Inequality x*x + y*y < n

Given a positive number n, count all distinct Non-Negative Integer pairs (x, y) that satisfy the inequality x*x + y*y < n.

Examples:

```
Input:  n = 5
Output: 6
The pairs are (0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (0, 2)

Input: n = 6
Output: 8
The pairs are (0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (0, 2),
              (1, 2), (2, 1)
```

A **Simple Solution** is to run two loops. The outer loop goes for all possible values of x (from 0 to √n). The inner loops picks all possible values of y for current value of x (picked by outer loop). Following is C++ implementation of simple solution.

```cpp
#include <iostream>
using namespace std;

// This function counts number of pairs (x, y) that satisfy
// the inequality x*x + y*y < n.
int countSolutions(int n)
{
   int res = 0;
   for (int x = 0; x*x < n; x++)
      for (int y = 0; x*x + y*y < n; y++)
         res++;
   return res;
}

// Driver program to test above function
int main()
{
    cout << "Total Number of distinct Non-Negative pairs is "
         << countSolutions(6) << endl;
    return 0;
}
```

Output:

```
Total Number of distinct Non-Negative pairs is 8
```

An upper bound for time complexity of the above solution is O(n). The outer loop runs √n times. The inner loop runs at most √n times.

Using an **Efficient Solution**, we can find the count in O(√n) time. The idea is to first find the count of all y values corresponding the 0 value of x. Let count of distinct y values be yCount. We can find yCount by running a loop and comparing yCount*yCount with n. After we have initial yCount, we can one by one increase value of x and find the next value of yCount by reducing yCount.

```
// An efficient C program to find different (x, y) pairs that
// satisfy x*x + y*y < n.
#include <iostream>
using namespace std;

// This function counts number of pairs (x, y) that satisfy
// the inequality x*x + y*y < n.
int countSolutions(int n)
{
   int x = 0, yCount, res = 0;

   // Find the count of different y values for x = 0.
   for (yCount = 0; yCount*yCount < n; yCount++) ;

   // One by one increase value of x, and find yCount for
   // current x.  If yCount becomes 0, then we have reached
   // maximum possible value of x.
   while (yCount != 0)
   {
       // Add yCount (count of different possible values of y
       // for current x) to result
       res  +=  yCount;

       // Increment x
       x++;

       // Update yCount for current x. Keep reducing yCount while
       // the inequality is not satisfied.
       while (yCount != 0 && (x*x + (yCount-1)*(yCount-1) >= n))
         yCount--;
   }

   return res;
}

// Driver program to test above function
int main()
{
    cout << "Total Number of distinct Non-Negative pairs is "
         << countSolutions(6) << endl;
    return 0;
}
```

Output:

```
Total Number of distinct Non-Negative pairs is 8
```

Time Complexity of the above solution seems more but if we take a closer look, we can see that it is O(√n). In every step inside the inner loop, value of yCount is decremented by 1. The value yCount can decrement at most O(√n) times as yCount is count y values for x = 0. In the outer loop, the value of x is incremented. The value of x can also increment at most O(√n) times as the last x is for yCount equals to 1.

This article is contributed by **Sachin Gupta**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

53. Birthday Paradox

How many people must be there in a room to make the probability 100% that two people in the room have same birthday?
Answer: 367 (since there are 366 possible birthdays, including February 29).
The above question was simple. Try the below question yourself.

How many people must be there in a room to make the probability 50% that two people in the room have same birthday?
Answer: 23
The number is surprisingly very low. In fact, we need only 70 people to make the probability 99.9 %.

Let us discuss the generalized formula.

What is the probability that two persons among n have same birthday?
Let the probability that two people in a room with n have same birthday be P(same).
P(Same) can be easily evaluated in terms of P(different) where P(different) is the probability that all of them have different birthday.

P(same) = 1 – P(different)

P(different) can be written as 1 x (364/365) x (363/365) x (362/365) x …. x (1 – (n-1)/365)

How did we get the above expression?
Persons from first to last can get birthdays in following order for all birthdays to be distinct:
The first person can have any birthday among 365
The second person should have a birthday which is not same as first person
The third person should have a birthday which is not same as first two persons.
…………….
……………
The n'th person should have a birthday which is not same as any of the earlier considered (n-1) persons.

Approximation of above expression
The above expression can be approximated using Taylor's Series.

