1. Given an array A[] and a number x, check for pair in A[] with sum as x

Write a C program that, given an array A[] of n numbers and another number x, determines whether or not there exist two elements in S whose sum is exactly x.

**METHOD 1 (Use Sorting)**

Algorithm:

```
hasArrayTwoCandidates (A[], ar_size, sum)
1) Sort the array in non-decreasing order.
2) Initialize two index variables to find the candidate
   elements in the sorted array.
      (a) Initialize first to the leftmost index: l = 0
      (b) Initialize second  the rightmost index:  r = ar_size-1
3) Loop while l < r.
      (a) If (A[l] + A[r] == sum)  then return 1
      (b) Else if( A[l] + A[r] <  sum )  then l++
      (c) Else r--
4) No candidates in whole array - return 0
```

Time Complexity: Depends on what sorting algorithm we use. If we use Merge Sort or Heap Sort then (-)(nlogn) in worst case. If we use Quick Sort then O(n^2) in worst case.
Auxiliary Space : Again, depends on sorting algorithm. For example auxiliary space is O(n) for merge sort and O(1) for Heap Sort.

Example:
Let Array be {1, 4, 45, 6, 10, -8} and sum to find be 16

Sort the array
A = {-8, 1, 4, 6, 10, 45}

Initialize l = 0, r = 5
A[l] + A[r] ( -8 + 45) > 16 => decrement r. Now r = 10
A[l] + A[r] ( -8 + 10) < 2 => increment l. Now l = 1
A[l] + A[r] ( 1 + 10) < 16 => increment l. Now l = 2
A[l] + A[r] ( 4 + 10) < 14 => increment l. Now l = 3
A[l] + A[r] ( 6 + 10) == 16 => Found candidates (return 1)

Note: If there are more than one pair having the given sum then this algorithm reports only one. Can be easily extended for this though.

Implementation:

```
# include <stdio.h>
# define bool int

void quickSort(int *, int, int);

bool hasArrayTwoCandidates(int A[], int arr_size, int sum)
{
    int l, r;

    /* Sort the elements */
    quickSort(A, 0, arr_size-1);

    /* Now look for the two candidates in the sorted
       array*/
    l = 0;
    r = arr_size-1;
    while(l < r)
    {
        if(A[l] + A[r] == sum)
            return 1;
        else if(A[l] + A[r] < sum)
            l++;
        else // A[i] + A[j] > sum
            r--;
    }
    return 0;
}

/* Driver program to test above function */
int main()
{
    int A[] = {1, 4, 45, 6, 10, -8};
    int n = 16;
    int arr_size = 6;

    if( hasArrayTwoCandidates(A, arr_size, n))
        printf("Array has two elements with sum 16");
    else
        printf("Array doesn't have two elements with sum 16 ");

    getchar();
    return 0;
}
```

```
/* FOLLOWING FUNCTIONS ARE ONLY FOR SORTING
   PURPOSE */
void exchange(int *a, int *b)
{
   int temp;
   temp = *a;
   *a   = *b;
   *b   = temp;
}

int partition(int A[], int si, int ei)
{
   int x = A[ei];
   int i = (si - 1);
   int j;

   for (j = si; j <= ei - 1; j++)
   {
      if(A[j] <= x)
      {
         i++;
         exchange(&A[i], &A[j]);
      }
   }
   exchange (&A[i + 1], &A[ei]);
   return (i + 1);
}

/* Implementation of Quick Sort
A[] --> Array to be sorted
si  --> Starting index
ei  --> Ending index
*/
void quickSort(int A[], int si, int ei)
{
   int pi;    /* Partitioning index */
   if(si < ei)
   {
      pi = partition(A, si, ei);
      quickSort(A, si, pi - 1);
      quickSort(A, pi + 1, ei);
   }
}
```

**METHOD 2 (Use Hash Map)**

Thanks to Bindu for suggesting this method and thanks to Shekhu for providing code.
This method works in O(n) time if range of numbers is known.
Let sum be the given sum and A[] be the array in which we need to find pair.

```
1) Initialize Binary Hash Map M[] = {0, 0, â€¦}
2) Do following for each element A[i] in A[]
   (a) If M[x  - A[i]] is set then print the pair (A[i], x â€“ A[i])
   (b) Set M[A[i]]
```

Implementation:

```
#include <stdio.h>
#define MAX 100000

void printPairs(int arr[], int arr_size, int sum)
{
  int i, temp;
  bool binMap[MAX] = {0}; /*initialize hash map as 0*/

  for(i = 0; i < arr_size; i++)
  {
    temp = sum - arr[i];
    if(temp >= 0 && binMap[temp] == 1)
    {
      printf("Pair with given sum %d is (%d, %d) \n", sum, arr[i], temp);
    }
    binMap[arr[i]] = 1;
  }
}

/* Driver program to test above function */
int main()
{
    int A[] = {1, 4, 45, 6, 10, 8};
    int n = 16;
    int arr_size = 6;

    printPairs(A, arr_size, n);

    getchar();
    return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(R) where R is range of integers.

If range of numbers include negative numbers then also it works. All we have to do for negative numbers is to make everything positive by adding the absolute value of smallest negative integer to all numbers.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

2. Majority Element

**Majority Element:** A majority element in an array A[] of size n is an element that appears more than n/2 times (and hence there is at most one such element).

Write a function which takes an array and emits the majority element (if it exists), otherwise prints NONE as follows:

```
I/P : 3 3 4 2 4 4 2 4 4
O/P : 4

I/P : 3 3 4 2 4 4 2 4
O/P : NONE
```

**METHOD 1 (Basic)**
The basic solution is to have two loops and keep track of maximum count for all different elements. If maximum count becomes greater than n/2 then break the loops and return the element having maximum count. If maximum count doesnâ€™t become more than n/2 then majority element doesnâ€™t exist.

**Time Complexity:** O(n*n).
**Auxiliary Space :** O(1).

**METHOD 2 (Using Binary Search Tree)**
Thanks to Sachin Midha for suggesting this solution.

Node of the Binary Search Tree (used in this approach) will be as follows.

```
struct tree
{
  int element;
  int count;
}BST;
```

Insert elements in BST one by one and if an element is already present then increment the count of the node. At any stage, if count of a node becomes more than n/2 then return.
The method works well for the cases where n/2+1 occurrences of the majority element is present in the starting of the array, for example {1, 1, 1, 1, 1, 2, 3, 4}.

**Time Complexity:** If a binary search tree is used then time complexity will be O(n^2). If a self-balancing-binary-search tree is used then O(nlogn)
**Auxiliary Space:** O(n)

**METHOD 3 (Using Mooreâ€™s Voting Algorithm)**

This is a two step process.
1. Get an element occurring most of the time in the array. This phase will make sure that if there is a majority element then it will return that only.
2. Check if the element obtained from above step is majority element.

*1. Finding a Candidate:*
The algorithm for first phase that works in O(n) is known as Mooreâ€™s Voting Algorithm. Basic idea of the algorithm is if we cancel out each occurrence of an element e with all the other elements that are different from e then e will exist till end if it is a majority element.

```
findCandidate(a[], size)
1.  Initialize index and count of majority element
     maj_index = 0, count = 1
2.  Loop for i = 1 to size â€“ 1
    (a)If a[maj_index] == a[i]
         count++
    (b)Else
         count--;
    (c)If count == 0
         maj_index = i;
         count = 1
3.  Return a[maj_index]
```

Above algorithm loops through each element and maintains a count of a[maj_index], If next element is same then increments the count, if next element is not same then decrements the count, and if the count reaches 0 then changes the maj_index to the current element and sets count to 1.
First Phase algorithm gives us a candidate element. In second phase we need to check if the candidate is really a majority element. Second phase is simple and can be easily done in O(n). We just need to check if count of the candidate element is greater than n/2.

Example:
A[] = 2, 2, 3, 5, 2, 2, 6
Initialize:
maj_index = 0, count = 1 â€“> candidate â€˜2?
2, 2, 3, 5, 2, 2, 6

Same as a[maj_index] => count = 2
2, 2, 3, 5, 2, 2, 6

Different from a[maj_index] => count = 1
2, 2, 3, 5, 2, 2, 6

Different from a[maj_index] => count = 0
Since count = 0, change candidate for majority element to 5 => maj_index = 3, count = 1
2, 2, 3, 5, 2, 2, 6

Different from a[maj_index] => count = 0
Since count = 0, change candidate for majority element to 2 => maj_index = 4
2, 2, 3, 5, 2, 2, 6

Same as a[maj_index] => count = 2
2, 2, 3, 5, 2, 2, 6

Different from a[maj_index] => count = 1

Finally candidate for majority element is 2.

First step uses Mooreâ€™s Voting Algorithm to get a candidate for majority element.

2. *Check if the element obtained in step 1 is majority*

```
printMajority (a[], size)
1.  Find the candidate for majority
2.  If candidate is majority. i.e., appears more than n/2 times.
      Print the candidate
3.  Else
      Print "NONE"
```

**Implementation of method 3:**

```
/* Program for finding out majority element in an array */
# include<stdio.h>
# define bool int

int findCandidate(int *, int);
bool isMajority(int *, int, int);
```

```
/* Function to print Majority Element */
void printMajority(int a[], int size)
{
  /* Find the candidate for Majority*/
  int cand = findCandidate(a, size);

  /* Print the candidate if it is Majority*/
  if(isMajority(a, size, cand))
    printf(" %d ", cand);
  else
    printf("NO Majority Element");
}

/* Function to find the candidate for Majority */
int findCandidate(int a[], int size)
{
    int maj_index = 0, count = 1;
    int i;
    for(i = 1; i < size; i++)
    {
        if(a[maj_index] == a[i])
            count++;
        else
            count--;
        if(count == 0)
        {
            maj_index = i;
            count = 1;
        }
    }
    return a[maj_index];
}

/* Function to check if the candidate occurs more than n/2 times */
bool isMajority(int a[], int size, int cand)
{
    int i, count = 0;
    for (i = 0; i < size; i++)
      if(a[i] == cand)
         count++;
    if (count > size/2)
       return 1;
    else
       return 0;
```

```
}

/* Driver function to test above functions */
int main()
{
   int a[] = {1, 3, 3, 1, 2};
   printMajority(a, 5);
   getchar();
   return 0;
}
```

**Time Complexity:** O(n)
**Auxiliary Space :** O(1)

Now give a try to below question
Given an array of 2n elements of which n elements are same and the remaining n elements are all different. Write a C program to find out the value which is present n times in the array. There is no restriction on the elements in the array. They are random (In particular they not sequential).

Â  Â

### 3. Find the Number Occurring Odd Number of Times

Given an array of positive integers. All numbers occur even number of times except one number which occurs odd number of times. Find the number in O(n) time & constant space.

**Example:**
I/P = [1, 2, 3, 2, 3, 1, 3]
O/P = 3

**Algorithm:**
Do bitwise XOR of all the elements. Finally we get the number which has odd occurrences.

**Program:**

```
#include <stdio.h>

int getOddOccurrence(int ar[], int ar_size)
{
    int i;
```

```
    int res = 0;
    for (i=0; i < ar_size; i++)
       res = res ^ ar[i];

    return res;
}

/* Diver function to test above function */
int main()
{
    int ar[] = {2, 3, 5, 4, 5, 2, 4, 3, 5, 2, 4, 4, 2};
    int n = sizeof(ar)/sizeof(ar[0]);
    printf("%d", getOddOccurrence(ar, n));
    return 0;
}
```

**Time Complexity:** O(n)

Â Â

4. Largest Sum Contiguous Subarray

Write an efficient C program to find the sum of contiguous subarray within a one-dimensional array of numbers which has the largest sum.

**Kadaneâ€™s Algorithm:**

```
Initialize:
   max_so_far = 0
   max_ending_here = 0

Loop for each element of the array
  (a) max_ending_here = max_ending_here + a[i]
  (b) if(max_ending_here < 0)
            max_ending_here = 0
  (c) if(max_so_far < max_ending_here)
            max_so_far = max_ending_here
return max_so_far
```

**Explanation:**
Simple idea of the Kadane's algorithm is to look for all positive contiguous segments of the array (max_ending_here is used for this). And keep track of maximum sum contiguous segment among all positive segments (max_so_far is used for this). Each

time we get a positive sum compare it with max_so_far and update max_so_far if it is greater than max_so_far

Lets take the example:
{-2, -3, 4, -1, -2, 1, 5, -3}

max_so_far = max_ending_here = 0

for i=0, a[0] = -2
max_ending_here = max_ending_here + (-2)
Set max_ending_here = 0 because max_ending_here < 0

for i=1, a[1] = -3
max_ending_here = max_ending_here + (-3)
Set max_ending_here = 0 because max_ending_here < 0

for i=2, a[2] = 4
max_ending_here = max_ending_here + (4)
max_ending_here = 4
max_so_far is updated to 4 because max_ending_here greater than max_so_far which was 0 till now

for i=3, a[3] = -1
max_ending_here = max_ending_here + (-1)
max_ending_here = 3

for i=4, a[4] = -2
max_ending_here = max_ending_here + (-2)
max_ending_here = 1

for i=5, a[5] = 1
max_ending_here = max_ending_here + (1)
max_ending_here = 2

for i=6, a[6] = 5
max_ending_here = max_ending_here + (5)
max_ending_here = 7
max_so_far is updated to 7 because max_ending_here is greater than max_so_far

for i=7, a[7] = -3
max_ending_here = max_ending_here + (-3)
max_ending_here = 4

**Program:**

```
#include<stdio.h>
int maxSubArraySum(int a[], int size)
{
```

```
    int max_so_far = 0, max_ending_here = 0;
    int i;
    for(i = 0; i < size; i++)
    {
      max_ending_here = max_ending_here + a[i];
      if(max_ending_here < 0)
         max_ending_here = 0;
      if(max_so_far < max_ending_here)
         max_so_far = max_ending_here;
     }
     return max_so_far;
}

/*Driver program to test maxSubArraySum*/
int main()
{
    int a[] = {-2, -3, 4, -1, -2, 1, 5, -3};
    int n = sizeof(a)/sizeof(a[0]);
    int max_sum = maxSubArraySum(a, n);
    printf("Maximum contiguous sum is %d\n", max_sum);
    getchar();
    return 0;
}
```

**Notes:**
Algorithm doesn't work for all negative numbers. It simply returns 0 if all numbers are negative. For handling this we can add an extra phase before actual implementation. The phase will look if all numbers are negative, if they are it will return maximum of them (or smallest in terms of absolute value). There may be other ways to handle it though.

Above program can be optimized further, if we compare max_so_far with max_ending_here only if max_ending_here is greater than 0.

```
int maxSubArraySum(int a[], int size)
{
    int max_so_far = 0, max_ending_here = 0;
    int i;
    for(i = 0; i < size; i++)
    {
      max_ending_here = max_ending_here + a[i];
      if(max_ending_here < 0)
          max_ending_here = 0;

      /* Do not compare for all elements. Compare only
```

```
          when  max_ending_here > 0 */
        else if (max_so_far < max_ending_here)
            max_so_far = max_ending_here;
      }
      return max_so_far;
   }
```

**Time Complexity:** O(n)
**Algorithmic Paradigm:** Dynamic Programming

Following is another simple implementation suggested by **Mohit Kumar**. The implementation handles the case when all numbers in array are negative.

```
#include<stdio.h>

int max(int x, int y)
{ return (y > x)? y : x; }

int maxSubArraySum(int a[], int size)
{
   int max_so_far = a[0], i;
   int curr_max = a[0];

   for (i = 1; i < size; i++)
   {
       curr_max = max(a[i], curr_max+a[i]);
       max_so_far = max(max_so_far, curr_max);
   }
   return max_so_far;
}

/* Driver program to test maxSubArraySum */
int main()
{
   int a[] =  {-2, -3, 4, -1, -2, 1, 5, -3};
   int n = sizeof(a)/sizeof(a[0]);
   int max_sum = maxSubArraySum(a, n);
   printf("Maximum contiguous sum is %d\n", max_sum);
   return 0;
}
```

Now try below question
Given an array of integers (possibly some of the elements negative), write a C program to find out the *maximum product* possible by adding 'n' consecutive integers in the array, n <= ARRAY_SIZE. Also give where in the array this sequence of n

integers starts.

**References:**
http://en.wikipedia.org/wiki/Kadane%27s_Algorithm

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

5. Find the Missing Number

You are given a list of n-1 integers and these integers are in the range of 1 to n. There are no duplicates in list. One of the integers is missing in the list. Write an efficient code to find the missing integer.

**Example:**
I/P [1, 2, 4, ,6, 3, 7, 8]
O/P 5

**METHOD 1(Use sum formula)**
Algorithm:

```
1. Get the sum of numbers
      total = n*(n+1)/2
2  Subtract all the numbers from sum and
   you will get the missing number.
```

Program:

```
#include<stdio.h>

/* getMissingNo takes array and size of array as arguments*/
int getMissingNo (int a[], int n)
{
    int i, total;
    total  = (n+1)*(n+2)/2;
    for ( i = 0; i< n; i++)
       total -= a[i];
    return total;
}

/*program to test above function */
```

```
int main()
{
    int a[] = {1,2,4,5,6};
    int miss = getMissingNo(a,5);
    printf("%d", miss);
    getchar();
}
```

Time Complexity: O(n)

**METHOD 2(Use XOR)**

```
 1) XOR all the array elements, let the result of XOR be X1.
 2) XOR all numbers from 1 to n, let XOR be X2.
 3) XOR of X1 and X2 gives the missing number.
```

```
#include<stdio.h>

/* getMissingNo takes array and size of array as arguments*/
int getMissingNo(int a[], int n)
{
    int i;
    int x1 = a[0]; /* For xor of all the elemets in arary */
    int x2 = 1; /* For xor of all the elemets from 1 to n+1 */

    for (i = 1; i< n; i++)
        x1 = x1^a[i];

    for ( i = 2; i <= n+1; i++)
        x2 = x2^i;

    return (x1^x2);
}

/*program to test above function */
int main()
{
    int a[] = {1, 2, 4, 5, 6};
    int miss = getMissingNo(a, 5);
    printf("%d", miss);
    getchar();
}
```

Time Complexity: O(n)

In method 1, if the sum of the numbers goes beyond maximum allowed integer, then there can be integer overflow and we may not get correct answer. Method 2 has no such problems.

Â Â

6. Search an element in a sorted and pivoted array

**Question:**
An element in a sorted array can be found in O(log n) time via binary search. But suppose I rotate the sorted array at some pivot unknown to you beforehand. So for instance, 1 2 3 4 5 might become 3 4 5 1 2. Devise a way to find an element in the rotated array in O(log n) time.


**Solution:**
Thanks to Ajay Mishra for initial solution.

**Algorithm:**
Find the pivot point, divide the array in two sub-arrays and call binary search.
The main idea for finding pivot is â€“ for a sorted (in increasing order) and pivoted array, pivot element is the only only element for which next element to it is smaller than it.
Using above criteria and binary search methodology we can get pivot element in O(logn) time

```
Input arr[] = {3, 4, 5, 1, 2}
Element to Search = 1
 1) Find out pivot point and divide the array in two
    sub-arrays. (pivot = 2) /*Index of 5*/
 2) Now call binary search for one of the two sub-arrays.
    (a) If element is greater than 0th element then
           search in left array
    (b) Else Search in right array
          (1 will go in else as 1 < 0th element(3))
 3) If element is found in selected sub-array then return index
    Else return -1.
```

**Implementation:**

```
/* Program to search an element in a sorted and pivoted array*/
```

```
#include <stdio.h>

int findPivot(int[], int, int);
int binarySearch(int[], int, int, int);

/* Searches an element no in a pivoted sorted array arrp[]
   of size arr_size */
int pivotedBinarySearch(int arr[], int arr_size, int no)
{
   int pivot = findPivot(arr, 0, arr_size-1);

   // If we didn't find a pivot, then array is not rotated at all
   if (pivot == -1)
     return binarySearch(arr, 0, arr_size-1, no);

   // If we found a pivot, then first compare with pivot and then
   // search in two subarrays around pivot
   if (arr[pivot] == no)
     return pivot;
   if (arr[0] <= no)
     return binarySearch(arr, 0, pivot-1, no);
   else
     return binarySearch(arr, pivot+1, arr_size-1, no);
}

/* Function to get pivot. For array 3, 4, 5, 6, 1, 2 it will
   return 3. If array is not rotated at all, then it returns -1 */
int findPivot(int arr[], int low, int high)
{
   // base cases
   if (high < low)  return -1;
   if (high == low) return low;

   int mid = (low + high)/2;   /*low + (high - low)/2;*/
   if (mid < high && arr[mid] > arr[mid + 1])
     return mid;
   if (mid > low && arr[mid] < arr[mid - 1])
     return (mid-1);
   if (arr[low] >= arr[mid])
     return findPivot(arr, low, mid-1);
   else
     return findPivot(arr, mid + 1, high);
}
```

```
/* Standard Binary Search function*/
int binarySearch(int arr[], int low, int high, int no)
{
  if (high < low)
      return -1;
  int mid = (low + high)/2;  /*low + (high - low)/2;*/
  if (no == arr[mid])
    return mid;
  if (no > arr[mid])
    return binarySearch(arr, (mid + 1), high, no);
  else
    return binarySearch(arr, low, (mid -1), no);
}


/* Driver program to check above functions */
int main()
{
  // Let us search 3 in below array
  int arr1[] = {5, 6, 7, 8, 9, 10, 1, 2, 3};
  int arr_size = sizeof(arr1)/sizeof(arr1[0]);
  int no = 3;
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr1, arr_size, no));

   // Let us search 3 in below array
  int arr2[] = {3, 4, 5, 1, 2};
  arr_size = sizeof(arr2)/sizeof(arr2[0]);
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr2, arr_size, no));

  // Let us search for 4 in above array
  no = 4;
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr2, arr_size, no));

  // Let us search 0 in below array
  int arr3[] = {1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1};
  no = 0;
  arr_size = sizeof(arr3)/sizeof(arr3[0]);
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr3, arr_size, no));

  // Let us search 3 in below array
  int arr4[] = {2, 3, 0, 2, 2, 2, 2, 2, 2, 2};
  no = 3;
  arr_size = sizeof(arr4)/sizeof(arr4[0]);
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr4, arr_size, no));
```

```
  // Let us search 2 in above array
  no = 2;
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr4, arr_size, no));

  // Let us search 3 in below array
  int arr5[] = {1, 2, 3, 4};
  no = 3;
  arr_size = sizeof(arr5)/sizeof(arr5[0]);
  printf("Index of the element is %d\n",  pivotedBinarySearch(arr5, arr_size, no));

  return 0;
}
```

Output:

```
Index of the element is 8
Index of the element is 0
Index of the element is 1
Index of the element is 3
Index of the element is 1
Index of the element is 0
Index of the element is 2
```

Please note that the solution may not work for cases where the input array has duplicates.

**Time Complexity** O(logn)

Please write comments if you find any bug in above codes/algorithms, or find other ways to solve the same problem.

Â  Â

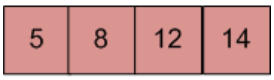
7.  Merge an array of size n into another array of size m+n

Asked by Binod
**Question:**
There are two sorted arrays. First one is of size m+n containing only m elements. Another one is of size n and contains n elements. Merge these two arrays into the first array of size m+n such that the output is sorted.

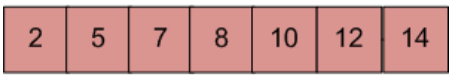
Input: array with m+n elements (mPlusN[]).

NA => Value is not filled/available in array mPlusN[]. There should be n such array blocks.

Input: array with n elements (N[]).




Output: N[] merged into mPlusN[] (Modified mPlusN[])




### Algorithm:

```
Let first array be mPlusN[] and other array be N[]
1) Move m elements of mPlusN[] to end.
2) Start from nth element of mPlusN[] and 0th element of N[] and merge them
   into mPlusN[].
```

### Implementation:

```
#include <stdio.h>

/* Assuming -1 is filled for the places where element
   is not available */
#define NA -1

/* Function to move m elements at the end of array mPlusN[] */
void moveToEnd(int mPlusN[], int size)
{
  int i = 0, j = size - 1;
  for (i = size-1; i >= 0; i--)
    if (mPlusN[i] != NA)
    {
      mPlusN[j] = mPlusN[i];
      j--;
    }
}

/* Merges array N[] of size n into array mPlusN[]
   of size m+n*/
int merge(int mPlusN[], int N[], int m, int n)
{
  int i = n;  /* Current index of i/p part of mPlusN[]*/
  int j = 0; /* Current index of N[]*/
```

```
  int k = 0; /* Current index of of output mPlusN[]*/
  while (k < (m+n))
  {
    /* Take an element from mPlusN[] if
       a) value of the picked element is smaller and we have
          not reached end of it
       b) We have reached end of N[] */
    if ((i < (m+n) && mPlusN[i] <= N[j]) || (j == n))
    {
      mPlusN[k] = mPlusN[i];
      k++;
      i++;
    }
    else  // Otherwise take emenet from N[]
    {
      mPlusN[k] = N[j];
      k++;
      j++;
    }
  }
}

/* Utility that prints out an array on a line */
void printArray(int arr[], int size)
{
  int i;
  for (i=0; i < size; i++)
    printf("%d ", arr[i]);

  printf("\n");
}

/* Driver function to test above functions */
int main()
{
  /* Initialize arrays */
  int mPlusN[] = {2, 8, NA, NA, NA, 13, NA, 15, 20};
  int N[] = {5, 7, 9, 25};
  int n = sizeof(N)/sizeof(N[0]);
  int m = sizeof(mPlusN)/sizeof(mPlusN[0]) - n;

  /*Move the m elements at the end of mPlusN*/
  moveToEnd(mPlusN, m+n);
```

```
  /*Merge N[] into mPlusN[] */
  merge(mPlusN, N, m, n);

  /* Print the resultant mPlusN */
  printArray(mPlusN, m+n);

  return 0;
}
```

Output:

```
2 5 7 8 9 13 15 20 25
```

**Time Complexity:** O(m+n)

Please write comment if you find any bug in the above program or a better way to solve the same problem.

Â  Â

8. Median of two sorted arrays

*Question:* There are 2 sorted arrays A and B of size n each. Write an algorithm to find the median of the array obtained after merging the above 2 arrays(i.e. array of length 2n). The complexity should be O(log(n))

*Median:* In probability theory and statistics, a median is described as the number separating the higher half of a sample, a population, or a probability distribution, from the lower half.
The median of a finite list of numbers can be found by arranging all the numbers from lowest value to highest value and picking the middle one.

For getting the median of input array { 12, 11, 15, 10, 20 }, first sort the array. We get { 10, 11, 12, 15, 20 } after sorting. Median is the middle element of the sorted array which is 12.

There are different conventions to take median of an array with even number of elements, one can take the mean of the two middle values, or first middle value, or second middle value.

Let us see different methods to get the median of two sorted arrays of size n each. Since size of the set for which we are looking for median is even (2n), we are taking average of middle two numbers in all below solutions.

**Method 1 (Simply count while Merging)**

Use merge procedure of merge sort. Keep track of count while comparing elements of two arrays. If count becomes n(For 2n elements), we have reached the median. Take the average of the elements at indexes n-1 and n in the merged array. See the below implementation.

Implementation:

```
#include <stdio.h>

/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    int i = 0;  /* Current index of i/p array ar1[] */
    int j = 0; /* Current index of i/p array ar2[] */
    int count;
    int m1 = -1, m2 = -1;

    /* Since there are 2n elements, median will be average
     of elements at index n-1 and n in the array obtained after
     merging ar1 and ar2 */
    for (count = 0; count <= n; count++)
    {
        /*Below is to handle case where all elements of ar1[] are
          smaller than smallest(or first) element of ar2[]*/
        if (i == n)
        {
            m1 = m2;
            m2 = ar2[0];
            break;
        }

        /*Below is to handle case where all elements of ar2[] are
          smaller than smallest(or first) element of ar1[]*/
        else if (j == n)
        {
            m1 = m2;
            m2 = ar1[0];
            break;
        }

        if (ar1[i] < ar2[j])
```

```
        {
            m1 = m2;  /* Store the prev median */
            m2 = ar1[i];
            i++;
        }
        else
        {
            m1 = m2;  /* Store the prev median */
            m2 = ar2[j];
            j++;
        }
    }

    return (m1 + m2)/2;
}

/* Driver program to test above function */
int main()
{
    int ar1[] = {1, 12, 15, 26, 38};
    int ar2[] = {2, 13, 17, 30, 45};

    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
        printf("Median is %d", getMedian(ar1, ar2, n1));
    else
        printf("Doesn't work for arrays of unequal size");
    getchar();
    return 0;
}
```

Time Complexity: O(n)

**Method 2 (By comparing the medians of two arrays)**
This method works by first getting medians of the two sorted arrays and then comparing them.

Let ar1 and ar2 be the input arrays.

Algorithm:

```
1) Calculate the medians m1 and m2 of the input arrays ar1[]
```

```
  and ar2[] respectively.
2) If m1 and m2 both are equal then we are done.
     return m1 (or m2)
3) If m1 is greater than m2, then median is present in one
   of the below two subarrays.
    a)  From first element of ar1 to m1 (ar1[0...|_n/2_|])
    b)  From m2 to last element of ar2  (ar2[|_n/2_|...n-1])
4) If m2 is greater than m1, then median is present in one
   of the below two subarrays.
   a)  From m1 to last element of ar1  (ar1[|_n/2_|...n-1])
   b)  From first element of ar2 to m2 (ar2[0...|_n/2_|])
5) Repeat the above process until size of both the subarrays
   becomes 2.
6) If size of the two arrays is 2 then use below formula to get
  the median.
    Median = (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1]))/2
```

Example:

```
  ar1[] = {1, 12, 15, 26, 38}
  ar2[] = {2, 13, 17, 30, 45}
```

For above two arrays m1 = 15 and m2 = 17

For the above ar1[] and ar2[], m1 is smaller than m2. So median is present in one of the following two subarrays.

```
  [15, 26, 38] and [2, 13, 17]
```

Let us repeat the process for above two subarrays:

```
   m1 = 26 m2 = 13.
```

m1 is greater than m2. So the subarrays become

```
  [15, 26] and [13, 17]
Now size is 2, so median = (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1]))/2
                         = (max(15, 13) + min(26, 17))/2
                         = (15 + 17)/2
                         = 16
```

Implementation:

```
#include<stdio.h>

int max(int, int); /* to get maximum of two integers */
int min(int, int); /* to get minimum of two integeres */
int median(int [], int); /* to get median of a sorted array */
```

```
/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    int m1; /* For median of ar1 */
    int m2; /* For median of ar2 */

    /* return -1  for invalid input */
    if (n <= 0)
        return -1;

    if (n == 1)
        return (ar1[0] + ar2[0])/2;

    if (n == 2)
        return (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1])) / 2;

    m1 = median(ar1, n); /* get the median of the first array */
    m2 = median(ar2, n); /* get the median of the second array */

    /* If medians are equal then return either m1 or m2 */
    if (m1 == m2)
        return m1;

     /* if m1 < m2 then median must exist in ar1[m1....] and ar2[....m2] */
    if (m1 < m2)
    {
        if (n % 2 == 0)
            return getMedian(ar1 + n/2 - 1, ar2, n - n/2 +1);
        else
            return getMedian(ar1 + n/2, ar2, n - n/2);
    }

    /* if m1 > m2 then median must exist in ar1[....m1] and ar2[m2...] */
    else
    {
        if (n % 2 == 0)
            return getMedian(ar2 + n/2 - 1, ar1, n - n/2 + 1);
        else
            return getMedian(ar2 + n/2, ar1, n - n/2);
    }
```

```
}

/* Function to get median of a sorted array */
int median(int arr[], int n)
{
    if (n%2 == 0)
        return (arr[n/2] + arr[n/2-1])/2;
    else
        return arr[n/2];
}

/* Driver program to test above function */
int main()
{
    int ar1[] = {1, 2, 3, 6};
    int ar2[] = {4, 6, 8, 10};
    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
      printf("Median is %d", getMedian(ar1, ar2, n1));
    else
     printf("Doesn't work for arrays of unequal size");

    getchar();
    return 0;
}

/* Utility functions */
int max(int x, int y)
{
    return x > y? x : y;
}

int min(int x, int y)
{
    return x > y? y : x;
}
```

Time Complexity: O(logn)
Algorithmic Paradigm: Divide and Conquer

**Method 3 (By doing binary search for the median):**
The basic idea is that if you are given two arrays ar1[] and ar2[] and know the length of

each, you can check whether an element ar1[i] is the median in constant time. Suppose that the median is ar1[i]. Since the array is sorted, it is greater than exactly i values in array ar1[]. Then if it is the median, it is also greater than exactly j = n â€“ i â€“ 1 elements in ar2[].
It requires constant time to check if ar2[j] <= ar1[i] <= ar2[j + 1]. If ar1[i] is not the median, then depending on whether ar1[i] is greater or less than ar2[j] and ar2[j + 1], you know that ar1[i] is either greater than or less than the median. Thus you can binary search for median in O(lg n) worst-case time.

For two arrays ar1 and ar2, first do binary search in ar1[]. If you reach at the end (left or right) of the first array and don't find median, start searching in the second array ar2[].

```
1) Get the middle element of ar1[] using array indexes left and right.
   Let index of the middle element be i.
2) Calculate the corresponding index j of ar2[]
     j = n â€“ i â€“ 1
3) If ar1[i] >= ar2[j] and ar1[i] <= ar2[j+1] then ar1[i] and ar2[j]
   are the middle elements.
     return average of ar2[j] and ar1[i]
4) If ar1[i] is greater than both ar2[j] and ar2[j+1] then
     do binary search in left half  (i.e., arr[left ... i-1])
5) If ar1[i] is smaller than both ar2[j] and ar2[j+1] then
     do binary search in right half (i.e., arr[i+1....right])
6) If you reach at any corner of ar1[] then do binary search in ar2[]
```

Example:

```
  ar1[] = {1, 5, 7, 10, 13}
  ar2[] = {11, 15, 23, 30, 45}
```

Middle element of ar1[] is 7. Let us compare 7 with 23 and 30, since 7 smaller than both 23 and 30, move to right in ar1[]. Do binary search in {10, 13}, this step will pick 10. Now compare 10 with 15 and 23. Since 10 is smaller than both 15 and 23, again move to right. Only 13 is there in right side now. Since 13 is greater than 11 and smaller than 15, terminate here. We have got the median as 12 (average of 11 and 13)

Implementation:

```
#include<stdio.h>

int getMedianRec(int ar1[], int ar2[], int left, int right, int n);

/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
```

```
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    return getMedianRec(ar1, ar2, 0, n-1, n);
}

/* A recursive function to get the median of ar1[] and ar2[]
   using binary search */
int getMedianRec(int ar1[], int ar2[], int left, int right, int n)
{
    int i, j;

    /* We have reached at the end (left or right) of ar1[] */
    if (left > right)
        return getMedianRec(ar2, ar1, 0, n-1, n);

    i = (left + right)/2;
    j = n - i - 1;  /* Index of ar2[] */

    /* Recursion terminates here.*/
    if (ar1[i] > ar2[j] && (j == n-1 || ar1[i] <= ar2[j+1]))
    {
        /* ar1[i] is decided as median 2, now select the median 1
           (element just before ar1[i] in merged array) to get the
           average of both*/
        if (i == 0 || ar2[j] > ar1[i-1])
            return (ar1[i] + ar2[j])/2;
        else
            return (ar1[i] + ar1[i-1])/2;
    }

    /*Search in left half of ar1[]*/
    else if (ar1[i] > ar2[j] && j != n-1 && ar1[i] > ar2[j+1])
        return getMedianRec(ar1, ar2, left, i-1, n);

    /*Search in right half of ar1[]*/
    else /* ar1[i] is smaller than both ar2[j] and ar2[j+1]*/
        return getMedianRec(ar1, ar2, i+1, right, n);
}

/* Driver program to test above function */
int main()
{
```

```
    int ar1[] = {1, 12, 15, 26, 38};
    int ar2[] = {2, 13, 17, 30, 45};
    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
        printf("Median is %d", getMedian(ar1, ar2, n1));
    else
        printf("Doesn't work for arrays of unequal size");

    getchar();
    return 0;
}
```

Time Complexity: O(logn)
Algorithmic Paradigm: Divide and Conquer

The above solutions can be optimized for the cases when all elements of one array are smaller than all elements of other array. For example, in method 3, we can change the getMedian() function to following so that these cases can be handled in O(1) time. Thanks to nutcracker for suggesting this optimization.

```
/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
   // If all elements of array 1 are smaller then
   // median is average of last element of ar1 and
   // first element of ar2
   if (ar1[n-1] < ar2[0])
     return (ar1[n-1]+ar2[0])/2;

   // If all elements of array 1 are smaller then
   // median is average of first element of ar1 and
   // last element of ar2
   if (ar2[n-1] < ar1[0])
     return (ar2[n-1]+ar1[0])/2;

   return getMedianRec(ar1, ar2, 0, n-1, n);
}
```

**References:**
http://en.wikipedia.org/wiki/Median

http://ocw.alfaisal.edu/NR/rdonlyres/Electrical-Engineering-and-Computer-Science/6-046JFall-2005/30C68118-E436-4FE3-8C79-6BAFBB07D935/0/ps9sol.pdf
ds3etph5wn

Asked by Snehal

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

9. Write a program to reverse an array

**Iterative way:**
1) Initialize start and end indexes.
start = 0, end = n-1
2) In a loop, swap arr[start] with arr[end] and change start and end as follows.
start = start +1; end = end â€“ 1

```
/* Function to reverse arr[] from start to end*/
void rvereseArray(int arr[], int start, int end)
{
  int temp;
  while(start < end)
  {
    temp = arr[start];
    arr[start] = arr[end];
    arr[end] = temp;
    start++;
    end--;
  }
}

/* Utility that prints out an array on a line */
void printArray(int arr[], int size)
{
  int i;
  for (i=0; i < size; i++)
    printf("%d ", arr[i]);

  printf("\n");
}

/* Driver function to test above functions */
```

```
int main()
{
  int arr[] = {1, 2, 3, 4, 5, 6};
  printArray(arr, 6);
  rvereseArray(arr, 0, 5);
  printf("Reversed array is \n");
  printArray(arr, 6);
  getchar();
  return 0;
}
```

Time Complexity: O(n)

**Recursive Way:**

1) Initialize start and end indexes
start = 0, end = n-1
2) Swap arr[start] with arr[end]
3) Recursively call reverse for rest of the array.

```
/* Function to reverse arr[] from start to end*/
void rvereseArray(int arr[], int start, int end)
{
   int temp;
   if(start >= end)
     return;
   temp = arr[start];
   arr[start] = arr[end];
   arr[end] = temp;
   rvereseArray(arr, start+1, end-1);
}

/* Utility that prints out an array on a line */
void printArray(int arr[], int size)
{
  int i;
  for (i=0; i < size; i++)
    printf("%d ", arr[i]);

  printf("\n");
}

/* Driver function to test above functions */
int main()
{
  int arr[] = {1, 2, 3, 4, 5};
```

```
  printArray(arr, 5);
  rvereseArray(arr, 0, 4);
  printf("Reversed array is \n");
  printArray(arr, 5);
  getchar();
  return 0;
}
```

Time Complexity: O(n)

Please write comments if you find any bug in the above programs or other ways to solve the same problem.

Â  Â

10.  Program for array rotation

Write a function rotate(ar[], d, n) that rotates arr[] of size n by d elements.


Rotation of the above array by 2 will make array


**METHOD 1 (Use temp array)**

```
Input arr[] = [1, 2, 3, 4, 5, 6, 7], d = 2, n =7
1) Store d elements in a temp array
   temp[] = [1, 2]
2) Shift rest of the arr[]
   arr[] = [3, 4, 5, 6, 7, 6, 7]
3) Store back the d elements
   arr[] = [3, 4, 5, 6, 7, 1, 2]
```

**Time complexity** O(n)
**Auxiliary Space:** O(d)

**METHOD 2 (Rotate one by one)**

```
leftRotate(arr[], d, n)
start
  For i = 0 to i < d
```

```
   Left rotate all elements of arr[] by one
end
```

To rotate by one, store arr[0] in a temporary variable temp, move arr[1] to arr[0], arr[2] to arr[1] â€¦and finally temp to arr[n-1]

Let us take the same example arr[] = [1, 2, 3, 4, 5, 6, 7], d = 2
Rotate arr[] by one 2 times
We get [2, 3, 4, 5, 6, 7, 1] after first rotation and [ 3, 4, 5, 6, 7, 1, 2] after second rotation.

```
/*Function to left Rotate arr[] of size n by 1*/
void leftRotatebyOne(int arr[], int n);

/*Function to left rotate arr[] of size n by d*/
void leftRotate(int arr[], int d, int n)
{
  int i;
  for (i = 0; i < d; i++)
    leftRotatebyOne(arr, n);
}

void leftRotatebyOne(int arr[], int n)
{
  int i, temp;
  temp = arr[0];
  for (i = 0; i < n-1; i++)
     arr[i] = arr[i+1];
  arr[i] = temp;
}

/* utility function to print an array */
void printArray(int arr[], int size)
{
  int i;
  for(i = 0; i < size; i++)
    printf("%d ", arr[i]);
}

/* Driver program to test above functions */
int main()
{
   int arr[] = {1, 2, 3, 4, 5, 6, 7};
   leftRotate(arr, 2, 7);
   printArray(arr, 7);
```

```
  getchar();
  return 0;
}
```

**Time complexity:** O(n*d)
**Auxiliary Space:** O(1)

**METHOD 3 (A Juggling Algorithm)**
This is an extension of method 2. Instead of moving one by one, divide the array in different sets
where number of sets is equal to GCD of n and d and move the elements within sets.
If GCD is 1 as is for the above example array (n = 7 and d =2), then elements will be moved within one set only, we just start with temp = arr[0] and keep moving arr[I+d] to arr[I] and finally store temp at the right place.

Here is an example for n =12 and d = 3. GCD is 3 and

```
Let arr[] be {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

a) Elements are first moved in first set â€“ (See below diagram for this movement)




        arr[] after this step --> {4 2 3 7 5 6 10 8 9 1 11 12}

b) Then in second set.
        arr[] after this step --> {4 5 3 7 8 6 10 11 9 1 2 12}

c) Finally in third set.
        arr[] after this step --> {4 5 6 7 8 9 10 11 12 1 2 3}
```

```
/* function to print an array */
void printArray(int arr[], int size);

/*Fuction to get gcd of a and b*/
int gcd(int a,int b);

/*Function to left rotate arr[] of siz n by d*/
void leftRotate(int arr[], int d, int n)
{
  int i, j, k, temp;
  for (i = 0; i < gcd(d, n); i++)
  {
    /* move i-th values of blocks */
    temp = arr[i];
```

```
    j = i;
    while(1)
    {
      k = j + d;
      if (k >= n)
        k = k - n;
      if (k == i)
        break;
      arr[j] = arr[k];
      j = k;
    }
    arr[j] = temp;
  }
}

/*UTILITY FUNCTIONS*/
/* function to print an array */
void printArray(int arr[], int size)
{
  int i;
  for(i = 0; i < size; i++)
    printf("%d ", arr[i]);
}

/*Fuction to get gcd of a and b*/
int gcd(int a,int b)
{
   if(b==0)
     return a;
   else
     return gcd(b, a%b);
}

/* Driver program to test above functions */
int main()
{
   int arr[] = {1, 2, 3, 4, 5, 6, 7};
   leftRotate(arr, 2, 7);
   printArray(arr, 7);
   getchar();
   return 0;
}
```

**Time complexity:** O(n)

**Auxiliary Space:** O(1)

Please see following posts for other methods of array rotation:
Block swap algorithm for array rotation
Reversal algorithm for array rotation

**References:**
http://www.cs.bell-labs.com/cm/cs/pearls/s02b.pdf

Please write comments if you find any bug in above programs/algorithms.

Â Â

11. Reversal algorithm for array rotation

Write a function rotate(arr[], d, n) that rotates arr[] of size n by d elements.


Rotation of the above array by 2 will make array


**Method 4(The Reversal Algorithm)**
Please read this for first three methods of array rotation.

**Algorithm:**

```
rotate(arr[], d, n)
  reverse(arr[], 1, d) ;
  reverse(arr[], d + 1, n);
  reverse(arr[], l, n);
```

Let AB are the two parts of the input array where A = arr[0..d-1] and B = arr[d..n-1].
The idea of the algorithm is:
Reverse A to get ArB. /* Ar is reverse of A */
Reverse B to get ArBr. /* Br is reverse of B */
Reverse all to get (ArBr) r = BA.

For arr[] = [1, 2, 3, 4, 5, 6, 7], d =2 and n = 7
A = [1, 2] and B = [3, 4, 5, 6, 7]
Reverse A, we get ArB = [2, 1, 3, 4, 5, 6, 7]
Reverse B, we get ArBr = [2, 1, 7, 6, 5, 4, 3]
Reverse all, we get (ArBr)r = [3, 4, 5, 6, 7, 1, 2]

### Implementation:

```
/*Utility function to print an array */
void printArray(int arr[], int size);

/* Utility function to reverse arr[] from start to end */
void rvereseArray(int arr[], int start, int end);

/* Function to left rotate arr[] of size n by d */
void leftRotate(int arr[], int d, int n)
{
  rvereseArray(arr, 0, d-1);
  rvereseArray(arr, d, n-1);
  rvereseArray(arr, 0, n-1);
}

/*UTILITY FUNCTIONS*/
/* function to print an array */
void printArray(int arr[], int size)
{
  int i;
  for(i = 0; i < size; i++)
    printf("%d ", arr[i]);
  printf("%\n ");
}

/*Function to reverse arr[] from index start to end*/
void rvereseArray(int arr[], int start, int end)
{
  int i;
  int temp;
  while(start < end)
  {
    temp = arr[start];
    arr[start] = arr[end];
    arr[end] = temp;
    start++;
    end--;
  }
}

/* Driver program to test above functions */
int main()
{
```

```
  int arr[] = {1, 2, 3, 4, 5, 6, 7};
  leftRotate(arr, 2, 7);
  printArray(arr, 7);
  getchar();
  return 0;
}
```

**Time Complexity:** O(n)

**References:**
http://www.cs.bell-labs.com/cm/cs/pearls/s02b.pdf

Â  Â

12. Block swap algorithm for array rotation

Write a function rotate(ar[], d, n) that rotates arr[] of size n by d elements.


Rotation of the above array by 2 will make array


**Algorithm:**

```
Initialize A = arr[0..d-1] and B = arr[d..n-1]
1) Do following until size of A is equal to size of B

  a)  If A is shorter, divide B into Bl and Br such that Br is of same
      length as A. Swap A and Br to change ABlBr into BrBlA. Now A
      is at its final place, so recur on pieces of B.

  b)  If A is longer, divide A into Al and Ar such that Al is of same
      length as B Swap Al and B to change AlArB into BArAl. Now B
      is at its final place, so recur on pieces of A.