$e^x = 1 + x + \frac{x^2}{2!} + \cdots$

provides a first-order approximation for ex for x << 1:

$e^x \approx 1 + x.$

To apply this approximation to the first expression derived for p(different), set x = -a / 365\ . Thus,

$e^{-a/365} \approx 1 - \frac{a}{365}.$

The above expression derived for p(different) can be written as
1 x (1 – 1/365) x (1 – 2/365) x (1 – 3/365) x …. x (1 – (n-1)/365)

By putting the value of $1 - \frac{a}{365}$ as $e^{-a/365}$, we get following.

$$\begin{aligned} &\approx 1 \times e^{-1/365} \times e^{-2/365} \cdots e^{-(n-1)/365} \\ &= 1 \times e^{-(1+2+\cdots+(n-1))/365} \\ &= e^{-(n(n-1)/2)/365}. \end{aligned} \tag{1}$$

Therefore,

p(same) = 1- p(different) $\approx 1 - e^{-n(n-1)/(2\times 365)}.$

An even coarser approximation is given by

p(same) $\approx 1 - e^{-n^2/(2\times 365)},$

By taking Log on both sides, we get the reverse formula.

$$n \approx \sqrt{2 \times 365 \ln\left(\frac{1}{1-p(same)}\right)}.$$

Using the above approximate formula, we can approximate number of people for a given probability. For example the following C++ function find() returns the smallest n for which the probability is greater than the given p.

C++ Implementation of approximate formula.
The following is C++ program to approximate number of people for a given probability.

```
// C++ program to approximate number of people in Birthday Paradox
// problem
#include <cmath>
#include <iostream>
using namespace std;

// Returns approximate number of people for a given probability
int find(double p)
{
    return ceil(sqrt(2*365*log(1/(1-p))));
}

int main()
{
   cout << find(0.70);
}
```

Output:

```
30
```

Source:
http://en.wikipedia.org/wiki/Birthday_problem

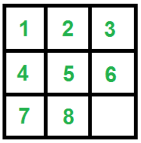
Applications:

1) Birthday Paradox is generally discussed with hashing to show importance of collision handling even for a small set of keys.
2) Birthday Attack

This article is contributed by **Shubham**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

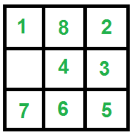
54. How to check if an instance of 8 puzzle is solvable?

What is 8 puzzle?
Given a 3×3 board with 8 tiles (every tile has one number from 1 to 8) and one empty space. The objective is to place the numbers on tiles in order using the empty space. We can slide four adjacent (left, right, above and below) tiles into the empty space.

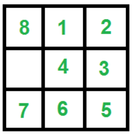


Goal State
Empty space can be anywhere

How to find if given state is solvable?
Following are two examples, the first example can reach goal state by a series of slides. The second example cannot.



Given State
Solvable
We can reach goal state by sliding tiles using blank space.



Given State
Not Solvable
We can not reach goal state by sliding tiles using blank space.

Following is simple rule to check if a 8 puzzle is solvable.
It is not possible to solve an instance of 8 puzzle if number of inversions is odd in the input state. In the examples given in above figure, the first example has 10 inversions, therefore solvable. The second example has 11 inversions, therefore unsolvable.

What is inversion?
A pair of tiles form an inversion if the the values on tiles are in reverse order of their appearance in goal state. For example, the following instance of 8 puzzle has two inversions, (8, 6) and (8, 7).

```
  1    2    3
  4    _    5
  8    6    7
```

Following is a simple C++ program to check whether a given instance of 8 puzzle is solvable or not. The idea is simple, we count inversions in the given 8 puzzle.

```
// C++ program to check if a given instance of 8 puzzle is solvable or
#include <iostream>
using namespace std;

// A utility function to count inversions in given array 'arr[]'
int getInvCount(int arr[])
{
    int inv_count = 0;
    for (int i = 0; i < 9 - 1; i++)
        for (int j = i+1; j < 9; j++)
             // Value 0 is used for empty space
             if (arr[j] && arr[i] &&  arr[i] > arr[j])
                  inv_count++;
    return inv_count;
}

// This function returns true if given 8 puzzle is solvable.
bool isSolvable(int puzzle[3][3])
{
    // Count inversions in given 8 puzzle
    int invCount = getInvCount((int *)puzzle);

    // return true if inversion count is even.
    return (invCount%2 == 0);
}

/* Driver progra to test above functions */
int main(int argv, int** args)
{
  int puzzle[3][3] =  {{1, 8, 2},
                      {0, 4, 3},  // Value 0 is used for empty space
                      {7, 6, 5}};
  isSolvable(puzzle)? cout << "Solvable":
                      cout << "Not Solvable";
  return 0;
}
```

Output:

```
Solvable
```

Note that the above implementation uses simple algorithm for inversion count. It is done this way for simplicity. The code can be optimized to O(nLogn) using the merge sort based algorithm for inversion count.