2)  Finally when A and B are of equal size, block swap them.
```

**Recursive Implementation:**

```
#include<stdio.h>

/*Prototype for utility functions */
```

```
void printArray(int arr[], int size);
void swap(int arr[], int fi, int si, int d);

void leftRotate(int arr[], int d, int n)
{
  /* Return If number of elements to be rotated is
     zero or equal to array size */
  if(d == 0 || d == n)
    return;

  /*If number of elements to be rotated is exactly
     half of array size */
  if(n-d == d)
  {
    swap(arr, 0, n-d, d);
    return;
  }

 /* If A is shorter*/
  if(d < n-d)
  {
    swap(arr, 0, n-d, d);
    leftRotate(arr, d, n-d);
  }
  else /* If B is shorter*/
  {
    swap(arr, 0, d, n-d);
    leftRotate(arr+n-d, 2*d-n, d); /*This is tricky*/
  }
}

/*UTILITY FUNCTIONS*/
/* function to print an array */
void printArray(int arr[], int size)
{
  int i;
  for(i = 0; i < size; i++)
    printf("%d ", arr[i]);
  printf("%\n ");
}

/*This function swaps d elements starting at index fi
  with d elements starting at index si */
void swap(int arr[], int fi, int si, int d)
```

```
{
  int i, temp;
  for(i = 0; i<d; i++)
  {
    temp = arr[fi + i];
    arr[fi + i] = arr[si + i];
    arr[si + i] = temp;
  }
}

/* Driver program to test above functions */
int main()
{
  int arr[] = {1, 2, 3, 4, 5, 6, 7};
  leftRotate(arr, 2, 7);
  printArray(arr, 7);
  getchar();
  return 0;
}
```

**Iterative Implementation:**

Here is iterative implementation of the same algorithm. Same utility function swap() is used here.

```
void leftRotate(int arr[], int d, int n)
{
 int i, j;
 if(d == 0 || d == n)
   return;
 i = d;
 j = n - d;
 while (i != j)
 {
   if(i < j) /*A is shorter*/
   {
     swap(arr, d-i, d+j-i, i);
     j -= i;
   }
   else /*B is shorter*/
   {
     swap(arr, d-i, d, j);
     i -= j;
   }
```

```
  // printArray(arr, 7);
 }
 /*Finally, block swap A and B*/
 swap(arr, d-i, d, i);
}
```

**Time Complexity:** O(n)

Please see following posts for other methods of array rotation:
http://geeksforgeeks.org/?p=2398
http://geeksforgeeks.org/?p=2838

**References:**
http://www.cs.bell-labs.com/cm/cs/pearls/s02b.pdf

Please write comments if you find any bug in the above programs/algorithms or want to share any additional information about the block swap algorithm.

Â  Â

13. Maximum sum such that no two elements are adjacent

**Question:** Given an array of positive numbers, find the maximum sum of a subsequence with the constraint that no 2 numbers in the sequence should be adjacent in the array. So 3 2 7 10 should return 13 (sum of 3 and 10) or 3 2 5 10 7 should return 15 (sum of 3, 5 and 7).Answer the question in most efficient way.

**Algorithm:**
Loop for all elements in arr[] and maintain two sums incl and excl where incl = Max sum including the previous element and excl = Max sum excluding the previous element.

Max sum excluding the current element will be max(incl, excl) and max sum including the current element will be excl + current element (Note that only excl is considered because elements cannot be adjacent).

At the end of the loop return max of incl and excl.

**Example:**

```
arr[] = {5,  5, 10, 40, 50, 35}

inc = 5
exc = 0

For i = 1 (current element is 5)
```

```
 incl =  (excl + arr[i])  = 5
 excl =  max(5, 0) = 5

 For i = 2 (current element is 10)
 incl =  (excl + arr[i]) = 15
 excl =  max(5, 5) = 5

 For i = 3 (current element is 40)
 incl = (excl + arr[i]) = 45
 excl = max(5, 15) = 15

 For i = 4 (current element is 50)
 incl = (excl + arr[i]) = 65
 excl =  max(45, 15) = 45

 For i = 5 (current element is 35)
 incl =  (excl + arr[i]) = 80
 excl = max(5, 15) = 65
```

And 35 is the last element. So, answer is max(incl, excl) =  80

Thanks to Debanjan for providing code.

**Implementation:**

```
#include<stdio.h>

/*Function to return max sum such that no two elements
 are adjacent */
int FindMaxSum(int arr[], int n)
{
  int incl = arr[0];
  int excl = 0;
  int excl_new;
  int i;

  for (i = 1; i < n; i++)
  {
     /* current max excluding i */
     excl_new = (incl > excl)? incl: excl;

     /* current max including i */
     incl = excl + arr[i];
     excl = excl_new;
  }
```

```
  /* return max of incl and excl */
  return ((incl > excl)? incl : excl);
}

/* Driver program to test above function */
int main()
{
  int arr[] = {5, 5, 10, 100, 10, 5};
  printf("%d \n", FindMaxSum(arr, 6));
  getchar();
  return 0;
}
```

**Time Complexity:** O(n)

Now try the same problem for array with negative numbers also.

Please write comments if you find any bug in the above program/algorithm or other ways to solve the same problem.

Â  Â

14. Leaders in an array

Write a program to print all the LEADERS in the array. An element is leader if it is greater than all the elements to its right side. And the rightmost element is always a leader. For example int the array {16, 17, 4, 3, 5, 2}, leaders are 17, 5 and 2.

Let the input array be arr[] and size of the array be *size*.

**Method 1 (Simple)**
Use two loops. The outer loop runs from 0 to size â€“ 1 and one by one picks all elements from left to right. The inner loop compares the picked element to all the elements to its right side. If the picked element is greater than all the elements to its right side, then the picked element is the leader.

```
/*Function to print leaders in an array */
void printLeaders(int arr[], int size)
{
  int i, j;

  for (i = 0; i < size; i++)
  {
    for (j = i+1; j < size; j++)
```

```
    {
        if(arr[i] <= arr[j])
          break;
    }
    if(j == size) // the loop didn't break
    {
        printf("%d ", arr[i]);
    }
  }
}

/*Driver program to test above function*/
int main()
{
  int arr[] = {16, 17, 4, 3, 5, 2};
  printLeaders(arr, 6);
  getchar();
}
// Output:  17 5 2
```

**Time Complexity:** O(n*n)

**Method 2 (Scan from right)**
Scan all the elements from right to left in array and keep track of maximum till now. When maximum changes itâ€™s value, print it.

```
/*Function to print leaders in an array */
void printLeaders(int arr[], int size)
{
  int max_from_right =  arr[size-1];
  int i;

  /* Rightmost element is always leader */
  printf("%d ", max_from_right);

  for(i = size-2; i >= 0; i--)
  {
    if(max_from_right < arr[i])
    {
        printf("%d ", arr[i]);
        max_from_right = arr[i];
    }
  }
}
```

```
/*Driver program to test above function*/
int main()
{
  int arr[] = {16, 17, 4, 3, 5, 2};
  printLeaders(arr, 6);
  getchar();
}
// Output:  2 5 17
```

**Time Complexity:** O(n)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

15.  Sort elements by frequency | Set 1

Asked By Binod

**Question:**
Print the elements of an array in the decreasing frequency if 2 numbers have same frequency then print the one which came 1st
E.g. 2 5 2 8 5 6 8 8 output: 8 8 8 2 2 5 5 6.

**METHOD 1 (Use Sorting)**

```
1) Use a sorting algorithm to sort the elements O(nlogn)
2) Scan the sorted array and construct a 2D array of element and count O(n).
3) Sort the 2D array according to count O(nlogn).
```

**Example:**

```
Input 2 5 2 8 5 6 8 8

After sorting we get
2 2 5 5 6 8 8 8

Now construct the 2D array as
2, 2
5, 2
6, 1
```

```
8, 3

Sort by count
8, 3
2, 2
5, 2
6, 1
```

There is one issue with above approach (thanks to ankit for pointing this out). If we modify the input to 5 2 2 8 5 6 8 8, then we should get 8 8 8 5 5 2 2 6 and not 8 8 8 2 2 5 5 6 as will be the case.
To handle this, we should use indexes in step 3, if two counts are same then we should first process(or print) the element with lower index. In step 1, we should store the indexes instead of elements.

```
Input 5  2  2  8  5  6  8  8

After sorting we get
Element 2 2 5 5 6 8 8 8
Index   1 2 0 4 5 3 6 7

Now construct the 2D array as
Index, Count
1,      2
0,      2
5,      1
3,      3

Sort by count (consider indexes in case of tie)
3, 3
0, 2
1, 2
5, 1

Print the elements using indexes in the above 2D array.
```

**METHOD 2(Use BST and Sorting)**
1. Insert elements in BST one by one and if an element is already present then increment the count of the node. Node of the Binary Search Tree (used in this approach) will be as follows.

```
struct tree
{
```

```
  int element;
  int first_index /*To handle ties in counts*/
  int count;
}BST;
```

2.Store the first indexes and corresponding counts of BST in a 2D array.
3 Sort the 2D array according to counts (and use indexes in case of tie).

**Time Complexity:** O(nlogn) if a Self Balancing Binary Search Tree is used.

**METHOD 3(Use Hashing and Sorting)**
Using a hashing mechanism, we can store the elements (also first index) and their counts in a hash. Finally, sort the hash elements according to their counts.

These are just our thoughts about solving the problem and may not be the optimal way of solving. We are open for better solutions.

**Related Links**

http://www.trunix.org/programlama/c/kandr2/krx604.html

http://drhanson.s3.amazonaws.com/storage/documents/common.pdf

http://www.cc.gatech.edu/classes/semantics/misc/pp2.pdf

Â  Â

16. Count Inversions in an array

*Inversion Count* for an array indicates â€“ how far (or close) the array is from being sorted. If array is already sorted then inversion count is 0. If array is sorted in reverse order that inversion count is the maximum.
Formally speaking, two elements a[i] and a[j] form an inversion if a[i] > a[j] and i < j

**Example:**
The sequence 2, 4, 1, 3, 5 has three inversions (2, 1), (4, 1), (4, 3).

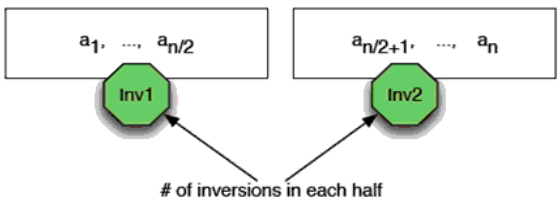
**METHOD 1 (Simple)**
For each element, count number of elements which are on right side of it and are smaller than it.

```
int getInvCount(int arr[], int n)
{
  int inv_count = 0;
```

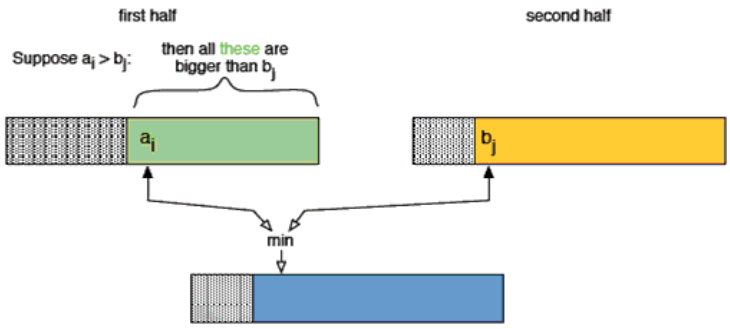
```
  int i, j;

  for(i = 0; i < n - 1; i++)
    for(j = i+1; j < n; j++)
      if(arr[i] > arr[j])
        inv_count++;

  return inv_count;
}

/* Driver progra to test above functions */
int main(int argv, char** args)
{
  int arr[] = {1, 20, 6, 4, 5};
  printf(" Number of inversions are %d \n", getInvCount(arr, 5));
  getchar();
  return 0;
}
```

**Time Complexity:** O(n^2)

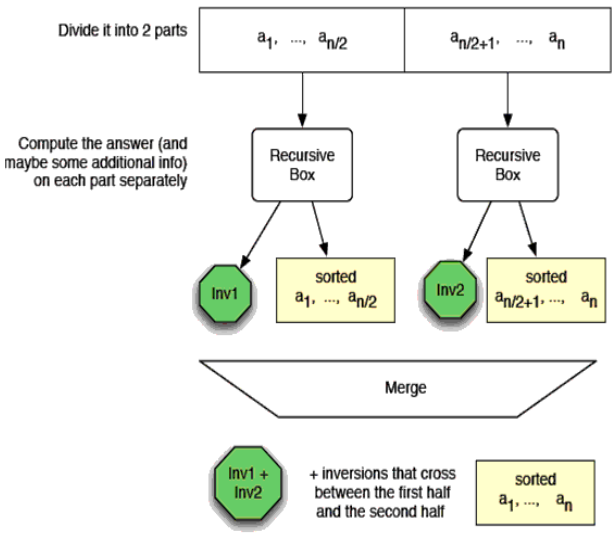
**METHOD 2(Enhance Merge Sort)**
Suppose we know the number of inversions in the left half and right half of the array (let be inv1 and inv2), what kinds of inversions are not accounted for in Inv1 + Inv2? The answer is â€“ the inversions we have to count during the merge step. Therefore, to get number of inversions, we need to add number of inversions in left subarray, right subarray and merge().




**How to get number of inversions in merge()?**
In merge process, let i is used for indexing left sub-array and j for right sub-array. At any step in merge(), if a[i] is greater than a[j], then there are (mid â€“ i) inversions. because left and right subarrays are sorted, so all the remaining elements in left-subarray (a[i+1], a[i+2] â€¦ a[mid]) will be greater than a[j]




### The complete picture:




### Implementation:

```
#include <stdio.h>
#include <stdlib.h>

int  _mergeSort(int arr[], int temp[], int left, int right);
int merge(int arr[], int temp[], int left, int mid, int right);

/* This function sorts the input array and returns the
   number of inversions in the array */
int mergeSort(int arr[], int array_size)
{
    int *temp = (int *)malloc(sizeof(int)*array_size);
    return _mergeSort(arr, temp, 0, array_size - 1);
}

/* An auxiliary recursive function that sorts the input array and
  returns the number of inversions in the array. */
int _mergeSort(int arr[], int temp[], int left, int right)
{
  int mid, inv_count = 0;
```

```
  if (right > left)
  {
    /* Divide the array into two parts and call _mergeSortAndCountInv()
       for each of the parts */
    mid = (right + left)/2;

    /* Inversion count will be sum of inversions in left-part, right-part
      and number of inversions in merging */
    inv_count  = _mergeSort(arr, temp, left, mid);
    inv_count += _mergeSort(arr, temp, mid+1, right);

    /*Merge the two parts*/
    inv_count += merge(arr, temp, left, mid+1, right);
  }
  return inv_count;
}

/* This funt merges two sorted arrays and returns inversion count in
   the arrays.*/
int merge(int arr[], int temp[], int left, int mid, int right)
{
  int i, j, k;
  int inv_count = 0;

  i = left; /* i is index for left subarray*/
  j = mid;  /* i is index for right subarray*/
  k = left; /* i is index for resultant merged subarray*/
  while ((i <= mid - 1) && (j <= right))
  {
    if (arr[i] <= arr[j])
    {
      temp[k++] = arr[i++];
    }
    else
    {
      temp[k++] = arr[j++];

     /*this is tricky -- see above explanation/diagram for merge()*/
      inv_count = inv_count + (mid - i);
    }
  }

  /* Copy the remaining elements of left subarray
   (if there are any) to temp*/
```

```
  while (i <= mid - 1)
    temp[k++] = arr[i++];

  /* Copy the remaining elements of right subarray
   (if there are any) to temp*/
  while (j <= right)
    temp[k++] = arr[j++];

  /*Copy back the merged elements to original array*/
  for (i=left; i <= right; i++)
    arr[i] = temp[i];

  return inv_count;
}

/* Driver progra to test above functions */
int main(int argv, char** args)
{
  int arr[] = {1, 20, 6, 4, 5};
  printf(" Number of inversions are %d \n", mergeSort(arr, 5));
  getchar();
  return 0;
}
```

Note that above code modifies (or sorts) the input array. If we want to count only inversions then we need to create a copy of original array and call mergeSort() on copy.

**Time Complexity:** O(nlogn)
**Algorithmic Paradigm:** Divide and Conquer

**References:**
http://www.cs.umd.edu/class/fall2009/cmsc451/lectures/Lec08-inversions.pdf
http://www.cp.eng.chula.ac.th/~piak/teaching/algo/algo2008/count-inv.htm

Please write comments if you find any bug in the above program/algorithm or other ways to solve the same problem.

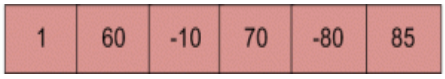
Â  Â

17. Two elements whose sum is closest to zero

**Question:** An Array of integers is given, both +ve and -ve. You need to find the two

elements such that their sum is closest to zero.

For the below array, program should print -80 and 85.




**METHOD 1 (Simple)**
For each element, find the sum of it with every other element in the array and compare sums. Finally, return the minimum sum.

**Implementation**

```
# include <stdio.h>
# include <stdlib.h> /* for abs() */
# include <math.h>
void minAbsSumPair(int arr[], int arr_size)
{
  int inv_count = 0;
  int l, r, min_sum, sum, min_l, min_r;

  /* Array should have at least two elements*/
  if(arr_size < 2)
  {
    printf("Invalid Input");
    return;
  }

  /* Initialization of values */
  min_l = 0;
  min_r = 1;
  min_sum = arr[0] + arr[1];

  for(l = 0; l < arr_size - 1; l++)
  {
    for(r = l+1; r < arr_size; r++)
    {
      sum = arr[l] + arr[r];
      if(abs(min_sum) > abs(sum))
      {
        min_sum = sum;
        min_l = l;
        min_r = r;
      }
    }
  }
```

```
  printf(" The two elements whose sum is minimum are %d and %d",
         arr[min_l], arr[min_r]);
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 60, -10, 70, -80, 85};
  minAbsSumPair(arr, 6);
  getchar();
  return 0;
}
```

**Time complexity:** O(n^2)

**METHOD 2 (Use Sorting)**
Thanks to baskin for suggesting this approach. We recommend to read this post for background of this approach.

**Algorithm**
1) Sort all the elements of the input array.
2) Use two index variables l and r to traverse from left and right ends respectively. Initialize l as 0 and r as n-1.
3) sum = a[l] + a[r]
4) If sum is -ve, then l++
5) If sum is +ve, then râ€“
6) Keep track of abs min sum.
7) Repeat steps 3, 4, 5 and 6 while l < r

**Implementation**

```
# include <stdio.h>
# include <math.h>
# include <limits.h>

void quickSort(int *, int, int);

/* Function to print pair of elements having minimum sum */
void minAbsSumPair(int arr[], int n)
{
  // Variables to keep track of current sum and minimum sum
  int sum, min_sum = INT_MAX;
```

```
  // left and right index variables
  int l = 0, r = n-1;

  // variable to keep track of the left and right pair for min_sum
  int min_l = l, min_r = n-1;

  /* Array should have at least two elements*/
  if(n < 2)
  {
    printf("Invalid Input");
    return;
  }

  /* Sort the elements */
  quickSort(arr, l, r);

  while(l < r)
  {
    sum = arr[l] + arr[r];

    /*If abs(sum) is less then update the result items*/
    if(abs(sum) < abs(min_sum))
    {
      min_sum = sum;
      min_l = l;
      min_r = r;
    }
    if(sum < 0)
      l++;
    else
      r--;
  }

  printf(" The two elements whose sum is minimum are %d and %d",
         arr[min_l], arr[min_r]);
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 60, -10, 70, -80, 85};
  int n = sizeof(arr)/sizeof(arr[0]);
  minAbsSumPair(arr, n);
  getchar();
```

```
  return 0;
}

/* FOLLOWING FUNCTIONS ARE ONLY FOR SORTING
    PURPOSE */
void exchange(int *a, int *b)
{
  int temp;
  temp = *a;
  *a   = *b;
  *b   = temp;
}

int partition(int arr[], int si, int ei)
{
  int x = arr[ei];
  int i = (si - 1);
  int j;

  for (j = si; j <= ei - 1; j++)
  {
    if(arr[j] <= x)
    {
      i++;
      exchange(&arr[i], &arr[j]);
    }
  }

  exchange (&arr[i + 1], &arr[ei]);
  return (i + 1);
}

/* Implementation of Quick Sort
arr[] --> Array to be sorted
si  --> Starting index
ei  --> Ending index
*/
void quickSort(int arr[], int si, int ei)
{
  int pi;    /* Partitioning index */
  if(si < ei)
  {
    pi = partition(arr, si, ei);
    quickSort(arr, si, pi - 1);
```

```
    quickSort(arr, pi + 1, ei);
  }
}
```

**Time Complexity:** complexity to sort + complexity of finding the optimum pair = O(nlogn) + O(n) = O(nlogn)

Asked by Vineet

Please write comments if you find any bug in the above program/algorithm or other ways to solve the same problem.

Â Â

18. Find the smallest and second smallest element in an array

**Question:** Write an efficient C program to find smallest and second smallest element in an array.

**Difficulty Level:** Rookie

**Algorithm:**

```
1) Initialize both first and second smallest as INT_MAX
   first = second = INT_MAX
2) Loop through all the elements.
   a) If the current element is smaller than first, then update first
      and second.
   b) Else if the current element is smaller than second then update
    second
```

**Implementation:**

```
#include <stdio.h>
#include <limits.h> /* For INT_MAX */

/* Function to print first smallest and second smallest elements */
void print2Smallest(int arr[], int arr_size)
{
   int i, first, second;

   /* There should be atleast two elements */
   if (arr_size < 2)
   {
```

```
        printf(" Invalid Input ");
        return;
    }

    first = second = INT_MAX;
    for (i = 0; i < arr_size ; i ++)
    {
        /* If current element is smaller than first then update both
          first and second */
        if (arr[i] < first)
        {
            second = first;
            first = arr[i];
        }

        /* If arr[i] is in between first and second then update second  */
        else if (arr[i] < second && arr[i] != first)
            second = arr[i];
    }
    if (second == INT_MAX)
        printf("There is no second smallest element\n");
    else
        printf("The smallest element is %d and second Smallest element is %d\n",
               first, second);
}


/* Driver program to test above function */
int main()
{
    int arr[] = {12, 13, 1, 10, 34, 1};
    int n = sizeof(arr)/sizeof(arr[0]);
    print2Smallest(arr, n);
    return 0;
}
```

Output:

```
The smallest element is 1 and second Smallest element is 10
```

The same approach can be used to find the largest and second largest elements in an array.

**Time Complexity:** O(n)

Please write comments if you find any bug in the above program/algorithm or other ways to solve the same problem.

Â Â

19. Check for Majority Element in a sorted array

**Question:** Write a C function to find if a given integer x appears more than n/2 times in a sorted array of n integers.

Basically, we need to write a function say isMajority() that takes an array (arr[] ), arrayâ€™s size (n) and a number to be searched (x) as parameters and returns true if x is a majority element (present more than n/2 times).

Examples:

```
Input: arr[] = {1, 2, 3, 3, 3, 3, 10}, x = 3
Output: True (x appears more than n/2 times in the given array)

Input: arr[] = {1, 1, 2, 4, 4, 4, 6, 6}, x = 4
Output: False (x doesn't appear more than n/2 times in the given array)

Input: arr[] = {1, 1, 1, 2, 2}, x = 1
Output: True (x appears more than n/2 times in the given array)
```

**METHOD 1 (Using Linear Search)**
Linearly search for the first occurrence of the element, once you find it (let at index i), check element at index i + n/2. If element is present at i+n/2 then return 1 else return 0.

```
/* Program to check for majority element in a sorted array */
# include <stdio.h>
# include <stdbool.h>

bool isMajority(int arr[], int n, int x)
{
  int i;

  /* get last index according to n (even or odd) */
  int last_index = n%2? (n/2+1): (n/2);

  /* search for first occurrence of x in arr[]*/
  for (i = 0; i < last_index; i++)
  {
```

```
    /* check if x is present and is present more than n/2 times */
    if (arr[i] == x && arr[i+n/2] == x)
        return 1;
  }
  return 0;
}

/* Driver program to check above function */
int main()
{
   int arr[] ={1, 2, 3, 4, 4, 4, 4};
   int n = sizeof(arr)/sizeof(arr[0]);
   int x = 4;
   if (isMajority(arr, n, x))
     printf("%d appears more than %d times in arr[]", x, n/2);
   else
    printf("%d does not appear more than %d times in arr[]", x, n/2);

   getchar();
   return 0;
}
```

**Time Complexity:** O(n)

**METHOD 2 (Using Binary Search)**

Use binary search methodology to find the first occurrence of the given number. The criteria for binary search is important here.

```
/* Program to check for majority element in a sorted array */
# include <stdio.h>;
# include <stdbool.h>

/* If x is present in arr[low...high] then returns the index of
  first occurrence of x, otherwise returns -1 */
int _binarySearch(int arr[], int low, int high, int x);

/* This function returns true if the x is present more than n/2
   times in arr[] of size n */
bool isMajority(int arr[], int n, int x)
{
   /* Find the index of first occurrence of x in arr[] */
   int i = _binarySearch(arr, 0, n-1, x);

   /* If element is not present at all, return false*/
```

```
  if (i == -1)
    return false;

  /* check if the element is present more than n/2 times */
  if (((i + n/2) <= (n -1)) && arr[i + n/2] == x)
    return true;
  else
    return false;
}

/* If x is present in arr[low...high] then returns the index of
  first occurrence of x, otherwise returns -1 */
int  _binarySearch(int arr[], int low, int high, int x)
{
  if (high >= low)
  {
    int mid = (low + high)/2;  /*low + (high - low)/2;*/

    /* Check if arr[mid] is the first occurrence of x.
       arr[mid] is first occurrence if x is one of the following
       is true:
       (i)  mid == 0 and arr[mid] == x
       (ii) arr[mid-1] < x and arr[mid] == x
     */
    if ( (mid == 0 || x > arr[mid-1]) && (arr[mid] == x) )
      return mid;
    else if (x > arr[mid])
      return _binarySearch(arr, (mid + 1), high, x);
    else
      return _binarySearch(arr, low, (mid -1), x);
  }

  return -1;
}

/* Driver program to check above functions */
int main()
{
  int arr[] = {1, 2, 3, 3, 3, 3, 10};
  int n = sizeof(arr)/sizeof(arr[0]);
  int x = 3;
  if(isMajority(arr, n, x))
    printf("%d appears more than %d times in arr[]", x, n/2);
  else
```

```
   printf("%d does not appear more than %d times in arr[]", x, n/2);

  return 0;
}
```

**Time Complexity:** O(Logn)
**Algorithmic Paradigm:** Divide and Conquer

Please write comments if you find any bug in the above program/algorithm or a better way to solve the same problem.

Â  Â

20. Maximum and minimum of an array using minimum number of comparisons

**Write a C function to return minimum and maximum in an array. You program should make minimum number of comparisons.**

First of all, how do we return multiple values from a C function? We can do it either using structures or pointers.

We have created a structure named pair (which contains min and max) to return multiple values.

```
struct pair
{
  int min;
  int max;
};
```

And the function declaration becomes: struct pair getMinMax(int arr[], int n) where arr[] is the array of size n whose minimum and maximum are needed.

**METHOD 1 (Simple Linear Search)**
Initialize values of min and max as minimum and maximum of the first two elements respectively. Starting from 3rd, compare each element with max and min, and change max and min accordingly (i.e., if the element is smaller than min then change min, else if the element is greater than max then change max, else ignore the element)

```
/* structure is used to return two values from minMax() */
#include<stdio.h>
struct pair
{
  int min;
  int max;
```

```
};

struct pair getMinMax(int arr[], int n)
{
  struct pair minmax;
  int i;

  /*If there is only one element then return it as min and max both*/
  if (n == 1)
  {
     minmax.max = arr[0];
     minmax.min = arr[0];
     return minmax;
  }

  /* If there are more than one elements, then initialize min
      and max*/
  if (arr[0] > arr[1])
  {
      minmax.max = arr[0];
      minmax.min = arr[1];
  }
  else
  {
      minmax.max = arr[1];
      minmax.min = arr[0];
  }

  for (i = 2; i<n; i++)
  {
    if (arr[i] >  minmax.max)
      minmax.max = arr[i];

    else if (arr[i] <  minmax.min)
      minmax.min = arr[i];
  }

  return minmax;
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1000, 11, 445, 1, 330, 3000};
```

```
  int arr_size = 6;
  struct pair minmax = getMinMax (arr, arr_size);
  printf("\nMinimum element is %d", minmax.min);
  printf("\nMaximum element is %d", minmax.max);
  getchar();
}
```

Time Complexity: O(n)

In this method, total number of comparisons is 1 + 2(n-2) in worst case and 1 + n â€“ 2 in best case.
In the above implementation, worst case occurs when elements are sorted in descending order and best case occurs when elements are sorted in ascending order.

**METHOD 2 (Tournament Method)**
Divide the array into two parts and compare the maximums and minimums of the the two parts to get the maximum and the minimum of the the whole array.

```
Pair MaxMin(array, array_size)
   if array_size = 1
      return element as both max and min
   else if arry_size = 2
      one comparison to determine max and min
      return that pair
   else    /* array_size  > 2 */
      recur for max and min of left half
      recur for max and min of right half
      one comparison determines true max of the two candidates
      one comparison determines true min of the two candidates
      return the pair of max and min
```

Implementation

```
/* structure is used to return two values from minMax() */
#include<stdio.h>
struct pair
{
  int min;
  int max;
};

struct pair getMinMax(int arr[], int low, int high)
{
  struct pair minmax, mml, mmr;
  int mid;
```

```
 /* If there is only on element */
 if (low == high)
 {
    minmax.max = arr[low];
    minmax.min = arr[low];
    return minmax;
 }

 /* If there are two elements */
 if (high == low + 1)
 {
    if (arr[low] > arr[high])
    {
       minmax.max = arr[low];
       minmax.min = arr[high];
    }
    else
    {
       minmax.max = arr[high];
       minmax.min = arr[low];
    }
    return minmax;
 }

 /* If there are more than 2 elements */
 mid = (low + high)/2;
 mml = getMinMax(arr, low, mid);
 mmr = getMinMax(arr, mid+1, high);

 /* compare minimums of two parts*/
 if (mml.min < mmr.min)
   minmax.min = mml.min;
 else
   minmax.min = mmr.min;

 /* compare maximums of two parts*/
 if (mml.max > mmr.max)
   minmax.max = mml.max;
 else
   minmax.max = mmr.max;

 return minmax;
}
```

```
/* Driver program to test above function */
int main()
{
  int arr[] = {1000, 11, 445, 1, 330, 3000};
  int arr_size = 6;
  struct pair minmax = getMinMax(arr, 0, arr_size-1);
  printf("\nMinimum element is %d", minmax.min);
  printf("\nMaximum element is %d", minmax.max);
  getchar();
}
```

Time Complexity: O(n)
Total number of comparisons: let number of comparisons be T(n). T(n) can be written as follows:
Algorithmic Paradigm: Divide and Conquer

```
 T(n) = T(floor(n/2)) + T(ceil(n/2)) + 2
 T(2) = 1
 T(1) = 0
```

If n is a power of 2, then we can write T(n) as:

```
  T(n) = 2T(n/2) + 2
```

After solving above recursion, we get

```
 T(n)  = 3/2n -2
```

Thus, the approach does 3/2n -2 comparisons if n is a power of 2. And it does more than 3/2n -2 comparisons if n is not a power of 2.

**METHOD 3 (Compare in Pairs)**
If n is odd then initialize min and max as first element.
If n is even then initialize min and max as minimum and maximum of the first two elements respectively.
For rest of the elements, pick them in pairs and compare their
maximum and minimum with max and min respectively.

```
#include<stdio.h>

/* structure is used to return two values from minMax() */
struct pair
{
  int min;
```

```
 int max;
};

struct pair getMinMax(int arr[], int n)
{
 struct pair minmax;
 int i;

 /* If array has even number of elements then
  initialize the first two elements as minimum and
  maximum */
 if (n%2 == 0)
 {
  if (arr[0] > arr[1])
  {
   minmax.max = arr[0];
   minmax.min = arr[1];
  }
  else
  {
   minmax.min = arr[0];
   minmax.max = arr[1];
  }
  i = 2;  /* set the startung index for loop */
 }

 /* If array has odd number of elements then
  initialize the first element as minimum and
  maximum */
 else
 {
  minmax.min = arr[0];
  minmax.max = arr[0];
  i = 1;  /* set the startung index for loop */
 }

 /* In the while loop, pick elements in pair and
   compare the pair with max and min so far */
 while (i < n-1)
 {
  if (arr[i] > arr[i+1])
  {
   if(arr[i] > minmax.max)
    minmax.max = arr[i];
```

```
    if(arr[i+1] < minmax.min)
      minmax.min = arr[i+1];
  }
  else
  {
    if (arr[i+1] > minmax.max)
      minmax.max = arr[i+1];
    if (arr[i] < minmax.min)
      minmax.min = arr[i];
  }
  i += 2; /* Increment the index by 2 as two
             elements are processed in loop */
 }

 return minmax;
}

/* Driver program to test above function */
int main()
{
 int arr[] = {1000, 11, 445, 1, 330, 3000};
 int arr_size = 6;
 struct pair minmax = getMinMax (arr, arr_size);
 printf("\nMinimum element is %d", minmax.min);
 printf("\nMaximum element is %d", minmax.max);
 getchar();
}
```

Time Complexity: O(n)

Total number of comparisons: Different for even and odd n, see below:

```
    If n is odd:    3*(n-1)/2
    If n is even:   1 Initial comparison for initializing min and max,
                    and 3(n-2)/2 comparisons for rest of the elements
                =  1 + 3*(n-2)/2 = 3n/2 -2
```

Second and third approaches make equal number of comparisons when n is a power of 2.

In general, method 3 seems to be the best.

Please write comments if you find any bug in the above programs/algorithms or a better way to solve the same problem.

Â  Â

21. Segregate 0s and 1s in an array

Asked by kapil.

You are given an array of 0s and 1s in random order. Segregate 0s on left side and 1s on right side of the array. Traverse array only once.

```
Input array   =  [0, 1, 0, 1, 0, 0, 1, 1, 1, 0]
Output array =  [0, 0, 0, 0, 0, 1, 1, 1, 1, 1]
```

**Method 1 (Count 0s or 1s)**
Thanks to Naveen for suggesting this method.
1) Count the number of 0s. Let count be C.
2) Once we have count, we can put C 0s at the beginning and 1s at the remaining n â€“ C positions in array.

Time Complexity: O(n)

The method 1 traverses the array two times. Method 2 does the same in a single pass.

**Method 2 (Use two indexes to traverse)**
Maintain two indexes. Initialize first index *left* as 0 and second index *right* as n-1.

Do following while *left* < *right*
a) Keep incrementing index *left* while there are 0s at it
b) Keep decrementing index *right* while there are 1s at it
c) If left < right then exchange arr[left] and arr[right]

Implementation:

```
#include<stdio.h>

/*Function to put all 0s on left and all 1s on right*/
void segregate0and1(int arr[], int size)
{
  /* Initialize left and right indexes */
  int left = 0, right = size-1;

  while(left < right)
  {
    /* Increment left index while we see 0 at left */
    while(arr[left] == 0 && left < right)
      left++;
```

```
    /* Decrement right index while we see 1 at right */
    while(arr[right] == 1 && left < right)
      rightâ€“;

   /* If left is smaller than right then there is a 1 at left
     and a 0 at right.  Exchange arr[left] and arr[right]*/
    if(left < right)
    {
      arr[left] = 0;
      arr[right] = 1;
      left++;
      rightâ€“;
    }
  }
}

/* driver program to test */
int main()
{
  int arr[] = {0, 1, 0, 1, 1, 1};
  int arr_size = 6, i = 0;

  segregate0and1(arr, arr_size);

  printf("array after segregation ");
  for(i = 0; i < 6; i++)
    printf("%d ", arr[i]);

  getchar();
  return 0;
}
```

Time Complexity: O(n)

Please write comments if you find any of the above algorithms/code incorrect, or a better ways to solve the same problem.

Â  Â

22.  k largest(or smallest) elements in an array | added Min Heap method

**Question:** Write an efficient program for printing k largest elements in an array.

Elements in array can be in any order.

For example, if given array is [1, 23, 12, 9, 30, 2, 50] and you are asked for the largest 3 elements i.e., k = 3 then your program should print 50, 30 and 23.

**Method 1 (Use Bubble k times)**
Thanks to Shailendra for suggesting this approach.
1) Modify Bubble Sort to run the outer loop at most k times.
2) Print the last k elements of the array obtained in step 1.

Time Complexity: O(nk)

Like Bubble sort, other sorting algorithms like Selection Sort can also be modified to get the k largest elements.

**Method 2 (Use temporary array)**
K largest elements from arr[0..n-1]

1) Store the first k elements in a temporary array temp[0..k-1].
2) Find the smallest element in temp[], let the smallest element be *min*.
3) For each element *x* in arr[k] to arr[n-1]
If *x* is greater than the min then remove *min* from temp[] and insert *x*.
4) Print final k elements of *temp[]*

Time Complexity: O((n-k)*k). If we want the output sorted then O((n-k)*k + klogk)

Thanks to nesamani1822 for suggesting this method.

**Method 3(Use Sorting)**
1) Sort the elements in descending order in O(nLogn)
2) Print the first k numbers of the sorted array O(k).

Time complexity: O(nlogn)

**Method 4 (Use Max Heap)**
1) Build a Max Heap tree in O(n)
2) Use Extract Max k times to get k maximum elements from the Max Heap O(klogn)

Time complexity: O(n + klogn)

**Method 5(Use Oder Statistics)**
1) Use order statistic algorithm to find the kth largest element. Please see the topic selection in worst-case linear time O(n)
2) Use QuickSort Partition algorithm to partition around the kth largest number O(n).
3) Sort the k-1 elements (elements greater than the kth largest element) O(kLogk). This step is needed only if sorted output is required.

Time complexity: O(n) if we donâ€™t need the sorted output, otherwise O(n+kLogk)

Thanks to Shilpi for suggesting the first two approaches.

**Method 6 (Use Min Heap)**
This method is mainly an optimization of method 1. Instead of using temp[] array, use Min Heap.

Thanks to geek4u for suggesting this method.

1) Build a Min Heap MH of the first k elements (arr[0] to arr[k-1]) of the given array. O(k)

2) For each element, after the kth element (arr[k] to arr[n-1]), compare it with root of MH.
â€¦â€¦a) If the element is greater than the root then make it root and call heapify for MH
â€¦â€¦b) Else ignore it.
// The step 2 is O((n-k)*logk)

3) Finally, MH has k largest elements and root of the MH is the kth largest element.

Time Complexity: O(k + (n-k)Logk) without sorted output. If sorted output is needed then O(k + (n-k)Logk + kLogk)

All of the above methods can also be used to find the kth largest (or smallest) element.

Please write comments if you find any of the above explanations/algorithms incorrect, or find better ways to solve the same problem.

**References:**
http://en.wikipedia.org/wiki/Selection_algorithm

Asked by geek4u

Â Â

23. Maximum size square sub-matrix with all 1s

Given a binary matrix, find out the maximum size square sub-matrix with all 1s.

For example, consider the below binary matrix.

```
0 1 1 0 1
1 1 0 1 0
```

```
0  1  1  1  0
1  1  1  1  0
1  1  1  1  1
0  0  0  0  0
```

The maximum square sub-matrix with all set bits is

```
1  1  1
1  1  1
1  1  1
```

Algorithm:
Let the given binary matrix be M[R][C]. The idea of the algorithm is to construct an auxiliary size matrix S[][] in which each entry S[i][j] represents size of the square sub-matrix with all 1s including M[i][j] where M[i][j] is the rightmost and bottommost entry in sub-matrix.

```
1) Construct a sum matrix S[R][C] for the given M[R][C].
    a) Copy first row and first columns as it is from M[][] to S[][]
    b) For other entries, use following expressions to construct S[][]
       If M[i][j] is 1 then
         S[i][j] = min(S[i][j-1], S[i-1][j], S[i-1][j-1]) + 1
       Else /*If M[i][j] is 0*/
         S[i][j] = 0
2) Find the maximum entry in S[R][C]
3) Using the value and coordinates of maximum entry in S[i], print
  sub-matrix of M[][]
```

For the given M[R][C] in above example, constructed S[R][C] would be:

```
0  1  1  0  1
1  1  0  1  0
0  1  1  1  0
1  1  2  2  0
1  2  2  3  1
0  0  0  0  0
```

The value of maximum entry in above matrix is 3 and coordinates of the entry are (4, 3). Using the maximum value and its coordinates, we can find out the required sub-matrix.

```
#include<stdio.h>
#define bool int
#define R 6
#define C 5

void printMaxSubSquare(bool M[R][C])
```

```
{
 int i,j;
 int S[R][C];
 int max_of_s, max_i, max_j;

 /* Set first column of S[][]*/
 for(i = 0; i < R; i++)
    S[i][0] = M[i][0];

 /* Set first row of S[][]*/
 for(j = 0; j < C; j++)
    S[0][j] = M[0][j];

 /* Construct other entries of S[][]*/
 for(i = 1; i < R; i++)
 {
   for(j = 1; j < C; j++)
   {
     if(M[i][j] == 1)
       S[i][j] = min(S[i][j-1], S[i-1][j], S[i-1][j-1]) + 1;
     else
       S[i][j] = 0;
   }
 }

 /* Find the maximum entry, and indexes of maximum entry
    in S[][] */
 max_of_s = S[0][0]; max_i = 0; max_j = 0;
 for(i = 0; i < R; i++)
 {
   for(j = 0; j < C; j++)
   {
     if(max_of_s < S[i][j])
     {
        max_of_s = S[i][j];
        max_i = i;
        max_j = j;
     }
   }
 }

 printf("\n Maximum size sub-matrix is: \n");
 for(i = max_i; i > max_i - max_of_s; i--)
 {
```

```
    for(j = max_j; j > max_j - max_of_s; j--)
    {
      printf("%d ", M[i][j]);
    }
    printf("\n");
  }
}

/* UTILITY FUNCTIONS */
/* Function to get minimum of three values */
int min(int a, int b, int c)
{
  int m = a;
  if (m > b)
    m = b;
  if (m > c)
    m = c;
  return m;
}

/* Driver function to test above functions */
int main()
{
  bool M[R][C] =  {{0, 1, 1, 0, 1},
                   {1, 1, 0, 1, 0},
                   {0, 1, 1, 1, 0},
                   {1, 1, 1, 1, 0},
                   {1, 1, 1, 1, 1},
                   {0, 0, 0, 0, 0}};

  printMaxSubSquare(M);
  getchar();
}
```

Time Complexity: O(m*n) where m is number of rows and n is number of columns in the given matrix.
Auxiliary Space: O(m*n) where m is number of rows and n is number of columns in the given matrix.
Algorithmic Paradigm: Dynamic Programming

Please write comments if you find any bug in above code/algorithm, or find other ways to solve the same problem

Â  Â

24. Maximum difference between two elements such that larger element appears after the smaller number

Given an array arr[] of integers, find out the difference between any two elements **such that larger element appears after the smaller number** in arr[].

Examples: If array is [2, 3, 10, 6, 4, 8, 1] then returned value should be 8 (Diff between 10 and 2). If array is [ 7, 9, 5, 6, 3, 2 ] then returned value should be 2 (Diff between 7 and 9)

**Method 1 (Simple)**
Use two loops. In the outer loop, pick elements one by one and in the inner loop calculate the difference of the picked element with every other element in the array and compare the difference with the maximum difference calculated so far.

```
#include<stdio.h>

/* The function assumes that there are at least two
   elements in array.
   The function returns a negative value if the array is
   sorted in decreasing order.
   Returns 0 if elements are equal */
int maxDiff(int arr[], int arr_size)
{
  int max_diff = arr[1] - arr[0];
  int i, j;
  for(i = 0; i < arr_size; i++)
  {
    for(j = i+1; j < arr_size; j++)
    {
      if(arr[j] - arr[i] > max_diff)
         max_diff = arr[j] - arr[i];
    }
  }
  return max_diff;
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 2, 90, 10, 110};
  printf("Maximum difference is %d",  maxDiff(arr, 5));
  getchar();
```

```
  return 0;
}
```

Time Complexity: O(n^2)
Auxiliary Space: O(1)

**Method 2 (Tricky and Efficient)**
In this method, instead of taking difference of the picked element with every other element, we take the difference with the minimum element found so far. So we need to keep track of 2 things:
1) Maximum difference found so far (max_diff).
2) Minimum number visited so far (min_element).

```
#include<stdio.h>

/* The function assumes that there are at least two
   elements in array.
   The function returns a negative value if the array is
   sorted in decreasing order.
   Returns 0 if elements are equal  */
int maxDiff(int arr[], int arr_size)
{
  int max_diff = arr[1] - arr[0];
  int min_element = arr[0];
  int i;
  for(i = 1; i < arr_size; i++)
  {
    if(arr[i] - min_element > max_diff)
      max_diff = arr[i] - min_element;
    if(arr[i] < min_element)
         min_element = arr[i];
  }
  return max_diff;
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 2, 6, 80, 100};
  int size = sizeof(arr)/sizeof(arr[0]);
  printf("Maximum difference is %d",  maxDiff(arr, size));
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(1)

**Method 3 (Another Tricky Solution)**
First find the difference between the adjacent elements of the array and store all differences in an auxiliary array diff[] of size n-1. Now this problems turns into finding the maximum sum subarray of this difference array.
Thanks to Shubham Mittal for suggesting this solution.

```
#include<stdio.h>

int maxDiff(int arr[], int n)
{
    // Create a diff array of size n-1. The array will hold
    //  the difference of adjacent elements
    int diff[n-1];
    for (int i=0; i < n-1; i++)
        diff[i] = arr[i+1] - arr[i];

    // Now find the maximum sum subarray in diff array
    int max_diff = diff[0];
    for (int i=1; i<n-1; i++)
    {
        if (diff[i-1] > 0)
            diff[i] += diff[i-1];
        if (max_diff < diff[i])
            max_diff = diff[i];
    }
    return max_diff;
}

/* Driver program to test above function */
int main()
{
    int arr[] = {80, 2, 6, 3, 100};
    int size = sizeof(arr)/sizeof(arr[0]);
    printf("Maximum difference is %d",  maxDiff(arr, size));
    return 0;
}
```

Output:

```
98
```

This method is also O(n) time complexity solution, but it requires O(n) extra space

Time Complexity: O(n)
Auxiliary Space: O(n)

We can modify the above method to work in O(1) extra space. Instead of creating an auxiliary array, we can calculate diff and max sum in same loop. Following is the space optimized version.

```
int maxDiff (int arr[], int n)
{
    // Initialize diff, current sum and max sum
    int diff = arr[1]-arr[0];
    int curr_sum = diff;
    int max_sum = curr_sum;

    for(int i=1; i<n-1; i++)
    {
        // Calculate current diff
        diff = arr[i+1]-arr[i];

        // Calculate current sum
        if (curr_sum > 0)
            curr_sum += diff;
        else
            curr_sum = diff;

        // Update max sum, if needed
        if (curr_sum > max_sum)
            max_sum = curr_sum;
    }

    return max_sum;
}
```

Time Complexity: O(n)
Auxiliary Space: O(1)

Please write comments if you find any bug in above codes/algorithms, or find other ways to solve the same problem

Â  Â

25. Union and Intersection of two sorted arrays

For example, if the input arrays are:

arr1[] = {1, 3, 4, 5, 7}
arr2[] = {2, 3, 5, 6}
Then your program should print Union as {1, 2, 3, 4, 5, 6, 7} and Intersection as {3, 5}.

**Algorithm Union(arr1[], arr2[]):**
For union of two arrays, follow the following merge procedure.
1) Use two index variables i and j, initial values i = 0, j = 0
2) If arr1[i] is smaller than arr2[j] then print arr1[i] and increment i.
3) If arr1[i] is greater than arr2[j] then print arr2[j] and increment j.
4) If both are same then print any of them and increment both i and j.
5) Print remaining elements of the larger array.

```
#include<stdio.h>

/* Function prints union of arr1[] and arr2[]
   m is the number of elements in arr1[]
   n is the number of elements in arr2[] */
int printUnion(int arr1[], int arr2[], int m, int n)
{
  int i = 0, j = 0;
  while(i < m && j < n)
  {
    if(arr1[i] < arr2[j])
      printf(" %d ", arr1[i++]);
    else if(arr2[j] < arr1[i])
      printf(" %d ", arr2[j++]);
    else
    {
      printf(" %d ", arr2[j++]);
      i++;
    }
  }

  /* Print remaining elements of the larger array */
  while(i < m)
   printf(" %d ", arr1[i++]);
  while(j < n)
   printf(" %d ", arr2[j++]);
}

/* Driver program to test above function */
int main()
{
  int arr1[] = {1, 2, 4, 5, 6};
```

```
  int arr2[] = {2, 3, 5, 7};
  int m = sizeof(arr1)/sizeof(arr1[0]);
  int n = sizeof(arr2)/sizeof(arr2[0]);
  printUnion(arr1, arr2, m, n);
  getchar();
  return 0;
}
```

Time Complexity: O(m+n)

**Algorithm Intersection(arr1[], arr2[]):**
For Intersection of two arrays, print the element only if the element is present in both arrays.
1) Use two index variables i and j, initial values i = 0, j = 0
2) If arr1[i] is smaller than arr2[j] then increment i.
3) If arr1[i] is greater than arr2[j] then increment j.
4) If both are same then print any of them and increment both i and j.

```
#include<stdio.h>

/* Function prints Intersection of arr1[] and arr2[]
   m is the number of elements in arr1[]
   n is the number of elements in arr2[] */
int printIntersection(int arr1[], int arr2[], int m, int n)
{
  int i = 0, j = 0;
  while(i < m && j < n)
  {
    if(arr1[i] < arr2[j])
      i++;
    else if(arr2[j] < arr1[i])
      j++;
    else /* if arr1[i] == arr2[j] */
    {
      printf(" %d ", arr2[j++]);
      i++;
    }
  }
}

/* Driver program to test above function */
int main()
{
  int arr1[] = {1, 2, 4, 5, 6};
  int arr2[] = {2, 3, 5, 7};
```

```
  int m = sizeof(arr1)/sizeof(arr1[0]);
  int n = sizeof(arr2)/sizeof(arr2[0]);
  printIntersection(arr1, arr2, m, n);
  getchar();
  return 0;
}
```

Time Complexity: O(m+n)

Please write comments if you find any bug in above codes/algorithms, or find other ways to solve the same problem.

Â  Â

26.  Floor and Ceiling in a sorted array

Given a sorted array and a value x, the ceiling of x is the smallest element in array greater than or equal to x, and the floor is the greatest element smaller than or equal to x. Assume than the array is sorted in non-decreasing order. Write efficient functions to find floor and ceiling of x.

```
For example, let the input array be {1, 2, 8, 10, 10, 12, 19}
For x = 0:    floor doesn't exist in array,  ceil  = 1
For x = 1:    floor  = 1,  ceil  = 1
For x = 5:    floor  = 2,  ceil  = 8
For x = 20:   floor  = 19,  ceil doesn't exist in array
```

In below methods, we have implemented only ceiling search functions. Floor search can be implemented in the same way.

**Method 1 (Linear Search)**
Algorithm to search ceiling of x:
1) If x is smaller than or equal to the first element in array then return 0(index of first element)
2) Else Linearly search for an index i such that x lies between arr[i] and arr[i+1].
3) If we do not find an index i in step 2, then return -1

```
#include<stdio.h>

/* Function to get index of ceiling of x in arr[low..high] */
int ceilSearch(int arr[], int low, int high, int x)
{
  int i;

  /* If x is smaller than or equal to first element,
```

```
   then return the first element */
  if(x <= arr[low])
    return low;

  /* Otherwise, linearly search for ceil value */
  for(i = low; i < high; i++)
  {
    if(arr[i] == x)
      return i;

    /* if x lies between arr[i] and arr[i+1] including
       arr[i+1], then return arr[i+1] */
    if(arr[i] < x && arr[i+1] >= x)
       return i+1;
  }

  /* If we reach here then x is greater than the last element
    of the array,  return -1 in this case */
  return -1;
}



/* Driver program to check above functions */
int main()
{
   int arr[] = {1, 2, 8, 10, 10, 12, 19};
   int n = sizeof(arr)/sizeof(arr[0]);
   int x = 3;
   int index = ceilSearch(arr, 0, n-1, x);
   if(index == -1)
     printf("Ceiling of %d doesn't exist in array ", x);
   else
     printf("ceiling of %d is %d", x, arr[index]);
   getchar();
   return 0;
}
```

Time Complexity: O(n)

**Method 2 (Binary Search)**
Instead of using linear search, binary search is used here to find out the index. Binary search reduces time complexity to O(Logn).

```
#include<stdio.h>
```

```
/* Function to get index of ceiling of x in arr[low..high]*/
int ceilSearch(int arr[], int low, int high, int x)
{
  int mid;

  /* If x is smaller than or equal to the first element,
    then return the first element */
  if(x <= arr[low])
    return low;

  /* If x is greater than the last element, then return -1 */
  if(x > arr[high])
    return -1;

  /* get the index of middle element of arr[low..high]*/
  mid = (low + high)/2;  /* low + (high - low)/2 */

  /* If x is same as middle element, then return mid */
  if(arr[mid] == x)
    return mid;

  /* If x is greater than arr[mid], then either arr[mid + 1]
    is ceiling of x or ceiling lies in arr[mid+1...high] */
  else if(arr[mid] < x)
  {
    if(mid + 1 <= high && x <= arr[mid+1])
      return mid + 1;
    else
      return ceilSearch(arr, mid+1, high, x);
  }

  /* If x is smaller than arr[mid], then either arr[mid]
     is ceiling of x or ceiling lies in arr[mid-1...high] */
  else
  {
    if(mid - 1 >= low && x > arr[mid-1])
      return mid;
    else
      return ceilSearch(arr, low, mid - 1, x);
  }
}

/* Driver program to check above functions */
int main()
```

```
{
  int arr[] = {1, 2, 8, 10, 10, 12, 19};
  int n = sizeof(arr)/sizeof(arr[0]);
  int x = 20;
  int index = ceilSearch(arr, 0, n-1, x);
  if(index == -1)
    printf("Ceiling of %d doesn't exist in array ", x);
  else
    printf("ceiling of %d is %d", x, arr[index]);
  getchar();
  return 0;
}
```

Time Complexity: O(Logn)

Please write comments if you find any of the above codes/algorithms incorrect, or find better ways to solve the same problem, or want to share code for floor implementation.

Â  Â

27. A Product Array Puzzle

Given an array arr[] of n integers, construct a Product Array prod[] (of same size) such that prod[i] is equal to the product of all the elements of arr[] except arr[i]. Solve it **without division operator and in O(n)**.

Example:
arr[] = {10, 3, 5, 6, 2}
prod[] = {180, 600, 360, 300, 900}

Algorithm:
1) Construct a temporary array left[] such that left[i] contains product of all elements on left of arr[i] excluding arr[i].
2) Construct another temporary array right[] such that right[i] contains product of all elements on on right of arr[i] excluding arr[i].
3) To get prod[], multiply left[] and right[].

Implementation:

```
#include<stdio.h>
#include<stdlib.h>

/* Function to print product array for a given array
 arr[] of size n */
void productArray(int arr[], int n)
```

```
{
  /* Allocate memory for temporary arrays left[] and right[] */
  int *left = (int *)malloc(sizeof(int)*n);
  int *right = (int *)malloc(sizeof(int)*n);

  /* Allocate memory for the product array */
  int *prod = (int *)malloc(sizeof(int)*n);

  int i, j;

  /* Left most element of left array is always 1 */
  left[0] = 1;

  /* Rightmost most element of right array is always 1 */
  right[n-1] = 1;

  /* Construct the left array */
  for(i = 1; i < n; i++)
    left[i] = arr[i-1]*left[i-1];

  /* Construct the right array */
  for(j = n-2; j >=0; j--)
    right[j] = arr[j+1]*right[j+1];

  /* Construct the product array using
    left[] and right[] */
  for (i=0; i<n; i++)
    prod[i] = left[i] * right[i];

  /* print the constructed prod array */
  for (i=0; i<n; i++)
    printf("%d ", prod[i]);

  return;
}

/* Driver program to test above functions */
int main()
{
  int arr[] = {10, 3, 5, 6, 2};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("The product array is: \n");
  productArray(arr, n);
  getchar();
```

```
}
```

Time Complexity: O(n)
Space Complexity: O(n)
Auxiliary Space: O(n)

**The above method can be optimized to work in space complexity O(1)**. Thanks to Dileep for suggesting the below solution.

```
void productArray(int arr[], int n)
{
  int i, temp = 1;

  /* Allocate memory for the product array */
  int *prod = (int *)malloc(sizeof(int)*n);

  /* Initialize the product array as 1 */
  memset(prod, 1, n);

  /* In this loop, temp variable contains product of
    elements on left side excluding arr[i] */
  for(i=0; i<n; i++)
  {
    prod[i] = temp;
    temp *= arr[i];
  }

  /* Initialize temp to 1 for product on right side */
  temp = 1;

  /* In this loop, temp variable contains product of
    elements on right side excluding arr[i] */
  for(i= n-1; i>=0; i--)
  {
    prod[i] *= temp;
    temp *= arr[i];
  }

  /* print the constructed prod array */
  for (i=0; i<n; i++)
    printf("%d ", prod[i]);

  return;
}
```

Time Complexity: O(n)
Space Complexity: O(n)
Auxiliary Space: O(1)

Please write comments if you find the above code/algorithm incorrect, or find better ways to solve the same problem.

Â  Â

28. Segregate Even and Odd numbers

Given an array A[], write a function that segregates even and odd numbers. The functions should put all even numbers first, and then odd numbers.

Example
Input = {12, 34, 45, 9, 8, 90, 3}
Output = {12, 34, 8, 90, 45, 9, 3}

In the output, order of numbers can be changed, i.e., in the above example 34 can come before 12 and 3 can come before 9.

The problem is very similar to our old post Segregate 0s and 1s in an array, and both of these problems are variation of famous Dutch national flag problem.

**Algorithm: segregateEvenOdd()**
1) Initialize two index variables left and right:
left = 0, right = size -1
2) Keep incrementing left index until we see an odd number.
3) Keep decrementing right index until we see an even number.
4) If lef < right then swap arr[left] and arr[right]

### Implementation:

```
#include<stdio.h>

/* Function to swap *a and *b */
void swap(int *a, int *b);

void segregateEvenOdd(int arr[], int size)
{
  /* Initialize left and right indexes */
  int left = 0, right = size-1;
  while(left < right)
  {
```

```
    /* Increment left index while we see 0 at left */
    while(arr[left]%2 == 0 && left < right)
       left++;

    /* Decrement right index while we see 1 at right */
    while(arr[right]%2 == 1 && left < right)
       right--;

    if(left < right)
    {
      /* Swap arr[left] and arr[right]*/
      swap(&arr[left], &arr[right]);
      left++;
      right--;
    }
  }
}

/* UTILITY FUNCTIONS */
void swap(int *a, int *b)
{
  int temp = *a;
  *a = *b;
  *b = temp;
}

/* driver program to test */
int main()
{
  int arr[] = {12, 34, 45, 9, 8, 90, 3};
  int arr_size = 7, i = 0;

  segregateEvenOdd(arr, arr_size);

  printf("array after segregation ");
  for(i = 0; i < arr_size; i++)
    printf("%d ", arr[i]);

  getchar();
  return 0;
}
```

Time Complexity: O(n)

References:
http://www.csse.monash.edu.au/~lloyd/tildeAlgDS/Sort/Flag/

Please write comments if you find the above code/algorithm incorrect, or find better ways to solve the same problem.

Â  Â

29. Find the two repeating elements in a given array

You are given an array of n+2 elements. All elements of the array are in range 1 to n. And all elements occur once except two numbers which occur twice. Find the two repeating numbers.

For example, array = {4, 2, 4, 5, 2, 3, 1} and n = 5

The above array has n + 2 = 7 elements with all elements occurring once except 2 and 4 which occur twice. So the output should be 4 2.

**Method 1 (Basic)**
Use two loops. In the outer loop, pick elements one by one and count the number of occurrences of the picked element in the inner loop.

This method doesnâ€™t use the other useful data provided in questions like range of numbers is between 1 to n and there are only two repeating elements.

```
#include<stdio.h>
#include<stdlib.h>
void printRepeating(int arr[], int size)
{
  int i, j;
  printf(" Repeating elements are ");
  for(i = 0; i < size; i++)
    for(j = i+1; j < size; j++)
      if(arr[i] == arr[j])
        printf(" %d ", arr[i]);
}

int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
```

```
}
```

Time Complexity: O(n*n)
Auxiliary Space: O(1)

**Method 2 (Use Count array)**
Traverse the array once. While traversing, keep track of count of all elements in the array using a temp array count[] of size n, when you see an element whose count is already set, print it as duplicate.

This method uses the range given in the question to restrict the size of count[], but doesnâ€™t use the data that there are only two repeating elements.

```
#include<stdio.h>
#include<stdlib.h>

void printRepeating(int arr[], int size)
{
  int *count = (int *)calloc(sizeof(int), (size - 2));
  int i;

  printf(" Repeating elements are ");
  for(i = 0; i < size; i++)
  {
    if(count[arr[i]] == 1)
      printf(" %d ", arr[i]);
    else
     count[arr[i]]++;
  }
}

int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

**Method 3 (Make two equations)**
Let the numbers which are being repeated are X and Y. We make two equations for X and Y and the simple task left is to solve the two equations.

We know the sum of integers from 1 to n is n(n+1)/2 and product is n!. We calculate the sum of input array, when this sum is subtracted from n(n+1)/2, we get X + Y because X and Y are the two numbers missing from set [1..n]. Similarly calculate product of input array, when this product is divided from n!, we get X*Y. Given sum and product of X and Y, we can find easily out X and Y.

Let summation of all numbers in array be S and product be P

X + Y = S â€“ n(n+1)/2
XY = P/n!

Using above two equations, we can find out X and Y. For array = 4 2 4 5 2 3 1, we get S = 21 and P as 960.

X + Y = 21 â€“ 15 = 6

XY = 960/5! = 8

X â€“ Y = sqrt((X+Y)^2 â€“ 4*XY) = sqrt(4) = 2

Using below two equations, we easily get X = (6 + 2)/2 and Y = (6-2)/2
X + Y = 6
X â€“ Y = 2

Thanks to geek4u for suggesting this method. As pointed by Beginer , there can be addition and multiplication overflow problem with this approach.
The methods 3 and 4 use all useful information given in the question 🙂

```
#include<stdio.h>
#include<stdlib.h>
#include<math.h>

/* function to get factorial of n */
int fact(int n);

void printRepeating(int arr[], int size)
{
  int S = 0;  /* S is for sum of elements in arr[] */
  int P = 1;  /* P is for product of elements in arr[] */
  int x,  y;   /* x and y are two repeating elements */
  int D;      /* D is for difference of x and y, i.e., x-y*/
  int n = size - 2,  i;

  /* Calculate Sum and Product of all elements in arr[] */
  for(i = 0; i < size; i++)
  {
    S = S + arr[i];
```

```
   P = P*arr[i];
  }

  S = S - n*(n+1)/2;  /* S is x + y now */
  P = P/fact(n);         /* P is x*y now */

  D = sqrt(S*S - 4*P); /* D is x - y now */

  x = (D + S)/2;
  y = (S - D)/2;

  printf("The two Repeating elements are %d & %d", x, y);
}

int fact(int n)
{
   return (n == 0)? 1 : n*fact(n-1);
}

int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(1)

**Method 4 (Use XOR)**
Thanks to neophyte for suggesting this method.
The approach used here is similar to method 2 of this post.
Let the repeating numbers be X and Y, if we xor all the elements in the array and all integers from 1 to n, then the result is X xor Y.
The 1â€™s in binary representation of X xor Y is corresponding to the different bits between X and Y. Suppose that the kth bit of X xor Y is 1, we can xor all the elements in the array and all integers from 1 to n, whose kth bits are 1. The result will be one of X and Y.

```
void printRepeating(int arr[], int size)
{
  int xor = arr[0]; /* Will hold xor of all elements */
  int set_bit_no;  /* Will have only single set bit of xor */
```

```
  int i;
  int n = size - 2;
  int x = 0, y = 0;

  /* Get the xor of all elements in arr[] and {1, 2 .. n} */
  for(i = 1; i < size; i++)
    xor ^= arr[i];
  for(i = 1; i <= n; i++)
    xor ^= i;

  /* Get the rightmost set bit in set_bit_no */
  set_bit_no = xor & ~(xor-1);

  /* Now divide elements in two sets by comparing rightmost set
   bit of xor with bit at same position in each element. */
  for(i = 0; i < size; i++)
  {
    if(arr[i] & set_bit_no)
      x = x ^ arr[i]; /*XOR of first set in arr[] */
    else
      y = y ^ arr[i]; /*XOR of second set in arr[] */
  }
  for(i = 1; i <= n; i++)
  {
    if(i & set_bit_no)
      x = x ^ i; /*XOR of first set in arr[] and {1, 2, ...n }*/
    else
      y = y ^ i; /*XOR of second set in arr[] and {1, 2, ...n } */
  }

  printf("\n The two repeating elements are %d & %d ", x, y);
}



int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

### Method 5 (Use array elements as index)

Thanks to Manish K. Aasawat for suggesting this method.

```
Traverse the array. Do following for every index i of A[].
{
check for sign of A[abs(A[i])] ;
if positive then
   make it negative by   A[abs(A[i])]=-A[abs(A[i])];
else  // i.e., A[abs(A[i])] is negative
   this   element (ith element of list) is a repetition
}
```

```
Example: A[] =  {1, 1, 2, 3, 2}
i=0;
Check sign of A[abs(A[0])] which is A[1].  A[1] is positive, so make it negative.
Array now becomes {1, -1, 2, 3, 2}

i=1;
Check sign of A[abs(A[1])] which is A[1].  A[1] is negative, so A[1] is a repetition.

i=2;
Check sign of A[abs(A[2])] which is A[2].  A[2] is  positive, so make it negative. '
Array now becomes {1, -1, -2, 3, 2}

i=3;
Check sign of A[abs(A[3])] which is A[3].  A[3] is  positive, so make it negative.
Array now becomes {1, -1, -2, -3, 2}

i=4;
Check sign of A[abs(A[4])] which is A[2].  A[2] is negative, so A[4] is a repetition.
```

Note that this method modifies the original array and may not be a recommended method if we are not allowed to modify the array.

```
#include <stdio.h>
#include <stdlib.h>

void printRepeating(int arr[], int size)
{
  int i;

  printf("\n The repeating elements are");

  for(i = 0; i < size; i++)
  {
    if(arr[abs(arr[i])] > 0)
```

```
      arr[abs(arr[i])] = -arr[abs(arr[i])];
    else
      printf(" %d ", abs(arr[i]));
  }
}

int main()
{
  int arr[] = {1, 3, 2, 2, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

The problem can be solved in linear time using other method also, please see this and this comments

Please write comments if you find the above codes/algorithms incorrect, or find better ways to solve the same problem.

Â  Â

You are given an array of n+2 elements. All elements of the array are in range 1 to n. And all elements occur once except two numbers which occur twice. Find the two repeating numbers.

For example, array = {4, 2, 4, 5, 2, 3, 1} and n = 5

The above array has n + 2 = 7 elements with all elements occurring once except 2 and 4 which occur twice. So the output should be 4 2.

**Method 1 (Basic)**
Use two loops. In the outer loop, pick elements one by one and count the number of occurrences of the picked element in the inner loop.

This method doesnâ€™t use the other useful data provided in questions like range of numbers is between 1 to n and there are only two repeating elements.

```
#include<stdio.h>
#include<stdlib.h>
void printRepeating(int arr[], int size)
{
  int i, j;
  printf(" Repeating elements are ");
```

```
  for(i = 0; i < size; i++)
    for(j = i+1; j < size; j++)
      if(arr[i] == arr[j])
        printf(" %d ", arr[i]);
}

int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Time Complexity: O(n*n)
Auxiliary Space: O(1)

**Method 2 (Use Count array)**
Traverse the array once. While traversing, keep track of count of all elements in the array using a temp array count[] of size n, when you see an element whose count is already set, print it as duplicate.

This method uses the range given in the question to restrict the size of count[], but doesnâ€™t use the data that there are only two repeating elements.

```
#include<stdio.h>
#include<stdlib.h>

void printRepeating(int arr[], int size)
{
  int *count = (int *)calloc(sizeof(int), (size - 2));
  int i;

  printf(" Repeating elements are ");
  for(i = 0; i < size; i++)
  {
    if(count[arr[i]] == 1)
      printf(" %d ", arr[i]);
    else
     count[arr[i]]++;
  }
}

int main()
```

```
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

**Method 3 (Make two equations)**
Let the numbers which are being repeated are X and Y. We make two equations for X and Y and the simple task left is to solve the two equations.
We know the sum of integers from 1 to n is n(n+1)/2 and product is n!. We calculate the sum of input array, when this sum is subtracted from n(n+1)/2, we get X + Y because X and Y are the two numbers missing from set [1..n]. Similarly calculate product of input array, when this product is divided from n!, we get X*Y. Given sum and product of X and Y, we can find easily out X and Y.

Let summation of all numbers in array be S and product be P

X + Y = S â€“ n(n+1)/2
XY = P/n!

Using above two equations, we can find out X and Y. For array = 4 2 4 5 2 3 1, we get S = 21 and P as 960.

X + Y = 21 â€“ 15 = 6

XY = 960/5! = 8

X â€“ Y = sqrt((X+Y)^2 â€“ 4*XY) = sqrt(4) = 2

Using below two equations, we easily get X = (6 + 2)/2 and Y = (6-2)/2
X + Y = 6
X â€“ Y = 2

Thanks to geek4u for suggesting this method. As pointed by Beginer , there can be addition and multiplication overflow problem with this approach.
The methods 3 and 4 use all useful information given in the question 😃

```
#include<stdio.h>
#include<stdlib.h>
#include<math.h>

/* function to get factorial of n */
int fact(int n);
```

```
void printRepeating(int arr[], int size)
{
  int S = 0;  /* S is for sum of elements in arr[] */
  int P = 1;  /* P is for product of elements in arr[] */
  int x,  y;   /* x and y are two repeating elements */
  int D;      /* D is for difference of x and y, i.e., x-y*/
  int n = size - 2,  i;

  /* Calculate Sum and Product of all elements in arr[] */
  for(i = 0; i < size; i++)
  {
    S = S + arr[i];
    P = P*arr[i];
  }

  S = S - n*(n+1)/2;  /* S is x + y now */
  P = P/fact(n);         /* P is x*y now */

  D = sqrt(S*S - 4*P); /* D is x - y now */

  x = (D + S)/2;
  y = (S - D)/2;

  printf("The two Repeating elements are %d & %d", x, y);
}

int fact(int n)
{
   return (n == 0)? 1 : n*fact(n-1);
}

int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(1)

**Method 4 (Use XOR)**

Thanks to neophyte for suggesting this method.

The approach used here is similar to method 2 of this post.

Let the repeating numbers be X and Y, if we xor all the elements in the array and all integers from 1 to n, then the result is X xor Y.

The 1â€™s in binary representation of X xor Y is corresponding to the different bits between X and Y. Suppose that the kth bit of X xor Y is 1, we can xor all the elements in the array and all integers from 1 to n, whose kth bits are 1. The result will be one of X and Y.

```
void printRepeating(int arr[], int size)
{
  int xor = arr[0]; /* Will hold xor of all elements */
  int set_bit_no;  /* Will have only single set bit of xor */
  int i;
  int n = size - 2;
  int x = 0, y = 0;

  /* Get the xor of all elements in arr[] and {1, 2 .. n} */
  for(i = 1; i < size; i++)
    xor ^= arr[i];
  for(i = 1; i <= n; i++)
    xor ^= i;

  /* Get the rightmost set bit in set_bit_no */
  set_bit_no = xor & ~(xor-1);

  /* Now divide elements in two sets by comparing rightmost set
   bit of xor with bit at same position in each element. */
  for(i = 0; i < size; i++)
  {
    if(arr[i] & set_bit_no)
      x = x ^ arr[i]; /*XOR of first set in arr[] */
    else
      y = y ^ arr[i]; /*XOR of second set in arr[] */
  }
  for(i = 1; i <= n; i++)
  {
    if(i & set_bit_no)
      x = x ^ i; /*XOR of first set in arr[] and {1, 2, ...n }*/
    else
      y = y ^ i; /*XOR of second set in arr[] and {1, 2, ...n } */
  }
```

```
  printf("\n The two repeating elements are %d & %d ", x, y);
}



int main()
{
  int arr[] = {4, 2, 4, 5, 2, 3, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

### Method 5 (Use array elements as index)

Thanks to Manish K. Aasawat for suggesting this method.

```
Traverse the array. Do following for every index i of A[].
{
check for sign of A[abs(A[i])] ;
if positive then
   make it negative by   A[abs(A[i])]=-A[abs(A[i])];
else  // i.e., A[abs(A[i])] is negative
   this   element (ith element of list) is a repetition
}
```

```
Example: A[] =  {1, 1, 2, 3, 2}
i=0;
Check sign of A[abs(A[0])] which is A[1].  A[1] is positive, so make it negative.
Array now becomes {1, -1, 2, 3, 2}

i=1;
Check sign of A[abs(A[1])] which is A[1].  A[1] is negative, so A[1] is a repetition.

i=2;
Check sign of A[abs(A[2])] which is A[2].  A[2] is  positive, so make it negative. '
Array now becomes {1, -1, -2, 3, 2}

i=3;
Check sign of A[abs(A[3])] which is A[3].  A[3] is  positive, so make it negative.
Array now becomes {1, -1, -2, -3, 2}

i=4;
Check sign of A[abs(A[4])] which is A[2].  A[2] is negative, so A[4] is a repetition.
```

Note that this method modifies the original array and may not be a recommended

method if we are not allowed to modify the array.

```
#include <stdio.h>
#include <stdlib.h>

void printRepeating(int arr[], int size)
{
  int i;

  printf("\n The repeating elements are");

  for(i = 0; i < size; i++)
  {
    if(arr[abs(arr[i])] > 0)
      arr[abs(arr[i])] = -arr[abs(arr[i])];
    else
      printf(" %d ", abs(arr[i]));
  }
}

int main()
{
  int arr[] = {1, 3, 2, 2, 1};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

The problem can be solved in linear time using other method also, please see this and this comments

Please write comments if you find the above codes/algorithms incorrect, or find better ways to solve the same problem.

Â  Â

31. Find the Minimum length Unsorted Subarray, sorting which makes the complete array sorted

Given an unsorted array arr[0..n-1] of size n, find the minimum length subarray arr[s..e] such that sorting this subarray makes the whole array sorted.

**Examples:**
1) If the input array is [10, 12, 20, 30, 25, 40, 32, 31, 35, 50, 60], your program should

be able to find that the subarray lies between the indexes 3 and 8.

2) If the input array is [0, 1, 15, 25, 6, 7, 30, 40, 50], your program should be able to find that the subarray lies between the indexes 2 and 5.

**Solution:**

**1) Find the candidate unsorted subarray**

a) Scan from left to right and find the first element which is greater than the next element. Let *s* be the index of such an element. In the above example 1, *s* is 3 (index of 30).

b) Scan from right to left and find the first element (first in right to left order) which is smaller than the next element (next in right to left order). Let *e* be the index of such an element. In the above example 1, e is 7 (index of 31).

**2) Check whether sorting the candidate unsorted subarray makes the complete array sorted or not. If not, then include more elements in the subarray.**

a) Find the minimum and maximum values in *arr[s..e]*. Let minimum and maximum values be *min* and *max*. *min* and *max* for [30, 25, 40, 32, 31] are 25 and 40 respectively.

b) Find the first element (if there is any) in *arr[0..s-1]* which is greater than *min*, change *s* to index of this element. There is no such element in above example 1.

c) Find the last element (if there is any) in *arr[e+1..n-1]* which is smaller than max, change *e* to index of this element. In the above example 1, e is changed to 8 (index of 35)

**3) Print *s* and *e*.**

**Implementation:**

```
#include<stdio.h>

void printUnsorted(int arr[], int n)
{
  int s = 0, e = n-1, i, max, min;

  // step 1(a) of above algo
  for (s = 0; s < n-1; s++)
  {
    if (arr[s] > arr[s+1])
      break;
  }
  if (s == n-1)
  {
    printf("The complete array is sorted");
    return;
```

```
 }

 // step 1(b) of above algo
 for(e = n - 1; e > 0; e--)
 {
   if(arr[e] < arr[e-1])
     break;
 }

 // step 2(a) of above algo
 max = arr[s]; min = arr[s];
 for(i = s + 1; i <= e; i++)
 {
   if(arr[i] > max)
     max = arr[i];
   if(arr[i] < min)
     min = arr[i];
 }

 // step 2(b) of above algo
 for( i = 0; i < s; i++)
 {
   if(arr[i] > min)
   {
     s = i;
     break;
   }
 }

 // step 2(c) of above algo
 for( i = n -1; i >= e+1; i--)
 {
   if(arr[i] < max)
   {
     e = i;
     break;
   }
 }

 // step 3 of above algo
 printf(" The unsorted subarray which makes the given array "
        " sorted lies between the indees %d and %d", s, e);
 return;
}
```

```
int main()
{
  int arr[] = {10, 12, 20, 30, 25, 40, 32, 31, 35, 50, 60};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printUnsorted(arr, arr_size);
  getchar();
  return 0;
}
```

**Time Complexity:** O(n)

Please write comments if you find the above code/algorithm incorrect, or find better ways to solve the same problem.

Â  Â

32.  Find duplicates in O(n) time and O(1) extra space

Given an array of n elements which contains elements from 0 to n-1, with any of these numbers appearing any number of times. Find these repeating numbers in O(n) and using only constant memory space.

For example, let n be 7 and array be {1, 2, 3, 1, 3, 6, 6}, the answer should be 1, 3 and 6.

This problem is an extended version of following problem.

Find the two repeating elements in a given array

Method 1 and Method 2 of the above link are not applicable as the question says O(n) time complexity and O(1) constant space. Also, Method 3 and Method 4 cannot be applied here because there can be more than 2 repeating elements in this problem. Method 5 can be extended to work for this problem. Below is the solution that is similar to the Method 5.

**Algorithm:**

```
traverse the list for i= 0 to n-1 elements
{
  check for sign of A[abs(A[i])] ;
  if positive then
     make it negative by   A[abs(A[i])]=-A[abs(A[i])];
  else  // i.e., A[abs(A[i])] is negative
     this   element (ith element of list) is a repetition
}
```

**Implementation:**

```
#include <stdio.h>
#include <stdlib.h>

void printRepeating(int arr[], int size)
{
  int i;
  printf("The repeating elements are: \n");
  for (i = 0; i < size; i++)
  {
    if (arr[abs(arr[i])] >= 0)
      arr[abs(arr[i])] = -arr[abs(arr[i])];
    else
      printf(" %d ", abs(arr[i]));
  }
}

int main()
{
  int arr[] = {1, 2, 3, 1, 3, 6, 6};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printRepeating(arr, arr_size);
  getchar();
  return 0;
}
```

Note: The above program doesnâ€™t handle 0 case (If 0 is present in array). The program can be easily modified to handle that also. It is not handled to keep the code simple.

Output:
*The repeating elements are:*
*1 3 6*

Time Complexity: O(n)
Auxiliary Space: O(1)

Please write comments if you find the above codes/algorithms incorrect, or find better ways to solve the same problem.

Â  Â

Equilibrium index of an array is an index such that the sum of elements at lower indexes is equal to the sum of elements at higher indexes. For example, in an arrya A:

A[0] = -7, A[1] = 1, A[2] = 5, A[3] = 2, A[4] = -4, A[5] = 3, A[6]=0

3 is an equilibrium index, because:
A[0] + A[1] + A[2] = A[4] + A[5] + A[6]

6 is also an equilibrium index, because sum of zero elements is zero, i.e., A[0] + A[1] + A[2] + A[3] + A[4] + A[5]=0

7 is not an equilibrium index, because it is not a valid index of array A.

Write a function *int equilibrium(int[] arr, int n)*; that given a sequence arr[] of size n, returns an equilibrium index (if any) or -1 if no equilibrium indexes exist.

**Method 1 (Simple but inefficient)**
Use two loops. Outer loop iterates through all the element and inner loop finds out whether the current index picked by the outer loop is equilibrium index or not. Time complexity of this solution is O(n^2).

```
#include <stdio.h>

int equilibrium(int arr[], int n)
{
  int i, j;
  int leftsum, rightsum;

  /* Check for indexes one by one until an equilibrium
    index is found */
  for ( i = 0; i < n; ++i)
  {
    leftsum = 0;  // initialize left sum for current index i
    rightsum = 0; // initialize right sum for current index i

    /* get left sum */
    for ( j = 0; j < i; j++)
      leftsum  += arr[j];

    /* get right sum */
    for( j = i+1; j < n; j++)
      rightsum += arr[j];

    /* if leftsum and rightsum are same, then we are done */
    if (leftsum == rightsum)
```

```
      return i;
    }

  /* return -1 if no equilibrium index is found */
  return -1;
}

int main()
{
  int arr[] = {-7, 1, 5, 2, -4, 3, 0};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printf("%d\n", equilibrium(arr, arr_size));

  getchar();
  return 0;
}
```

Time Complexity: O(n^2)

**Method 2 (Tricky and Efficient)**

The idea is to get total sum of array first. Then Iterate through the array and keep updating the left sum which is initialized as zero. In the loop, we can get right sum by subtracting the elements one by one. Thanks to Sambasiva for suggesting this solution and providing code for this.

```
1) Initialize leftsum  as 0
2) Get the total sum of the array as sum
3) Iterate through the array and for each index i, do following.
    a)  Update sum to get the right sum.
          sum = sum - arr[i]
       // sum is now right sum
    b) If leftsum is equal to sum, then return current index.
    c) leftsum = leftsum + arr[i] // update leftsum for next iteration.
4) return -1 // If we come out of loop without returning then
             // there is no equilibrium index
```

```
#include <stdio.h>

int equilibrium(int arr[], int n)
{
   int sum = 0;      // initialize sum of whole array
   int leftsum = 0; // initialize leftsum
   int i;
```

```
  /* Find sum of the whole array */
  for (i = 0; i < n; ++i)
       sum += arr[i];

  for( i = 0; i < n; ++i)
  {
     sum -= arr[i]; // sum is now right sum for index i

     if(leftsum == sum)
       return i;

     leftsum += arr[i];
  }

   /* If no equilibrium index found, then return 0 */
   return -1;
}

int main()
{
  int arr[] = {-7, 1, 5, 2, -4, 3, 0};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  printf("First equilibrium index is %d\n", equilibrium(arr, arr_size));

  getchar();
  return 0;
}
```

Time Complexity: O(n)

As pointed out by Sameer, we can remove the return statement and add a print statement to print all equilibrium indexes instead of returning only one.

Please write comments if you find the above codes/algorithms incorrect, or find better ways to solve the same problem.

Â  Â

34. Linked List vs Array

**Difficulty Level:** Rookie

Both Arrays and Linked List can be used to store linear data of similar types, but they both have some advantages and disadvantages over each other.

Following are the points in favour of Linked Lists.

(1) The size of the arrays is fixed: So we must know the upper limit on the number of elements in advance. Also, generally, the allocated memory is equal to the upper limit irrespective of the usage, and in practical uses, upper limit is rarely reached.

(2) Inserting a new element in an array of elements is expensive, because room has to be created for the new elements and to create room existing elements have to shifted.

For example, suppose we maintain a sorted list of IDs in an array id[].

id[] = [1000, 1010, 1050, 2000, 2040, â€¦].

And if we want to insert a new ID 1005, then to maintain the sorted order, we have to move all the elements after 1000 (excluding 1000).

Deletion is also expensive with arrays until unless some special techniques are used. For example, to delete 1010 in id[], everything after 1010 has to be moved.

So Linked list provides following two advantages over arrays
1) Dynamic size
2) Ease of insertion/deletion

Linked lists have following drawbacks:
1) Random access is not allowed. We have to access elements sequentially starting from the first node. So we cannot do binary search with linked lists.
2) Extra memory space for a pointer is required with each element of the list.
3) Arrays have better cache locality that can make a pretty big difference in performance.

Please also see this thread.

References:
http://cslibrary.stanford.edu/103/LinkedListBasics.pdf

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

35. Which sorting algorithm makes minimum number of memory writes?

Minimizing the number of writes is useful when making writes to some huge data set is very expensive, such as with EEPROMs or Flash memory, where each write reduces the lifespan of the memory.

Among the sorting algorithms that we generally study in our data structure and

algorithm courses,Â  Selection Sort makes least number of writes (it makes O(n) swaps).Â  But, Cycle Sort almost always makes less number of writes compared to Selection Sort.Â  In Cycle Sort, each value is either written zero times, if itâ€™s already in its correct position, or written one time to its correct position. This matches the minimal number of overwrites required for a completed in-place sort.

Sources:
http://en.wikipedia.org/wiki/Cycle_sort
http://en.wikipedia.org/wiki/Selection_sort

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

36.  Turn an image by 90 degree

Given an image, how will you turn it by 90 degrees?Â A vagueÂ question. Minimize the browser and try your solution before going further.

An image can be treated as 2D matrix which can be stored in a buffer. We are provided with matrix dimensions and itâ€™s base address. How can we turn it?

For example see the below picture,

```
* * * ^ * * *
* * * | * * *
* * * | * * *
* * * | * * *
```

After rotating right, it appears (observe arrow direction)

```
* * * *
* * * *
* * * *
-- - - >
* * * *
* * * *
* * * *
```

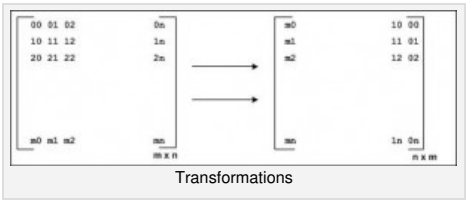
The idea is simple. Transform each row of source matrix into requiredÂ columnÂ of final image. We will use anÂ auxiliaryÂ buffer to transform the image.

From the above picture, we can observe that

```
first row of source ------> last column of destination
```

```
second row of source ------> last but-one column of destination
so ... on
last row of sourceÂ ------> first column of destination
```

In pictorial form, we can represent the above transformations of an (m x n) matrix into (n x m) matrix,



Transformations

If you have not attempted, atleast try your pseudo code now.

It will be easy to write our pseudo code. In C/C++ we will usually traverse matrix on row major order. Each row is transformed into different column of final image. We need to construct columns of final image. See the following algorithm (transformation)

```
for(r = 0; r < m; r++)
{
Â Â  for(c = 0; c < n; c++)
Â Â  {
Â Â  Â  Â // Hint:Â Map each source element indices into
     // indices of destination matrix element.
Â Â  Â  Â  dest_buffer [ c ] [ m - r - 1 ] = source_buffer [ r ] [ c ];
Â Â  }
}
```

Note that there are various ways to implement the algorithm based on traversal of matrix, row major or column major order. We have two matrices and two ways (row and column major) to traverse each matrix. Hence, there can atleast be 4 different ways of transformation of source matrix into final matrix.

**Code:**

```
#include <stdio.h>
#include <stdlib.h>

void displayMatrix(unsigned int const *p, unsigned int row, unsigned int col);
void rotate(unsigned int *pS, unsigned int *pD, unsigned int row, unsigned int col);

int main()
{
   // declarations
   unsigned int image[][4] = {{1,2,3,4}, {5,6,7,8}, {9,10,11,12}};
   unsigned int *pSource;
   unsigned int *pDestination;
```

```
    unsigned int m, n;

    // setting initial values and memory allocation
    m = 3, n = 4, pSource = (unsigned int *)image;
    pDestination = (unsigned int *)malloc(sizeof(int)*m*n);

    // process each buffer
    displayMatrix(pSource, m, n);

    rotate(pSource, pDestination, m, n);

    displayMatrix(pDestination, n, m);

    free(pDestination);

    getchar();
    return 0;
}

void displayMatrix(unsigned int const *p, unsigned int r, unsigned int c)
{
    unsigned int row, col;
    printf("\n\n");

    for(row = 0; row < r; row++)
    {
        for(col = 0; col < c; col++)
        {
            printf("%d\t", *(p + row * c + col));
        }
        printf("\n");
    }

    printf("\n\n");
}

void rotate(unsigned int *pS, unsigned int *pD, unsigned int row, unsigned int col)
{
    unsigned int r, c;
    for(r = 0; r < row; r++)
    {
        for(c = 0; c < col; c++)
        {
            *(pD + c * row + (row - r - 1)) = *(pS + r * col + c);
```

```
    }
  }
}
```

Compiled byÂ **Venki**.Â Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

37. Search in a row wise and column wise sorted matrix

Given an n x n matrix, where every row and column is sorted in increasing order. Given a number x, how to decide whether this x is in the matrix. The designed algorithm should have linear time complexity.

Thanks to devendraiiit for suggesting below approach.

1) Start with top right element
2) Loop: compare this element e with x
â€¦.i) if they are equal then return its position
â€¦ii) e < x then move it to down (if out of bound of matrix then break return false)
..iii) e > x then move it to left (if out of bound of matrix then break return false)
3) repeat the i), ii) and iii) till you find element or returned false

**Implementation:**

```
#include<stdio.h>

/* Searches the element x in mat[][]. If the element is found,
   then prints its position and returns true, otherwise prints
   "not found" and returns false */
int search(int mat[4][4], int n, int x)
{
  int i = 0, j = n-1;  //set indexes for top right element
  while ( i < n && j >= 0 )
  {
    if ( mat[i][j] == x )
    {
      printf("\n Found at %d, %d", i, j);
      return 1;
    }
    if ( mat[i][j] > x )
      j--;
    else //  if mat[i][j] < x
```

```
      i++;
  }

   printf("\n Element not found");
   return 0;  // if ( i==n || j== -1 )
}

// driver program to test above function
int main()
{
  int mat[4][4] = { {10, 20, 30, 40},
                    {15, 25, 35, 45},
                    {27, 29, 37, 48},
                    {32, 33, 39, 50},
                  };
  search(mat, 4, 29);
  getchar();
  return 0;
}
```

Time Complexity: O(n)

The above approach will also work for m x n matrix (not only for n x n). Complexity would be O(m + n).

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

38.  Next Greater Element

Given an array, print the Next Greater Element (NGE) for every element. The Next greater Element for an element x is the first greater element on the right side of x in array. Elements for which no greater element exist, consider next greater element as -1.

Examples:
**a)** For any array, rightmost element always has next greater element as -1.
**b)** For an array which is sorted in decreasing order, all elements have next greater element as -1.
**c)** For the input array [4, 5, 2, 25}, the next greater elements for each element are as follows.