How does this work?

The idea is based on the fact the parity of inversions remains same after a set of moves, i.e., if the inversion count is odd in initial stage, then it remain odd after any sequence of moves and if the inversion count is even, then it remains even after any sequence of moves. In the goal state, there are 0 inversions. So we can reach goal state only from a state which has even inversion count.

How parity of inversion count is invariant?
When we slide a tile, we either make a row move (moving a left or right tile into the blank space), or make a column move (moving a up or down tile to the blank space).
a) A row move doesn't change the inversion count. See following example

```
 1   2   3    Row Move     1   2   3
 4   _   5   ---------->   _   4   5
 8   6   7                 8   6   7
Inversion count remains 2 after the move

 1   2   3    Row Move     1   2   3
 4   _   5   ---------->   4   5   _
 8   6   7                 8   6   7
Inversion count remains 2 after the move
```

b) A column move does one of the following three.
…..(i) Increases inversion count by 2. See following example.

```
       1   2   3   Column Move     1   _   3
       4   _   5   ----------->    4   2   5
       8   6   7                   8   6   7
    Inversion count increases by 2 (changes from 2 to 4)
```

…..(ii) Decreases inversion count by 2

```
       1   3   4    Column Move     1   3   4
       5   _   6   ------------>    5   2   6
       7   2   8                    7   _   8
    Inversion count decreases by 2 (changes from 5  to 3)
```

…..(iii) Keeps the inversion count same.

```
       1   2   3    Column Move     1   2   3
       4   _   5   ------------>    4   6   5
       7   6   8                    7   _   8
      Inversion count remains 1 after the move
```

So if a move either increases/decreases inversion count by 2, or keeps the inversion count same, then it is not possible to change parity of a state by any sequence of row/column moves.

Exercise: How to check if a given instance of 15 puzzle is solvable or not. In a 15 puzzle, we have 4×4 board where 15 tiles have a number and one empty space. Note that the above simple rules of inversion count don't directly work for 15 puzzle, the rules need to be modified for 15 puzzle.

This article is contributed by Ishan. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

55. Factorial of a large number

How to compute factorial of 100 using a C/C++ program?
Factorial of 100 has 158 digits. It is not possible to store these many digits even if we use long long int. Following is a simple solution where we use an array to store individual digits of the result. The idea is to use basic mathematics for multiplication.

The following is detailed algorithm for finding factorial.

factorial(n)
1) Create an array 'res[]' of MAX size where MAX is number of maximum digits in output.
2) Initialize value stored in 'res[]' as 1 and initialize 'res_size' (size of 'res[]') as 1.
3) Do following for all numbers from x = 2 to n.
……a) Multiply x with res[] and update res[] and res_size to store the multiplication result.

How to multiply a number 'x' with the number stored in res[]?
The idea is to use simple school mathematics. We one by one multiply x with every digit of res[]. The important point to note here is digits are multiplied from rightmost digit to leftmost digit. If we store digits in same order in res[], then it becomes difficult to update res[] without extra space. That is why res[] is maintained in reverse way, i.e., digits from right to left are stored.

multiply(res[], x)
1) Initialize carry as 0.
2) Do following for i = 0 to res_size – 1
….a) Find value of res[i] * x + carry. Let this value be prod.
….b) Update res[i] by storing last digit of prod in it.
….c) Update carry by storing remaining digits in carry.
3) Put all digits of carry in res[] and increase res_size by number of digits in carry.

```
Example to show working of multiply(res[], x)
A number 5189 is stored in res[] as following.
res[] = {9, 8, 1, 5}
x = 10

Initialize carry = 0;
```

```
i = 0, prod = res[0]*x + carry = 9*10 + 0 = 90.
res[0] = 0, carry = 9

i = 1, prod = res[1]*x + carry = 8*10 + 9 = 89
res[1] = 9, carry = 8

i = 2, prod = res[2]*x + carry = 1*10 + 8 = 18
res[2] = 8, carry = 1

i = 3, prod = res[3]*x + carry = 5*10 + 1 = 51
res[3] = 1, carry = 5

res[4] = carry = 5

res[] = {0, 9, 8, 1, 5}
```

Below is C++ implementation of above algorithm.

```cpp
// C++ program to compute factorial of big numbers
#include<iostream>
using namespace std;

// Maximum number of digits in output
#define MAX 500

int multiply(int x, int res[], int res_size)

// This function finds factorial of large numbers and prints them
void factorial(int n)
{
    int res[MAX];

    // Initialize result
    res[0] = 1;
    int res_size = 1;

    // Apply simple factorial formula n! = 1 * 2 * 3 * 4...*n
    for (int x=2; x<=n; x++)
        res_size = multiply(x, res, res_size);

    cout << "Factorial of given number is \n";
    for (int i=res_size-1; i>=0; i--)
        cout << res[i];
}

// This function multiplies x with the number represented by res[].
// res_size is size of res[] or number of digits in the number represe
// by res[]. This function uses simple school mathematics for multipli
// This function may value of res_size and returns the new value of re
int multiply(int x, int res[], int res_size)
{
    int carry = 0;  // Initialize carry

    // One by one multiply n with individual digits of res[]
    for (int i=0; i<res_size; i++)
    {
        int prod = res[i] * x + carry;
        res[i] = prod % 10;  // Store last digit of 'prod' in res[]
        carry  = prod/10;    // Put rest in carry
    }

    // Put carry in res and increase result size
    while (carry)
    {
        res[res_size] = carry%10;
        carry = carry/10;
        res_size++;
    }
    return res_size;
}

// Driver program
int main()
{
    factorial(100);
    return 0;
}
```

Output:

```
Factorial of given number is
93326215443944152681699238856266700490715968264381621468592963895
21759999322991560894146397615651828625369792082722375825118521091
6864000000000000000000000000
```

The above approach can be optimized in many ways. We will soon be discussing optimized solution for same.

This article is contributed by **Harshit Agrawal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

56. Find length of period in decimal value of 1/n

Given a positive integer n, find the period in decimal value of 1/n. Period in decimal value is number of digits (somewhere after decimal point) that keep repeating.

Examples:

```
Input:  n = 3
Output: 1
The value of 1/3 is 0.333333...

Input: n = 7
Output: 6
The value of 1/7 is 0.142857142857142857.....

Input: n = 210
Output: 6
The value of 1/210 is 0.0047619047619048.....
```

Let us first discuss a simpler problem of finding individual digits in value of 1/n.

How to find individual digits in value of 1/n?
Let us take an example to understand the process. For example for n = 7. The first digit can be obtained by doing 10/7. Second digit can be obtained by 30/7 (3 is remainder in previous division). Third digit can be obtained by 20/7 (2 is remainder of previous division). So the idea is to get the first digit, then keep taking value of (remainder * 10)/n as next digit and keep updating remainder as (remainder * 10) % 10. The complete program is discussed here.

How to find the period?
The period of 1/n is equal to the period in sequence of remainders used in the above

process. This can be easily proved from the fact that digits are directly derived from remainders.
One interesting fact about sequence of remainders is, all terns in period of this remainder sequence are distinct. The reason for this is simple, if a remainder repeats, then it's beginning of new period.

The following is C++ implementation of above idea.

```
// C++ program to find length of period of 1/n
#include <iostream>
#include <map>
using namespace std;

// Function to find length of period in 1/n
int getPeriod(int n)
{
   // Create a map to store mapping from remainder
   // its position
   map<int, int> mymap;
   map<int, int>::iterator it;

   // Initialize remainder and position of remainder
   int rem = 1, i = 1;

   // Keep finding remainders till a repeating remainder
   // is found
   while (true)
   {
        // Find next remainder
        rem = (10*rem) % n;

        // If remainder exists in mymap, then the difference
        // between current and previous position is length of
        // period
        it = mymap.find(rem);
        if (it != mymap.end())
            return (i - it->second);

        // If doesn't exist, then add 'i' to mymap
        mymap[rem] = i;
        i++;
   }

   // This code should never be reached
   return INT_MAX;
}

// Driver program to test above function
int main()
{
    cout <<  getPeriod(3) << endl;
    cout <<  getPeriod(7) << endl;
    return 0;
}
```

Output:

```
1
6
```

We can avoid the use of map or hash using the following fact. For a number n, there can be at most n distinct remainders. Also, the period may not begin from the first remainder as some initial remainders may be non-repetitive (not part of any period). So to make sure that a remainder from a period is picked, start from the (n+1)th remainder and keep looking for its next occurrence. The distance between (n+1)'th remainder and its next occurrence is the length of the period.

```cpp
// C++ program to find length of period of 1/n without
// using map or hash
#include <iostream>
using namespace std;

// Function to find length of period in 1/n
int getPeriod(int n)
{
   // Find the (n+1)th remainder after decimal point
   // in value of 1/n
   int rem = 1; // Initialize remainder
   for (int i = 1; i <= n+1; i++)
         rem = (10*rem) % n;

   // Store (n+1)th remainder
   int d = rem;

   // Count the number of remainders before next
   // occurrence of (n+1)'th remainder 'd'
   int count = 0;
   do {
      rem = (10*rem) % n;
      count++;
   } while(rem != d);

   return count;
}

// Driver program to test above function
int main()
{
    cout <<  getPeriod(3) << endl;
    cout <<  getPeriod(7) << endl;
    return 0;
}
```