```
Element       NGE
  4      -->   5
  5      -->   25
  2      -->   25
  25     -->   -1
```

**d)** For the input array [13, 7, 6, 12}, the next greater elements for each element are as follows.

```
 Element       NGE
  13     -->    -1
  7      -->    12
  6      -->    12
  12     -->    -1
```

**Method 1 (Simple)**
Use two loops: The outer loop picks all the elements one by one. The inner loop looks for the first greater element for the element picked by outer loop. If a greater element is found then that element is printed as next, otherwise -1 is printed.

Thanks to Sachin for providing following code.

```
#include<stdio.h>
/* prints element and NGE pair for all elements of
arr[] of size n */
void printNGE(int arr[], int n)
{
    int next = -1;
    int i = 0;
    int j = 0;
    for (i=0; i<n; i++)
    {
        next = -1;
        for (j = i+1; j<n; j++)
        {
            if (arr[i] < arr[j])
            {
                next = arr[j];
                break;
            }
        }
        printf("%d -- %d\n", arr[i], next);
    }
}
```

```
int main()
{
    int arr[]= {11, 13, 21, 3};
    int n = sizeof(arr)/sizeof(arr[0]);
    printNGE(arr, n);
    getchar();
    return 0;
}
```

Output:

```
11 -- 13
13 -- 21
21 -- -1
3 -- -1
```

Time Complexity: O(n^2). The worst case occurs when all elements are sorted in decreasing order.

**Method 2 (Using Stack)**
Thanks to pchild for suggesting following approach.
1) Push the first element to stack.
2) Pick rest of the elements one by one and follow following steps in loop.
â€¦.a) Mark the current element as *next*.
â€¦.b) If stack is not empty, then pop an element from stack and compare it with *next*.
â€¦.c) If next is greater than the popped element, then *next* is the next greater element fot the popped element.
â€¦.d) Keep poppoing from the stack while the popped element is smaller than *next*. *next* becomes the next greater element for all such popped elements
â€¦.g) If *next* is smaller than the popped element, then push the popped element back.
3) After the loop in step 2 is over, pop all the elements from stack and print -1 as next element for them.

```
#include<stdio.h>
#include<stdlib.h>
#define STACKSIZE 100

// stack structure
struct stack
{
    int top;
    int items[STACKSIZE];
};
```

```
// Stack Functions to be used by printNGE()
void push(struct stack *ps, int x)
{
    if (ps->top == STACKSIZE-1)
    {
        printf("Error: stack overflow\n");
        getchar();
        exit(0);
    }
    else
    {
        ps->top += 1;
        int top = ps->top;
        ps->items [top] = x;
    }
}

bool isEmpty(struct stack *ps)
{
    return (ps->top == -1)? true : false;
}

int pop(struct stack *ps)
{
    int temp;
    if (ps->top == -1)
    {
        printf("Error: stack underflow \n");
        getchar();
        exit(0);
    }
    else
    {
        int top = ps->top;
        temp = ps->items [top];
        ps->top -= 1;
        return temp;
    }
}

/* prints element and NGE pair for all elements of
arr[] of size n */
void printNGE(int arr[], int n)
```

```
{
   int i = 0;
   struct stack s;
   s.top = -1;
   int element, next;

   /* push the first element to stack */
   push(&s, arr[0]);

   // iterate for rest of the elements
   for (i=1; i<n; i++)
   {
      next = arr[i];

      if (isEmpty(&s) == false)
      {
         // if stack is not empty, then pop an element from stack
         element = pop(&s);

         /* If the popped element is smaller than next, then
            a) print the pair
            b) keep popping while elements are smaller and
            stack is not empty */
         while (element < next)
         {
            printf("\n %d --> %d", element, next);
            if(isEmpty(&s) == true)
               break;
            element = pop(&s);
         }

         /* If element is greater than next, then push
           the element back */
         if (element > next)
            push(&s, element);
      }

      /* push next to stack so that we can find
         next greater for it */
      push(&s, next);
   }

   /* After iterating over the loop, the remaining
      elements in stack do not have the next greater
```

```
      element, so print -1 for them */
    while (isEmpty(&s) == false)
    {
        element = pop(&s);
        next = -1;
        printf("\n %d -- %d", element, next);
    }
}

/* Driver program to test above functions */
int main()
{
    int arr[]= {11, 13, 21, 3};
    int n = sizeof(arr)/sizeof(arr[0]);
    printNGE(arr, n);
    getchar();
    return 0;
}
```

Output:

```
11 -- 13
13 -- 21
3 -- -1
21 -- -1
```

Time Complexity: O(n). The worst case occurs when all elements are sorted in decreasing order. If elements are sorted in decreasing order, then every element is processed at most 4 times.
a) Initialy pushed to the stack.
b) Popped from the stack when next element is being processed.
c) Pushed back to the stack because next element is smaller.
d) Popped from the stack in step 3 of algo.

Source:
http://geeksforgeeks.org/forum/topic/next-greater-element#post-60

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

Given an unsorted array of numbers, write a function that returns true if array consists of consecutive numbers.

Examples:
**a)** If array is {5, 2, 3, 1, 4}, then the function should return true because the array has consecutive numbers from 1 to 5.

**b)** If array is {83, 78, 80, 81, 79, 82}, then the function should return true because the array has consecutive numbers from 78 to 83.

**c)** If the array is {34, 23, 52, 12, 3 }, then the function should return false because the elements are not consecutive.

**d)** If the array is {7, 6, 5, 5, 3, 4}, then the function should return false because 5 and 5 are not consecutive.

**Method 1 (Use Sorting)**
1) Sort all the elements.
2) Do a linear scan of the sorted array. If the difference between current element and next element is anything other than 1, then return false. If all differences are 1, then return true.

Time Complexity: O(nLogn)

**Method 2 (Use visited array)**
The idea is to check for following two conditions. If following two conditions are true, then return true.
1) *max â€“ min + 1 = n* where max is the maximum element in array, min is minimum element in array and n is the number of elements in array.
2) All elements are distinct.

To check if all elements are distinct, we can create a visited[] array of size n. We can map the ith element of input array arr[] to visited array by using arr[i] â€“ min as index in visited[].

```
#include<stdio.h>
#include<stdlib.h>

/* Helper functions to get minimum and maximum in an array */
int getMin(int arr[], int n);
int getMax(int arr[], int n);

/* The function checks if the array elements are consecutive
  If elements are consecutive, then returns true, else returns
  false */
bool areConsecutive(int arr[], int n)
```

```
{
  if ( n <  1 )
    return false;

  /* 1) Get the minimum element in array */
  int min = getMin(arr, n);

  /* 2) Get the maximum element in array */
  int max = getMax(arr, n);

  /* 3) max - min + 1 is equal to n,  then only check all elements */
  if (max - min  + 1 == n)
  {
      /* Create a temp array to hold visited flag of all elements.
         Note that, calloc is used here so that all values are initialized
         as false */
      bool *visited = (bool *) calloc (n, sizeof(bool));
      int i;
      for (i = 0; i < n; i++)
      {
          /* If we see an element again, then return false */
          if ( visited[arr[i] - min] != false )
            return false;

          /* If visited first time, then mark the element as visited */
          visited[arr[i] - min] = true;
      }

      /* If all elements occur once, then return true */
      return true;
  }

  return false; // if (max - min  + 1 != n)
}

/* UTILITY FUNCTIONS */
int getMin(int arr[], int n)
{
  int min = arr[0];
  for (int i = 1; i < n; i++)
   if (arr[i] < min)
     min = arr[i];
  return min;
}
```

```
int getMax(int arr[], int n)
{
  int max = arr[0];
  for (int i = 1; i < n; i++)
   if (arr[i] > max)
     max = arr[i];
  return max;
}

/* Driver program to test above functions */
int main()
{
    int arr[]= {5, 4, 2, 3, 1, 6};
    int n = sizeof(arr)/sizeof(arr[0]);
    if(areConsecutive(arr, n) == true)
        printf(" Array elements are consecutive ");
    else
        printf(" Array elements are not consecutive ");
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Extra Space: O(n)

**Method 3 (Mark visited array elements as negative)**
This method is O(n) time complexity and O(1) extra space, but it changes the original array and it works only if all numbers are positive. We can get the original array by adding an extra step though. It is an extension of method 2 and it has the same two steps.
1) *max â€“ min + 1 = n* where max is the maximum element in array, min is minimum element in array and n is the number of elements in array.
2) All elements are distinct.

In this method, the implementation of step 2 differs from method 2. Instead of creating a new array, we modify the input array arr[] to keep track of visited elements. The idea is to traverse the array and for each index i (where 0 <= i < n), make arr[arr[i] - min]] as a negative value. If we see a negative value again then there is repetition.

```
#include<stdio.h>
#include<stdlib.h>

/* Helper functions to get minimum and maximum in an array */
int getMin(int arr[], int n);
```

```
int getMax(int arr[], int n);

/* The function checks if the array elements are consecutive
  If elements are consecutive, then returns true, else returns
  false */
bool areConsecutive(int arr[], int n)
{

    if ( n <  1 )
        return false;

    /* 1) Get the minimum element in array */
    int min = getMin(arr, n);

    /* 2) Get the maximum element in array */
    int max = getMax(arr, n);

    /* 3) max â€“ min + 1 is equal to n then only check all elements */
    if (max â€“ min  + 1 == n)
    {
        int i;
        for(i = 0; i < n; i++)
        {
            int j;

            if (arr[i] < 0)
                j = -arr[i] â€“ min;
            else
                j = arr[i] â€“ min;

            // if the value at index j is negative then
            // there is repitition
            if (arr[j] > 0)
                arr[j] = -arr[j];
            else
                return false;
        }

        /* If we do not see a negative value then all elements
           are distinct */
        return true;
    }

    return false; // if (max â€“ min  + 1 != n)
```

```
}

/* UTILITY FUNCTIONS */
int getMin(int arr[], int n)
{
    int min = arr[0];
    for (int i = 1; i < n; i++)
        if (arr[i] < min)
            min = arr[i];
    return min;
}

int getMax(int arr[], int n)
{
    int max = arr[0];
    for (int i = 1; i < n; i++)
        if (arr[i] > max)
            max = arr[i];
    return max;
}

/* Driver program to test above functions */
int main()
{
    int arr[]= {1, 4, 5, 3, 2, 6};
    int n = sizeof(arr)/sizeof(arr[0]);
    if(areConsecutive(arr, n) == true)
        printf(" Array elements are consecutive ");
    else
        printf(" Array elements are not consecutive ");
    getchar();
    return 0;
}
```

Note that this method might not work for negative numbers. For example, it returns false for {2, 1, 0, -3, -1, -2}.

Time Complexity: O(n)
Extra Space: O(1)

Source: http://geeksforgeeks.org/forum/topic/amazon-interview-question-for-software-engineerdeveloper-fresher-9

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â Â

40. Find the smallest missing number

Given a **sorted** array of n integers where each integer is in the range from 0 to m-1 and m > n. Find the smallest number that is missing from the array.

Examples
Input: {0, 1, 2, 6, 9}, n = 5, m = 10
Output: 3

Input: {4, 5, 10, 11}, n = 4, m = 12
Output: 0

Input: {0, 1, 2, 3}, n = 4, m = 5
Output: 4

Input: {0, 1, 2, 3, 4, 5, 6, 7, 10}, n = 9, m = 11
Output: 8

Thanks to Ravichandra for suggesting following two methods.

**Method 1 (Use Binary Search)**
For i = 0 to m-1, do binary search for i in the array. If i is not present in the array then return i.

Time Complexity: O(m log n)

**Method 2 (Linear Search)**
If arr[0] is not 0, return 0. Otherwise traverse the input array starting from index 1, and for each pair of elements a[i] and a[i+1], find the difference between them. if the difference is greater than 1 then a[i]+1 is the missing number.

Time Complexity: O(n)

**Method 3 (Use Modified Binary Search)**
Thanks to yasein and Jams for suggesting this method.
In the standard Binary Search process, the element to be searched is compared with the middle element and on the basis of comparison result, we decide whether to search is over or to go to left half or right half.
In this method, we modify the standard Binary Search algorithm to compare the middle element with its index and make decision on the basis of this comparison.

â€¦1) If the first element is not same as its index then return first index
â€¦2) Else get the middle index say mid
â€¦â€¦â€¦a) If arr[mid] greater than mid then the required element lies in left half.

â€¦â€¦â€¦b) Else the required element lies in right half.

```
#include<stdio.h>

int findFirstMissing(int array[], int start, int end) {

   if(start  > end)
     return end + 1;

   if (start != array[start])
     return start;

   int mid = (start + end) / 2;

   if (array[mid] > mid)
     return findFirstMissing(array, start, mid);
   else
     return findFirstMissing(array, mid + 1, end);
}

// driver program to test above function
int main()
{
  int arr[] = {0, 1, 2, 3, 4, 5, 6, 7, 10};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf(" First missing element is %d",
           findFirstMissing(arr, 0, n-1));
  getchar();
  return 0;
}
```

**Note:** This method doesnâ€™t work if there are duplicate elements in the array.

Time Complexity: O(Logn)

Source: http://geeksforgeeks.org/forum/topic/commvault-interview-question-for-software-engineerdeveloper-2-5-years-about-algorithms

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

Given a **sorted** array of n integers where each integer is in the range from 0 to m-1

and m > n. Find the smallest number that is missing from the array.

Examples
Input: {0, 1, 2, 6, 9}, n = 5, m = 10
Output: 3

Input: {4, 5, 10, 11}, n = 4, m = 12
Output: 0

Input: {0, 1, 2, 3}, n = 4, m = 5
Output: 4

Input: {0, 1, 2, 3, 4, 5, 6, 7, 10}, n = 9, m = 11
Output: 8

Thanks to Ravichandra for suggesting following two methods.

**Method 1 (Use Binary Search)**
For i = 0 to m-1, do binary search for i in the array. If i is not present in the array then return i.

Time Complexity: O(m log n)

**Method 2 (Linear Search)**
If arr[0] is not 0, return 0. Otherwise traverse the input array starting from index 1, and for each pair of elements a[i] and a[i+1], find the difference between them. if the difference is greater than 1 then a[i]+1 is the missing number.

Time Complexity: O(n)

**Method 3 (Use Modified Binary Search)**
Thanks to yasein and Jams for suggesting this method.
In the standard Binary Search process, the element to be searched is compared with the middle element and on the basis of comparison result, we decide whether to search is over or to go to left half or right half.
In this method, we modify the standard Binary Search algorithm to compare the middle element with its index and make decision on the basis of this comparison.

â€¦1) If the first element is not same as its index then return first index
â€¦2) Else get the middle index say mid
â€¦â€¦â€¦a) If arr[mid] greater than mid then the required element lies in left half.
â€¦â€¦â€¦b) Else the required element lies in right half.

```
#include<stdio.h>

int findFirstMissing(int array[], int start, int end) {

   if(start  > end)
```

```
    return end + 1;

  if (start != array[start])
    return start;

  int mid = (start + end) / 2;

  if (array[mid] > mid)
    return findFirstMissing(array, start, mid);
  else
    return findFirstMissing(array, mid + 1, end);
}

// driver program to test above function
int main()
{
 int arr[] = {0, 1, 2, 3, 4, 5, 6, 7, 10};
 int n = sizeof(arr)/sizeof(arr[0]);
 printf(" First missing element is %d",
         findFirstMissing(arr, 0, n-1));
 getchar();
 return 0;
}
```

**Note:** This method doesnâ€™t work if there are duplicate elements in the array.

Time Complexity: O(Logn)

Source: http://geeksforgeeks.org/forum/topic/commvault-interview-question-for-software-engineerdeveloper-2-5-years-about-algorithms

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â Â

42. Interpolation search vs Binary search

Interpolation search works better than Binary Search for a sorted and uniformly distributed array.

On average the interpolation search makes about log(log(n)) comparisons (if the elements are uniformly distributed), where n is the number of elements to be searched. In the worst case (for instance where the numerical values of the keys

increase exponentially) it can make up to O(n) comparisons.

**Sources:**
http://en.wikipedia.org/wiki/Interpolation_search

Â Â

43. Given an array arr[], find the maximum j – i such that arr[j] > arr[i]

Given an array arr[], find the maximum j â€“ i such that arr[j] > arr[i].

Examples:

```
Input: {34, 8, 10, 3, 2, 80, 30, 33, 1}
Output: 6  (j = 7, i = 1)

Input: {9, 2, 3, 4, 5, 6, 7, 8, 18, 0}
Output: 8 ( j = 8, i = 0)

Input:  {1, 2, 3, 4, 5, 6}
Output: 5  (j = 5, i = 0)

Input:  {6, 5, 4, 3, 2, 1}
Output: -1
```

**Method 1 (Simple but Inefficient)**
Run two loops. In the outer loop, pick elements one by one from left. In the inner loop, compare the picked element with the elements starting from right side. Stop the inner loop when you see an element greater than the picked element and keep updating the maximum j-i so far.

```
#include <stdio.h>
/* For a given array arr[], returns the maximum j â€“ i such that
   arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
   int maxDiff = -1;
   int i, j;

   for (i = 0; i < n; ++i)
   {
      for (j = n-1; j > i; --j)
      {
         if(arr[j] > arr[i] && maxDiff < (j - i))
```

```
            maxDiff = j - i;
        }
    }

    return maxDiff;
}

int main()
{
    int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    int maxDiff = maxIndexDiff(arr, n);
    printf("\n %d", maxDiff);
    getchar();
    return 0;
}
```

Time Complexity: O(n^2)

**Method 2 (Efficient)**

To solve this problem, we need to get two optimum indexes of arr[]: left index i and right index j. For an element arr[i], we do not need to consider arr[i] for left index if there is an element smaller than arr[i] on left side of arr[i]. Similarly, if there is a greater element on right side of arr[j] then we do not need to consider this j for right index. So we construct two auxiliary arrays LMin[] and RMax[] such that LMin[i] holds the smallest element on left side of arr[i] including arr[i], and RMax[j] holds the greatest element on right side of arr[j] including arr[j]. After constructing these two auxiliary arrays, we traverse both of these arrays from left to right. While traversing LMin[] and RMa[] if we see that LMin[i] is greater than RMax[j], then we must move ahead in LMin[] (or do i++) because all elements on left of LMin[i] are greater than or equal to LMin[i]. Otherwise we must move ahead in RMax[j] to look for a greater j â€“ i value.

Thanks to celicom for suggesting the algorithm for this method.

```
#include <stdio.h>

/* Utility Functions to get max and minimum of two integers */
int max(int x, int y)
{
    return x > y? x : y;
}

int min(int x, int y)
{
    return x < y? x : y;
```

```
}

/* For a given array arr[], returns the maximum j â€“ i such that
    arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
    int maxDiff;
    int i, j;

    int *LMin = (int *)malloc(sizeof(int)*n);
    int *RMax = (int *)malloc(sizeof(int)*n);

   /* Construct LMin[] such that LMin[i] stores the minimum value
       from (arr[0], arr[1], ... arr[i]) */
    LMin[0] = arr[0];
    for (i = 1; i < n; ++i)
        LMin[i] = min(arr[i], LMin[i-1]);

    /* Construct RMax[] such that RMax[j] stores the maximum value
       from (arr[j], arr[j+1], ..arr[n-1]) */
    RMax[n-1] = arr[n-1];
    for (j = n-2; j >= 0; --j)
        RMax[j] = max(arr[j], RMax[j+1]);

    /* Traverse both arrays from left to right to find optimum j - i
        This process is similar to merge() of MergeSort */
    i = 0, j = 0, maxDiff = -1;
    while (j < n && i < n)
    {
        if (LMin[i] < RMax[j])
        {
            maxDiff = max(maxDiff, j-i);
            j = j + 1;
        }
        else
            i = i+1;
    }

    return maxDiff;
}

/* Driver program to test above functions */
int main()
{
```

```
    int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    int maxDiff = maxIndexDiff(arr, n);
    printf("\n %d", maxDiff);
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

Given an array arr[], find the maximum j â€“ i such that arr[j] > arr[i].

Examples:

```
Input: {34, 8, 10, 3, 2, 80, 30, 33, 1}
Output: 6  (j = 7, i = 1)

Input: {9, 2, 3, 4, 5, 6, 7, 8, 18, 0}
Output: 8 ( j = 8, i = 0)

Input:  {1, 2, 3, 4, 5, 6}
Output: 5  (j = 5, i = 0)

Input:  {6, 5, 4, 3, 2, 1}
Output: -1
```

**Method 1 (Simple but Inefficient)**
Run two loops. In the outer loop, pick elements one by one from left. In the inner loop, compare the picked element with the elements starting from right side. Stop the inner loop when you see an element greater than the picked element and keep updating the maximum j-i so far.

```
#include <stdio.h>
/* For a given array arr[], returns the maximum j â€“ i such that
    arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
    int maxDiff = -1;
    int i, j;
```

```
    for (i = 0; i < n; ++i)
    {
        for (j = n-1; j > i; --j)
        {
            if(arr[j] > arr[i] && maxDiff < (j - i))
                maxDiff = j - i;
        }
    }

    return maxDiff;
}

int main()
{
    int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    int maxDiff = maxIndexDiff(arr, n);
    printf("\n %d", maxDiff);
    getchar();
    return 0;
}
```

Time Complexity: O(n^2)

**Method 2 (Efficient)**

To solve this problem, we need to get two optimum indexes of arr[]: left index i and right index j. For an element arr[i], we do not need to consider arr[i] for left index if there is an element smaller than arr[i] on left side of arr[i]. Similarly, if there is a greater element on right side of arr[j] then we do not need to consider this j for right index. So we construct two auxiliary arrays LMin[] and RMax[] such that LMin[i] holds the smallest element on left side of arr[i] including arr[i], and RMax[j] holds the greatest element on right side of arr[j] including arr[j]. After constructing these two auxiliary arrays, we traverse both of these arrays from left to right. While traversing LMin[] and RMa[] if we see that LMin[i] is greater than RMax[j], then we must move ahead in LMin[] (or do i++) because all elements on left of LMin[i] are greater than or equal to LMin[i]. Otherwise we must move ahead in RMax[j] to look for a greater j â€“ i value.

Thanks to celicom for suggesting the algorithm for this method.

```
#include <stdio.h>

/* Utility Functions to get max and minimum of two integers */
int max(int x, int y)
{
```

```
    return x > y? x : y;
}

int min(int x, int y)
{
    return x < y? x : y;
}

/* For a given array arr[], returns the maximum j â€“ i such that
    arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
    int maxDiff;
    int i, j;

    int *LMin = (int *)malloc(sizeof(int)*n);
    int *RMax = (int *)malloc(sizeof(int)*n);

   /* Construct LMin[] such that LMin[i] stores the minimum value
       from (arr[0], arr[1], ... arr[i]) */
    LMin[0] = arr[0];
    for (i = 1; i < n; ++i)
        LMin[i] = min(arr[i], LMin[i-1]);

    /* Construct RMax[] such that RMax[j] stores the maximum value
       from (arr[j], arr[j+1], ..arr[n-1]) */
    RMax[n-1] = arr[n-1];
    for (j = n-2; j >= 0; --j)
        RMax[j] = max(arr[j], RMax[j+1]);

    /* Traverse both arrays from left to right to find optimum j - i
        This process is similar to merge() of MergeSort */
    i = 0, j = 0, maxDiff = -1;
    while (j < n && i < n)
    {
        if (LMin[i] < RMax[j])
        {
            maxDiff = max(maxDiff, j-i);
            j = j + 1;
        }
        else
            i = i+1;
    }
```

```
    return maxDiff;
}

/* Driver program to test above functions */
int main()
{
    int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    int maxDiff = maxIndexDiff(arr, n);
    printf("\n %d", maxDiff);
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

Given an array arr[], find the maximum j â€“ i such that arr[j] > arr[i].

Examples:

```
Input: {34, 8, 10, 3, 2, 80, 30, 33, 1}
Output: 6  (j = 7, i = 1)

Input: {9, 2, 3, 4, 5, 6, 7, 8, 18, 0}
Output: 8 ( j = 8, i = 0)

Input:  {1, 2, 3, 4, 5, 6}
Output: 5  (j = 5, i = 0)

Input:  {6, 5, 4, 3, 2, 1}
Output: -1
```

**Method 1 (Simple but Inefficient)**
Run two loops. In the outer loop, pick elements one by one from left. In the inner loop, compare the picked element with the elements starting from right side. Stop the inner loop when you see an element greater than the picked element and keep updating the maximum j-i so far.

```
#include <stdio.h>
```

```
/* For a given array arr[], returns the maximum j â€“ i such that
   arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
   int maxDiff = -1;
   int i, j;

   for (i = 0; i < n; ++i)
   {
      for (j = n-1; j > i; --j)
      {
         if(arr[j] > arr[i] && maxDiff < (j - i))
            maxDiff = j - i;
      }
   }

   return maxDiff;
}

int main()
{
   int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
   int n = sizeof(arr)/sizeof(arr[0]);
   int maxDiff = maxIndexDiff(arr, n);
   printf("\n %d", maxDiff);
   getchar();
   return 0;
}
```

Time Complexity: O(n^2)

**Method 2 (Efficient)**

To solve this problem, we need to get two optimum indexes of arr[]: left index i and right index j. For an element arr[i], we do not need to consider arr[i] for left index if there is an element smaller than arr[i] on left side of arr[i]. Similarly, if there is a greater element on right side of arr[j] then we do not need to consider this j for right index. So we construct two auxiliary arrays LMin[] and RMax[] such that LMin[i] holds the smallest element on left side of arr[i] including arr[i], and RMax[j] holds the greatest element on right side of arr[j] including arr[j]. After constructing these two auxiliary arrays, we traverse both of these arrays from left to right. While traversing LMin[] and RMa[] if we see that LMin[i] is greater than RMax[j], then we must move ahead in LMin[] (or do i++) because all elements on left of LMin[i] are greater than or equal to LMin[i]. Otherwise we must move ahead in RMax[j] to look for a greater j â€“ i value.

Thanks to celicom for suggesting the algorithm for this method.

```
#include <stdio.h>

/* Utility Functions to get max and minimum of two integers */
int max(int x, int y)
{
    return x > y? x : y;
}

int min(int x, int y)
{
    return x < y? x : y;
}

/* For a given array arr[], returns the maximum j â€“ i such that
    arr[j] > arr[i] */
int maxIndexDiff(int arr[], int n)
{
    int maxDiff;
    int i, j;

    int *LMin = (int *)malloc(sizeof(int)*n);
    int *RMax = (int *)malloc(sizeof(int)*n);

   /* Construct LMin[] such that LMin[i] stores the minimum value
       from (arr[0], arr[1], ... arr[i]) */
    LMin[0] = arr[0];
    for (i = 1; i < n; ++i)
        LMin[i] = min(arr[i], LMin[i-1]);

    /* Construct RMax[] such that RMax[j] stores the maximum value
       from (arr[j], arr[j+1], ..arr[n-1]) */
    RMax[n-1] = arr[n-1];
    for (j = n-2; j >= 0; --j)
        RMax[j] = max(arr[j], RMax[j+1]);

    /* Traverse both arrays from left to right to find optimum j - i
        This process is similar to merge() of MergeSort */
    i = 0, j = 0, maxDiff = -1;
    while (j < n && i < n)
    {
        if (LMin[i] < RMax[j])
        {
            maxDiff = max(maxDiff, j-i);
```

```
            j = j + 1;
        }
        else
            i = i+1;
    }

    return maxDiff;
}

/* Driver program to test above functions */
int main()
{
    int arr[] = {9, 2, 3, 4, 5, 6, 7, 8, 18, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    int maxDiff = maxIndexDiff(arr, n);
    printf("\n %d", maxDiff);
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

46. Find the minimum distance between two numbers

Given an unsorted array arr[] and two numbers x and y, find the minimum distance between x and y in arr[]. The array might also contain duplicates. You may assume that both x and y are different and present in arr[].

Examples:
Input: arr[] = {1, 2}, x = 1, y = 2
Output: Minimum distance between 1 and 2 is 1.

Input: arr[] = {3, 4, 5}, x = 3, y = 5
Output: Minimum distance between 3 and 5 is 2.

Input: arr[] = {3, 5, 4, 2, 6, 5, 6, 6, 5, 4, 8, 3}, x = 3, y = 6
Output: Minimum distance between 3 and 6 is 4.

Input: arr[] = {2, 5, 3, 5, 4, 4, 2, 3}, x = 3, y = 2

Output: Minimum distance between 3 and 2 is 1.

**Method 1 (Simple)**
Use two loops: The outer loop picks all the elements of arr[] one by one. The inner loop picks all the elements after the element picked by outer loop. If the elements picked by outer and inner loops have same values as x or y then if needed update the minimum distance calculated so far.

```
#include <stdio.h>
#include <stdlib.h> // for abs()
#include <limits.h> // for INT_MAX

int minDist(int arr[], int n, int x, int y)
{
  int i, j;
  int min_dist = INT_MAX;
  for (i = 0; i < n; i++)
  {
    for (j = i+1; j < n; j++)
    {
       if( (x == arr[i] && y == arr[j] ||
            y == arr[i] && x == arr[j]) && min_dist > abs(i-j))
       {
            min_dist = abs(i-j);
       }
    }
  }
  return min_dist;
}

/* Driver program to test above fnction */
int main()
{
   int arr[] = {3, 5, 4, 2, 6, 5, 6, 6, 5, 4, 8, 3};
   int n = sizeof(arr)/sizeof(arr[0]);
   int x = 3;
   int y = 6;

   printf("Minimum distance between %d and %d is %d\n", x, y,
            minDist(arr, n, x, y));
   return 0;
}
```

Output: *Minimum distance between 3 and 6 is 4*

Time Complexity: O(n^2)

**Method 2 (Tricky)**
1) Traverse array from left side and stop if either *x* or *y* are found. Store index of this first occurrrence in a variable say *prev*
2) Now traverse *arr[]* after the index *prev*. If the element at current index *i* matches with either x or y then check if it is different from *arr[prev]*. If it is different then update the minimum distance if needed. If it is same then update *prev* i.e., make *prev = i*.

Thanks to wgpshashank for suggesting this approach.

```
#include <stdio.h>
#include <limits.h>  // For INT_MAX

int minDist(int arr[], int n, int x, int y)
{
  int i = 0;
  int min_dist = INT_MAX;
  int prev;

  // Find the first occurence of any of the two numbers (x or y)
  // and store the index of this occurence in prev
  for (i = 0; i < n; i++)
  {
    if (arr[i] == x || arr[i] == y)
    {
      prev = i;
      break;
    }
  }

  // Traverse after the first occurence
  for ( ; i < n; i++)
  {
     if (arr[i] == x || arr[i] == y)
     {
        // If the current element matches with any of the two then
        // check if current element and prev element are different
        // Also check if this value is smaller than minimm distance so far
        if ( arr[prev] != arr[i] && (i - prev) < min_dist )
        {
           min_dist = i - prev;
           prev = i;
        }
        else
```

```
            prev = i;
        }
    }

    return min_dist;
}

/* Driver program to test above fnction */
int main()
{
    int arr[] ={3, 5, 4, 2, 6, 3, 0, 0, 5, 4, 8, 3};
    int n = sizeof(arr)/sizeof(arr[0]);
    int x = 3;
    int y = 6;

    printf("Minimum distance between %d and %d is %d\n", x, y,
            minDist(arr, n, x, y));
    return 0;
}
```

Output: *Minimum distance between 3 and 6 is 1*

Time Complexity: O(n)

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

47. Find the repeating and the missing | Added 3 new methods

Given an unsorted array of size n. Array elements are in range from 1 to n. One number from set {1, 2, â€¦n} is missing and one number occurs twice in array. Find these two numbers.

Examples:

```
arr[] = {3, 1, 3}
Output: 2, 3   // 2 is missing and 3 occurs twice

arr[] = {4, 3, 6, 2, 1, 1}
Output: 1, 5  // 5 is missing and 1 occurs twice
```

**Method 1 (Use Sorting)**

1) Sort the input array.
2) Traverse the array and check for missing and repeating.

Time Complexity: O(nLogn)

Thanks to LoneShadow for suggesting this method.

**Method 2 (Use count array)**
1) Create a temp array temp[] of size n with all initial values as 0.
2) Traverse the input array arr[], and do following for each arr[i]
â€¦â€¦a) if(temp[arr[i]] == 0) temp[arr[i]] = 1;
â€¦â€¦b) if(temp[arr[i]] == 1) output â€œarr[i]â€ //repeating
3) Traverse temp[] and output the array element having value as 0 (This is the missing element)

Time Complexity: O(n)
Auxiliary Space: O(n)

**Method 3 (Use elements as Index and mark the visited places)**
Traverse the array. While traversing, use absolute value of every element as index and make the value at this index as negative to mark it visited. If something is already marked negative then this is the repeating element. To find missing, traverse the array again and look for a positive value.

```
#include<stdio.h>
#include<stdlib.h>

void printTwoElements(int arr[], int size)
{
   int i;
   printf("\n The repeating element is");

   for(i = 0; i < size; i++)
   {
      if(arr[abs(arr[i])-1] > 0)
         arr[abs(arr[i])-1] = -arr[abs(arr[i])-1];
      else
         printf(" %d ", abs(arr[i]));
   }

   printf("\nand the missing element is ");
   for(i=0; i<size; i++)
   {
      if(arr[i]>0)
         printf("%d",i+1);
   }
```

```
}

/* Driver program to test above function */
int main()
{
    int arr[] = {7, 3, 4, 5, 5, 6, 2};
    int  n = sizeof(arr)/sizeof(arr[0]);
    printTwoElements(arr, n);
    return 0;
}
```

Time Complexity: O(n)

Thanks to Manish Mishra for suggesting this method.

**Method 4 (Make two equations)**
Let x be the missing and y be the repeating element.

1) Get sum of all numbers.
Sum of array computed S = n(n+1)/2 â€“ x + y
2) Get product of all numbers.
Product of array computed P = 1*2*3*â€¦*n * y / x
3) The above two steps give us two equations, we can solve the equations and get the values of x and y.

Time Complexity: O(n)

Thanks to disappearedng for suggesting this solution.

This method can cause arithmetic overflow as we calculate product and sum of all array elements. See this for changes suggested by john to reduce the chances of overflow.

**Method 5 (Use XOR)**
Let x and y be the desired output elements.
Calculate XOR of all the array elements.

```
    xor1 = arr[0]^arr[1]^arr[2].....arr[n-1]
```

XOR the result with all numbers from 1 to n

```
    xor1 = xor1^1^2^.....^n
```

In the result *xor1*, all elements would nullify each other except x and y. All the bits that are set in *xor1* will be set in either x or y. So if we take any set bit (We have chosen the rightmost set bit in code) of *xor1* and divide the elements of the array in two sets â€“ one set of elements with same bit set and other set with same bit not set. By doing so, we will get x in one set and y in another set. Now if we do XOR of all the elements

in first set, we will get x, and by doing same in other set we will get y.

```
#include <stdio.h>
#include <stdlib.h>

/* The output of this function is stored at *x and *y */
void getTwoElements(int arr[], int n, int *x, int *y)
{
  int xor1;   /* Will hold xor of all elements and numbers from 1 to n */
  int set_bit_no;  /* Will have only single set bit of xor1 */
  int i;
  *x = 0;
  *y = 0;

  xor1 = arr[0];

  /* Get the xor of all array elements elements */
  for(i = 1; i < n; i++)
     xor1 = xor1^arr[i];

  /* XOR the previous result with numbers from 1 to n*/
  for(i = 1; i <= n; i++)
     xor1 = xor1^i;

  /* Get the rightmost set bit in set_bit_no */
  set_bit_no = xor1 & ~(xor1-1);

  /* Now divide elements in two sets by comparing rightmost set
   bit of xor1 with bit at same position in each element. Also, get XORs
   of two sets. The two XORs are the output elements.
   The following two for loops serve the purpose */
  for(i = 0; i < n; i++)
  {
    if(arr[i] & set_bit_no)
      *x = *x ^ arr[i]; /* arr[i] belongs to first set */
    else
      *y = *y ^ arr[i]; /* arr[i] belongs to second set*/
  }
  for(i = 1; i <= n; i++)
  {
    if(i & set_bit_no)
      *x = *x ^ i; /* i belongs to first set */
    else
      *y = *y ^ i; /* i belongs to second set*/
```

```
  }

  /* Now *x and *y hold the desired output elements */
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 3, 4, 5, 5, 6, 2};
  int *x = (int *)malloc(sizeof(int));
  int *y = (int *)malloc(sizeof(int));
  int  n = sizeof(arr)/sizeof(arr[0]);
  getTwoElements(arr, n, x, y);
  printf(" The two elements are %d and %d", *x, *y);
  getchar();
}
```

Time Complexity: O(n)

This method doesnâ€™t cause overflow, but it doesnâ€™t tell which one occurs twice and which one is missing. We can add one more step that checks which one is missing and which one is repeating. This can be easily done in O(n) time.

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â Â

48. Print a given matrix in spiral form

Given a 2D array, print it in spiral form. See the following examples.

```
Input:
      1    2   3   4
      5    6   7   8
      9   10  11  12
      13  14  15  16
Output:
1 2 3 4 8 12 16 15 14 13 9 5 6 7 11 10


Input:
      1   2   3   4  5   6
      7   8   9  10  11  12
```

```
      13  14  15 16  17  18
```

Output:

```
1 2 3 4 5 6 12 18 17 16 15 14 13 7 8 9 10 11
```

## Solution:

```
/* This code is adopted from the solution given
   @ http://effprog.blogspot.com/2011/01/spiral-printing-of-two-dimensional.html */

#include <stdio.h>
#define R 3
#define C 6

void spiralPrint(int m, int n, int a[R][C])
{
    int i, k = 0, l = 0;

    /*  k - starting row index
        m - ending row index
        l - starting column index
        n - ending column index
        i - iterator
    */

    while (k < m && l < n)
    {
        /* Print the first row from the remaining rows */
        for (i = l; i < n; ++i)
        {
            printf("%d ", a[k][i]);
        }
        k++;

        /* Print the last column from the remaining columns */
        for (i = k; i < m; ++i)
        {
            printf("%d ", a[i][n-1]);
        }
        n--;

        /* Print the last row from the remaining rows */
        if ( k < m)
        {
            for (i = n-1; i >= l; --i)
```

```
        {
            printf("%d ", a[m-1][i]);
        }
        m--;
    }

    /* Print the first column from the remaining columns */
    if (l < n)
    {
        for (i = m-1; i >= k; --i)
        {
            printf("%d ", a[i][l]);
        }
        l++;
    }
  }
}

/* Driver program to test above functions */
int main()
{
  int a[R][C] = { {1,  2,  3,  4,  5,  6},
    {7,  8,  9,  10, 11, 12},
    {13, 14, 15, 16, 17, 18}
  };

  spiralPrint(R, C, a);
  return 0;
}

/* OUTPUT:
 1 2 3 4 5 6 12 18 17 16 15 14 13 7 8 9 10 11
*/
```

Time Complexity: Time complexity of the above solution is O(mn).

Please write comments if you find the above code incorrect, or find other ways to solve the same problem.

Â  Â

Given a boolean matrix mat[M][N] of size M X N, modify it such that if a matrix cell mat[i][j] is 1 (or true) then make all the cells of ith row and jth column as 1.

```
Example 1
The matrix
1 0
0 0
should be changed to following
1 1
1 0

Example 2
The matrix
0 0 0
0 0 1
should be changed to following
0 0 1
1 1 1

Example 3
The matrix
1 0 0 1
0 0 1 0
0 0 0 0
should be changed to following
1 1 1 1
1 1 1 1
1 0 1 1
```

**Method 1 (Use two temporary arrays)**
1) Create two temporary arrays row[M] and col[N]. Initialize all values of row[] and col[] as 0.
2) Traverse the input matrix mat[M][N]. If you see an entry mat[i][j] as true, then mark row[i] and col[j] as true.
3) Traverse the input matrix mat[M][N] again. For each entry mat[i][j], check the values of row[i] and col[j]. If any of the two values (row[i] or col[j]) is true, then mark mat[i][j] as true.

Thanks to Dixit Sethi for suggesting this method.

```
#include <stdio.h>
#define R 3
#define C 4
```

```
void modifyMatrix(bool mat[R][C])
{
   bool row[R];
   bool col[C];

   int i, j;


   /* Initialize all values of row[] as 0 */
   for (i = 0; i < R; i++)
   {
      row[i] = 0;
   }


   /* Initialize all values of col[] as 0 */
   for (i = 0; i < C; i++)
   {
      col[i] = 0;
   }


   /* Store the rows and columns to be marked as 1 in row[] and col[]
      arrays respectively */
   for (i = 0; i < R; i++)
   {
       for (j = 0; j < C; j++)
       {
           if (mat[i][j] == 1)
           {
               row[i] = 1;
               col[j] = 1;
           }
       }
   }

   /* Modify the input matrix mat[] using the above constructed row[] and
      col[] arrays */
   for (i = 0; i < R; i++)
   {
       for (j = 0; j < C; j++)
       {
           if ( row[i] == 1 || col[j] == 1 )
```

```
            {
                mat[i][j] = 1;
            }
        }
    }
}

/* A utility function to print a 2D matrix */
void printMatrix(bool mat[R][C])
{
    int i, j;
    for (i = 0; i < R; i++)
    {
        for (j = 0; j < C; j++)
        {
            printf("%d ", mat[i][j]);
        }
        printf("\n");
    }
}

/* Driver program to test above functions */
int main()
{
    bool mat[R][C] = { {1, 0, 0, 1},
        {0, 0, 1, 0},
        {0, 0, 0, 0},
    };

    printf("Input Matrix \n");
    printMatrix(mat);

    modifyMatrix(mat);

    printf("Matrix after modification \n");
    printMatrix(mat);

    return 0;
}
```

Output:

```
Input Matrix
1 0 0 1
```

```
0 0 1 0
0 0 0 0
Matrix after modification
1 1 1 1
1 1 1 1
1 0 1 1
```

Time Complexity: O(M*N)
Auxiliary Space: O(M + N)

**Method 2 (A Space Optimized Version of Method 1)**
This method is a space optimized version of above method 1. This method uses the first row and first column of the input matrix in place of the auxiliary arrays row[] and col[] of method 1. So what we do is: first take care of first row and column and store the info about these two in two flag variables rowFlag and colFlag. Once we have this info, we can use first row and first column as auxiliary arrays and apply method 1 for submatrix (matrix excluding first row and first column) of size (M-1)*(N-1).

1) Scan the first row and set a variable rowFlag to indicate whether we need to set all 1s in first row or not.
2) Scan the first column and set a variable colFlag to indicate whether we need to set all 1s in first column or not.
3) Use first row and first column as the auxiliary arrays row[] and col[] respectively, consider the matrix as submatrix starting from second row and second column and apply method 1.
4) Finally, using rowFlag and colFlag, update first row and first column if needed.

Time Complexity: O(M*N)
Auxiliary Space: O(1)

Thanks to Sidh for suggesting this method.

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

50.  Median in a stream of integers (running integers)

Given that integers are read from a data stream. Find median of elements read so for in efficient way. For simplicity assume there are no duplicates. For example, let us consider the stream 5, 15, 1, 3 â€¦

```
After reading 1st element of stream - 5 -> median - 5
```

```
After reading 2nd element of stream - 5, 15 -> median - 10
After reading 3rd element of stream - 5, 15, 1 -> median - 5
After reading 4th element of stream - 5, 15, 1, 3 -> median - 4, so on...
```

Making it clear, when the input size is odd, we take the middle element of sorted data. If the input size is even, we pick average of middle two elements in sorted stream.

Note that output is *effective median* of integers read from the stream so far. Such an algorithm is called online algorithm. Any algorithm that can guarantee output of *i*-elements after processing *i*-th element, is said to be ***online algorithm***. Let us discuss three solutions for the above problem.

**Method 1:** Insertion Sort

If we can sort the data as it appears, we can easily locate median element. *Insertion Sort* is one such online algorithm that sorts the data appeared so far. At any instance of sorting, say after sorting *i*-th element, the first *i* elements of array are sorted. The insertion sort doesnâ€™t depend on future data to sort data input till that point. In other words, insertion sort considers data sorted so far while inserting next element. This is the key part of insertion sort that makes it anÂ online algorithm.

However, insertion sort takes $O(n^2)$ time to sort *n* elements. Perhaps we can useÂ *binary search* on *insertion sort* to find location of next element in O(log n) time. Yet, we canâ€™t do data movement in O(log n) time. No matter how efficient the implementation is, it takes polynomial time in case of insertion sort.

Interested reader can try implementation of Method 1.

**Method 2:**Â AugmentedÂ self balanced binary search tree (AVL, RB, etcâ€¦)

At every node of BST, maintain number of elements in the subtree rooted at that node. We can use a node as root of simple binary tree, whose left child is self balancing BST with elements less than root andÂ right child is self balancing BST with elements greater than root. The root element always holds *effective median*.

If left and right subtrees contain same number of elements, root node holds average of left and right subtree root data. Otherwise, root contains same data as the root of subtree which is having more elements. After processing an incoming element, the left and right subtrees (BST) are differed utmost byÂ 1.

Self balancing BST is costly in managing balancing factor of BST. However, they provide sorted data which we donâ€™t need. We need median only. The next method make use of Heaps to trace median.

**Method 3:**Â Heaps

Similar to balancing BST in Method 2 above, we can use a max heap on left side to represent elements that are less than *effective median*, and a min heap on right side

to represent elements that are greater than *effective median*.

After processing an incoming element, the number of elements in heaps differ utmost by 1 element.Â When both heaps contain same number of elements, we pick average of heaps root data as *effective median*.Â When the heaps are not balanced, we select *effective median* from the root of heap containing more elements.

Given below is implementation of above method.Â For algorithm to build these heaps, please read the highlighted code.

```
#include <iostream>
using namespace std;

// Heap capacity
#define MAX_HEAP_SIZE (128)
#define ARRAY_SIZE(a) sizeof(a)/sizeof(a[0])

//// Utility functions

// exchange a and b
inline
void Exch(int &a, int &b)
{
   int aux = a;
   a = b;
   b = aux;
}

// Greater and Smaller are used as comparators
bool Greater(int a, int b)
{
   return a > b;
}

bool Smaller(int a, int b)
{
   return a < b;
}

int Average(int a, int b)
{
   return (a + b) / 2;
}

// Signum function
```

```
// = 0  if a == b  - heaps are balanced
// = -1 if a < b   - left contains less elements than right
// = 1  if a > b   - left contains more elements than right
int Signum(int a, int b)
{
   if( a == b )
      return 0;

   return a < b ? -1 : 1;
}

// Heap implementation
// The functionality is embedded into
// Heap abstract class to avoid code duplication
class Heap
{
public:
   // Initializes heap array and comparator required
   // in heapification
   Heap(int *b, bool (*c)(int, int)) : A(b), comp(c)
   {
      heapSize = -1;
   }

   // Frees up dynamic memory
   virtual ~Heap()
   {
      if( A )
      {
         delete[] A;
      }
   }

   // We need only these four interfaces of Heap ADT
   virtual bool Insert(int e) = 0;
   virtual int  GetTop() = 0;
   virtual int  ExtractTop() = 0;
   virtual int  GetCount() = 0;

protected:

   // We are also using location 0 of array
   int left(int i)
   {
```

```
    return 2 * i + 1;
}

int right(int i)
{
    return 2 * (i + 1);
}

int parent(int i)
{
    if( i <= 0 )
    {
        return -1;
    }

    return (i - 1)/2;
}

// Heap array
int   *A;
// Comparator
bool  (*comp)(int, int);
// Heap size
int   heapSize;

// Returns top element of heap data structure
int top(void)
{
    int max = -1;

    if( heapSize >= 0 )
    {
        max = A[0];
    }

    return max;
}

// Returns number of elements in heap
int count()
{
    return heapSize + 1;
}
```

```
// Heapification
// Note that, for the current median tracing problem
// we need to heapify only towards root, always
void heapify(int i)
{
    int p = parent(i);

    // comp - differentiate MaxHeap and MinHeap
    // percolates up
    if( p >= 0 && comp(A[i], A[p]) )
    {
        Exch(A[i], A[p]);
        heapify(p);
    }
}

// Deletes root of heap
int deleteTop()
{
    int del = -1;

    if( heapSize > -1)
    {
        del = A[0];

        Exch(A[0], A[heapSize]);
        heapSize--;
        heapify(parent(heapSize+1));
    }

    return del;
}

// Helper to insert key into Heap
bool insertHelper(int key)
{
    bool ret = false;

    if( heapSize < MAX_HEAP_SIZE )
    {
        ret = true;
        heapSize++;
        A[heapSize] = key;
        heapify(heapSize);
```

```
        }

        return ret;
    }
};

// Specilization of Heap to define MaxHeap
class MaxHeap : public Heap
{
private:

public:
    MaxHeap() : Heap(new int[MAX_HEAP_SIZE], &Greater)  {  }

    ~MaxHeap()  { }

    // Wrapper to return root of Max Heap
    int GetTop()
    {
        return top();
    }

    // Wrapper to delete and return root of Max Heap
    int ExtractTop()
    {
        return deleteTop();
    }

    // Wrapper to return # elements of Max Heap
    int  GetCount()
    {
        return count();
    }

    // Wrapper to insert into Max Heap
    bool Insert(int key)
    {
        return insertHelper(key);
    }
};

// Specilization of Heap to define MinHeap
class MinHeap : public Heap
{
```

```
private:

public:

    MinHeap() : Heap(new int[MAX_HEAP_SIZE], &Smaller) { }

    ~MinHeap() { }

    // Wrapper to return root of Min Heap
    int GetTop()
    {
        return top();
    }

    // Wrapper to delete and return root of Min Heap
    int ExtractTop()
    {
        return deleteTop();
    }

    // Wrapper to return # elements of Min Heap
    int  GetCount()
    {
        return count();
    }

    // Wrapper to insert into Min Heap
    bool Insert(int key)
    {
        return insertHelper(key);
    }
};

// Function implementing algorithm to find median so far.
int getMedian(int e, int &m, Heap &l, Heap &r)
{
    // Are heaps balanced? If yes, sig will be 0
    int sig = Signum(l.GetCount(), r.GetCount());
    switch(sig)
    {
    case 1: // There are more elements in left (max) heap

        if( e < m ) // current element fits in left (max) heap
        {
```

```
        // Remore top element from left heap and
        // insert into right heap
        r.Insert(l.ExtractTop());

        // current element fits in left (max) heap
        l.Insert(e);
    }
    else
    {
        // current element fits in right (min) heap
        r.Insert(e);
    }

    // Both heaps are balanced
    m = Average(l.GetTop(), r.GetTop());

    break;

case 0: // The left and right heaps contain same number of elements

    if( e < m ) // current element fits in left (max) heap
    {
        l.Insert(e);
        m = l.GetTop();
    }
    else
    {
        // current element fits in right (min) heap
        r.Insert(e);
        m = r.GetTop();
    }

    break;

case -1: // There are more elements in right (min) heap

    if( e < m ) // current element fits in left (max) heap
    {
        l.Insert(e);
    }
    else
    {
        // Remove top element from right heap and
        // insert into left heap
```

```
            l.Insert(r.ExtractTop());

            // current element fits in right (min) heap
            r.Insert(e);
        }

        // Both heaps are balanced
        m = Average(l.GetTop(), r.GetTop());

        break;
    }

    // No need to return, m already updated
    return m;
}

void printMedian(int A[], int size)
{
    int m = 0; // effective median
    Heap *left  = new MaxHeap();
    Heap *right = new MinHeap();

    for(int i = 0; i < size; i++)
    {
        m = getMedian(A[i], m, *left, *right);

        cout << m << endl;
    }

    // C++ more flexible, ensure no leaks
    delete left;
    delete right;
}
// Driver code
int main()
{
    int A[] = {5, 15, 1, 3, 2, 8, 7, 9, 10, 6, 11, 4};
    int size = ARRAY_SIZE(A);

    // In lieu of A, we can also use data read from a stream
    printMedian(A, size);

    return 0;
}
```

**Time Complexity:** If we omit the way how stream was read, complexity of median finding is ***O(N log N)***, as we need to read the stream, and due to heap insertions/deletions.

At first glance the above code may look complex. If you read the code carefully, it is simple algorithm.

â€”Â **Venki**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

51. Find a Fixed Point in a given array

Given an array of n distinct integers sorted in ascending order, write a function that returns a Fixed Point in the array, if there is any Fixed Point present in array, else returns -1. Fixed Point in an array is an index i such that arr[i] is equal to i. Note that integers in array can be negative.

Examples:

```
Input: arr[] = {-10, -5, 0, 3, 7}
Output: 3  // arr[3] == 3

Input: arr[] = {0, 2, 5, 8, 17}
Output: 0  // arr[0] == 0



Input: arr[] = {-10, -5, 3, 4, 7, 9}
Output: -1  // No Fixed Point
```

Asked by rajk

**Method 1 (Linear Search)**
Linearly search for an index i such that arr[i] == i. Return the first such index found. Thanks to pm for suggesting this solution.

```
int linearSearch(int arr[], int n)
{
   int i;
   for(i = 0; i < n; i++)
   {
      if(arr[i] == i)
         return i;
   }
```

```
    /* If no fixed point present then return -1 */
    return -1;
}

/* Driver program to check above functions */
int main()
{
    int arr[] = {-10, -1, 0, 3, 10, 11, 30, 50, 100};
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Fixed Point is %d", linearSearch(arr, n));
    getchar();
    return 0;
}
```

Time Complexity: O(n)

**Method 2 (Binary Search)**
First check whether middle element is Fixed Point or not. If it is, then return it; otherwise check whether index of middle element is greater than value at the index. If index is greater, then Fixed Point(s) lies on the right side of the middle point (obviously only if there is a Fixed Point). Else the Fixed Point(s) lies on left side.

```
int binarySearch(int arr[], int low, int high)
{
    if(high >= low)
    {
        int mid = (low + high)/2;  /*low + (high - low)/2;*/
        if(mid == arr[mid])
            return mid;
        if(mid > arr[mid])
            return binarySearch(arr, (mid + 1), high);
        else
            return binarySearch(arr, low, (mid -1));
    }

    /* Return -1 if there is no Fixed Point */
    return -1;
}

/* Driver program to check above functions */
int main()
{
    int arr[10] = {-10, -1, 0, 3, 10, 11, 30, 50, 100};
```

```
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Fixed Point is %d", binarySearch(arr, 0, n-1));
    getchar();
    return 0;
}
```

Algorithmic Paradigm: Divide & Conquer
Time Complexity: O(Logn)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

52. Maximum Length Bitonic Subarray

Given an array A[0 â€¦ n-1] containing n positive integers, a subarray A[i â€¦ j] is bitonic if there is a k with i <= k <= j such that A[i] <= A[i + 1] ... <= A[k] >= A[k + 1] >= .. A[j â€“ 1] > = A[j]. Write a function that takes an array as argument and returns the length of the maximum length bitonic subarray.
Expected time complexity of the solution is O(n)

*Simple Examples*
**1)** A[] = {12, 4, 78, 90, 45, 23}, the maximum length bitonic subarray is {4, 78, 90, 45, 23} which is of length 5.

**2)** A[] = {20, 4, 1, 2, 3, 4, 2, 10}, the maximum length bitonic subarray is {1, 2, 3, 4, 2} which is of length 5.

*Extreme Examples*
**1)** A[] = {10}, the single element is bitnoic, so output is 1.

**2)** A[] = {10, 20, 30, 40}, the complete array itself is bitonic, so output is 4.

**3)** A[] = {40, 30, 20, 10}, the complete array itself is bitonic, so output is 4.

**Solution**
Let us consider the array {12, 4, 78, 90, 45, 23} to understand the soultion.
1) Construct an auxiliary array inc[] from left to right such that inc[i] contains length of the nondecreaing subarray ending at arr[i].
For for A[] = {12, 4, 78, 90, 45, 23}, inc[] is {1, 1, 2, 3, 1, 1}

2) Construct another array dec[] from right to left such that dec[i] contains length of nonincreasing subarray starting at arr[i].
For A[] = {12, 4, 78, 90, 45, 23}, dec[] is {2, 1, 1, 3, 2, 1}.

3) Once we have the inc[] and dec[] arrays, all we need to do is find the maximum value of (inc[i] + dec[i] â€“ 1).
For {12, 4, 78, 90, 45, 23}, the max value of (inc[i] + dec[i] â€“ 1) is 5 for i = 3.

```
#include<stdio.h>
#include<stdlib.h>

int bitonic(int arr[], int n)
{
   int i;
   int *inc = new int[n];
   int *dec = new int[n];
   int max;
   inc[0] = 1; // The length of increasing sequence ending at first index is 1
   dec[n-1] = 1; // The length of increasing sequence starting at first index is 1

   // Step 1) Construct increasing sequence array
   for(i = 1; i < n; i++)
   {
      if (arr[i] > arr[i-1])
         inc[i] = inc[i-1] + 1;
      else
         inc[i] = 1;
   }

   // Step 2) Construct decreasing sequence array
   for (i = n-2; i >= 0; i--)
   {
      if (arr[i] > arr[i+1])
         dec[i] = dec[i+1] + 1;
      else
         dec[i] = 1;
   }

   // Step 3) Find the length of maximum length bitonic sequence
   max = inc[0] + dec[0] - 1;
   for (i = 1; i < n; i++)
   {
      if (inc[i] + dec[i] - 1 > max)
      {
         max = inc[i] + dec[i] - 1;
      }
   }
```

```
    // free dynamically allocated memory
    delete [] inc;
    delete [] dec;

    return max;
}

/* Driver program to test above function */
int main()
{
    int arr[] = {12, 4, 78, 90, 45, 23};
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("\n Length of max length Bitnoic Subarray is %d", bitonic(arr, n));
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Auxiliary Space: O(n)

As an exercise, extend the above implementation to print the longest bitonic subarray also. The above implementation only returns the length of such subarray.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

53. Find the maximum element in an array which is first increasing and then decreasing

Given an array of integers which is initially increasing and then decreasing, find the maximum value in the array.

```
Input: arr[] = {8, 10, 20, 80, 100, 200, 400, 500, 3, 2, 1}
Output: 500

Input: arr[] = {1, 3, 50, 10, 9, 7, 6}
Output: 50

Corner case (No decreasing part)
Input: arr[] = {10, 20, 30, 40, 50}
Output: 50

Corner case (No increasing part)
```

```
Input: arr[] = {120, 100, 80, 20, 0}
Output: 120
```

**Method 1 (Linear Search)**
We can traverse the array and keep track of maximum and element. And finally return the maximum element.

```
#include <stdio.h>

int findMaximum(int arr[], int low, int high)
{
  int max = arr[low];
  int i;
  for (i = low; i <= high; i++)
  {
     if (arr[i] > max)
        max = arr[i];
  }
  return max;
}

/* Driver program to check above functions */
int main()
{
  int arr[] = {1, 30, 40, 50, 60, 70, 23, 20};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("The maximum element is %d", findMaximum(arr, 0, n-1));
  getchar();
  return 0;
}
```

Time Complexity: O(n)

**Method 2 (Binary Search)**
We can modify the standard Binary Search algorithm for the given type of arrays.
i) If the mid element is greater than both of its adjacent elements, then mid is the maximum.
ii) If mid element is greater than its next element and smaller than the previous element then maximum lies on left side of mid. Example array: {3, 50, 10, 9, 7, 6}
iii) If mid element is smaller than its next element and greater than the previous element then maximum lies on right side of mid. Example array: {2, 4, 6, 8, 10, 3, 1}

```
#include <stdio.h>
```

```
int findMaximum(int arr[], int low, int high)
{

  /* Base Case: Only one element is present in arr[low..high]*/
  if (low == high)
    return arr[low];

  /* If there are two elements and first is greater then
     the first element is maximum */
  if ((high == low + 1) && arr[low] >= arr[high])
     return arr[low];

  /* If there are two elements and second is greater then
     the second element is maximum */
  if ((high == low + 1) && arr[low] < arr[high])
     return arr[high];

  int mid = (low + high)/2;   /*low + (high - low)/2;*/

  /* If we reach a point where arr[mid] is greater than both of
    its adjacent elements arr[mid-1] and arr[mid+1], then arr[mid]
    is the maximum element*/
  if ( arr[mid] > arr[mid + 1] && arr[mid] > arr[mid - 1])
     return arr[mid];

  /* If arr[mid] is greater than the next element and smaller than the previous
   element then maximum lies on left side of mid */
  if (arr[mid] > arr[mid + 1] && arr[mid] < arr[mid - 1])
    return findMaximum(arr, low, mid-1);
  else // when arr[mid] is greater than arr[mid-1] and smaller than arr[mid+1]
    return findMaximum(arr, mid + 1, high);
}

/* Driver program to check above functions */
int main()
{
  int arr[] = {1, 3, 50, 10, 9, 7, 6};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("The maximum element is %d", findMaximum(arr, 0, n-1));
  getchar();
  return 0;
}
```

Time Complexity: O(Logn)

This method works only for distinct numbers. For example, it will not work for an array like {0, 1, 1, 2, 2, 2, 2, 2, 3, 4, 4, 5, 3, 3, 2, 2, 1, 1}.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

Given an array of integers which is initially increasing and then decreasing, find the maximum value in the array.

```
Input: arr[] = {8, 10, 20, 80, 100, 200, 400, 500, 3, 2, 1}
Output: 500

Input: arr[] = {1, 3, 50, 10, 9, 7, 6}
Output: 50

Corner case (No decreasing part)
Input: arr[] = {10, 20, 30, 40, 50}
Output: 50

Corner case (No increasing part)
Input: arr[] = {120, 100, 80, 20, 0}
Output: 120
```

**Method 1 (Linear Search)**
We can traverse the array and keep track of maximum and element. And finally return the maximum element.

```
#include <stdio.h>

int findMaximum(int arr[], int low, int high)
{
  int max = arr[low];
  int i;
  for (i = low; i <= high; i++)
  {
    if (arr[i] > max)
      max = arr[i];
  }
  return max;
}

/* Driver program to check above functions */
```

```
int main()
{
  int arr[] = {1, 30, 40, 50, 60, 70, 23, 20};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("The maximum element is %d", findMaximum(arr, 0, n-1));
  getchar();
  return 0;
}
```

Time Complexity: O(n)

**Method 2 (Binary Search)**
We can modify the standard Binary Search algorithm for the given type of arrays.
i) If the mid element is greater than both of its adjacent elements, then mid is the maximum.
ii) If mid element is greater than its next element and smaller than the previous element then maximum lies on left side of mid. Example array: {3, 50, 10, 9, 7, 6}
iii) If mid element is smaller than its next element and greater than the previous element then maximum lies on right side of mid. Example array: {2, 4, 6, 8, 10, 3, 1}

```
#include <stdio.h>

int findMaximum(int arr[], int low, int high)
{

  /* Base Case: Only one element is present in arr[low..high]*/
  if (low == high)
    return arr[low];

  /* If there are two elements and first is greater then
     the first element is maximum */
  if ((high == low + 1) && arr[low] >= arr[high])
     return arr[low];

  /* If there are two elements and second is greater then
     the second element is maximum */
  if ((high == low + 1) && arr[low] < arr[high])
     return arr[high];

  int mid = (low + high)/2;   /*low + (high - low)/2;*/

  /* If we reach a point where arr[mid] is greater than both of
    its adjacent elements arr[mid-1] and arr[mid+1], then arr[mid]
    is the maximum element*/
```

```
  if ( arr[mid] > arr[mid + 1] && arr[mid] > arr[mid - 1])
     return arr[mid];

  /* If arr[mid] is greater than the next element and smaller than the previous
   element then maximum lies on left side of mid */
  if (arr[mid] > arr[mid + 1] && arr[mid] < arr[mid - 1])
    return findMaximum(arr, low, mid-1);
  else // when arr[mid] is greater than arr[mid-1] and smaller than arr[mid+1]
    return findMaximum(arr, mid + 1, high);
}

/* Driver program to check above functions */
int main()
{
  int arr[] = {1, 3, 50, 10, 9, 7, 6};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("The maximum element is %d", findMaximum(arr, 0, n-1));
  getchar();
  return 0;
}
```

Time Complexity: O(Logn)

This method works only for distinct numbers. For example, it will not work for an array like {0, 1, 1, 2, 2, 2, 2, 2, 3, 4, 4, 5, 3, 3, 2, 2, 1, 1}.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

55. Minimum number of jumps to reach end

Given an array of integers where each element represents the max number of steps that can be made forward from that element. Write a function to return the minimum number of jumps to reach the end of the array (starting from the first element). If an element is 0, then cannot move through that element.

Example:

```
Input: arr[] = {1, 3, 5, 8, 9, 2, 6, 7, 6, 8, 9}
Output: 3 (1-> 3 -> 8 ->9)
```

First element is 1, so can only go to 3. Second element is 3, so can make at most 3 steps eg to 5 or 8 or 9.

**Method 1 (Naive Recursive Approach)**
A naive approach is to start from the first element and recursively call for all the elements reachable from first element. The minimum number of jumps to reach end from first can be calculated using minimum number of jumps needed to reach end from the elements reachable from first.

*minJumps(start, end) = Min ( minJumps(k, end) ) for all k reachable from start*

```
#include <stdio.h>
#include <limits.h>

// Returns minimum number of jumps to reach arr[h] from arr[l]
int minJumps(int arr[], int l, int h)
{
  // Base case: when source and destination are same
  if (h == l)
    return 0;

  // When nothing is reachable from the given source
  if (arr[l] == 0)
    return INT_MAX;

  // Traverse through all the points reachable from arr[l]. Recursively
  // get the minimum number of jumps needed to reach arr[h] from these
  // reachable points.
  int min = INT_MAX;
  for (int i = l+1; i <= h && i <= l + arr[l]; i++)
  {
      int jumps = minJumps(arr, i, h);
      if(jumps != INT_MAX && jumps + 1 < min)
          min = jumps + 1;
  }

  return min;
}

// Driver program to test above function
int main()
{
 int arr[] = {1, 3, 6, 3, 2, 3, 6, 8, 9, 5};
 int n = sizeof(arr)/sizeof(arr[0]);
 printf("Minimum number of jumps to reach end is %d ", minJumps(arr, 0, n-1));
 return 0;
}
```

If we trace the execution of this method, we can see that there will be overlapping subproblems. For example, minJumps(3, 9) will be called two times as arr[3] is reachable from arr[1] and arr[2]. So this problem has both properties (optimal substructure and overlapping subproblems) of Dynamic Programming.

**Method 2 (Dynamic Programming)**
In this method, we build a jumps[] array from left to right such that jumps[i] indicates the minimum number of jumps needed to reach arr[i] from arr[0]. Finally, we return jumps[n-1].

```
#include <stdio.h>
#include <limits.h>

int min(int x, int y) { return (x < y)? x: y; }

// Returns minimum number of jumps to reach arr[n-1] from arr[0]
int minJumps(int arr[], int n)
{
    int *jumps = new int[n];  // jumps[n-1] will hold the result
    int i, j;

    if (n == 0 || arr[0] == 0)
        return INT_MAX;

    jumps[0] = 0;

    // Find the minimum number of jumps to reach arr[i]
    // from arr[0], and assign this value to jumps[i]
    for (i = 1; i < n; i++)
    {
        jumps[i] = INT_MAX;
        for (j = 0; j < i; j++)
        {
            if (i <= j + arr[j] && jumps[j] != INT_MAX)
            {
                jumps[i] = min(jumps[i], jumps[j] + 1);
                break;
            }
        }
    }
    return jumps[n-1];
}
```

```
// Driver program to test above function
int main()
{
    int arr[] = {1, 3, 6, 1, 0, 9};
    int size = sizeof(arr)/sizeof(int);
    printf("Minimum number of jumps to reach end is %d ", minJumps(arr,size));
    return 0;
}
```

Output:

```
Minimum number of jumps to reach end is 3
```

Thanks to paras for suggesting this method.

Time Complexity: O(n^2)

**Method 3 (Dynamic Programming)**
In this method, we build jumps[] array from right to left such that jumps[i] indicates the minimum number of jumps needed to reach arr[n-1] from arr[i]. Finally, we return arr[0].

```
int minJumps(int arr[], int n)
{
    int *jumps = new int[n];  // jumps[0] will hold the result
    int min;

    // Minimum number of jumps needed to reach last element
    // from last elements itself is always 0
    jumps[n-1] = 0;

    int i, j;

    // Start from the second element, move from right to left
    // and construct the jumps[] array where jumps[i] represents
    // minimum number of jumps needed to reach arr[m-1] from arr[i]
    for (i = n-2; i >=0; i--)
    {
        // If arr[i] is 0 then arr[n-1] can't be reached from here
        if (arr[i] == 0)
            jumps[i] = INT_MAX;

        // If we can direcly reach to the end point from here then
        // jumps[i] is 1
        else if (arr[i] >= n - i - 1)
```

```
            jumps[i] = 1;

        // Otherwise, to find out the minimum number of jumps needed
        // to reach arr[n-1], check all the points reachable from here
        // and jumps[] value for those points
        else
        {
            min = INT_MAX;  // initialize min value

            // following loop checks with all reachable points and
            // takes the minimum
            for (j = i+1; j < n && j <= arr[i] + i; j++)
            {
                if (min > jumps[j])
                    min = jumps[j];
            }

            // Handle overflow
            if (min != INT_MAX)
              jumps[i] = min + 1;
            else
              jumps[i] = min; // or INT_MAX
        }
    }

    return jumps[0];
}
```

Time Complexity: O(n^2) in worst case.

Thanks to Ashish for suggesting this solution.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

56. Implement two stacks in an array

Create a data structure *twoStacks* that represents two stacks. Implementation of *twoStacks* should use only one array, i.e., both stacks should use the same array for storing elements. Following functions must be supported by *twoStacks*.

push1(int x) â€“> pushes x to first stack

push2(int x) â€“> pushes x to second stack

pop1() â€“> pops an element from first stack and return the popped element
pop2() â€“> pops an element from second stack and return the popped element

Implementation of *twoStack* should be space efficient.

**Method 1 (Divide the space in two halves)**
A simple way to implement two stacks is to divide the array in two halves and assign the half half space to two stacks, i.e., use arr[0] to arr[n/2] for stack1, and arr[n/2+1] to arr[n-1] for stack2 where arr[] is the array to be used to implement two stacks and size of array be n.

The problem with this method is inefficient use of array space. A stack push operation may result in stack overflow even if there is space available in arr[]. For example, say the array size is 6 and we push 3 elements to stack1 and do not push anything to second stack2. When we push 4th element to stack1, there will be overflow even if we have space for 3 more elements in array.

**Method 2 (A space efficient implementation)**
This method efficiently utilizes the available space. It doesnâ€™t cause an overflow if there is space available in arr[]. The idea is to start two stacks from two extreme corners of arr[]. stack1 starts from the leftmost element, the first element in stack1 is pushed at index 0. The stack2 starts from the rightmost corner, the first element in stack2 is pushed at index (n-1). Both stacks grow (or shrink) in opposite direction. To check for overflow, all we need to check is for space between top elements of both stacks. This check is highlighted in the below code.

```
#include<iostream>
#include<stdlib.h>

using namespace std;

class twoStacks
{
   int *arr;
   int size;
   int top1, top2;
public:
  twoStacks(int n)  // constructor
  {
     size = n;
     arr = new int[n];
     top1 = -1;
     top2 = size;
  }
```

```
// Method to push an element x to stack1
void push1(int x)
{
    // There is at least one empty space for new element
    if (top1 < top2 - 1)
    {
        top1++;
        arr[top1] = x;
    }
    else
    {
        cout << "Stack Overflow";
        exit(1);
    }
}

// Method to push an element x to stack2
void push2(int x)
{
    // There is at least one empty space for new element
    if (top1 < top2 - 1)
    {
        top2--;
        arr[top2] = x;
    }
    else
    {
        cout << "Stack Overflow";
        exit(1);
    }
}

// Method to pop an element from first stack
int pop1()
{
    if (top1 >= 0 )
    {
       int x = arr[top1];
       top1--;
       return x;
    }
    else
    {
```

```
            cout << "Stack UnderFlow";
            exit(1);
        }
    }

    // Method to pop an element from second stack
    int pop2()
    {
        if (top2 < size)
        {
            int x = arr[top2];
            top2++;
            return x;
        }
        else
        {
            cout << "Stack UnderFlow";
            exit(1);
        }
    }
};


/* Driver program to test twStacks class */
int main()
{
    twoStacks ts(5);
    ts.push1(5);
    ts.push2(10);
    ts.push2(15);
    ts.push1(11);
    ts.push2(7);
    cout << "Popped element from stack1 is " << ts.pop1();
    ts.push2(40);
    cout << "\nPopped element from stack2 is " << ts.pop2();
    return 0;
}
```

Output:

```
Popped element from stack1 is 11
Popped element from stack2 is 40
```

Time complexity of all 4 operations of *twoStack* is O(1).
We will extend to 3 stacks in an array in a separate post.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

57. Find subarray with given sum

Given an unsorted array of nonnegative integers, find a continous subarray which adds to a given number.

Examples:

```
Input: arr[] = {1, 4, 20, 3, 10, 5}, sum = 33
Ouptut: Sum found between indexes 2 and 4

Input: arr[] = {1, 4, 0, 0, 3, 10, 5}, sum = 7
Ouptut: Sum found between indexes 1 and 4

Input: arr[] = {1, 4}, sum = 0
Output: No subarray found
```

There may be more than one subarrays with sum as the given sum. The following solutions print first such subarray.

Source: Google Interview Question

**Method 1 (Simple)**
A simple solution is to consider all subarrays one by one and check the sum of every subarray. Following program implements the simple solution. We run two loops: the outer loop picks a starting point i and the inner loop tries all subarrays starting from i.

```
/* A simple program to print subarray with sum as given sum */
#include<stdio.h>

/* Returns true if the there is a subarray of arr[] with sum equal to 'sum'
   otherwise returns false.  Also, prints the result */
int subArraySum(int arr[], int n, int sum)
{
    int curr_sum, i, j;

    // Pick a starting point
    for (i = 0; i < n; i++)
    {
        curr_sum = arr[i];
```

```
        // try all subarrays starting with 'i'
        for (j = i+1; j <= n; j++)
        {
            if (curr_sum == sum)
            {
                printf ("Sum found between indexes %d and %d", i, j-1);
                return 1;
            }
            if (curr_sum > sum || j == n)
                break;
            curr_sum = curr_sum + arr[j];
        }
    }

    printf("No subarray found");
    return 0;
}

// Driver program to test above function
int main()
{
    int arr[] = {15, 2, 4, 8, 9, 5, 10, 23};
    int n = sizeof(arr)/sizeof(arr[0]);
    int sum = 23;
    subArraySum(arr, n, sum);
    return 0;
}
```

Output:

```
Sum found between indexes 1 and 4
```

Time Complexity: O(n^2) in worst case.

**Method 2 (Efficient)**
Initialize a variable curr_sum as first element. curr_sum indicates the sum of current subarray. Start from the second element and add all elements one by one to the curr_sum. If curr_sum becomes equal to sum, then print the solution. If curr_sum exceeds the sum, then remove trailing elemnents while curr_sum is greater than sum.

Following is C implementation of the above approach.

```
/* An efficient program to print subarray with sum as given sum */
#include<stdio.h>
```

```
/* Returns true if the there is a subarray of arr[] with sum equal to 'sum'
   otherwise returns false.  Also, prints the result */
int subArraySum(int arr[], int n, int sum)
{
    /* Initialize curr_sum as value of first element
       and starting point as 0 */
    int curr_sum = arr[0], start = 0, i;

    /* Add elements one by one to curr_sum and if the curr_sum exceeds the
       sum, then remove starting element */
    for (i = 1; i <= n; i++)
    {
        // If curr_sum exceeds the sum, then remove the starting elements
        while (curr_sum > sum && start < i-1)
        {
            curr_sum = curr_sum - arr[start];
            start++;
        }

        // If curr_sum becomes equal to sum, then return true
        if (curr_sum == sum)
        {
            printf ("Sum found between indexes %d and %d", start, i-1);
            return 1;
        }

        // Add this element to curr_sum
        if (i < n)
          curr_sum = curr_sum + arr[i];
    }

    // If we reach here, then no subarray
    printf("No subarray found");
    return 0;
}

// Driver program to test above function
int main()
{
    int arr[] = {15, 2, 4, 8, 9, 5, 10, 23};
    int n = sizeof(arr)/sizeof(arr[0]);
    int sum = 23;
    subArraySum(arr, n, sum);
    return 0;
```

```
}
```

Output:

```
Sum found between indexes 1 and 4
```

Time complexity of method 2 looks more than O(n), but if we take a closer look at the program, then we can figure out the time complexity is O(n). We can prove it by counting the number of operations performed on every element of arr[] in worst case. There are at most 2 operations performed on every element: (a) the element is added to the curr_sum (b) the element is subtracted from curr_sum. So the upper bound on number of operations is 2n which is O(n).

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

58. Dynamic Programming | Set 14 (Maximum Sum Increasing Subsequence)

Given an array of n positive integers. Write a program to find the sum of maximum sum subsequence of the given array such that the intgers in the subsequence are sorted in increasing order. For example, if input is {1, 101, 2, 3, 100, 4, 5}, then output should be 106 (1 + 2 + 3 + 100), if the input array is {3, 4, 5, 10}, then output should be 22 (3 + 4 + 5 + 10) and if the input array is {10, 5, 4, 3}, then output should be 10

**Solution**
This problem is a variation of standard Longest Increasing Subsequence (LIS) problem. We need a slight change in the Dynamic Programming solution of LIS problem. All we need to change is to use sum as a criteria instead of length of increasing subsequence.

Following is C implementation for Dynamic Programming solution of the problem.

```
/* Dynamic Programming implementation of Maximum Sum Increasing
   Subsequence (MSIS) problem */
#include<stdio.h>

/* maxSumIS() returns the maximum sum of increasing subsequence in arr[] of
   size n */
int maxSumIS( int arr[], int n )
{
   int *msis, i, j, max = 0;
   msis = (int*) malloc ( sizeof( int ) * n );
```

```
  /* Initialize msis values for all indexes */
  for ( i = 0; i < n; i++ )
     msis[i] = arr[i];

  /* Compute maximum sum values in bottom up manner */
  for ( i = 1; i < n; i++ )
     for ( j = 0; j < i; j++ )
        if ( arr[i] > arr[j] && msis[i] < msis[j] + arr[i])
           msis[i] = msis[j] + arr[i];

  /* Pick maximum of all msis values */
  for ( i = 0; i < n; i++ )
     if ( max < msis[i] )
        max = msis[i];

  /* Free memory to avoid memory leak */
  free( msis );

  return max;
}

/* Driver program to test above function */
int main()
{
  int arr[] = {1, 101, 2, 3, 100, 4, 5};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("Sum of maximum sum increasing subsequence is %d\n",
        maxSumIS( arr, n ) );

  getchar();
  return 0;
}
```

Time Complexity: O(n^2)

Source: Maximum Sum Increasing Subsequence Problem

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

After few months of gap posting an algo. The current post is pending from long time, and many readers (e.g.Â here, here, hereÂ may be few more, I am not keeping track of all) are posting requests for explanation of the below problem.

**Given an array of random numbers. Find *longest monotonically increasing subsequence*Â (LIS) in the array. I know many of you might have read recursive and dynamic programming (DP) solutions. There are few requests for O(N log N) algo in the forum posts.**

For the time being, forget about recursive and DP solutions. Let us take small samples and extend the solution to large instances. Even though it may look complex at first time, once if we understood the logic, coding is simple.

Consider an input array A = {2, 5, 3}. I will extend the array during explanation.

By observation we know that the LIS is either {2, 3} or {2, 5}. ***Note that I am considering only strictly increasing monotone sequences***.

Let us add two more elements, say 7, 11 to the array. These elements will extend the existing sequences. Now the increasing sequences are {2, 3, 7, 11} and {2, 5, 7, 11} for the input array {2, 5, 3, 7, 11}.

Further, we add one more element, say 8 to the array i.e. input array becomes {2, 5, 3, 7, 11, 8}. Note that the latest element 8 is greater than smallest element of any active sequence (*will discussÂ shortly about active sequences*). How can we extend the existing sequences with 8? First of all, can 8 be part of LIS? If yes, how? If we want to add 8, it should come after 7 (by replacing 11).

Since the approach isÂ *offline (what we mean by offline?)*, we are not sure whether adding 8 will extend the series or not. Assume there is 9 in the input array, sayÂ {2, 5, 3, 7, 11, 8, 7, 9 â€¦}. We can replace 11 with 8, as there is potentiallyÂ *best*Â candidate (9) that can extend the new series {2, 3, 7, 8} or {2, 5, 7, 8}.

Our observation is, assume that the end element of largest sequence is E. We can add (replace) current element A[i] to the existing sequence if there is an element A[j] (j > i) such that E < A[i] < A[j] or (E > A[i] < A[j] â€“ for replace). In the above example, E = 11, A[i] = 8 and A[j] = 9.

In case of our original array {2, 5, 3}, note that we face same situation when we are adding 3 to increasing sequence {2, 5}. I just created two increasing sequences to make explanation simple. Instead of two sequences, 3 can replace 5 in the sequence {2, 5}.

I know it will be confusing, I will clear it shortly!

*The question is, when will it be safe to add or replace an element in the existing*

*sequence?*

Let us consider another sampleÂ A = {2, 5, 3}. Say, the next element is 1. How can it extend the current sequences {2,3} or {2, 5}. Obviously, it canâ€™t extend either. Yet, there is a potential that the new smallest element can be start of an LIS. To make it clear, consider the array is {2, 5, 3, 1, 2, 3, 4, 5, 6}. Making 1 as new sequence will create new sequence which is largest.

*The observation is, when we encounter new smallest element in the array, it can be a potential candidate to start new sequence.*

From the observations, we need to maintain lists of increasing sequences.

In general, we have set ofÂ **active lists** of varying length. We are adding an element A[i] to these lists. We scan the lists (for end elements) in decreasing order of their length. We will verify the end elements of all the lists to find a list whose end element is smaller than A[i] (*floor* value).

Our strategy determined by the following conditions,

**1. If A[i] is smallest among all *end* candidates of active lists, we will *start* new active list of length 1.**

**2. If A[i] is largest among all *end* candidates of active lists, we will clone the *largest* active list, and extend it by A[i].**

**3. If A[i] is in between, we will find a list with *largest end element that is smaller than* A[i]. Clone and extend this list by A[i]. We will discard all other lists of same length as that of this modified list.**

Note that at any instance during our construction of active lists, the following condition is maintained.

*â€œend element of smaller list is smaller than end elements of larger listsâ€.*

It will be clear with an example, let us take example from wiki {0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15}.

```
A[0] = 0. Case 1. There are no active lists, create one.
0.
-------------------------------------------------------------------------------
A[1] = 8. Case 2. Clone and extend.
0.
0, 8.
-------------------------------------------------------------------------------
A[2] = 4. Case 3. Clone, extend and discard.
0.
0, 4.
```

~~0, 8~~. Discarded

---

A[3] = 12. Case 2. Clone and extend.

0.

0, 4.

0, 4, 12.

---

A[4] = 2. Case 3. Clone, extend and discard.

0.

0, 2.

~~0, 4~~. Discarded.

0, 4, 12.

---

A[5] = 10. Case 3. Clone, extend and discard.

0.

0, 2.

0, 2, 10.

~~0, 4, 12~~. Discarded.

---

A[6] = 6. Case 3. Clone, extend and discard.

0.

0, 2.

0, 2, 6.

~~0, 2, 10~~. Discarded.

---

A[7] = 14. Case 2. Clone and extend.

0.

0, 2.

0, 2, 6.

0, 2, 6, 14.

---

A[8] = 1. Case 3. Clone, extend and discard.

0.

0, 1.

~~0, 2~~. Discarded.

0, 2, 6.

0, 2, 6, 14.

---

A[9] = 9. Case 3. Clone, extend and discard.

0.

0, 1.

0, 2, 6.

0, 2, 6, 9.

~~0, 2, 6, 14~~. Discarded.

---

A[10] = 5. Case 3. Clone, extend and discard.

0.

0, 1.

0, 1, 5.

~~0, 2, 6~~. Discarded.

0, 2, 6, 9.

---

A[11] = 13. Case 2. Clone and extend.

0.

0, 1.

0, 1, 5.

0, 2, 6, 9.

0, 2, 6, 9, 13.

---

A[12] = 3. Case 3. Clone, extend and discard.

0.

0, 1.

0, 1, 3.

~~0, 1, 5~~. Discarded.

0, 2, 6, 9.

0, 2, 6, 9, 13.

---

A[13] = 11. Case 3. Clone, extend and discard.

0.

0, 1.

0, 1, 3.

0, 2, 6, 9.

0, 2, 6, 9, 11.

~~0, 2, 6, 9, 13~~. Discarded.

---

A[14] = 7. Case 3. Clone, extend and discard.

0.

0, 1.

0, 1, 3.

0, 1, 3, 7.

~~0, 2, 6, 9~~. Discarded.

0, 2, 6, 9, 11.

---

A[15] = 15. Case 2. Clone and extend.

0.

0, 1.

0, 1, 3.

0, 1, 3, 7.

```
0, 2, 6, 9, 11.
0, 2, 6, 9, 11, 15. <-- LIS List
--------------------------------------------------------------------------------
```

It is required to understand above strategy to devise an algorithm. Also, ensure we have maintained the condition, â€œ*end element of smaller list is smaller than end elements of larger lists*â€œ. Try with few other examples, before reading further. It is important to understand what happening to end elements.

**Algorithm:**

Querying length of longest is fairly easy. Note that we are dealing with end elements only. We need not to maintain all the lists. We can store the end elements in an array. Discarding operation can be simulated with replacement, and extending a list isÂ analogousÂ to adding more elements to array.

We will use anÂ auxiliaryÂ array to keep end elements. The maximum length of this array is that of input. In the worst case the array divided into N lists of size one (*note that it doesâ€™t lead to worst case complexity*). To discard an element, we will trace ceil value of A[i] in auxiliary array (again observe the end elements in your rough work), and replace ceil value with A[i]. We extend a list by adding element to auxiliary array. We also maintain a counter to keep track of auxiliary array length.

**Bonus:**Â You have learntÂ Patience SortingÂ technique partially :).

Here is a proverb, â€œ*Tell me and I will forget. Show me and I will remember. Involve me and I will understand.*â€ So, pick a suit from deck of cards. Find the longest increasing sub-sequence of cards from the shuffled suit. You will never forget the approach. 😀

Given below is code to find length of LIS,

```
#include <iostream>
#include <string.h>
#include <stdio.h>

using namespace std;

#define ARRAY_SIZE(A) sizeof(A)/sizeof(A[0])
// Binary search (note boundaries in the caller)
// A[] is ceilIndex in the caller
int CeilIndex(int A[], int l, int r, int key) {
   int m;

   while( r - l > 1 ) {
      m = l + (r - l)/2;
      (A[m] >= key ? r : l) = m; // ternary expression returns an l-value
```

```
    }

    return r;
}

int LongestIncreasingSubsequenceLength(int A[], int size) {
    // Add boundary case, when array size is one

    int *tailTable   = new int[size];
    int len; // always points empty slot

    memset(tailTable, 0, sizeof(tailTable[0])*size);

    tailTable[0] = A[0];
    len = 1;
    for( int i = 1; i < size; i++ ) {
        if( A[i] < tailTable[0] )
            // new smallest value
            tailTable[0] = A[i];
        else if( A[i] > tailTable[len-1] )
            // A[i] wants to extend largest subsequence
            tailTable[len++] = A[i];
        else
            // A[i] wants to be current end candidate of an existing subsequence
            // It will replace ceil value in tailTable
            tailTable[CeilIndex(tailTable, -1, len-1, A[i])] = A[i];
    }

    delete[] tailTable;

    return len;
}

int main() {
    int A[] = { 2, 5, 3, 7, 11, 8, 10, 13, 6 };
    int n = ARRAY_SIZE(A);

    printf("Length of Longest Increasing Subsequence is %d\n",
            LongestIncreasingSubsequenceLength(A, n));

    return 0;
}
```

**Complexity:**

The loop runs for N elements. In the worst case (what is worst case input?), we may end up querying ceil value using binary search (log *i*) for many A[i].

Therefore, T(n) <Â O( log N! )Â Â =Â O(N log N). Analyse to ensure that the upper and lower bounds are also O( N log N ). The complexity is THETA (N log N).

**Exercises:**

1. Design an algorithm to construct the longest increasing list. Also, model your solution using DAGs.

2. Design an algorithm to construct **all**Â monotonically increasing list**s of equal longest size**.

3. Is the above algorithm an *online* algorithm?

4. Design an algorithm to construct the longest *decreasing* list..

â€" Venki.Â Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

60. Find a triplet that sum to a given value

Given an array and a value, find if there is a triplet in array whose sum is equal to the given value. If there is such a triplet present in array, then print the triplet and return true. Else return false. For example, if the given array is {12, 3, 4, 1, 6, 9} and given sum is 24, then there is a triplet (12, 3 and 9) present in array whose sum is 24.