Output:

```
1
6
```

Reference:
Algorithms And Programming: Problems And Solutions by Alexander Shen

This article is contributed by **Sachin**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

57. Greedy Algorithm for Egyptian Fraction

Every positive fraction can be represented as sum of unique unit fractions. A fraction is unit fraction if numerator is 1 and denominator is a positive integer, for example 1/3 is a unit fraction. Such a representation is called Egyptial Fraction as it was used by ancient Egyptians.

Following are few examples:

```
Egyptian Fraction Representation of 2/3 is 1/2 + 1/6
Egyptian Fraction Representation of 6/14 is 1/3 + 1/11 + 1/231
Egyptian Fraction Representation of 12/13 is 1/2 + 1/3 + 1/12 + 1/156
```

We can generate Egyptian Fractions using Greedy Algorithm. For a given number of the form 'nr/dr' where dr > nr, first find the greatest possible unit fraction, then recur for the remaining part. For example, consider 6/14, we first find ceiling of 14/6, i.e., 3. So the first unit fraction becomes 1/3, then recur for (6/14 – 1/3) i.e., 4/42.

Below is C++ implementation of above idea.

```
// C++ program to print a fraction in Egyptian Form using Greedy
// Algorithm
#include <iostream>
using namespace std;

void printEgyptian(int nr, int dr)
{
    // If either numerator or denominator is 0
    if (dr == 0 || nr == 0)
        return;

    // If numerator divides denominator, then simple division
    // makes the fraction in 1/n form
    if (dr%nr == 0)
    {
        cout << "1/" << dr/nr;
        return;
    }

    // If denominator divides numerator, then the given number
    // is not fraction
    if (nr%dr == 0)
    {
        cout << nr/dr;
        return;
    }

    // If numerator is more than denominator
    if (nr > dr)
    {
        cout << nr/dr << " + ";
        printEgyptian(nr%dr, dr);
        return;
    }

    // We reach here dr > nr and dr%nr is non-zero
    // Find ceiling of dr/nr and print it as first
    // fraction
    int n = dr/nr + 1;
    cout << "1/" << n << " + ";

    // Recur for remaining part
    printEgyptian(nr*n-dr, dr*n);
 }

// Driver Program
int main()
{
    int nr = 6, dr = 14;
    cout << "Egyptian Fraction Representation of "
         << nr << "/" << dr << " is\n ";
    printEgyptian(nr, dr);
    return 0;
}
```

Output:

```
Egyptian Fraction Representation of 6/14 is
 1/3 + 1/11 + 1/231
```

The Greedy algorithm works because a fraction is always reduced to a form where denominator is greater than numerator and numerator doesn't divide denominator. For such reduced forms, the highlighted recursive call is made for reduced numerator. So the recursive calls keep on reducing the numerator till it reaches 1.

References:
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html

This article is contributed by **Shubham**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

58. Write an iterative O(Log y) function for pow(x, y)

Given an integer x and a positive number y, write a function that computes x^y under following conditions.
a) Time complexity of the function should be O(Log y)
b) Extra Space is O(1)

Examples:

```
Input: x = 3, y = 5
Output: 243

Input: x = 2, y = 5
Output: 32
```

We strongly recommend to minimize your browser and try this yourself first.

We have discussed recursive O(Log y) solution for power. The recursive solutions are generally not preferred as they require space on call stack and they involve function call overhead.

Following is C function to compute x^y.

```
#include <stdio.h>

/* Iterative Function to calculate x raised to the power y in O(logy)
int power(int x, unsigned int y)
{
    // Initialize result
    int res = 1;

    while (y > 0)
    {
        // If y is even, simply do x square
        if (y%2 == 0)
        {
            y = y/2;
            x = x*x;
        }

        // Else multiply x with result.  Note that this else
        // is always executed in the end when y becomes 1
        else
        {
            y--;
            res = res*x;
        }
    }
    return res;
}

// Driver program to test above functions
int main()
{
    int x = 3;
    unsigned int y = 5;

    printf("Power is %d", power(x, y));

    return 0;
}
```

Output:

```
Power is 243
```

This article is contributed by **Udit Gupta**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above