**Method 1 (Naive)**
A simple method is to generate all possible triplets and compare the sum of every triplet with the given value. The following code implements this simple method using three nested loops.

```
# include <stdio.h>

// returns true if there is triplet with sum equal
// to 'sum' present in A[]. Also, prints the triplet
bool find3Numbers(int A[], int arr_size, int sum)
{
   int l, r;

   // Fix the first element as A[i]
   for (int i = 0; i < arr_size-2; i++)
   {
```

```
    // Fix the second element as A[j]
    for (int j = i+1; j < arr_size-1; j++)
    {
        // Now look for the third number
        for (int k = j+1; k < arr_size; k++)
        {
            if (A[i] + A[j] + A[k] == sum)
            {
              printf("Triplet is %d, %d, %d", A[i], A[j], A[k]);
              return true;
            }
        }
    }
  }

  // If we reach here, then no triplet was found
  return false;
}

/* Driver program to test above function */
int main()
{
  int A[] = {1, 4, 45, 6, 10, 8};
  int sum = 22;
  int arr_size = sizeof(A)/sizeof(A[0]);

  find3Numbers(A, arr_size, sum);

  getchar();
  return 0;
}
```

Output:

```
Triplet is 4, 10, 8
```

Time Complexity: O(n^3)

**Method 2 (Use Sorting)**

Time complexity of the method 1 is O(n^3). The complexity can be reduced to O(n^2) by sorting the array first, and then using method 1 of this post in a loop.

1) Sort the input array.

2) Fix the first element as A[i] where i is from 0 to array size â€“ 2. After fixing the first element of triplet, find the other two elements using method 1 of this post.

```
# include <stdio.h>

// A utility function to sort an array using Quicksort
void quickSort(int *, int, int);

// returns true if there is triplet with sum equal
// to 'sum' present in A[]. Also, prints the triplet
bool find3Numbers(int A[], int arr_size, int sum)
{
    int l, r;

    /* Sort the elements */
    quickSort(A, 0, arr_size-1);

    /* Now fix the first element one by one and find the
       other two elements */
    for (int i = 0; i < arr_size - 2; i++)
    {

        // To find the other two elements, start two index variables
        // from two corners of the array and move them toward each
        // other
        l = i + 1; // index of the first element in the remaining elements
        r = arr_size-1; // index of the last element
        while (l < r)
        {
            if( A[i] + A[l] + A[r] == sum)
            {
                printf("Triplet is %d, %d, %d", A[i], A[l], A[r]);
                return true;
            }
            else if (A[i] + A[l] + A[r] < sum)
                l++;
            else // A[i] + A[l] + A[r] > sum
                r--;
        }
    }

    // If we reach here, then no triplet was found
    return false;
}

/* FOLLOWING 2 FUNCTIONS ARE ONLY FOR SORTING
    PURPOSE */
```

```
void exchange(int *a, int *b)
{
   int temp;
   temp = *a;
   *a   = *b;
   *b   = temp;
}

int partition(int A[], int si, int ei)
{
   int x = A[ei];
   int i = (si - 1);
   int j;

   for (j = si; j <= ei - 1; j++)
   {
      if(A[j] <= x)
      {
         i++;
         exchange(&A[i], &A[j]);
      }
   }
   exchange (&A[i + 1], &A[ei]);
   return (i + 1);
}

/* Implementation of Quick Sort
A[] --> Array to be sorted
si  --> Starting index
ei  --> Ending index
*/
void quickSort(int A[], int si, int ei)
{
   int pi;    /* Partitioning index */
   if(si < ei)
   {
      pi = partition(A, si, ei);
      quickSort(A, si, pi - 1);
      quickSort(A, pi + 1, ei);
   }
}

/* Driver program to test above function */
int main()
```

```
{
    int A[] = {1, 4, 45, 6, 10, 8};
    int sum = 22;
    int arr_size = sizeof(A)/sizeof(A[0]);

    find3Numbers(A, arr_size, sum);

    getchar();
    return 0;
}
```

Output:

```
Triplet is 4, 8, 10
```

Time Complexity: O(n^2)

Note that there can be more than one triplet with the given sum. We can easily modify the above methods to print all triplets.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

61. Find the smallest positive number missing from an unsorted array

You are given an unsorted array with both positive and negative elements. You have to find the smallest positive number missing from the array in O(n) time using constant extra space. You can modify the original array.

Examples

```
Input:  {2, 3, 7, 6, 8, -1, -10, 15}
Output: 1

Input:  { 2, 3, -7, 6, 8, 1, -10, 15 }
Output: 4

Input: {1, 1, 0, -1, -2}
Output: 2
```

Source: To find the smallest positive no missing from an unsorted array

A **naive method** to solve this problem is to search all positive integers, starting from 1 in the given array. We may have to search at most n+1 numbers in the given array. So

this solution takes O(n^2) in worst case.

We can **use sorting** to solve it in lesser time complexity. We can sort the array in O(nLogn) time. Once the array is sorted, then all we need to do is a linear scan of the array. So this approach takes O(nLogn + n) time which is O(nLogn).

We can also **use hashing**. We can build a hash table of all positive elements in the given array. Once the hash table is built. We can look in the hash table for all positive integers, starting from 1. As soon as we find a number which is not there in hash table, we return it. This approach may take O(n) time on average, but it requires O(n) extra space.

**A O(n) time and O(1) extra space solution:**
The idea is similar to this post. We use array elements as index. To mark presence of an element x, we change the value at the index x to negative. But this approach doesnâ€™t work if there are non-positive (-ve and 0) numbers. So we segregate positive from negative numbers as first step and then apply the approach.

Following is the two step algorithm.
1) Segregate positive numbers from others i.e., move all non-positive numbers to left side. In the following code, segregate() function does this part.
2) Now we can ignore non-positive elements and consider only the part of array which contains all positive elements. We traverse the array containing all positive numbers and to mark presence of an element x, we change the sign of value at index x to negative. We traverse the array again and print the first index which has positive value. In the following code, findMissingPositive() function does this part. Note that in findMissingPositive, we have subtracted 1 from the values as indexes start from 0 in C.

```
/* Program to find the smallest positive missing number */
#include <stdio.h>
#include <stdlib.h>

/* Utility to swap to integers */
void swap(int *a, int *b)
{
    int temp;
    temp = *a;
    *a   = *b;
    *b   = temp;
}

/* Utility function that puts all non-positive (0 and negative) numbers on left
  side of arr[] and return count of such numbers */
int segregate (int arr[], int size)
```

```
{
    int j = 0, i;
    for(i = 0; i < size; i++)
    {
        if (arr[i] <= 0)
        {
            swap(&arr[i], &arr[j]);
            j++;  // increment count of non-positive integers
        }
    }

    return j;
}

/* Find the smallest positive missing number in an array that contains
  all positive integers */
int findMissingPositive(int arr[], int size)
{
  int i;

  // Mark arr[i] as visited by making arr[arr[i] - 1] negative. Note that
  // 1 is subtracted because index start from 0 and positive numbers start from 1
  for(i = 0; i < size; i++)
  {
    if(abs(arr[i]) - 1 < size && arr[abs(arr[i]) - 1] > 0)
      arr[abs(arr[i]) - 1] = -arr[abs(arr[i]) - 1];
  }

  // Return the first index value at which is positive
  for(i = 0; i < size; i++)
    if (arr[i] > 0)
      return i+1;  // 1 is added becuase indexes start from 0

  return size+1;
}

/* Find the smallest positive missing number in an array that contains
  both positive and negative integers */
int findMissing(int arr[], int size)
{
   // First separate positive and negative numbers
   int shift = segregate (arr, size);

   // Shift the array and call findMissingPositive for
```

```
  // positive part
  return findMissingPositive(arr+shift, size-shift);
}

int main()
{
  int arr[] = {0, 10, 2, -10, -20};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  int missing = findMissing(arr, arr_size);
  printf("The smallest positive missing number is %d ", missing);
  getchar();
  return 0;
}
```

Output:

```
The smallest positive missing number is 1
```

Note that this method modifies the original array. We can change the sign of elements in the segregated array to get the same set of elements back. But we still loose the order of elements. If we want to keep the original array as it was, then we can create a copy of the array and run this approach on the temp array.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

You are given an unsorted array with both positive and negative elements. You have to find the smallest positive number missing from the array in O(n) time using constant extra space. You can modify the original array.

Examples

```
Input:  {2, 3, 7, 6, 8, -1, -10, 15}
Output: 1

Input:  { 2, 3, -7, 6, 8, 1, -10, 15 }
Output: 4

Input: {1, 1, 0, -1, -2}
Output: 2
```

Source: To find the smallest positive no missing from an unsorted array

A **naive method** to solve this problem is to search all positive integers, starting from 1 in the given array. We may have to search at most n+1 numbers in the given array. So this solution takes O(n^2) in worst case.

We can **use sorting** to solve it in lesser time complexity. We can sort the array in O(nLogn) time. Once the array is sorted, then all we need to do is a linear scan of the array. So this approach takes O(nLogn + n) time which is O(nLogn).

We can also **use hashing**. We can build a hash table of all positive elements in the given array. Once the hash table is built. We can look in the hash table for all positive integers, starting from 1. As soon as we find a number which is not there in hash table, we return it. This approach may take O(n) time on average, but it requires O(n) extra space.

**A O(n) time and O(1) extra space solution:**
The idea is similar to this post. We use array elements as index. To mark presence of an element x, we change the value at the index x to negative. But this approach doesnâ€™t work if there are non-positive (-ve and 0) numbers. So we segregate positive from negative numbers as first step and then apply the approach.

Following is the two step algorithm.
1) Segregate positive numbers from others i.e., move all non-positive numbers to left side. In the following code, segregate() function does this part.
2) Now we can ignore non-positive elements and consider only the part of array which contains all positive elements. We traverse the array containing all positive numbers and to mark presence of an element x, we change the sign of value at index x to negative. We traverse the array again and print the first index which has positive value. In the following code, findMissingPositive() function does this part. Note that in findMissingPositive, we have subtracted 1 from the values as indexes start from 0 in C.

```
/* Program to find the smallest positive missing number */
#include <stdio.h>
#include <stdlib.h>

/* Utility to swap to integers */
void swap(int *a, int *b)
{
   int temp;
   temp = *a;
   *a   = *b;
   *b   = temp;
}

/* Utility function that puts all non-positive (0 and negative) numbers on left
```

```
  side of arr[] and return count of such numbers */
int segregate (int arr[], int size)
{
    int j = 0, i;
    for(i = 0; i < size; i++)
    {
       if (arr[i] <= 0)
       {
           swap(&arr[i], &arr[j]);
           j++;  // increment count of non-positive integers
       }
    }

    return j;
}

/* Find the smallest positive missing number in an array that contains
  all positive integers */
int findMissingPositive(int arr[], int size)
{
  int i;

  // Mark arr[i] as visited by making arr[arr[i] - 1] negative. Note that
  // 1 is subtracted because index start from 0 and positive numbers start from 1
  for(i = 0; i < size; i++)
  {
    if(abs(arr[i]) - 1 < size && arr[abs(arr[i]) - 1] > 0)
      arr[abs(arr[i]) - 1] = -arr[abs(arr[i]) - 1];
  }

  // Return the first index value at which is positive
  for(i = 0; i < size; i++)
    if (arr[i] > 0)
      return i+1;  // 1 is added becuase indexes start from 0

  return size+1;
}

/* Find the smallest positive missing number in an array that contains
  both positive and negative integers */
int findMissing(int arr[], int size)
{
   // First separate positive and negative numbers
   int shift = segregate (arr, size);
```

```
  // Shift the array and call findMissingPositive for
  // positive part
  return findMissingPositive(arr+shift, size-shift);
}

int main()
{
  int arr[] = {0, 10, 2, -10, -20};
  int arr_size = sizeof(arr)/sizeof(arr[0]);
  int missing = findMissing(arr, arr_size);
  printf("The smallest positive missing number is %d ", missing);
  getchar();
  return 0;
}
```

Output:

```
The smallest positive missing number is 1
```

Note that this method modifies the original array. We can change the sign of elements in the segregated array to get the same set of elements back. But we still loose the order of elements. If we want to keep the original array as it was, then we can create a copy of the array and run this approach on the temp array.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

63. The Celebrity Problem

Another classical problem.

*In a party of N people, only one person is known to everyone. Such a person* ***mayÂ be present****Â in the party, if yes, (s)he doesnâ€™t know anyone in the party. We can only ask questions like â€œ****does A know B?*** *â€œ. Find the stranger (celebrity) in minimum number of questions.*

We can describe the problem input as an array of numbers/characters representing persons in the party. We also have a hypothetical function *HaveAcquaintance(A, B)* which returns *true* if A knows B, *false* otherwise. How can we solve the problem, try yourself first.

We measure the complexity in terms of calls made toÂ *HaveAcquaintance().*

**Graph:**

We can model the solution using graphs. Initialize indegree and outdegree of every vertex as 0. If A knows B, draw a directed edge from A to B, increase indegree of B and outdegree of A by 1. Construct all possible edges of the graph for every possible pair [i, j]. We have $N_{C_2}$ pairs. If celebrity is present in the party, we will have one sink node in the graph with outdegree of zero, and indegree of N-1. We can find the sink node in (N) time, but the overall complexity is $O(N^2)$ as we need to construct the graph first.

**Recursion:**

We can decompose the problem into combination of smaller instances. Say, if we know celebrity of N-1 persons, can we extend the solution to N? We have two possibilities, Celebrity(N-1) may know N, or N already knew Celebrity(N-1). In the former case, N will be celebrity if N doesnâ€™t know anyone else. In the later case we need to check that Celebrity(N-1) doesnâ€™t know N.

Solve the problem of smaller instance during divide step. On the way back, we may find a celebrity from the smaller instance. During combine stage, check whether the returned celebrity is known to everyone and he doesnâ€™t know anyone. The recurrence of the recursive decomposition is,

$T(N) = T(N-1) + O(N)$

$T(N) = O(N^2)$. You may try Writing pseudo code to check your recursion skills.

**Using Stack:**

The graph construction takes $O(N^2)$ time, it is similar to brute force search. In case of recursion, we reduce the problem instance by not more than one, and also combine step may examine M-1 persons (M â€“ instance size).

We have following observation based on elimination technique (Refer *Polyaâ€™s How to Solve It* book).

- If A knows B, then A canâ€™t be celebrity. Discard A,Â and *B may be celebrity*.
- If A doesnâ€™t know B, then B canâ€™t be celebrity. Discard B, and *A may be celebrity*.
- Repeat above two steps till we left with only one person.
- Ensure the remained person is celebrity. (Why do we need this step?)

We can use stack to verity celebrity.

1. Push all the celebrities into a stack.
2. Pop off top two persons from the stack, discard one person based on return status of *HaveAcquaintance(A, B)*.

3. Push the remained person onto stack.
4. Repeat step 2 and 3 until only one person remains in the stack.
5. Check the remained person in stack doesnâ€™t have acquaintance with anyone else.

We will discard N elements utmost (Why?). If the celebrity is present in the party, we will call *HaveAcquaintance()* 3(N-1) times. Here is code using stack.

```
#include <iostream>
#include <list>
using namespace std;

// Max # of persons in the party
#define N 8

// Celebrities identified with numbers from 0 through size-1
int size = 4;
// Person with 2 is celebrity
bool MATRIX[N][N] = {{0, 0, 1, 0}, {0, 0, 1, 0}, {0, 0, 0, 0}, {0, 0, 1, 0}};

bool HaveAcquiantance(int a, int b) { return MATRIX[a][b]; }

int CelebrityUsingStack(int size)
{
  // Handle trivial case of size = 2

  list<int> stack; // Careful about naming
  int i;
  int C; // Celebrity

  i = 0;
  while( i < size )
  {
    stack.push_back(i);
    i = i + 1;
  }

  int A = stack.back();
  stack.pop_back();

  int B = stack.back();
  stack.pop_back();

  while( stack.size() != 1 )
```

```
{
  if( HaveAcquiantance(A, B) )
  {
    A = stack.back();
    stack.pop_back();
  }
  else
  {
    B = stack.back();
    stack.pop_back();
  }
}

// Potential candidate?
C = stack.back();
stack.pop_back();

// Last candidate was not examined, it leads one excess comparison (optimise)
if( HaveAcquiantance(C, B) )
  C = B;

if( HaveAcquiantance(C, A) )
  C = A;

// I know these are redundant,
// we can simply check i against C
i = 0;
while( i < size )
{
  if( C != i )
  stack.push_back(i);
  i = i + 1;
}

while( !stack.empty() )
{
  i = stack.back();
  stack.pop_back();

  // C must not know i
  if( HaveAcquiantance(C, i) )
    return -1;

  // i must know C
```

```
    if( !HaveAcquiantance(i, C) )
      return -1;
  }

  return C;
}

int main()
{
  int id = CelebrityUsingStack(size);
  id == -1 ? cout << "No celebrity" : cout << "Celebrity ID " << id;
  return 0;
}
```

Output

```
Celebrity ID 2
```

Complexity O(N). Total comparisons 3(N-1). Try the above code for successful MATRIXÂ {{0, 0, 0, 1}, {0, 0, 0, 1}, {0, 0, 0, 1}, {0, 0, 0, 1}}.

**A Note:**

You may think that why do we need a new graph as we already have access to input matrix. Note that the matrix MATRIXÂ used to help the hypothetical functionÂ *HaveAcquaintance(A, B),*Â but never accessed via usual notation MATRIX[i, j]. We have access to the input only through the functionÂ *HaveAcquiantance(A, B)*. Matrix is just a way to code the solution. We can assume the cost of hypothetical function as O(1).

If still not clear, assume that the functionÂ *HaveAcquiantanceÂ* accessing information stored in a set of linked lists arranged in levels. List node will haveÂ *next*Â andÂ *nextLevel*Â pointers. Every level will have N nodes i.e. an N element list,Â *next*Â points to next node in the current level list and theÂ *nextLevel*Â pointer in last node of every list will point to head of next level list. For example the linked list representation of above matrix looks like,

```
L0 0->0->1->0
          |
L1         0->0->1->0
                  |
L2                  0->0->1->0
                          |
L3                          0->0->1->0
```

The function *HaveAcquanintance(i, j)* will search in the listÂ for *j-th* node in the *i-th*

level.Â Out goal is to minimize calls toÂ *HaveAcquanintance*Â function.

**Exercises:**

1. Write code to find celebrity. Donâ€™t use any data structures like graphs, stack, etcâ€¦ you have access to *N* and *HaveAcquaintance(int, int)* only.

2. Implement the algorithm using Queues. What is your observation? Compare your solution with Finding Maximum and MinimumÂ in an arrayÂ andÂ TournamentÂ Tree. What are minimum number of comparisons do we need (optimal number of calls to *HaveAcquaintance()*)?

â€”Â Venki.Â Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

64. Dynamic Programming | Set 15 (Longest Bitonic Subsequence)

Given an array arr[0 â€¦ n-1] containing n positive integers, a subsequence of arr[] is called Bitonic if it is first increasing, then decreasing. Write a function that takes an array as argument and returns the length of the longest bitonic subsequence.
A sequence, sorted in increasing order is considered Bitonic with the decreasing part as empty. Similarly, decreasing order sequence is considered Bitonic with the increasing part as empty.

**Examples:**

```
Input arr[] = {1, 11, 2, 10, 4, 5, 2, 1};
Output: 6 (A Longest Bitonic Subsequence of length 6 is 1, 2, 10, 4, 2, 1)

Input arr[] = {12, 11, 40, 5, 3, 1}
Output: 5 (A Longest Bitonic Subsequence of length 5 is 12, 11, 5, 3, 1)

Input arr[] = {80, 60, 30, 40, 20, 10}
Output: 5 (A Longest Bitonic Subsequence of length 5 is 80, 60, 30, 20, 10)
```

Source: Microsoft Interview Question

**Solution**
This problem is a variation of standard Longest Increasing Subsequence (LIS) problem. Let the input array be arr[] of length n. We need to construct two arrays lis[] and lds[] using Dynamic Programming solution of LIS problem. lis[i] stores the length of the Longest Increasing subsequence ending with arr[i]. lds[i] stores the length of the longest Decreasing subsequence starting from arr[i]. Finally, we need to return the

max value of lis[i] + lds[i] â€“ 1 where i is from 0 to n-1.

Following is C++ implementation of the above Dynamic Programming solution.

```
/* Dynamic Programming implementation of longest bitonic subsequence problem */
#include<stdio.h>
#include<stdlib.h>

/* lbs() returns the length of the Longest Bitonic Subsequence in
    arr[] of size n. The function mainly creates two temporary arrays
    lis[] and lds[] and returns the maximum lis[i] + lds[i] - 1.

    lis[i] ==> Longest Increasing subsequence ending with arr[i]
    lds[i] ==> Longest decreasing subsequence starting with arr[i]
*/
int lbs( int arr[], int n )
{
   int i, j;

   /* Allocate memory for LIS[] and initialize LIS values as 1 for
      all indexes */
   int *lis = new int[n];
   for ( i = 0; i < n; i++ )
      lis[i] = 1;

   /* Compute LIS values from left to right */
   for ( i = 1; i < n; i++ )
      for ( j = 0; j < i; j++ )
         if ( arr[i] > arr[j] && lis[i] < lis[j] + 1)
            lis[i] = lis[j] + 1;

   /* Allocate memory for lds and initialize LDS values for
      all indexes */
   int *lds = new int [n];
   for ( i = 0; i < n; i++ )
      lds[i] = 1;

   /* Compute LDS values from right to left */
   for ( i = n-2; i >= 0; i-- )
      for ( j = n-1; j > i; j-- )
         if ( arr[i] > arr[j] && lds[i] < lds[j] + 1)
            lds[i] = lds[j] + 1;
```

```
  /* Return the maximum value of lis[i] + lds[i] - 1*/
  int max = lis[0] + lds[0] - 1;
  for (i = 1; i < n; i++)
    if (lis[i] + lds[i] - 1 > max)
        max = lis[i] + lds[i] - 1;
  return max;
}

/* Driver program to test above function */
int main()
{
  int arr[] = {0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15};
  int n = sizeof(arr)/sizeof(arr[0]);
  printf("Length of LBS is %d\n", lbs( arr, n ) );

  getchar();
  return 0;
}
```

Output:

```
Length of LBS is 7
```

Time Complexity: O(n^2)
Auxiliary Space: O(n)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

65. Find a sorted subsequence of size 3 in linear time

Given an array of n integers, find the 3 elements such that a[i] < a[j] < a[k] and i < j < k in 0(n) time. If there are multiple such triplets, then print any one of them.

Examples:

```
Input:  arr[] = {12, 11, 10, 5, 6, 2, 30}
Output: 5, 6, 30

Input:  arr[] = {1, 2, 3, 4}
Output: 1, 2, 3 OR 1, 2, 4 OR 2, 3, 4

Input:  arr[] = {4, 3, 2, 1}
```

```
Output: No such triplet
```

Source: Amazon Interview Question

Hint: Use Auxiliary Space

**Solution:**
1) Create an auxiliary array smaller[0..n-1]. smaller[i] should store the index of a number which is smaller than arr[i] and is on left side of arr[i]. smaller[i] should contain -1 if there is no such element.
2) Create another auxiliary array greater[0..n-1]. greater[i] should store the index of a number which is greater than arr[i] and is on right side of arr[i]. greater[i] should contain -1 if there is no such element.
3) Finally traverse both smaller[] and greater[] and find the index i for which both smaller[i] and greater[i] are not -1.

```
#include<stdio.h>

// A function to fund a sorted subsequence of size 3
void find3Numbers(int arr[], int n)
{
  int max = n-1; //Index of maximum element from right side
  int min = 0; //Index of minimum element from left side
  int i;

  // Create an array that will store index of a smaller
  // element on left side. If there is no smaller element
  // on left side, then smaller[i] will be -1.
  int *smaller = new int[n];
  smaller[0] = -1;  // first entry will always be -1
  for (i = 1; i < n; i++)
  {
     if (arr[i] <= arr[min])
     {
        min = i;
        smaller[i] = -1;
     }
     else
        smaller[i] = min;
  }

  // Create another array that will store index of a
  // greater element on right side. If there is no greater
  // element on right side, then greater[i] will be -1.
  int *greater = new int[n];
```

```
  greater[n-1] = -1;  // last entry will always be -1
  for (i = n-2; i >= 0; i--)
  {
     if (arr[i] >= arr[max])
     {
        max = i;
        greater[i] = -1;
     }
     else
        greater[i] = max;
  }

  // Now find a number which has both a greater number on
  // right side and smaller number on left side
  for (i = 0; i < n; i++)
  {
     if (smaller[i] != -1 && greater[i] != -1)
     {
        printf("%d %d %d", arr[smaller[i]],
              arr[i], arr[greater[i]]);
        return;
     }
  }

  // If we reach number, then there are no such 3 numbers
  printf("No such triplet found");

  // Free the dynamically alloced memory to avoid memory leak
  delete [] smaller;
  delete [] greater;

  return;
}

// Driver program to test above function
int main()
{
   int arr[] = {12, 11, 10, 5, 6, 2, 30};
   int n = sizeof(arr)/sizeof(arr[0]);
   find3Numbers(arr, n);
   return 0;
}
```

Output:

```
5 6 30
```

Time Complexity: O(n)
Auxliary Space: O(n)

Source: How to find 3 numbers in increasing order and increasing indices in an array in linear time

**Exercise:**
**1.** Find a subsequence of size 3 such that arr[i] < arr[j] > arr[k].
**2.** Find a sorted subsequence of size 4 in linear time

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

66. Largest subarray with equal number of 0s and 1s

Given an array containing only 0s and 1s, find the largest subarray which contain equal no of 0s and 1s. Expected time complexity is O(n).

Examples:

```
Input: arr[] = {1, 0, 1, 1, 1, 0, 0}
Output: 1 to 6 (Starting and Ending indexes of output subarray)

Input: arr[] = {1, 1, 1, 1}
Output: No such subarray

Input: arr[] = {0, 0, 1, 1, 0}
Output: 0 to 3 Or 1 to 4
```

Source: Largest subarray with equal number of 0s and 1s

**Method 1 (Simple)**
A simple method is to use two nested loops. The outer loop picks a starting point i. The inner loop considers all subarrays starting from i. If size of a subarray is greater than maximum size so far, then update the maximum size.
In the below code, 0s are considered as -1 and sum of all values from i to j is calculated. If sum becomes 0, then size of this subarray is compared with largest size so far.

```
// A simple program to find the largest subarray with equal number of 0s and 1s
#include <stdio.h>
```

```
// This function Prints the starting and ending indexes of the largest subarray
// with equal number of 0s and 1s. Also returns the size of such subarray.
int findSubArray(int arr[], int n)
{
    int sum = 0;
    int maxsize = -1, startindex;

    // Pick a starting point as i
    for (int i = 0; i < n-1; i++)
    {
        sum = (arr[i] == 0)? -1 : 1;

        // Consider all subarrays starting from i
        for (int j = i+1; j < n; j++)
        {
            (arr[j] == 0)? sum += -1: sum += 1;

            // If this is a 0 sum subarray, then compare it with
            // maximum size subarray calculated so far
            if(sum == 0 && maxsize < j-i+1)
            {
                maxsize = j - i + 1;
                startindex = i;
            }
        }
    }
    if ( maxsize == -1 )
        printf("No such subarray");
    else
        printf("%d to %d", startindex, startindex+maxsize-1);

    return maxsize;
}

/* Driver program to test above functions*/
int main()
{
    int arr[] =  {1, 0, 0, 1, 0, 1, 1};
    int size = sizeof(arr)/sizeof(arr[0]);

    findSubArray(arr, size);
    return 0;
}
```

Output:

```
0 to 5
```

Time Complexity: O(n^2)
Auxiliary Space: O(1)

**Method 2 (Tricky)**
Following is a solution that uses O(n) extra space and solves the problem in O(n) time complexity.
Let input array be arr[] of size n and maxsize be the size of output subarray.
**1)** Consider all 0 values as -1. The problem now reduces to find out the maximum length subarray with sum = 0.
**2)** Create a temporary array sumleft[] of size n. Store the sum of all elements from arr[0] to arr[i] in sumleft[i]. This can be done in O(n) time.
**3)** There are two cases, the output subarray may start from 0th index or may start from some other index. We will return the max of the values obtained by two cases.
**4)** To find the maximum length subarray starting from 0th index, scan the sumleft[] and find the maximum i where sumleft[i] = 0.
**5)** Now, we need to find the subarray where subarray sum is 0 and start index is not 0. This problem is equivalent to finding two indexes i & j in sumleft[] such that sumleft[i] = sumleft[j] and j-i is maximum. To solve this, we can create a hash table with size = max-min+1 where min is the minimum value in the sumleft[] and max is the maximum value in the sumleft[]. The idea is to hash the leftmost occurrences of all different values in sumleft[]. The size of hash is chosen as max-min+1 because there can be these many different possible values in sumleft[]. Initialize all values in hash as -1
**6)** To fill and use hash[], traverse sumleft[] from 0 to n-1. If a value is not present in hash[], then store its index in hash. If the value is present, then calculate the difference of current index of sumleft[] and previously stored value in hash[]. If this difference is more than maxsize, then update the maxsize.
**7)** To handle corner cases (all 1s and all 0s), we initialize maxsize as -1. If the maxsize remains -1, then print there is no such subarray.

```
// A O(n) program to find the largest subarray with equal number of 0s and 1s
#include <stdio.h>
#include <stdlib.h>

// A utility function to get maximum of two integers
int max(int a, int b) { return a>b? a: b; }

// This function Prints the starting and ending indexes of the largest subarray
// with equal number of 0s and 1s. Also returns the size of such subarray.
int findSubArray(int arr[], int n)
{
```

```
int maxsize = -1, startindex;  // variables to store result values

// Create an auxiliary array sunmleft[]. sumleft[i] will be sum of array
// elements from arr[0] to arr[i]
int sumleft[n];
int min, max; // For min and max values in sumleft[]
int i;

// Fill sumleft array and get min and max values in it.
// Consider 0 values in arr[] as -1
sumleft[0] = ((arr[0] == 0)? -1: 1);
min = arr[0]; max = arr[0];
for (i=1; i<n; i++)
{
    sumleft[i] = sumleft[i-1] + ((arr[i] == 0)? -1: 1);
    if (sumleft[i] < min)
        min = sumleft[i];
    if (sumleft[i] > max)
        max = sumleft[i];
}

// Now calculate the max value of j - i such that sumleft[i] = sumleft[j].
// The idea is to create a hash table to store indexes of all visited values.
// If you see a value again, that it is a case of sumleft[i] = sumleft[j]. Check
// if this j-i is more than maxsize.
// The optimum size of hash will be max-min+1 as these many different values
// of sumleft[i] are possible. Since we use optimum size, we need to shift
// all values in sumleft[] by min before using them as an index in hash[].
int hash[max-min+1];

// Initialize hash table
for (i=0; i<max-min+1; i++)
    hash[i] = -1;

for (i=0; i<n; i++)
{
    // Case 1: when the subarray starts from index 0
    if (sumleft[i] == 0)
    {
       maxsize = i+1;
       startindex = 0;
    }

    // Case 2: fill hash table value. If already filled, then use it
```

```
        if (hash[sumleft[i]-min] == -1)
            hash[sumleft[i]-min] = i;
        else
        {
            if ( (i - hash[sumleft[i]-min]) > maxsize )
            {
                maxsize = i - hash[sumleft[i]-min];
                startindex = hash[sumleft[i]-min] + 1;
            }
        }
    }
    if ( maxsize == -1 )
        printf("No such subarray");
    else
        printf("%d to %d", startindex, startindex+maxsize-1);

    return maxsize;
}

/* Driver program to test above functions */
int main()
{
    int arr[] =  {1, 0, 0, 1, 0, 1, 1};
    int size = sizeof(arr)/sizeof(arr[0]);

    findSubArray(arr, size);
    return 0;
}
```

Output:

```
0 to 5
```

Time Complexity: O(n)
Auxiliary Space: O(n)

Thanks to Aashish Barnwal for suggesting this solution.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

Partition problem is to determine whether a given set can be partitioned into two subsets such that the sum of elements in both subsets is same.

Examples

```
arr[] = {1, 5, 11, 5}
Output: true
The array can be partitioned as {1, 5, 5} and {11}

arr[] = {1, 5, 3}
Output: false
The array cannot be partitioned into equal sum sets.
```

Following are the two main steps to solve this problem:
1) Calculate sum of the array. If sum is odd, there can not be two subsets with equal sum, so return false.
2) If sum of array elements is even, calculate sum/2 and find a subset of array with sum equal to sum/2.

The first step is simple. The second step is crucial, it can be solved either using recursion or Dynamic Programming.

**Recursive Solution**
Following is the recursive property of the second step mentioned above.

```
Let isSubsetSum(arr, n, sum/2) be the function that returns true if
there is a subset of arr[0..n-1] with sum equal to sum/2

The isSubsetSum problem can be divided into two subproblems
 a) isSubsetSum() without considering last element
    (reducing n to n-1)
 b) isSubsetSum considering the last element
    (reducing sum/2 by arr[n-1] and n to n-1)
If any of the above the above subproblems return true, then return true.
isSubsetSum (arr, n, sum/2) = isSubsetSum (arr, n-1, sum/2) ||
                              isSubsetSum (arr, n-1, sum/2 - arr[n-1])
```

```
// A recursive solution for partition problem
#include <stdio.h>

// A utility function that returns true if there is a subset of arr[]
// with sun equal to given sum
bool isSubsetSum (int arr[], int n, int sum)
{
   // Base Cases
   if (sum == 0)
```

```
    return true;
  if (n == 0 && sum != 0)
    return false;

  // If last element is greater than sum, then ignore it
  if (arr[n-1] > sum)
    return isSubsetSum (arr, n-1, sum);

  /* else, check if sum can be obtained by any of the following
     (a) including the last element
     (b) excluding the last element
  */
  return isSubsetSum (arr, n-1, sum) || isSubsetSum (arr, n-1, sum-arr[n-1]);
}

// Returns true if arr[] can be partitioned in two subsets of
// equal sum, otherwise false
bool findPartiion (int arr[], int n)
{
   // Calculate sum of the elements in array
   int sum = 0;
   for (int i = 0; i < n; i++)
      sum += arr[i];

   // If sum is odd, there cannot be two subsets with equal sum
   if (sum%2 != 0)
      return false;

   // Find if there is subset with sum equal to half of total sum
   return isSubsetSum (arr, n, sum/2);
}

// Driver program to test above function
int main()
{
  int arr[] = {3, 1, 5, 9, 12};
  int n = sizeof(arr)/sizeof(arr[0]);
  if (findPartiion(arr, n) == true)
     printf("Can be divided into two subsets of equal sum");
  else
     printf("Can not be divided into two subsets of equal sum");
  getchar();
  return 0;
}
```

Output:

```
Can be divided into two subsets of equal sum
```

Time Complexity: O(2^n) In worst case, this solution tries two possibilities (whether to include or exclude) for every element.

**Dynamic Programming Solution**

The problem can be solved using dynamic programming when the sum of the elements is not too big. We can create a 2D array part[][] of size (sum/2)*(n+1). And we can construct the solution in bottom up manner such that every filled entry has following property

```
part[i][j] = true if a subset of {arr[0], arr[1], ..arr[j-1]} has sum
             equal to i, otherwise false
```

```
// A Dynamic Programming solution to partition problem
#include <stdio.h>

// Returns true if arr[] can be partitioned in two subsets of
// equal sum, otherwise false
bool findPartiion (int arr[], int n)
{
   int sum = 0;
   int i, j;

   // Caculcate sun of all elements
   for (i = 0; i < n; i++)
     sum += arr[i];

   if (sum%2 != 0)
      return false;

   bool part[sum/2+1][n+1];

   // initialize top row as true
   for (i = 0; i <= n; i++)
     part[0][i] = true;

   // initialize leftmost column, except part[0][0], as 0
   for (i = 1; i <= sum/2; i++)
     part[i][0] = false;

    // Fill the partition table in botton up manner
```

```
    for (i = 1; i <= sum/2; i++)
    {
      for (j = 1; j <= n; j++)
      {
        part[i][j] = part[i][j-1];
        if (i >= arr[j-1])
          part[i][j] = part[i][j] || part[i - arr[j-1]][j-1];
      }
    }

   /* // uncomment this part to print table
    for (i = 0; i <= sum/2; i++)
    {
      for (j = 0; j <= n; j++)
         printf ("%4d", part[i][j]);
      printf("\n");
    } */

    return part[sum/2][n];
}

// Driver program to test above funtion
int main()
{
  int arr[] = {3, 1, 1, 2, 2, 1};
  int n = sizeof(arr)/sizeof(arr[0]);
  if (findPartiion(arr, n) == true)
     printf("Can be divided into two subsets of equal sum");
  else
     printf("Can not be divided into two subsets of equal sum");
  getchar();
  return 0;
}
```

Output:

```
Can be divided into two subsets of equal sum
```

Following diagram shows the values in partition table. The diagram is taken form the wiki page of partition problem.

The entry part[i][j] indicates whether there is a subset of {arr[0], arr[1], .. arr[j-1]} that sums to i

| | {} | {3} | {3,1} | {3,1,1} | {3,1,1,2} | {3,1,1,2,2} | {3,1,1,2,2,1} |
|---|---|---|---|---|---|---|---|
| 0 | True | True | True | True | True | True | True |
| 1 | False | False | True | True | True | True | True |
| 2 | False | False | False | True | True | True | True |
| 3 | False | True | True | True | True | True | True |
| 4 | False | False | True | True | True | True | True |
| 5 | False | False | False | True | True | True | True |

Dynamic Programming table for
arr[] = {3,1,1,2,2,1}

Time Complexity: O(sum*n)
Auxiliary Space: O(sum*n)
Please note that this solution will not be feasible for arrays with big sum.

**References:**
http://en.wikipedia.org/wiki/Partition_problem

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

68. Maximum Product Subarray

Given an array that contains both positive and negative integers, find the product of the maximum product subarray. Expected Time complexity is O(n) and only O(1) extra space can be used.

**Examples:**

```
Input: arr[] = {6, -3, -10, 0, 2}
Output:   180  // The subarray is {6, -3, -10}

Input: arr[] = {-1, -3, -10, 0, 60}
Output:   60  // The subarray is {60}

Input: arr[] = {-2, -3, 0, -2, -40}
Output:   80  // The subarray is {-2, -40}
```

The following solution assumes that the given input array always has a positive ouput. The solution works for all cases mentioned above. It doesnâ€™t work for arrays like {0, 0, -20, 0}, {0, 0, 0}.. etc. The solution can be easily modified to handle this case.

It is similar to Largest Sum Contiguous Subarray problem. The only thing to note here is, maximum product can also be obtained by minimum (negative) product ending with the previous element multiplied by this element. For example, in array {12, 2, -3, -5, -6, -2}, when we are at element -2, the maximum product is multiplication of, minimum product ending with -6 and -2.

```
#include <stdio.h>

// Utility functions to get minimum of two integers
int min (int x, int y) {return x < y? x : y; }

// Utility functions to get maximum of two integers
int max (int x, int y) {return x > y? x : y; }

/* Returns the product of max product subarray.  Assumes that the
   given array always has a subarray with product more than 1 */
int maxSubarrayProduct(int arr[], int n)
{
    // max positive product ending at the current position
    int max_ending_here = 1;

    // min negative product ending at the current position
    int min_ending_here = 1;

    // Initialize overall max product
    int max_so_far = 1;

    /* Traverse throught the array. Following values are maintained after the ith iteration:
       max_ending_here is always 1 or some positive product ending with arr[i]
       min_ending_here is always 1 or some negative product ending with arr[i] */
    for (int i = 0; i < n; i++)
    {
        /* If this element is positive, update max_ending_here. Update
           min_ending_here only if min_ending_here is negative */
        if (arr[i] > 0)
        {
            max_ending_here = max_ending_here*arr[i];
            min_ending_here = min (min_ending_here * arr[i], 1);
        }

        /* If this element is 0, then the maximum product cannot
           end here, make both max_ending_here and min_ending_here 0
           Assumption: Output is alway greater than or equal to 1. */
        else if (arr[i] == 0)
```

```
        {
            max_ending_here = 1;
            min_ending_here = 1;
        }

        /* If element is negative. This is tricky
           max_ending_here can either be 1 or positive. min_ending_here can either be 1
           or negative.
           next min_ending_here will always be prev. max_ending_here * arr[i]
           next max_ending_here will be 1 if prev min_ending_here is 1, otherwise
           next max_ending_here will be prev min_ending_here * arr[i] */
        else
        {
            int temp = max_ending_here;
            max_ending_here = max (min_ending_here * arr[i], 1);
            min_ending_here = temp * arr[i];
        }

        // update max_so_far, if needed
        if (max_so_far <  max_ending_here)
          max_so_far  =  max_ending_here;
    }

    return max_so_far;
}

// Driver Program to test above function
int main()
{
    int arr[] = {1, -2, -3, 0, 7, -8, -2};
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Maximum Sub array product is %d", maxSubarrayProduct(arr, n));
    return 0;
}
```

Output:

```
Maximum Sub array product is 112
```

Time Complexity: O(n)
Auxiliary Space: O(1)

This article is compiled by **Dheeraj Jain** and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

69. Find a pair with the given difference

Given an unsorted array and a number n, find if there exists a pair of elements in the array whose difference is n.

Examples:
Input: arr[] = {5, 20, 3, 2, 50, 80}, n = 78
Output: Pair Found: (2, 80)

Input: arr[] = {90, 70, 20, 80, 50}, n = 45
Output: No Such Pair

Source: find pair

The simplest method is to run two loops, the outer loop picks the first element (smaller element) and the inner loop looks for the element picked by outer loop plus n. Time complexity of this method is O(n^2).

We can use sorting and Binary Search to improve time complexity to O(nLogn). The first step is to sort the array in ascending order. Once the array is sorted, traverse the array from left to right, and for each element arr[i], binary search for arr[i] + n in arr[i+1..n-1]. If the element is found, return the pair.
Both first and second steps take O(nLogn). So overall complexity is O(nLogn).

The second step of the above algorithm can be improved to O(n). The first step remain same. The idea for second step is take two index variables i and j, initialize them as 0 and 1 respectively. Now run a linear loop. If arr[j] â€“ arr[i] is smaller than n, we need to look for greater arr[j], so increment j. If arr[j] â€“ arr[i] is greater than n, we need to look for greater arr[i], so increment i. Thanks to Aashish Barnwal for suggesting this approach.
The following code is only for the second step of the algorithm, it assumes that the array is already sorted.

```
#include <stdio.h>

// The function assumes that the array is sorted
bool findPair(int arr[], int size, int n)
{
   // Initialize positions of two elements
   int i = 0;
   int j = 1;
```

```
    // Search for a pair
    while (i<size && j<size)
    {
        if (i != j && arr[j]-arr[i] == n)
        {
            printf("Pair Found: (%d, %d)", arr[i], arr[j]);
            return true;
        }
        else if (arr[j]-arr[i] < n)
            j++;
        else
            i++;
    }

    printf("No such pair");
    return false;
}

// Driver program to test above function
int main()
{
    int arr[] = {1, 8, 30, 40, 100};
    int size = sizeof(arr)/sizeof(arr[0]);
    int n = 60;
    findPair(arr, size, n);
    return 0;
}
```

Output:

```
Pair Found: (40, 100)
```

Hashing can also be used to solve this problem. Create an empty hash table HT. Traverse the array, use array elements as hash keys and enter them in HT. Traverse the array again look for value n + arr[i] in HT.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

Given an unsorted array and a number n, find if there exists a pair of elements in the array whose difference is n.

Examples:
Input: arr[] = {5, 20, 3, 2, 50, 80}, n = 78
Output: Pair Found: (2, 80)

Input: arr[] = {90, 70, 20, 80, 50}, n = 45
Output: No Such Pair

Source: find pair

The simplest method is to run two loops, the outer loop picks the first element (smaller element) and the inner loop looks for the element picked by outer loop plus n. Time complexity of this method is O(n^2).

We can use sorting and Binary Search to improve time complexity to O(nLogn). The first step is to sort the array in ascending order. Once the array is sorted, traverse the array from left to right, and for each element arr[i], binary search for arr[i] + n in arr[i+1..n-1]. If the element is found, return the pair.
Both first and second steps take O(nLogn). So overall complexity is O(nLogn).

The second step of the above algorithm can be improved to O(n). The first step remain same. The idea for second step is take two index variables i and j, initialize them as 0 and 1 respectively. Now run a linear loop. If arr[j] â€“ arr[i] is smaller than n, we need to look for greater arr[j], so increment j. If arr[j] â€“ arr[i] is greater than n, we need to look for greater arr[i], so increment i. Thanks to Aashish Barnwal for suggesting this approach.
The following code is only for the second step of the algorithm, it assumes that the array is already sorted.

```
#include <stdio.h>

// The function assumes that the array is sorted
bool findPair(int arr[], int size, int n)
{
    // Initialize positions of two elements
    int i = 0;
    int j = 1;

    // Search for a pair
    while (i<size && j<size)
    {
        if (i != j && arr[j]-arr[i] == n)
        {
            printf("Pair Found: (%d, %d)", arr[i], arr[j]);
            return true;
        }
```

```
        else if (arr[j]-arr[i] < n)
            j++;
        else
            i++;
    }

    printf("No such pair");
    return false;
}

// Driver program to test above function
int main()
{
    int arr[] = {1, 8, 30, 40, 100};
    int size = sizeof(arr)/sizeof(arr[0]);
    int n = 60;
    findPair(arr, size, n);
    return 0;
}
```

Output:

```
Pair Found: (40, 100)
```

Hashing can also be used to solve this problem. Create an empty hash table HT. Traverse the array, use array elements as hash keys and enter them in HT. Traverse the array again look for value n + arr[i] in HT.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

71. Dynamic Programming | Set 20 (Maximum Length Chain of Pairs)

You are given n pairs of numbers. In every pair, the first number is always smaller than the second number. A pair (c, d) can follow another pair (a, b) if b < c. Chain of pairs can be formed in this fashion. Find the longest chain which can be formed from a given set of pairs.
Source: Amazon Interview | Set 2

For example, if the given pairs are {{5, 24}, {39, 60}, {15, 28}, {27, 40}, {50, 90} }, then the longest chain that can be formed is of length 3, and the chain is {{5, 24}, {27, 40}, {50, 90}}

This problem is a variation of standard Longest Increasing Subsequence problem. Following is a simple two step process.
1) Sort given pairs in increasing order of first (or smaller) element.
2) Now run a modified LIS process where we compare the second element of already finalized LIS with the first element of new LIS being constructed.

The following code is a slight modification of method 2 of this post.

```
#include<stdio.h>
#include<stdlib.h>

// Structure for a pair
struct pair
{
  int a;
  int b;
};

// This function assumes that arr[] is sorted in increasing order
// according the first (or smaller) values in pairs.
int maxChainLength( struct pair arr[], int n)
{
   int i, j, max = 0;
   int *mcl = (int*) malloc ( sizeof( int ) * n );

   /* Initialize MCL (max chain length) values for all indexes */
   for ( i = 0; i < n; i++ )
      mcl[i] = 1;

   /* Compute optimized chain length values in bottom up manner */
   for ( i = 1; i < n; i++ )
      for ( j = 0; j < i; j++ )
         if ( arr[i].a > arr[j].b && mcl[i] < mcl[j] + 1)
            mcl[i] = mcl[j] + 1;

   // mcl[i] now stores the maximum chain length ending with pair i

   /* Pick maximum of all MCL values */
   for ( i = 0; i < n; i++ )
      if ( max < mcl[i] )
         max = mcl[i];

   /* Free memory to avoid memory leak */
   free( mcl );
```

```
  return max;
}



/* Driver program to test above function */
int main()
{
    struct pair arr[] = { {5, 24}, {15, 25},
                          {27, 40}, {50, 60} };
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Length of maximum size chain is %d\n",
           maxChainLength( arr, n ));
    return 0;
}
```

Output:

```
Length of maximum size chain is 3
```

Time Complexity: O(n^2) where n is the number of pairs.

The given problem is also a variation of Activity Selection problem and can be solved in (nLogn) time. To solve it as a activity selection problem, consider the first element of a pair as start time in activity selection problem, and the second element of pair as end time. Thanks to Palash for suggesting this approach.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

72.  Find four elements that sum to a given value | Set 1 (n^3 solution)

Given an array of integers, find all combination of four elements in the array whose sum is equal to a given value X.
For example, if the given array is {10, 2, 3, 4, 5, 9, 7, 8} and X = 23, then your function should print â€œ3 5 7 8â€³ (3 + 5 + 7 + 8 = 23).

**Sources:** Find Specific Sum and Amazon Interview Question

A **Naive Solution** is to generate all possible quadruples and compare the sum of every quadruple with X. The following code implements this simple method using four nested loops

```
#include <stdio.h>
```

```
/* A naive solution to print all combination of 4 elements in A[]
  with sum equal to X */
void findFourElements(int A[], int n, int X)
{
  // Fix the first element and find other three
  for (int i = 0; i < n-3; i++)
  {
    // Fix the second element and find other two
    for (int j = i+1; j < n-2; j++)
    {
      // Fix the third element and find the fourth
      for (int k = j+1; k < n-1; k++)
      {
        // find the fourth
        for (int l = k+1; l < n; l++)
          if (A[i] + A[j] + A[k] + A[l] == X)
            printf("%d, %d, %d, %d", A[i], A[j], A[k], A[l]);
      }
    }
  }
}

// Driver program to test above funtion
int main()
{
    int A[] = {10, 20, 30, 40, 1, 2};
    int n = sizeof(A) / sizeof(A[0]);
    int X = 91;
    findFourElements (A, n, X);
    return 0;
}
```

Output:

```
20, 30, 40, 1
```

Time Complexity: O(n^4)

*The time complexity can be improved to O(n^3) with the* ***use of sorting*** *as a preprocessing step, and then using method 1 of this post to reduce a loop.*

Following are the detailed steps.
1) Sort the input array.
2) Fix the first element as A[i] where i is from 0 to nâ€“3. After fixing the first element of

quadruple, fix the second element as A[j] where j varies from i+1 to n-2. Find remaining two elements in O(n) time, using the method 1 of this post

Following is C implementation of O(n^3) solution.

```
# include <stdio.h>
# include <stdlib.h>

/* Following function is needed for library function qsort(). Refer
   http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */
int compare (const void *a, const void * b)
{  return ( *(int *)a - *(int *)b ); }

/* A sorting based solution to print all combination of 4 elements in A[]
   with sum equal to X */
void find4Numbers(int A[], int n, int X)
{
    int l, r;

    // Sort the array in increasing order, using library
    // function for quick sort
    qsort (A, n, sizeof(A[0]), compare);

    /* Now fix the first 2 elements one by one and find
       the other two elements */
    for (int i = 0; i < n - 3; i++)
    {
        for (int j = i+1; j < n - 2; j++)
        {
            // Initialize two variables as indexes of the first and last
            // elements in the remaining elements
            l = j + 1;
            r = n-1;

            // To find the remaining two elements, move the index
            // variables (l & r) toward each other.
            while (l < r)
            {
                if( A[i] + A[j] + A[l] + A[r] == X)
                {
                   printf("%d, %d, %d, %d", A[i], A[j],
                                        A[l], A[r]);
                   l++; r--;
                }
```

```
            else if (A[i] + A[j] + A[l] + A[r] < X)
                l++;
            else // A[i] + A[j] + A[l] + A[r] > X
                r--;
        } // end of while
    } // end of inner for loop
  } // end of outer for loop
}

/* Driver program to test above function */
int main()
{
  int A[] = {1, 4, 45, 6, 10, 12};
  int X = 21;
  int n = sizeof(A)/sizeof(A[0]);
  find4Numbers(A, n, X);
  return 0;
}
```

Output:

```
1, 4, 6, 10
```

Time Complexity: O(n^3)

This problem can also be solved in O(n^2Logn) complexity. We will soon be publishing the O(n^2Logn) solution as a separate post.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

73.  Find four elements that sum to a given value | Set 2 ( O(n^2Logn) Solution)

Given an array of integers, find all combination of four elements in the array whose sum is equal to a given value X.
For example, if the given array is {10, 2, 3, 4, 5, 9, 7, 8} and X = 23, then your function should print â€œ3 5 7 8â€³ (3 + 5 + 7 + 8 = 23).

**Sources:** Find Specific Sum and Amazon Interview Question

We have discussed a O(n^3) algorithm in the previous post on this topic. The problem can be solved in O(n^2Logn) time with the help of auxiliary space.

Thanks to itsnimish for suggesting this method. Following is the detailed process.

Let the input array be A[].

**1)** Create an auxiliary array aux[] and store sum of all possible pairs in aux[]. The size of aux[] will be n*(n-1)/2 where n is the size of A[].

**2)** Sort the auxiliary array aux[].

**3)** Now the problem reduces to find two elements in aux[] with sum equal to X. We can use method 1 of this post to find the two elements efficiently. There is following important point to note though. An element of aux[] represents a pair from A[]. While picking two elements from aux[], we must check whether the two elements have an element of A[] in common. For example, if first element sum of A[1] and A[2], and second element is sum of A[2] and A[4], then these two elements of aux[] donâ€™t represent four distinct elements of input array A[].

Following is C implementation of this method.

```
#include <stdio.h>
#include <stdlib.h>

// The following structure is needed to store pair sums in aux[]
struct pairSum
{
    int first; // index (int A[]) of first element in pair
    int sec; // index of second element in pair
    int sum;  // sum of the pair
};

// Following function is needed for library function qsort()
int compare (const void *a, const void * b)
{
    return ( (*(pairSum *)a).sum - (*(pairSum*)b).sum );
}

// Function to check if two given pairs have any common element or not
bool noCommon(struct pairSum a, struct pairSum b)
{
    if (a.first == b.first || a.first == b.sec ||
            a.sec == b.first || a.sec == b.sec)
        return false;
    return true;
}
```

```
// The function finds four elements with given sum X
void findFourElements (int arr[], int n, int X)
{
    int i, j;

    // Create an auxiliary array to store all pair sums
    int size = (n*(n-1))/2;
    struct pairSum aux[size];

    /* Generate all possible pairs from A[] and store sums
       of all possible pairs in aux[] */
    int k = 0;
    for (i = 0; i < n-1; i++)
    {
        for (j = i+1; j < n; j++)
        {
            aux[k].sum = arr[i] + arr[j];
            aux[k].first = i;
            aux[k].sec = j;
            k++;
        }
    }

    // Sort the aux[] array using library function for sorting
    qsort (aux, size, sizeof(aux[0]), compare);

    // Now start two index variables from two corners of array
    // and move them toward each other.
    i = 0;
    j = size-1;
    while (i < size && j >=0 )
    {
        if ((aux[i].sum + aux[j].sum == X) && noCommon(aux[i], aux[j]))
        {
            printf ("%d, %d, %d, %d\n", arr[aux[i].first], arr[aux[i].sec],
                                    arr[aux[j].first], arr[aux[j].sec]);
            return;
        }
        else if (aux[i].sum + aux[j].sum < X)
            i++;
        else
            j--;
    }
}
```

```
// Driver program to test above function
int main()
{
    int arr[] = {10, 20, 30, 40, 1, 2};
    int n = sizeof(arr) / sizeof(arr[0]);
    int X = 91;
    findFourElements (arr, n, X);
    return 0;
}
```

Output:

```
20, 1, 30, 40
```

Please note that the above code prints only one quadruple. If we remove the return statement and add statements â€œi++; jâ€“;â€, then it prints same quadruple five times. The code can modified to print all quadruples only once. It has been kept this way to keep it simple.

*Time complexity:* The step 1 takes O(n^2) time. The second step is sorting an array of size O(n^2). Sorting can be done in O(n^2Logn) time using merge sort or heap sort or any other O(nLogn) algorithm. The third step takes O(n^2) time. So overall complexity is O(n^2Logn).

*Auxiliary Space:* O(n^2). The big size of auxiliary array can be a concern in this method.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

74. Sort a nearly sorted (or K sorted) array

Given an array of n elements, where each element is at most k away from its target position, devise an algorithm that sorts in O(n log k) time.
For example, let us consider k is 2, an element at index 7 in the sorted array, can be at indexes 5, 6, 7, 8, 9 in the given array.

Source: Nearly sorted algorithm

We can **use Insertion Sort** to sort the elements efficiently. Following is the C code for standard Insertion Sort.

```
/* Function to sort an array using insertion sort*/
```

```
void insertionSort(int A[], int size)
{
  int i, key, j;
  for (i = 1; i < size; i++)
  {
     key = A[i];
     j = i-1;

     /* Move elements of A[0..i-1], that are greater than key, to one
        position ahead of their current position.
        This loop will run at most k times */
     while (j >= 0 && A[j] > key)
     {
         A[j+1] = A[j];
         j = j-1;
     }
     A[j+1] = key;
  }
}
```

The inner loop will run at most k times. To move every element to its correct place, at most k elements need to be moved. So overall *complexity will be O(nk)*

We can sort such arrays **more efficiently with the help of Heap data structure**. Following is the detailed process that uses Heap.
1) Create a Min Heap of size k+1 with first k+1 elements. This will take O(k) time (See this GFact)
2) One by one remove min element from heap, put it in result array, and add a new element to heap from remaining elements.

Removing an element and adding a new element to min heap will take Logk time. So overall complexity will be O(k) + O((n-k)*logK)

```
#include<iostream>
using namespace std;

// Prototype of a utility function to swap two integers
void swap(int *x, int *y);

// A class for Min Heap
class MinHeap
{
   int *harr; // pointer to array of elements in heap
   int heap_size; // size of min heap
public:
```

```
    // Constructor
    MinHeap(int a[], int size);

    // to heapify a subtree with root at given index
    void MinHeapify(int );

    // to get index of left child of node at index i
    int left(int i) { return (2*i + 1); }

    // to get index of right child of node at index i
    int right(int i) { return (2*i + 2); }

    // to remove min (or root), add a new value x, and return old root
    int replaceMin(int x);

    // to extract the root which is the minimum element
    int extractMin();
};

// Given an array of size n, where every element is k away from its target
// position, sorts the array in O(nLogk) time.
int sortK(int arr[], int n, int k)
{
    // Create a Min Heap of first (k+1) elements from
    // input array
    int *harr = new int[k+1];
    for (int i = 0; i<=k && i<n; i++) // i < n condition is needed when k > n
        harr[i] = arr[i];
    MinHeap hp(harr, k+1);

    // i is index for remaining elements in arr[] and ti
    // is target index of for cuurent minimum element in
    // Min Heapm 'hp'.
    for(int i = k+1, ti = 0; ti < n; i++, ti++)
    {
        // If there are remaining elements, then place
        // root of heap at target index and add arr[i]
        // to Min Heap
        if (i < n)
            arr[ti] = hp.replaceMin(arr[i]);

        // Otherwise place root at its target index and
        // reduce heap size
        else
```

```
            arr[ti] = hp.extractMin();
    }
}

// FOLLOWING ARE IMPLEMENTATIONS OF STANDARD MIN HEAP METHODS FROM CORMEN BOOK
// Constructor: Builds a heap from a given array a[] of given size
MinHeap::MinHeap(int a[], int size)
{
    heap_size = size;
    harr = a;  // store address of array
    int i = (heap_size - 1)/2;
    while (i >= 0)
    {
        MinHeapify(i);
        i--;
    }
}

// Method to remove minimum element (or root) from min heap
int MinHeap::extractMin()
{
    int root = harr[0];
    if (heap_size > 1)
    {
        harr[0] = harr[heap_size-1];
        heap_size--;
        MinHeapify(0);
    }
    return root;
}

// Method to change root with given value x, and return the old root
int MinHeap::replaceMin(int x)
{
    int root = harr[0];
    harr[0] = x;
    if (root < x)
        MinHeapify(0);
    return root;
}

// A recursive method to heapify a subtree with root at given index
// This method assumes that the subtrees are already heapified
void MinHeap::MinHeapify(int i)
```

```
{
    int l = left(i);
    int r = right(i);
    int smallest = i;
    if (l < heap_size && harr[l] < harr[i])
        smallest = l;
    if (r < heap_size && harr[r] < harr[smallest])
        smallest = r;
    if (smallest != i)
    {
        swap(&harr[i], &harr[smallest]);
        MinHeapify(smallest);
    }
}

// A utility function to swap two elements
void swap(int *x, int *y)
{
    int temp = *x;
    *x = *y;
    *y = temp;
}

// A utility function to print array elements
void printArray(int arr[], int size)
{
   for (int i=0; i < size; i++)
       cout << arr[i] << " ";
   cout << endl;
}

// Driver program to test above functions
int main()
{
    int k = 3;
    int arr[] = {2, 6, 3, 12, 56, 8};
    int n = sizeof(arr)/sizeof(arr[0]);
    sortK(arr, n, k);

    cout << "Following is sorted array\n";
    printArray (arr, n);

    return 0;
}
```

Output:

```
Following is sorted array
2 3 6 8 12 56
```

The Min Heap based method takes O(nLogk) time and uses O(k) auxiliary space.

We can also **use a Balanced Binary Search Tree** instead of Heap to store K+1 elements. The insert and delete operations on Balanced BST also take O(Logk) time. So Balanced BST based method will also take O(nLogk) time, but the Heap bassed method seems to be more efficient as the minimum element will always be at root. Also, Heap doesnâ€™t need extra space for left and right pointers.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â Â

75. Maximum circular subarray sum

Given n numbers (both +ve and -ve), arranged in a circle, fnd the maximum sum of consecutive number.

Examples:

```
Input: a[] = {8, -8, 9, -9, 10, -11, 12}
Output: 22 (12 + 8 - 8 + 9 - 9 + 10)

Input: a[] = {10, -3, -4, 7, 6, 5, -4, -1}
Output:  23 (7 + 6 + 5 - 4 -1 + 10)

Input: a[] = {-1, 40, -14, 7, 6, 5, -4, -1}
Output: 52 (7 + 6 + 5 - 4 - 1 - 1 + 40)
```

There can be two cases for the maximum sum:

**Case 1:** The elements that contribute to the maximum sum are arranged such that no wrapping is there. Examples: {-10, 2, -1, 5}, {-2, 4, -1, 4, -1}. In this case, Kadaneâ€™s algorithm will produce the result.

**Case 2:** The elements which contribute to the maximum sum are arranged such that wrapping is there. Examples: {10, -12, 11}, {12, -5, 4, -8, 11}. In this case, we change wrapping to non-wrapping. Let us see how. Wrapping of contributing elements implies non wrapping of non contributing elements, so find out the sum of non contributing elements and subtract this sum from the total sum. To find out the sum of non contributing, invert sign of each element and then run Kadaneâ€™s algorithm.

Our array is like a ring and we have to eliminate the maximum continuous negative that implies maximum continuous positive in the inverted arrays.

Finally we compare the sum obtained by both cases, and return the maximum of the two sums.

Thanks to ashishdey0 for suggesting this solution. Following is C implementation of the above method.

```
// Program for maximum contiguous circular sum problem
#include<stdio.h>

// Standard Kadane's algorithm to find maximum subarray sum
int kadane (int a[], int n);

// The function returns maximum circular contiguous sum in a[]
int maxCircularSum (int a[], int n)
{
  // Case 1: get the maximum sum using standard kadane's algorithm
  int max_kadane = kadane(a, n);

  // Case 2: Now find the maximum sum that includes corner elements.
  int max_wrap  =  0, i;
  for(i=0; i<n; i++)
  {
      max_wrap += a[i]; // Calculate array-sum
      a[i] = -a[i];  // invert the array (change sign)
  }

  // max sum with corner elements will be:
  // array-sum - (-max subarray sum of inverted array)
  max_wrap = max_wrap + kadane(a, n);

  // The maximum circular sum will be maximum of two sums
  return (max_wrap > max_kadane)? max_wrap: max_kadane;
}

// Standard Kadane's algorithm to find maximum subarray sum
// See http://www.geeksforgeeks.org/archives/576 for details
int kadane (int a[], int n)
{
   int max_so_far = 0, max_ending_here = 0;
   int i;
   for(i = 0; i < n; i++)
   {
```

```
        max_ending_here = max_ending_here + a[i];
        if(max_ending_here < 0)
            max_ending_here = 0;
        if(max_so_far < max_ending_here)
            max_so_far = max_ending_here;
    }
    return max_so_far;
}

/* Driver program to test maxCircularSum() */
int main()
{
    int a[] =  {11, 10, -20, 5, -3, -5, 8, -13, 10};
    int n = sizeof(a)/sizeof(a[0]);
    printf("Maximum circular sum is %d\n", maxCircularSum(a, n));
    return 0;
}
```

Output:

```
Maximum circular sum is 31
```

Time Complexity: O(n) where n is the number of elements in input array.

Note that the above algorithm doesnâ€™t work if all numbers are negative e.g., {-1, -2, -3}. It returns 0 in this case. This case can be handled by adding a pre-check to see if all the numbers are negative before running the above algorithm.

Please write comments if you find any of the above codes/algorithms incorrect, or find other ways to solve the same problem.

Â  Â

76.  Find the row with maximum number of 1s

Given a boolean 2D array, where each row is sorted. Find the row with the maximum number of 1s.

```
Example
Input matrix
0 1 1 1
0 0 1 1
1 1 1 1  // this row has maximum 1s
0 0 0 0
```

Output: 2

**A simple method** is to do a row wise traversal of the matrix, count the number of 1s in each row and compare the count with max. Finally, return the index of row with maximum 1s. The time complexity of this method is O(m*n) where m is number of rows and n is number of columns in matrix.

We can do better. Since each row is sorted, we can **use Binary Search** to count of 1s in each row. We find the index of first instance of 1 in each row. The count of 1s will be equal to total number of columns minus the index of first 1.

See the following code for implementation of the above approach.

```
#include <stdio.h>
#define R 4
#define C 4

/* A function to find the index of first index of 1 in a boolean array arr[] */
int first(bool arr[], int low, int high)
{
  if(high >= low)
  {
    // get the middle index
    int mid = low + (high - low)/2;

    // check if the element at middle index is first 1
    if ( ( mid == 0 || arr[mid-1] == 0) && arr[mid] == 1)
      return mid;

    // if the element is 0, recur for right side
    else if (arr[mid] == 0)
      return first(arr, (mid + 1), high);

    else // If element is not first 1, recur for left side
      return first(arr, low, (mid -1));
  }
  return -1;
}

// The main function that returns index of row with maximum number of 1s.
int rowWithMax1s(bool mat[R][C])
{
    int max_row_index = 0, max = -1; // Initialize max values

    // Traverse for each row and count number of 1s by finding the index
```

```
    // of first 1
    int i, index;
    for (i = 0; i < R; i++)
    {
        index = first (mat[i], 0, C-1);
        if (index != -1 && C-index > max)
        {
            max = C - index;
            max_row_index = i;
        }
    }

    return max_row_index;
}

/* Driver program to test above functions */
int main()
{
    bool mat[R][C] = { {0, 0, 0, 1},
        {0, 1, 1, 1},
        {1, 1, 1, 1},
        {0, 0, 0, 0}
    };

    printf("Index of row with maximum 1s is %d \n", rowWithMax1s(mat));

    return 0;
}
```

Output:

```
Index of row with maximum 1s is 2
```

Time Complexity: O(mLogn) where m is number of rows and n is number of columns in matrix.

The above solution **can be optimized further**. Instead of doing binary search in every row, we first check whether the row has more 1s than max so far. If the row has more 1s, then only count 1s in the row. Also, to count 1s in a row, we donâ€™t do binary search in complete row, we do search in before the index of last max.

Following is an optimized version of the above solution.

```
// The main function that returns index of row with maximum number of 1s.
int rowWithMax1s(bool mat[R][C])
{
```

```
    int i, index;

    // Initialize max using values from first row.
    int max_row_index = 0;
    int max = C - first(mat[0], 0, C-1);

    // Traverse for each row and count number of 1s by finding the index
    // of first 1
    for (i = 1; i < R; i++)
    {
        // Count 1s in this row only if this row has more 1s than
        // max so far
        if (mat[i][C-max-1] == 1)
        {
            // Note the optimization here also
            index = first (mat[i], 0, C-max);

            if (index != -1 && C-index > max)
            {
                max = C - index;
                max_row_index = i;
            }
        }
    }
    return max_row_index;
}
```

The worst case time complexity of the above optimized version is also O(mLogn), the will solution work better on average. Thanks to Naveen Kumar Singh for suggesting the above solution.

Sources: this and this

The worst case of the above solution occurs for a matrix like following.
0 0 0 â€¦ 0 1
0 0 0 ..0 1 1
0 â€¦ 0 1 1 1
â€¦.0 1 1 1 1

**Following method works in O(m+n) time complexity in worst case**.

Step1: Get the index of first (or leftmost) 1 in the first row.

Step2: Do following for every row after the first row
â€¦IF the element on left of previous leftmost 1 is 0, ignore this row.
â€¦ELSE Move left until a 0 is found. Update the leftmost index to this index and

max_row_index to be the current row.

The time complexity is O(m+n) because we can possibly go as far left as we came ahead in the first step.

Following is C++ implementation of this method.

```
// The main function that returns index of row with maximum number of 1s.
int rowWithMax1s(bool mat[R][C])
{
   // Initialize first row as row with max 1s
   int max_row_index = 0;

   // The function first() returns index of first 1 in row 0.
   // Use this index to initialize the index of leftmost 1 seen so far
   int j = first(mat[0], 0, C-1) - 1;
   if (j == -1) // if 1 is not present in first row
     j = C - 1;

   for (int i = 1; i < R; i++)
   {
      // Move left until a 0 is found
      while (j >= 0 && mat[i][j] == 1)
      {
         j = j-1;  // Update the index of leftmost 1 seen so far
         max_row_index = i;  // Update max_row_index
      }
   }
   return max_row_index;
}
```

Thanks to Tylor, Ankan and Palash for their inputs.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

77.  Median of two sorted arrays of different sizes

This is an extension of median of two sorted arrays of equal size problem. Here we handle arrays of unequal size also.

The approach discussed in this post is similar to method 2 of equal size post. The basic idea is same, we find the median of two arrays and compare the medians to

discard almost half of the elements in both arrays. Since the number of elements may differ here, there are many base cases that need to be handled separately. Before we proceed to complete solution, let us first talk about all base cases.

Let the two arrays be A[N] and B[M]. In the following explanation, it is assumed that N is smaller than or equal to M.

**Base cases:**
The smaller array has only one element
Case 1: N = 1, M = 1.
Case 2: N = 1, M is odd
Case 3: N = 1, M is even
The smaller array has only two elements
Case 4: N = 2, M = 2
Case 5: N = 2, M is odd
Case 6: N = 2, M is even

**Case 1:** There is only one element in both arrays, so output the average of A[0] and B[0].

**Case 2:** N = 1, M is odd
Let B[5] = {5, 10, 12, 15, 20}
First find the middle element of B[], which is 12 for above array. There are following 4 sub-cases.
â€¦**2.1** If A[0] is smaller than 10, the median is average of 10 and 12.
â€¦**2.2** If A[0] lies between 10 and 12, the median is average of A[0] and 12.
â€¦**2.3** If A[0] lies between 12 and 15, the median is average of 12 and A[0].
â€¦**2.4** If A[0] is greater than 15, the median is average of 12 and 15.
In all the sub-cases, we find that 12 is fixed. So, we need to find the median of B[ M / 2 â€“ 1 ], B[ M / 2 + 1], A[ 0 ] and take its average with B[ M / 2 ].

**Case 3:** N = 1, M is even
Let B[4] = {5, 10, 12, 15}
First find the middle items in B[], which are 10 and 12 in above example. There are following 3 sub-cases.
â€¦**3.1** If A[0] is smaller than 10, the median is 10.
â€¦**3.2** If A[0] lies between 10 and 12, the median is A[0].
â€¦**3.3** If A[0] is greater than 10, the median is 12.
So, in this case, find the median of three elements B[ M / 2 â€“ 1 ], B[ M / 2] and A[ 0 ].

**Case 4:** N = 2, M = 2
There are four elements in total. So we find the median of 4 elements.

**Case 5:** N = 2, M is odd
Let B[5] = {5, 10, 12, 15, 20}
The median is given by median of following three elements: B[M/2], max(A[0], B[M/2

â€“ 1]), min(A[1], B[M/2 + 1]).

**Case 6:** N = 2, M is even
Let B[4] = {5, 10, 12, 15}
The median is given by median of following four elements: B[M/2], B[M/2 â€“ 1], max(A[0], B[M/2 â€“ 2]), min(A[1], B[M/2 + 1])

**Remaining Cases:**
Once we have handled the above base cases, following is the remaining process.
**1)** Find the middle item of A[] and middle item of B[].
â€¦..**1.1)** If the middle item of A[] is greater than middle item of B[], ignore the last half of A[], let length of ignored part is idx. Also, cut down B[] by idx from the start.
â€¦..**1.2)** else, ignore the first half of A[], let length of ignored part is idx. Also, cut down B[] by idx from the last.

Following is C implementation of the above approach.

```
// A C program to find median of two sorted arrays of unequal size
#include <stdio.h>
#include <stdlib.h>

// A utility function to find maximum of two integers
int max( int a, int b )
{ return a > b ? a : b; }

// A utility function to find minimum of two integers
int min( int a, int b )
{ return a < b ? a : b; }

// A utility function to find median of two integers
float MO2( int a, int b )
{ return ( a + b ) / 2.0; }

// A utility function to find median of three integers
float MO3( int a, int b, int c )
{
   return a + b + c - max( a, max( b, c ) )
                    - min( a, min( b, c ) );
}

// A utility function to find median of four integers
float MO4( int a, int b, int c, int d )
{
   int Max = max( a, max( b, max( c, d ) ) );
   int Min = min( a, min( b, min( c, d ) ) );
```

```
    return ( a + b + c + d - Max - Min ) / 2.0;
}

// This function assumes that N is smaller than or equal to M
float findMedianUtil( int A[], int N, int B[], int M )
{
    // If the smaller array has only one element
    if( N == 1 )
    {
        // Case 1: If the larger array also has one element, simply call MO2()
        if( M == 1 )
            return MO2( A[0], B[0] );

        // Case 2: If the larger array has odd number of elements, then consider
        // the middle 3 elements of larger array and the only element of
        // smaller array. Take few examples like following
        // A = {9}, B[] = {5, 8, 10, 20, 30} and
        // A[] = {1}, B[] = {5, 8, 10, 20, 30}
        if( M & 1 )
            return MO2( B[M/2], MO3(A[0], B[M/2 - 1], B[M/2 + 1]) );

        // Case 3: If the larger array has even number of element, then median
        // will be one of the following 3 elements
        // ... The middle two elements of larger array
        // ... The only element of smaller array
        return MO3( B[M/2], B[M/2 - 1], A[0] );
    }

    // If the smaller array has two elements
    else if( N == 2 )
    {
        // Case 4: If the larger array also has two elements, simply call MO4()
        if( M == 2 )
            return MO4( A[0], A[1], B[0], B[1] );

        // Case 5: If the larger array has odd number of elements, then median
        // will be one of the following 3 elements
        // 1. Middle element of larger array
        // 2. Max of first element of smaller array and element just
        //    before the middle in bigger array
        // 3. Min of second element of smaller array and element just
        //    after the middle in bigger array
        if( M & 1 )
            return MO3 ( B[M/2],
```

```
                    max( A[0], B[M/2 - 1] ),
                    min( A[1], B[M/2 + 1] )
                  );

        // Case 6: If the larger array has even number of elements, then
        // median will be one of the following 4 elements
        // 1) & 2) The middle two elements of larger array
        // 3) Max of first element of smaller array and element
        //    just before the first middle element in bigger array
        // 4. Min of second element of smaller array and element
        //    just after the second middle in bigger array
        return MO4 ( B[M/2],
                     B[M/2 - 1],
                     max( A[0], B[M/2 - 2] ),
                     min( A[1], B[M/2 + 1] )
                   );
    }

    int idxA = ( N - 1 ) / 2;
    int idxB = ( M - 1 ) / 2;

     /* if A[idxA] <= B[idxB], then median must exist in
        A[idxA....] and B[....idxB] */
    if( A[idxA] <= B[idxB] )
        return findMedianUtil( A + idxA, N / 2 + 1, B, M - idxA );

    /* if A[idxA] > B[idxB], then median must exist in
       A[...idxA] and B[idxB....] */
    return findMedianUtil( A, N / 2 + 1, B + idxA, M - idxA );
}

// A wrapper function around findMedianUtil(). This function makes
// sure that smaller array is passed as first argument to findMedianUtil
float findMedian( int A[], int N, int B[], int M )
{
    if ( N > M )
       return findMedianUtil( B, M, A, N );

    return findMedianUtil( A, N, B, M );
}

// Driver program to test above functions
int main()
{
```

```
    int A[] = {900};
    int B[] = {5, 8, 10, 20};

    int N = sizeof(A) / sizeof(A[0]);
    int M = sizeof(B) / sizeof(B[0]);

    printf( "%f", findMedian( A, N, B, M ) );
    return 0;
}
```

Output:

```
10
```

Time Complexity: O(LogM + LogN)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

78. Print unique rows in a given boolean matrix

Given a binary matrix, print all unique rows of the given matrix.

```
Input:
 {0, 1, 0, 0, 1}
       {1, 0, 1, 1, 0}
       {0, 1, 0, 0, 1}
       {1, 1, 1, 0, 0}
Output:
 0 1 0 0 1
 1 0 1 1 0
 1 1 1 0 0
```

**Method 1 (Simple)**
A simple approach is to check each row with all processed rows. Print the first row. Now, starting from the second row, for each row, compare the row with already processed rows. If the row matches with any of the processed rows, donâ€™t print it. If the current row doesnâ€™t match with any row, print it.

Time complexity: O( ROW^2 x COL )
Auxiliary Space: O( 1 )

**Method 2 (Use Binary Search Tree)**

Find the decimal equivalent of each row and insert it into BST. Each node of the BST will contain two fields, one field for the decimal value, other for row number. Do not insert a node if it is duplicated. Finally, traverse the BST and print the corresponding rows.

Time complexity: O( ROW x COL + ROW x log( ROW ) )
Auxiliary Space: O( ROW )

This method will lead to Integer Overflow if number of columns is large.

**Method 3 (Use Trie data structure)**
Since the matrix is boolean, a variant of Trie data structure can be used where each node will be having two children one for 0 and other for 1. Insert each row in the Trie. If the row is already there, donâ€™t print the row. If row is not there in Trie, insert it in Trie and print it.

Below is C implementation of method 3.

```
//Given a binary matrix of M X N of integers, you need to return only unique rows of binary array
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>

#define ROW 4
#define COL 5

// A Trie node
typedef struct Node
{
    bool isEndOfCol;
    struct Node *child[2]; // Only two children needed for 0 and 1
} Node;



// A utility function to allocate memory for a new Trie node
Node* newNode()
{
    Node* temp = (Node *)malloc( sizeof( Node ) );
    temp->isEndOfCol = 0;
    temp->child[0] = temp->child[1] = NULL;
    return temp;
}

// Inserts a new matrix row to Trie.  If row is already
// present, then returns 0, otherwise insets the row and
```

```
// return 1
bool insert( Node** root, int (*M)[COL], int row, int col )
{
    // base case
    if ( *root == NULL )
        *root = newNode();

    // Recur if there are more entries in this row
    if ( col < COL )
        return insert ( &( (*root)->child[ M[row][col] ] ), M, row, col+1 );

    else // If all entries of this row are processed
    {
        // unique row found, return 1
        if ( !( (*root)->isEndOfCol ) )
            return (*root)->isEndOfCol = 1;

        // duplicate row found, return 0
        return 0;
    }
}

// A utility function to print a row
void printRow( int (*M)[COL], int row )
{
    int i;
    for( i = 0; i < COL; ++i )
        printf( "%d ", M[row][i] );
    printf("\n");
}

// The main function that prints all unique rows in a
// given matrix.
void findUniqueRows( int (*M)[COL] )
{
    Node* root = NULL; // create an empty Trie
    int i;

    // Iterate through all rows
    for ( i = 0; i < ROW; ++i )
        // insert row to TRIE
        if ( insert(&root, M, i, 0) )
            // unique row found, print it
            printRow( M, i );
```

```
}

// Driver program to test above functions
int main()
{
    int M[ROW][COL] = {{0, 1, 0, 0, 1},
        {1, 0, 1, 1, 0},
        {0, 1, 0, 0, 1},
        {1, 0, 1, 0, 0}
    };

    findUniqueRows( M );

    return 0;
}
```

Time complexity: O( ROW x COL )
Auxiliary Space: O( ROW x COL )

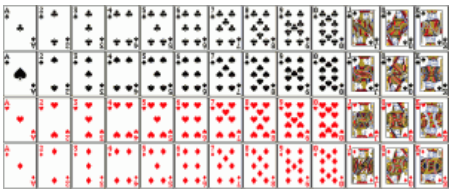
This method has better time complexity. Also, relative order of rows is maintained while printing.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

79. Shuffle a given array

Given an array, write a program to generate a random permutation of array elements. This question is also asked as â€œshuffle a deck of cardsâ€ or â€œrandomize a given arrayâ€.



Let the given array be *arr[]*. A simple solution is to create an auxiliary array *temp[]* which is initially a copy of *arr[]*. Randomly select an element from *temp[]*, copy the randomly selected element to *arr[0]* and remove the selected element from *temp[]*. Repeat the same process n times and keep copying elements to *arr[1], arr[2], â€¦* . The time complexity of this solution will be O(n^2).

Fisherâ€“Yates shuffle Algorithm works in O(n) time complexity. The assumption here

is, we are given a function rand() that generates random number in O(1) time.
The idea is to start from the last element, swap it with a randomly selected element from the whole array (including last). Now consider the array from 0 to n-2 (size reduced by 1), and repeat the process till we hit the first element.

Following is the detailed algorithm

```
To shuffle an array a of n elements (indices 0..n-1):
  for i from n - 1 downto 1 do
       j = random integer with 0 <= j <= i
       exchange a[j] and a[i]
```

Following is C++ implementation of this algorithm.

```
// C Program to shuffle a given array

#include <stdio.h>
#include <stdlib.h>
#include <time.h>

// A utility function to swap to integers
void swap (int *a, int *b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// A utility function to print an array
void printArray (int arr[], int n)
{
    for (int i = 0; i < n; i++)
        printf("%d ", arr[i]);
    printf("\n");
}

// A function to generate a random permutation of arr[]
void randomize ( int arr[], int n )
{
    // Use a different seed value so that we don't get same
    // result each time we run this program
    srand ( time(NULL) );

    // Start from the last element and swap one by one. We don't
    // need to run for the first element that's why i > 0
```

```
    for (int i = n-1; i > 0; i--)
    {
        // Pick a random index from 0 to i
        int j = rand() % (i+1);

        // Swap arr[i] with the element at random index
        swap(&arr[i], &arr[j]);
    }
}

// Driver program to test above function.
int main()
{
    int arr[] = {1, 2, 3, 4, 5, 6, 7, 8};
    int n = sizeof(arr)/ sizeof(arr[0]);
    randomize (arr, n);
    printArray(arr, n);

    return 0;
}
```

Output:

```
7 8 4 6 3 1 2 5
```

The above function assumes that rand() generates a random number.

Time Complexity: O(n), assuming that the function rand() takes O(1) time.

**How does this work?**
The probability that ith element (including the last one) goes to last position is 1/n, because we randomly pick an element in first iteration.

The probability that ith element goes to second last position can be proved to be 1/n by dividing it in two cases.
*Case 1: i = n-1 (index of last element)*:
The probability of last element going to second last position is = (probability that last element doesn't stay at its original position) x (probability that the index picked in previous step is picked again so that the last element is swapped)
So the probability = ((n-1)/n) x (1/(n-1)) = 1/n
*Case 2: 0 < i < n-1 (index of non-last)*:
The probability of ith element going to second position = (probability that ith element is not picked in previous iteration) x (probability that ith element is picked in this iteration)
So the probability = ((n-1)/n) x (1/(n-1)) = 1/n

We can easily generalize above proof for any other position.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

80. Count the number of possible triangles

Given an unsorted array of positive integers. Find the number of triangles that can be formed with three different array elements as three sides of triangles. For a triangle to be possible from 3 values, the sum of any two values (or sides) must be greater than the third value (or third side).
For example, if the input array is {4, 6, 3, 7}, the output should be 3. There are three triangles possible {3, 4, 6}, {4, 6, 7} and {3, 6, 7}. Note that {3, 4, 7} is not a possible triangle.
As another example, consider the array {10, 21, 22, 100, 101, 200, 300}. There can be 6 possible triangles: {10, 21, 22}, {21, 100, 101}, {22, 100, 101}, {10, 100, 101}, {100, 101, 200} and {101, 200, 300}

**Method 1 (Brute force)**
The brute force method is to run three loops and keep track of the number of triangles possible so far. The three loops select three different values from array, the innermost loop checks for the triangle property ( the sum of any two sides must be greater than the value of third side).

Time Complexity: O(N^3) where N is the size of input array.

**Method 2 (Tricky and Efficient)**
Let a, b and c be three sides. The below condition must hold for a triangle (Sum of two sides is greater than the third side)
i) a + b > c
ii) b + c > a
iii) a + c > b

Following are steps to count triangle.

**1.** Sort the array in non-decreasing order.

**2.** Initialize two pointers â€˜iâ€™ and â€˜jâ€™ to first and second elements respectively, and initialize count of triangles as 0.

**3.** Fix â€˜iâ€™ and â€˜jâ€™ and find the rightmost index â€˜kâ€™ (or largest â€˜arr[k]â€™) such that â€˜arr[i] + arr[j] > arr[k]â€™. The number of triangles that can be formed with â€˜arr[i]â€™ and â€˜arr[j]â€™ as two sides is â€˜k â€“ jâ€™. Add â€˜k â€“ jâ€™ to count of triangles.

Let us consider â€˜arr[i]â€™ as â€˜aâ€™, â€˜arr[j]â€™ as b and all elements between â€˜arr[j+1]â€™ and â€˜arr[k]â€™ as â€˜câ€™. The above mentioned conditions (ii) and (iii) are satisfied because â€˜arr[i] < arr[j] < arr[k]'. And we check for condition (i) when we pick 'k'

**4.** Increment â€˜jâ€™ to fix the second element again.

Note that in step 3, we can use the previous value of â€˜kâ€™. The reason is simple, if we know that the value of â€˜arr[i] + arr[j-1]â€™ is greater than â€˜arr[k]â€™, then we can say â€˜arr[i] + arr[j]â€™ will also be greater than â€˜arr[k]â€™, because the array is sorted in increasing order.

**5.** If â€˜jâ€™ has reached end, then increment â€˜iâ€™. Initialize â€˜jâ€™ as â€˜i + 1â€², â€˜kâ€™ as â€˜i+2â€² and repeat the steps 3 and 4.

Following is implementation of the above approach.

```
// Program to count number of triangles that can be formed from given array
#include <stdio.h>
#include <stdlib.h>

/* Following function is needed for library function qsort(). Refer
   http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */
int comp(const void* a, const void* b)
{  return *(int*)a > *(int*)b ; }

// Function to count all possible triangles with arr[] elements
int findNumberOfTriangles(int arr[], int n)
{
    // Sort the array elements in non-decreasing order
    qsort(arr, n, sizeof( arr[0] ), comp);

    // Initialize count of triangles
    int count = 0;

    // Fix the first element.  We need to run till n-3 as the other two elements are
    // selected from arr[i+1...n-1]
    for (int i = 0; i < n-2; ++i)
    {
        // Initialize index of the rightmost third element
        int k = i+2;

        // Fix the second element
        for (int j = i+1; j < n; ++j)
        {
```

```
            // Find the rightmost element which is smaller than the sum
            // of two fixed elements
            // The important thing to note here is, we use the previous
            // value of k. If value of arr[i] + arr[j-1] was greater than arr[k],
            // then arr[i] + arr[j] must be greater than k, because the
            // array is sorted.
            while (k < n && arr[i] + arr[j] > arr[k])
              ++k;

            // Total number of possible triangles that can be formed
            // with the two fixed elements is k - j - 1.  The two fixed
            // elements are arr[i] and arr[j].  All elements between arr[j+1]
            // to arr[k-1] can form a triangle with arr[i] and arr[j].
            // One is subtracted from k because k is incremented one extra
            // in above while loop.
            // k will always be greater than j. If j becomes equal to k, then
            // above loop will increment k, because arr[k] + arr[i] is always
            // greater than arr[k]
            count += k - j - 1;
        }
    }

    return count;
}

// Driver program to test above functionarr[j+1]
int main()
{
    int arr[] =   {10, 21, 22, 100, 101, 200, 300};
    int size = sizeof( arr ) / sizeof( arr[0] );

    printf("Total number of triangles possible is %d ",
           findNumberOfTriangles( arr, size ) );

    return 0;
}
```

Output:

```
Total number of triangles possible is 6
```

Time Complexity: O(n^2). The time complexity looks more because of 3 nested loops. If we take a closer look at the algorithm, we observe that k is initialized only once in the outermost loop. The innermost loop executes at most O(n) time for every iteration of outer most loop, because k starts from i+2 and goes upto n for all values of j.

Therefore, the time complexity is O(n^2).

Source: http://stackoverflow.com/questions/8110538/total-number-of-possible-triangles-from-n-numbers

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

81. Iterative Quick Sort

Following is a typical recursive implementation of Quick Sort that uses last element as pivot.

```
/* A typical recursive implementation of quick sort */

/* This function takes last element as pivot, places the pivot element at its
   correct position in sorted array, and places all smaller (smaller than pivot)
   to left of pivot and all greater elements to right of pivot */
int partition (int arr[], int l, int h)
{
    int x = arr[h];
    int i = (l - 1);

    for (int j = l; j <= h- 1; j++)
    {
        if (arr[j] <= x)
        {
            i++;
            swap (&arr[i], &arr[j]);
        }
    }
    swap (&arr[i + 1], &arr[h]);
    return (i + 1);
}

/* A[] --> Array to be sorted, l  --> Starting index, h  --> Ending index */
void quickSort(int A[], int l, int h)
{
    if (l < h)
    {
        int p = partition(A, l, h); /* Partitioning index */
        quickSort(A, l, p - 1);
```

```
        quickSort(A, p + 1, h);
    }
}
```

The above implementation can be optimized in many ways

1) The above implementation uses last index as pivot. This causes worst-case behavior on already sorted arrays, which is a commonly occurring case. The problem can be solved by choosing either a random index for the pivot, or choosing the middle index of the partition or choosing the median of the first, middle and last element of the partition for the pivot. (See this for details)

2) To reduce the recursion depth, recur first for the smaller half of the array, and use a tail call to recurse into the other.

3) Insertion sort works better for small subarrays. Insertion sort can be used for invocations on such small arrays (i.e. where the length is less than a threshold t determined experimentally). For example, this library implementation of qsort uses insertion sort below size 7.

Despite above optimizations, the function remains recursive and uses function call stack to store intermediate values of l and h. The function call stack stores other bookkeeping information together with parameters. Also, function calls involve overheads like storing activation record of the caller function and then resuming execution.

The above function can be easily converted to iterative version with the help of an auxiliary stack. Following is an iterative implementation of the above recursive code.

```
// An iterative implementation of quick sort
#include <stdio.h>

// A utility function to swap two elements
void swap ( int* a, int* b )
{
    int t = *a;
    *a = *b;
    *b = t;
}

/* This function is same in both iterative and recursive*/
int partition (int arr[], int l, int h)
{
    int x = arr[h];
    int i = (l - 1);
```

```
    for (int j = l; j <= h- 1; j++)
    {
        if (arr[j] <= x)
        {
            i++;
            swap (&arr[i], &arr[j]);
        }
    }
    swap (&arr[i + 1], &arr[h]);
    return (i + 1);
}

/* A[] --> Array to be sorted, l  --> Starting index, h  --> Ending index */
void quickSortIterative (int arr[], int l, int h)
{
    // Create an auxiliary stack
    int stack[ h - l + 1 ];

    // initialize top of stack
    int top = -1;

    // push initial values of l and h to stack
    stack[ ++top ] = l;
    stack[ ++top ] = h;

    // Keep popping from stack while is not empty
    while ( top >= 0 )
    {
        // Pop h and l
        h = stack[ top-- ];
        l = stack[ top-- ];

        // Set pivot element at its correct position in sorted array
        int p = partition( arr, l, h );

        // If there are elements on left side of pivot, then push left
        // side to stack
        if ( p-1 > l )
        {
            stack[ ++top ] = l;
            stack[ ++top ] = p - 1;
        }

        // If there are elements on right side of pivot, then push right
```

```
        // side to stack
        if ( p+1 < h )
        {
            stack[ ++top ] = p + 1;
            stack[ ++top ] = h;
        }
    }
}

// A utility function to print contents of arr
void printArr( int arr[], int n )
{
    int i;
    for ( i = 0; i < n; ++i )
        printf( "%d ", arr[i] );
}

// Driver program to test above functions
int main()
{
    int arr[] = {4, 3, 5, 2, 1, 3, 2, 3};
    int n = sizeof( arr ) / sizeof( *arr );
    quickSortIterative( arr, 0, n - 1 );
    printArr( arr, n );
    return 0;
}
```

Output:

```
1 2 2 3 3 3 4 5
```

The above mentioned optimizations for recursive quick sort can also be applied to iterative version.

1) Partition process is same in both recursive and iterative. The same techniques to choose optimal pivot can also be applied to iterative version.

2) To reduce the stack size, first push the indexes of smaller half.

3) Use insertion sort when the size reduces below a experimentally calculated threshold.

**References:**
http://en.wikipedia.org/wiki/Quicksort

This article is compiled by **Aashish Barnwal** and reviewed by GeeksforGeeks team.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

82. Inplace M x N size matrix transpose | Updated

About four months of gap (missing GFG), a new post. Given an M x N matrix, transpose the matrix without auxiliary memory.It is easy to transpose matrix using an auxiliary array. If the matrix is symmetric in size, we can transpose the matrix inplace by mirroring the 2D array across itâ€™s diagonal (try yourself). How to transpose an arbitrary size matrix inplace? See the following matrix,

```
a b c      a d g j
d e f  ==>  b e h k
g h i      c f i l
j k l
```

As per 2D numbering in C/C++, corresponding location mapping looks like,

```
Org element New
 0    a    0
 1    b    4
 2    c    8
 3    d    1
 4    e    5
 5    f    9
 6    g    2
 7    h    6
 8    i   10
 9    j    3
10    k    7
11    l   11
```

Note that the first and last elements stay in their original location. We can easily see theÂ transformationÂ forms few permutation cycles. 1->4->5->9->3->1 Â â€“ Total 5 elements form the cycle 2->8->10->7->6->2 â€“ Another 5 elements form the cycle 0 Â â€“ Self cycle 11 â€“ Self cycle From the above example, we can easilyÂ deviseÂ an algorithm to move the elements along these cycles. *How can we generate permutation cycles?* Number of elements in both the matrices are constant, given by N = R * C, where R is row count and C is column count. An element at location *ol* (old location in R x C matrix), moved to *nl* (new location in C x R matrix). We need to establish relation between *ol, nl, R* and *C*. Assume *ol = A[or][oc]*. In C/C++ we can calculate the element address as,

```
ol = or x C + oc (ignore base reference for simplicity)
```

It is to be moved to new location *nl* in the transposed matrix, say *nl = A[nr][nc]*, or in C/C++ terms

```
nl = nr x R + nc (R - column count, C is row count as the matrix is transposed)
```

Observe, *nr = oc* and *nc = or*, so replacing these for *nl*,

```
nl = oc x R + or -----> [eq 1]
```

after solving for relation between *ol* and *nl*, we get

```
ol     = or x C     + oc
ol x R = or x C x R + oc x R
       = or x N     + oc x R    (from the fact R * C = N)
       = or x N     + (nl - or) --- from [eq 1]
       = or x (N-1) + nl
```

OR,

```
nl = ol x R - or x (N-1)
```

Note that the values of *nl* and *ol* never go beyond *N-1*, so considering modulo division on both the sides by (*N-1*), we get the following based on properties ofÂ congruence,

```
nl mod (N-1) = (ol x R - or x (N-1)) mod (N-1)
             = (ol x R) mod (N-1) - or x (N-1) mod(N-1)
             = ol x R mod (N-1), since second term evaluates to zero
nl = (ol x R) mod (N-1), since nl is always less than N-1
```

**A curious reader might have observed the significance of above relation. Every location is scaled by a factor of R (row size). It is obvious from the matrix that every location is displaced by scaled factor of R. The actual multiplier depends on congruence class of (N-1), i.e. the multiplier can be both -ve and +ve value of the congruent class.**Hence every locationÂ transformationÂ is simple modulo division. These modulo divisions form cyclic permutations. We need some book keeping information to keep track of already moved elements. Here is code for inplace matrixÂ transformation,

```
// Program for in-place matrix transpose
#include <stdio.h>
#include <iostream>
#include <bitset>
#define HASH_SIZE 128

using namespace std;
```

```
// A utility function to print a 2D array of size nr x nc and base address A
void Print2DArray(int *A, int nr, int nc)
{
   for(int r = 0; r < nr; r++)
   {
      for(int c = 0; c < nc; c++)
         printf("%4d", *(A + r*nc + c));

      printf("\n");
   }

   printf("\n\n");
}

// Non-square matrix transpose of matrix of size r x c and base address A
void MatrixInplaceTranspose(int *A, int r, int c)
{
   int size = r*c - 1;
   int t; // holds element to be replaced, eventually becomes next element to move
   int next; // location of 't' to be moved
   int cycleBegin; // holds start of cycle
   int i; // iterator
   bitset<HASH_SIZE> b; // hash to mark moved elements

   b.reset();
   b[0] = b[size] = 1;
   i = 1; // Note that A[0] and A[size-1] won't move
   while (i < size)
   {
      cycleBegin = i;
      t = A[i];
      do
      {
         // Input matrix [r x c]
         // Output matrix 1
         // i_new = (i*r)%(N-1)
         next = (i*r)%size;
         swap(A[next], t);
         b[i] = 1;
         i = next;
      }
      while (i != cycleBegin);

      // Get Next Move (what about querying random location?)
```

```
        for (i = 1; i < size && b[i]; i++)
            ;
        cout << endl;
    }
}

// Driver program to test above function
int main(void)
{
    int r = 5, c = 6;
    int size = r*c;
    int *A = new int[size];

    for(int i = 0; i < size; i++)
        A[i] = i+1;

    Print2DArray(A, r, c);
    MatrixInplaceTranspose(A, r, c);
    Print2DArray(A, c, r);

    delete[] A;

    return 0;
}
```

Output:

```
 1   2   3   4   5   6
 7   8   9  10  11  12
13  14  15  16  17  18
19  20  21  22  23  24
25  26  27  28  29  30

 1   7  13  19  25
 2   8  14  20  26
 3   9  15  21  27
 4  10  16  22  28
 5  11  17  23  29
 6  12  18  24  30
```

**Extension: 17 â€“ March â€“ 2013** Some readers identified similarity between the matrix transpose and string transformation. Without much theory I am presenting the problem and solution. In given array of elements like [a1b2c3d4e5f6g7h8i9j1k2l3m4]. Convert it to [abcdefghijklm1234567891234]. The program should run inplace. What we need is an inplace transpose. Given below is code.

```
#include <stdio.h>
#include <iostream>
#include <bitset>
#define HASH_SIZE 128

using namespace std;

typedef char data_t;

void Print2DArray(char A[], int nr, int nc) {
  int size = nr*nc;
  for(int i = 0; i < size; i++)
    printf("%4c", *(A + i));

  printf("\n");
}

void MatrixTransposeInplaceArrangement(data_t A[], int r, int c) {
  int size = r*c - 1;
  data_t t; // holds element to be replaced, eventually becomes next element to move
  int next; // location of 't' to be moved
  int cycleBegin; // holds start of cycle
  int i; // iterator
  bitset<HASH_SIZE> b; // hash to mark moved elements

  b.reset();
  b[0] = b[size] = 1;
  i = 1; // Note that A[0] and A[size-1] won't move
  while( i < size ) {
    cycleBegin = i;
    t = A[i];
    do {
      // Input matrix [r x c]
      // Output matrix 1
      // i_new = (i*r)%size
      next = (i*r)%size;
      swap(A[next], t);
      b[i] = 1;
      i = next;
    } while( i != cycleBegin );

    // Get Next Move (what about querying random location?)
    for(i = 1; i < size && b[i]; i++)
      ;
```

```
      cout << endl;
    }
}

void Fill(data_t buf[], int size) {
    // Fill abcd ...
    for(int i = 0; i < size; i++)
    buf[i] = 'a'+i;

    // Fill 0123 ...
    buf += size;
    for(int i = 0; i < size; i++)
      buf[i] = '0'+i;
}

void TestCase_01(void) {
    int r = 2, c = 10;
    int size = r*c;
    data_t *A = new data_t[size];

    Fill(A, c);

    Print2DArray(A, r, c), cout << endl;
    MatrixTransposeInplaceArrangement(A, r, c);
    Print2DArray(A, c, r), cout << endl;

    delete[] A;
}

int main() {
    TestCase_01();

    return 0;
}
```

The post is incomplete without mentioning two links.

1. Aashish covered good theory behind cycle leader algorithm. See his post on string transformation.

2. As usual, Sambasiva demonstrated his exceptional skills in recursion to the problem. Ensure to understand his solution.

â€”Â Venki.Â Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

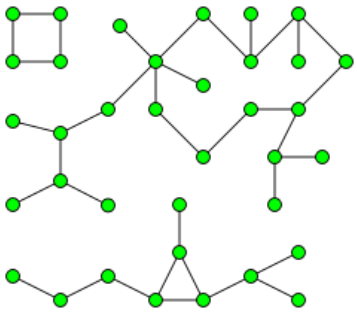
Â Â

83. Find the number of islands

Given a boolean 2D matrix, find the number of islands.

This is an variation of the standard problem: â€œCounting number of connected components in a undirected graphâ€.

Before we go to the problem, let us understand what is a connected component. A connected component of an undirected graph is a subgraph in which every two vertices are connected to each other by a path(s), and which is connected to no other vertices outside the subgraph.
For example, the graph shown below has three connected components.



A graph where all vertices are connected with each other, has exactly one connected component, consisting of the whole graph. Such graph with only one connected component is called as Strongly Connected Graph.

The problem can be easily solved by applying DFS() on each component. In each DFS() call, a component or a sub-graph is visited. We will call DFS on the next un-visited component. The number of calls to DFS() gives the number of connected components. BFS can also be used.

***What is an island?***
A group of connected 1s forms an island. For example, the below matrix contains 5 islands

```
{1, 1, 0, 0, 0},
    {0, 1, 0, 0, 1},
    {1, 0, 0, 1, 1},
    {0, 0, 0, 0, 0},
    {1, 0, 1, 0, 1}
```

A cell in 2D matrix can be connected to 8 neighbors. So, unlike standard DFS(), where we recursively call for all adjacent vertices, here we can recursive call for 8 neighbors only. We keep track of the visited 1s so that they are not visited again.

```
// Program to count islands in boolean 2D matrix

#include <stdio.h>
#include <string.h>
#include <stdbool.h>

#define ROW 5
#define COL 5

// A function to check if a given cell (row, col) can be included in DFS
int isSafe(int M[][COL], int row, int col, bool visited[][COL])
{
    return (row >= 0) && (row < ROW) &&     // row number is in range
           (col >= 0) && (col < COL) &&     // column number is in range
           (M[row][col] && !visited[row][col]); // value is 1 and not yet visited
}

// A utility function to do DFS for a 2D boolean matrix. It only considers
// the 8 neighbors as adjacent vertices
void DFS(int M[][COL], int row, int col, bool visited[][COL])
{
    // These arrays are used to get row and column numbers of 8 neighbors
    // of a given cell
    static int rowNbr[] = {-1, -1, -1,  0, 0,  1, 1, 1};
    static int colNbr[] = {-1,  0,  1, -1, 1, -1, 0, 1};

    // Mark this cell as visited
    visited[row][col] = true;

    // Recur for all connected neighbours
    for (int k = 0; k < 8; ++k)
        if (isSafe(M, row + rowNbr[k], col + colNbr[k], visited) )
            DFS(M, row + rowNbr[k], col + colNbr[k], visited);
}

// The main function that returns count of islands in a given boolean
// 2D matrix
int countIslands(int M[][COL])
{
    // Make a bool array to mark visited cells.
    // Initially all cells are unvisited
    bool visited[ROW][COL];
    memset(visited, 0, sizeof(visited));
```

```
    // Initialize count as 0 and travese through the all cells of
    // given matrix
    int count = 0;
    for (int i = 0; i < ROW; ++i)
        for (int j = 0; j < COL; ++j)
            if (M[i][j] && !visited[i][j]) // If a cell with value 1 is not
            {                              // visited yet, then new island found
                DFS(M, i, j, visited);     // Visit all cells in this island.
                ++count;                   // and increment island count
            }

    return count;
}

// Driver program to test above function
int main()
{
    int M[][COL]= {  {1, 1, 0, 0, 0},
        {0, 1, 0, 0, 1},
        {1, 0, 0, 1, 1},
        {0, 0, 0, 0, 0},
        {1, 0, 1, 0, 1}
    };

    printf("Number of islands is: %d\n", countIslands(M));

    return 0;
}
```

Output:

```
Number of islands is: 5
```

Time complexity: O(ROW x COL)

Reference:
http://en.wikipedia.org/wiki/Connected_component_%28graph_theory%29

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

In my previous post, I have explained about longest monotonically increasing sub-sequenceÂ (LIS) problem in detail. However, the post only covered code related to querying size of LIS, but not theÂ constructionÂ of LIS. I left it as an exercise. If you have solved, cheers. If not, you are not alone, here is code.

If you have not read my previous post, read here.Â Note that the below code prints LIS in reverse order. We can modify print order using a stack (explicit or system stack). I am leaving explanation as an exercise (easy).

```
#include <iostream>
#include <string.h>
#include <stdio.h>
using namespace std;

// Binary search
int GetCeilIndex(int A[], int T[], int l, int r, int key) {
  int m;

  while( r - l > 1 ) {
    m = l + (r - l)/2;
    if( A[T[m]] >= key )
      r = m;
    else
      l = m;
  }

  return r;
}

int LongestIncreasingSubsequence(int A[], int size) {
  // Add boundary case, when array size is zero
  // Depend on smart pointers

  int *tailIndices = new int[size];
  int *prevIndices = new int[size];
  int len;

  memset(tailIndices, 0, sizeof(tailIndices[0])*size);
  memset(prevIndices, 0xFF, sizeof(prevIndices[0])*size);

  tailIndices[0] = 0;
  prevIndices[0] = -1;
  len = 1; // it will always point to empty location
  for( int i = 1; i < size; i++ ) {
```

```
    if( A[i] < A[tailIndices[0]] ) {
      // new smallest value
      tailIndices[0] = i;
    } else if( A[i] > A[tailIndices[len-1]] ) {
      // A[i] wants to extend largest subsequence
      prevIndices[i] = tailIndices[len-1];
      tailIndices[len++] = i;
    } else {
      // A[i] wants to be a potential condidate of future subsequence
      // It will replace ceil value in tailIndices
      int pos = GetCeilIndex(A, tailIndices, -1, len-1, A[i]);

      prevIndices[i] = tailIndices[pos-1];
      tailIndices[pos] = i;
    }
  }
  cout << "LIS of given input" << endl;
  for( int i = tailIndices[len-1]; i >= 0; i = prevIndices[i] )
    cout << A[i] << "  ";
  cout << endl;

  delete[] tailIndices;
  delete[] prevIndices;

  return len;
}

int main() {
  int A[] = { 2, 5, 3, 7, 11, 8, 10, 13, 6 };
  int size = sizeof(A)/sizeof(A[0]);

  printf("LIS size %d\n", LongestIncreasingSubsequence(A, size));

  return 0;
}
```

**Exercises:**

1. You know Kadaneâ€˜s algorithm to findÂ maximum sum sub-array. Modify Kadaneâ€™s algorithm to trace starting and ending location of maximum sum sub-array.

2. ModifyÂ Kadaneâ€˜s algorithm to find maximum sum sub-array in a circular array. Refer GFG forum for many comments on the question.

3. Given two integers A and B as input. Find number of Fibonacci numbers existing in between these two numbers (including A and B). For example, A = 3 and B = 18, there are 4 Fibonacci numbers in between {3,Â 5, 8, 13}. Do it in O(log K) time, where K is max(A, B). What is your observation?

â€”Â Venki.Â Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

85. Find the first circular tour that visits all petrol pumps

Suppose there is a circle. There are n petrol pumps on that circle. You are given two sets of data.

**1.** The amount of petrol that petrol pump will give.
**2.** Distance from that petrol pump to the next petrol pump.

Calculate the first point from where a truck will be able to complete the circle (The truck will stop at each petrol pump and it has infinite capacity). Expected time complexity is O(n). Assume for 1 litre petrol, the truck can go 1 unit of distance.

For example, let there be 4 petrol pumps with amount of petrol and distance to next petrol pump value pairs as {4, 6}, {6, 5}, {7, 3} and {4, 5}. The first point from where truck can make a circular tour is 2nd petrol pump. Output should be â€œstart = 1â€³ (index of 2nd petrol pump).

A **Simple Solution** is to consider every petrol pumps as starting point and see if there is a possible tour. If we find a starting point with feasible solution, we return that starting point. The worst case time complexity of this solution is O(n^2).

We can **use a Queue** to store the current tour. We first enqueue first petrol pump to the queue, we keep enqueueing petrol pumps till we either complete the tour, or current amount of petrol becomes negative. If the amount becomes negative, then we keep dequeueing petrol pumps till the current amount becomes positive or queue becomes empty.

Instead of creating a separate queue, we use the given array itself as queue. We maintain two index variables start and end that represent rear and front of queue.

```
// C program to find circular tour for a truck
#include <stdio.h>

// A petrol pump has petrol and distance to next petrol pump
struct petrolPump
```

```
{
  int petrol;
  int distance;
};

// The function returns starting point if there is a possible solution,
// otherwise returns -1
int printTour(struct petrolPump arr[], int n)
{
    // Consider first petrol pump as a starting point
    int start = 0;
    int end =  1;

    int curr_petrol = arr[start].petrol - arr[start].distance;

    /* Run a loop while all petrol pumps are not visited.
      And we have reached first petrol pump again with 0 or more petrol */
    while (end != start || curr_petrol < 0)
    {
        // If curremt amount of petrol in truck becomes less than 0, then
        // remove the starting petrol pump from tour
        while (curr_petrol < 0 && start != end)
        {
            // Remove starting petrol pump. Change start
            curr_petrol -= arr[start].petrol - arr[start].distance;
            start = (start + 1)%n;

            // If 0 is being considered as start again, then there is no
            // possible solution
            if (start == 0)
               return -1;
        }

        // Add a petrol pump to current tour
        curr_petrol += arr[end].petrol - arr[end].distance;

        end = (end + 1)%n;
    }

    // Return starting point
    return start;
}

// Driver program to test above functions
```

```
int main()
{
    struct petrolPump arr[] = {{6, 4}, {3, 6}, {7, 3}};

    int n = sizeof(arr)/sizeof(arr[0]);
    int start = printTour(arr, n);

    (start == -1)? printf("No solution"): printf("Start = %d", start);

    return 0;
}
```

Output:

```
start = 2
```

**Time Complexity:** Seems to be more than linear at first look. If we consider the items between start and end as part of a circular queue, we can observe that every item is enqueued at most two times to the queue. The total number of operations is proportional to total number of enqueue operations. Therefore the time complexity is O(n).

**Auxiliary Space:** O(1)

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

86. Arrange given numbers to form the biggest number

Given an array of numbers, arrange them in a way that yields the largest value. For example, if the given numbers are {54, 546, 548, 60}, the arrangement 6054854654 gives the largest value. And if the given numbers are {1, 34, 3, 98, 9, 76, 45, 4}, then the arrangement 998764543431 gives the largest value.

A simple solution that comes to our mind is to sort all numbers in descending order, but simply sorting doesnâ€™t work. For example, 548 is greater than 60, but in output 60 comes before 548. As a second example, 98 is greater than 9, but 9 comes before 98 in output.

So how do we go about it? The idea is to use any comparison based sorting algorithm. In the used sorting algorithm, instead of using the default comparison, write a comparison function myCompare() and use it to sort numbers. Given two numbers X and Y, how should myCompare() decide which number to put first â€“ we compare two

numbers XY (Y appended at the end of X) and YX (X appended at the end of Y). If XY is larger, then X should come before Y in output, else Y should come before. For example, let X and Y be 542 and 60. To compare X and Y, we compare 54260 and 60542. Since 60542 is greater than 54260, we put Y first.

Following is C++ implementation of the above approach. To keep the code simple, numbers are considered as strings, and vector is used instead of normal array.

```
// Given an array of numbers, program to arrange the numbers to form the
// largest number
#include <iostream>
#include <string>
#include <vector>
#include <algorithm>
using namespace std;

// A comparison function which is used by sort() in printLargest()
int myCompare(string X, string Y)
{
    // first append Y at the end of X
    string XY = X.append(Y);

    // then append X at the end of Y
    string YX = Y.append(X);

    // Now see which of the two formed numbers is greater
    return XY.compare(YX) > 0 ? 1: 0;
}

// The main function that prints the arrangement with the largest value.
// The function accepts a vector of strings
void printLargest(vector<string> arr)
{
    // Sort the numbers using library sort funtion. The function uses
    // our comparison function myCompare() to compare two strings.
    // See http://www.cplusplus.com/reference/algorithm/sort/ for details
    sort(arr.begin(), arr.end(), myCompare);

    for (int i=0; i < arr.size() ; i++ )
        cout << arr[i];
}

// driverr program to test above functions
int main()
```

```
{
   vector<string> arr;

   //output should be 6054854654
   arr.push_back("54");
   arr.push_back("546");
   arr.push_back("548");
   arr.push_back("60");
   printLargest(arr);

   // output should be 777776
   /*arr.push_back("7");
   arr.push_back("776");
   arr.push_back("7");
   arr.push_back("7");*/

   //output should be 998764543431
   /*arr.push_back("1");
   arr.push_back("34");
   arr.push_back("3");
   arr.push_back("98");
   arr.push_back("9");
   arr.push_back("76");
   arr.push_back("45");
   arr.push_back("4");
   */

  return 0;
}
```

Output:

```
6054854654
```

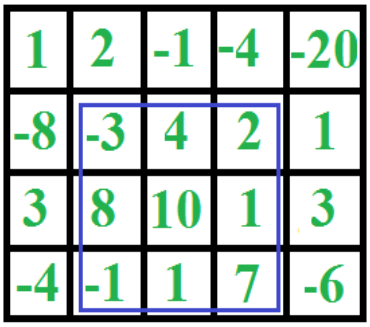
This article is compiled by **Ravi Chandra Enaganti**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

87. Dynamic Programming | Set 27 (Maximum sum rectangle in a 2D matrix)

Given a 2D array, find the maximum sum subarray in it. For example, in the following 2D array, the maximum sum subarray is highlighted with blue rectangle and sum of

this subarray is 29.




This problem is mainly an extension of Largest Sum Contiguous Subarray for 1D array.

The **naive solution** for this problem is to check every possible rectangle in given 2D array. This solution requires 4 nested loops and time complexity of this solution would be O(n^4).

**Kadaneâ€™s algorithm** for 1D array can be used to reduce the time complexity to O(n^3). The idea is to fix the left and right columns one by one and find the maximum sum contiguous rows for every left and right column pair. We basically find top and bottom row numbers (which have maximum sum) for every fixed left and right column pair. To find the top and bottom row numbers, calculate sun of elements in every row from left to right and store these sums in an array say temp[]. So temp[i] indicates sum of elements from left to right in row i. If we apply Kadaneâ€™s 1D algorithm on temp[], and get the maximum sum subarray of temp, this maximum sum would be the maximum possible sum with left and right as boundary columns. To get the overall maximum sum, we compare this sum with the maximum sum so far.

```
// Program to find maximum sum subarray in a given 2D array
#include <stdio.h>
#include <string.h>
#include <limits.h>
#define ROW 4
#define COL 5

// Implementation of Kadane's algorithm for 1D array. The function returns the
// maximum sum and stores starting and ending indexes of the maximum sum subarray
// at addresses pointed by start and finish pointers respectively.
int kadane(int* arr, int* start, int* finish, int n)
{
   // initialize sum, maxSum and
   int sum = 0, maxSum = INT_MIN, i;

   // Just some initial value to check for all negative values case
   *finish = -1;
```

```
    // local variable
    int local_start = 0;

    for (i = 0; i < n; ++i)
    {
        sum += arr[i];
        if (sum < 0)
        {
            sum = 0;
            local_start = i+1;
        }
        else if (sum > maxSum)
        {
            maxSum = sum;
            *start = local_start;
            *finish = i;
        }
    }

     // There is at-least one non-negative number
    if (*finish != -1)
        return maxSum;

    // Special Case: When all numbers in arr[] are negative
    maxSum = arr[0];
    *start = *finish = 0;

    // Find the maximum element in array
    for (i = 1; i < n; i++)
    {
        if (arr[i] > maxSum)
        {
            maxSum = arr[i];
            *start = *finish = i;
        }
    }
    return maxSum;
}

// The main function that finds maximum sum rectangle in M[][]
void findMaxSum(int M[][COL])
{
    // Variables to store the final output
```

```
    int maxSum = INT_MIN, finalLeft, finalRight, finalTop, finalBottom;

    int left, right, i;
    int temp[ROW], sum, start, finish;

    // Set the left column
    for (left = 0; left < COL; ++left)
    {
        // Initialize all elements of temp as 0
        memset(temp, 0, sizeof(temp));

        // Set the right column for the left column set by outer loop
        for (right = left; right < COL; ++right)
        {
            // Calculate sum between current left and right for every row 'i'
            for (i = 0; i < ROW; ++i)
                temp[i] += M[i][right];

            // Find the maximum sum subarray in temp[]. The kadane() function
            // also sets values of start and finish.  So 'sum' is sum of
            // rectangle between (start, left) and (finish, right) which is the
            //  maximum sum with boundary columns strictly as left and right.
            sum = kadane(temp, &start, &finish, ROW);

            // Compare sum with maximum sum so far. If sum is more, then update
            // maxSum and other output values
            if (sum > maxSum)
            {
                maxSum = sum;
                finalLeft = left;
                finalRight = right;
                finalTop = start;
                finalBottom = finish;
            }
        }
    }

    // Print final values
    printf("(Top, Left) (%d, %d)\n", finalTop, finalLeft);
    printf("(Bottom, Right) (%d, %d)\n", finalBottom, finalRight);
    printf("Max sum is: %d\n", maxSum);
}

// Driver program to test above functions
```

```
int main()
{
   int M[ROW][COL] = {{1, 2, -1, -4, -20},
                      {-8, -3, 4, 2, 1},
                      {3, 8, 10, 1, 3},
                      {-4, -1, 1, 7, -6}
                     };

   findMaxSum(M);

   return 0;
}
```

Output:

```
(Top, Left) (1, 1)
(Bottom, Right) (3, 3)
Max sum is: 29
```

Time Complexity: O(n^3)

This article is compiled by Aashish Barnwal. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

88. Pancake sorting

Given an an unsorted array, sort the given array. You are allowed to do only following operation on array.

```
flip(arr, i): Reverse array from 0 to i
```

Unlike a traditional sorting algorithm, which attempts to sort with the fewest comparisons possible, the goal is to sort the sequence in as few reversals as possible.

The idea is to do something similar to Selection Sort. We one by one place maximum element at the end and reduce the size of current array by one.

Following are the detailed steps. Let given array be arr[] and size of array be n.
1) Start from current size equal to n and reduce current size by one while itâ€™s greater than 1. Let the current size be curr_size. Do following for every curr_size
â€¦â€¦a) Find index of the maximum element in arr[0..curr_szie-1]. Let the index be â€˜miâ€™

â€¦â€¦b) Call flip(arr, mi)
â€¦â€¦c) Call flip(arr, curr_size-1)

See following video for visualization of the above algorithm.

```
/* A C++ program for Pancake Sorting */
#include <stdlib.h>
#include <stdio.h>

/* Reverses arr[0..i] */
void flip(int arr[], int i)
{
    int temp, start = 0;
    while (start < i)
    {
        temp = arr[start];
        arr[start] = arr[i];
        arr[i] = temp;
        start++;
        i--;
    }
}

/* Returns index of the maximum element in arr[0..n-1] */
int findMax(int arr[], int n)
{
   int mi, i;
   for (mi = 0, i = 0; i < n; ++i)
       if (arr[i] > arr[mi])
             mi = i;
   return mi;
}

// The main function that sorts given array using flip operations
int pancakeSort(int *arr, int n)
{
    // Start from the complete array and one by one reduce current size by one
    for (int curr_size = n; curr_size > 1; --curr_size)
    {
        // Find index of the maximum element in arr[0..curr_size-1]
        int mi = findMax(arr, curr_size);

        // Move the maximum element to end of current array if it's not
        // already at the end
```

```
        if (mi != curr_size-1)
        {
            // To move at the end, first move maximum number to beginning
            flip(arr, mi);

            // Now move the maximum number to end by reversing current array
            flip(arr, curr_size-1);
        }
    }
}

/* A utility function to print an array of size n */
void printArray(int arr[], int n)
{
    for (int i = 0; i < n; ++i)
        printf("%d ", arr[i]);
}

// Driver program to test above function
int main()
{
    int arr[] = {23, 10, 20, 11, 12, 6, 7};
    int n = sizeof(arr)/sizeof(arr[0]);

    pancakeSort(arr, n);

    puts("Sorted Array ");
    printArray(arr, n);

    return 0;
}
```

Output:

```
Sorted Array
6 7 10 11 12 20 23
```

Total O(n) flip operations are performed in above code. The overall time complexity is O(n^2).

**References:**
http://en.wikipedia.org/wiki/Pancake_sorting

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

89. A Pancake Sorting Problem

We have discussed Pancake Sorting in the previous post. Following is a problem based on Pancake Sorting.
Given an an unsorted array, sort the given array. You are allowed to do only following operation on array.

```
flip(arr, i): Reverse array from 0 to i
```

**Imagine a hypothetical machine where flip(i) always takes O(1) time. Write an efficient program for sorting a given array in O(nLogn) time on the given machine**. If we apply the same algorithm here, the time taken will be O(n^2) because the algorithm calls findMax() in a loop and find findMax() takes O(n) time even on this hypothetical machine.

We can use insertion sort that uses binary search. The idea is to run a loop from second element to last element (from i = 1 to n-1), and one by one insert arr[i] in arr[0..i-1] (like standard insertion sort algorithm). When we insert an element arr[i], we can use binary search to find position of arr[i] in O(Logi) time. Once we have the position, we can use some flip operations to put arr[i] at its new place. Following are abstract steps.

```
// Standard Insertion Sort Loop that starts from second element
for (i=1; i < n; i++) ----> O(n)
{
  int key = arr[i];

  // Find index of ceiling of arr[i] in arr[0..i-1] using binary search
  j = celiSearch(arr, key, 0, i-1); ----> O(logn) (See this)

  // Apply some flip operations to put arr[i] at correct place
}
```

Since flip operation takes O(1) on given hypothetical machine, total running time of above algorithm is O(nlogn). Thanks to Kumar for suggesting above problem and algorithm.

Let us see how does the above algorithm work. ceilSearch() actually returns the index of the smallest element which is greater than arr[i] in arr[0..i-1]. If there is no such element, it returns -1. Let the returned value be j. If j is -1, then we donâ€™t need to do anything as arr[i] is already the greatest element among arr[0..i]. Otherwise we need to put arr[i] just before arr[j].

So how to apply flip operations to put arr[i] just before arr[j] using values of i and j. Let us take an example to understand this. Let i be 6 and current array be {12, 15, 18, 30, 35, 40, **20**, 6, 90, 80}. To put 20 at its new place, the array should be changed to {12, 15, 18, **20**, 30, 35, 40, 6, 90, 80}. We apply following steps to put 20 at its new place.

1) Find j using ceilSearch (In the above example j is 3).
2) flip(arr, j-1) (array becomes {18, 15, 12, 30, 35, 40, **20**, 6, 90, 80})
3) flip(arr, i-1); (array becomes {40, 35, 30, 12, 15, 18, **20**, 6, 90, 80})
4) flip(arr, i); (array becomes {**20**, 18, 15, 12, 30, 35, 40, 6, 90, 80})
5) flip(arr, j); (array becomes {12, 15, 18, **20**, 30, 35, 40, 6, 90, 80})

Following is C implementation of the above algorithm.

```
#include <stdlib.h>
#include <stdio.h>

/* A Binary Search based function to get index of ceiling of x in
   arr[low..high] */
int ceilSearch(int arr[], int low, int high, int x)
{
    int mid;

    /* If x is smaller than or equal to the first element,
      then return the first element */
    if(x <= arr[low])
        return low;

    /* If x is greater than the last element, then return -1 */
    if(x > arr[high])
        return -1;

    /* get the index of middle element of arr[low..high]*/
    mid = (low + high)/2;  /* low + (high â€“ low)/2 */

    /* If x is same as middle element, then return mid */
    if(arr[mid] == x)
        return mid;

    /* If x is greater than arr[mid], then either arr[mid + 1]
      is ceiling of x, or ceiling lies in arr[mid+1...high] */
    if(arr[mid] < x)
    {
        if(mid + 1 <= high && x <= arr[mid+1])
            return mid + 1;
        else
```

```
        return ceilSearch(arr, mid+1, high, x);
    }

    /* If x is smaller than arr[mid], then either arr[mid]
       is ceiling of x or ceiling lies in arr[mid-1...high] */
    if (mid - 1 >= low && x > arr[mid-1])
        return mid;
    else
        return ceilSearch(arr, low, mid - 1, x);
}

/* Reverses arr[0..i] */
void flip(int arr[], int i)
{
    int temp, start = 0;
    while (start < i)
    {
        temp = arr[start];
        arr[start] = arr[i];
        arr[i] = temp;
        start++;
        i--;
    }
}

/* Function to sort an array using insertion sort, binary search and flip */
void insertionSort(int arr[], int size)
{
    int i, j;

    // Start from the second element and one by one insert arr[i]
    // in already sorted arr[0..i-1]
    for(i = 1; i < size; i++)
    {
        // Find the smallest element in arr[0..i-1] which is also greater than
        // or equal to arr[i]
        int j = ceilSearch(arr, 0, i-1, arr[i]);

        // Check if there was no element greater than arr[i]
        if (j != -1)
        {
            // Put arr[i] before arr[j] using following four flip operations
            flip(arr, j-1);
            flip(arr, i-1);
```

```
            flip(arr, i);
            flip(arr, j);
        }
    }
}

/* A utility function to print an array of size n */
void printArray(int arr[], int n)
{
    int i;
    for (i = 0; i < n; ++i)
        printf("%d ", arr[i]);
}

/* Driver program to test insertion sort */
int main()
{
    int arr[] = {18, 40, 35, 12, 30, 35, 20, 6, 90, 80};
    int n = sizeof(arr)/sizeof(arr[0]);
    insertionSort(arr, n);
    printArray(arr, n);
    return 0;
}
```

Output:

```
6 12 18 20 30 35 35 40 80 90
```

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

90. Tug of War

Given a set of n integers, divide the set in two subsets of n/2 sizes each such that the difference of the sum of two subsets is as minimum as possible. If n is even, then sizes of two subsets must be strictly n/2 and if n is odd, then size of one subset must be (n-1)/2 and size of other subset must be (n+1)/2.

For example, let given set be {3, 4, 5, -3, 100, 1, 89, 54, 23, 20}, the size of set is 10. Output for this set should be {4, 100, 1, 23, 20} and {3, 5, -3, 89, 54}. Both output subsets are of size 5 and sum of elements in both subsets is same (148 and 148). Let us consider another example where n is odd. Let given set be {23, 45, -34, 12, 0,

98, -99, 4, 189, -1, 4}. The output subsets should be {45, -34, 12, 98, -1} and {23, 0, -99, 4, 189, 4}. The sums of elements in two subsets are 120 and 121 respectively.

The following solution tries every possible subset of half size. If one subset of half size is formed, the remaining elements form the other subset. We initialize current set as empty and one by one build it. There are two possibilities for every element, either it is part of current set, or it is part of the remaining elements (other subset). We consider both possibilities for every element. When the size of current set becomes n/2, we check whether this solutions is better than the best solution available so far. If it is, then we update the best solution.

Following is C++ implementation for Tug of War problem. It prints the required arrays.

```
#include <iostream>
#include <stdlib.h>
#include <limits.h>
using namespace std;

// function that tries every possible solution by calling itself recursively
void TOWUtil(int* arr, int n, bool* curr_elements, int no_of_selected_elements,
             bool* soln, int* min_diff, int sum, int curr_sum, int curr_position)
{
    // checks whether the it is going out of bound
    if (curr_position == n)
        return;

    // checks that the numbers of elements left are not less than the
    // number of elements required to form the solution
    if ((n/2 - no_of_selected_elements) > (n - curr_position))
        return;

    // consider the cases when current element is not included in the solution
    TOWUtil(arr, n, curr_elements, no_of_selected_elements,
             soln, min_diff, sum, curr_sum, curr_position+1);

    // add the current element to the solution
    no_of_selected_elements++;
    curr_sum = curr_sum + arr[curr_position];
    curr_elements[curr_position] = true;

    // checks if a solution is formed
    if (no_of_selected_elements == n/2)
    {
        // checks if the solution formed is better than the best solution so far
        if (abs(sum/2 - curr_sum) < *min_diff)
```

```
        {
            *min_diff = abs(sum/2 - curr_sum);
            for (int i = 0; i<n; i++)
                soln[i] = curr_elements[i];
        }
    }
    else
    {
        // consider the cases where current element is included in the solution
        TOWUtil(arr, n, curr_elements, no_of_selected_elements, soln,
                min_diff, sum, curr_sum, curr_position+1);
    }

    // removes current element before returning to the caller of this function
    curr_elements[curr_position] = false;
}

// main function that generate an arr
void tugOfWar(int *arr, int n)
{
    // the boolen array that contains the inclusion and exclusion of an element
    // in current set. The number excluded automatically form the other set
    bool* curr_elements = new bool[n];

    // The inclusion/exclusion array for final solution
    bool* soln = new bool[n];

    int min_diff = INT_MAX;

    int sum = 0;
    for (int i=0; i<n; i++)
    {
        sum += arr[i];
        curr_elements[i] =  soln[i] = false;
    }

    // Find the solution using recursive function TOWUtil()
    TOWUtil(arr, n, curr_elements, 0, soln, &min_diff, sum, 0, 0);

    // Print the solution
    cout << "The first subset is: ";
    for (int i=0; i<n; i++)
    {
        if (soln[i] == true)
```

```
            cout << arr[i] << " ";
    }
    cout << "\nThe second subset is: ";
    for (int i=0; i<n; i++)
    {
        if (soln[i] == false)
            cout << arr[i] << " ";
    }
}

// Driver program to test above functions
int main()
{
    int arr[] = {23, 45, -34, 12, 0, 98, -99, 4, 189, -1, 4};
    int n = sizeof(arr)/sizeof(arr[0]);
    tugOfWar(arr, n);
    return 0;
}
```

Output:

```
The first subset is: 45 -34 12 98 -1
The second subset is: 23 0 -99 4 189 4
```

This article is compiled by Ashish Anand and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

91. Print Matrix Diagonally

Given a 2D matrix, print all elements of the given matrix in diagonal order. For example, consider the following 5 X 4 input matrix.

```
 1    2    3    4
 5    6    7    8
 9   10   11   12
13   14   15   16
17   18   19   20
```

Diagonal printing of the above matrix is

```
 1
 5    2
```

```
 9    6    3
13   10    7    4
17   14   11    8
18   15   12
19   16
20
```

Following is C++ code for diagonal printing.

The diagonal printing of a given matrix â€˜matrix[ROW][COL]â€™ always has â€˜ROW + COL â€“ 1â€² lines in output

```
#include <stdio.h>
#include <stdlib.h>

#define ROW 5
#define COL 4

// A utility function to find min of two integers
int min(int a, int b)
{ return (a < b)? a: b; }

// A utility function to find min of three integers
int min(int a, int b, int c)
{ return min(min(a, b), c);}

// A utility function to find max of two integers
int max(int a, int b)
{ return (a > b)? a: b; }

// The main function that prints given matrix in diagonal order
void diagonalOrder(int matrix[][COL])
{
    // There will be ROW+COL-1 lines in the output
    for (int line=1; line<=(ROW + COL -1); line++)
    {
        /* Get column index of the first element in this line of output.
           The index is 0 for first ROW lines and line - ROW for remaining
           lines  */
        int start_col =  max(0, line-ROW);

        /* Get count of elements in this line. The count of elements is
           equal to minimum of line number, COL-start_col and ROW */
         int count = min(line, (COL-start_col), ROW);
```

```
        /* Print elements of this line */
        for (int j=0; j<count; j++)
            printf("%5d ", matrix[min(ROW, line)-j-1][start_col+j]);

        /* Ptint elements of next diagonal on next line */
        printf("\n");
    }
}

// Utility function to print a matrix
void printMatrix(int matrix[ROW][COL])
{
    for (int i=0; i< ROW; i++)
    {
        for (int j=0; j<COL; j++)
            printf("%5d ", matrix[i][j]);
        printf("\n");
    }
}

// Driver program to test above functions
int main()
{
    int M[ROW][COL] = {{1, 2, 3, 4},
                       {5, 6, 7, 8},
                       {9, 10, 11, 12},
                       {13, 14, 15, 16},
                       {17, 18, 19, 20},
                      };
    printf ("Given matrix is \n");
    printMatrix(M);

    printf ("\nDiagonal printing of matrix is \n");
    diagonalOrder(M);
    return 0;
}
```

Output:

```
Given matrix is
    1     2     3     4
    5     6     7     8
    9    10    11    12
   13    14    15    16
```

```
 17   18   19   20

Diagonal printing of matrix is
 1
 5   2
 9   6   3
 13  10   7   4
 17  14   11   8
 18  15   12
 19  16
 20
```

This article is compiled by Ashish Anand and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

92. Divide and Conquer | Set 3 (Maximum Subarray Sum)

You are given a one dimensional array that may contain both positive and negative integers, find the sum of contiguous subarray of numbers which has the largest sum.

For example, if the given array is {-2, -5, **6, -2, -3, 1, 5**, -6}, then the maximum subarray sum is 7 (see highlighted elements).

**The naive method** is to run two loops. The outer loop picks the beginning element, the inner loop finds the maximum possible sum with first element picked by outer loop and compares this maximum with the overall maximum. Finally return the overall maximum. The time complexity of the Naive method is O(n^2).

Using **Divide and Conquer** approach, we can find the maximum subarray sum in O(nLogn) time. Following is the Divide and Conquer algorithm.

**1)** Divide the given array in two halves
**2)** Return the maximum of following three
â€¦.**a)** Maximum subarray sum in left half (Make a recursive call)
â€¦.**b)** Maximum subarray sum in right half (Make a recursive call)
â€¦.**c)** Maximum subarray sum such that the subarray crosses the midpoint

The lines 2.a and 2.b are simple recursive calls. How to find maximum subarray sum such that the subarray crosses the midpoint? We can easily find the crossing sum in linear time. The idea is simple, find the maximum sum starting from mid point and ending at some point on left of mid, then find the maximum sum starting from mid + 1 and ending with sum point on right of mid + 1. Finally, combine the two and return.

```
// A Divide and Conquer based program for maximum subarray sum problem
#include <stdio.h>
#include <limits.h>

// A utility funtion to find maximum of two integers
int max(int a, int b) { return (a > b)? a : b; }

// A utility funtion to find maximum of three integers
int max(int a, int b, int c) { return max(max(a, b), c); }

// Find the maximum possible sum in arr[] auch that arr[m] is part of it
int maxCrossingSum(int arr[], int l, int m, int h)
{
    // Include elements on left of mid.
    int sum = 0;
    int left_sum = INT_MIN;
    for (int i = m; i >= l; i--)
    {
        sum = sum + arr[i];
        if (sum > left_sum)
          left_sum = sum;
    }

    // Include elements on right of mid
    sum = 0;
    int right_sum = INT_MIN;
    for (int i = m+1; i <= h; i++)
    {
        sum = sum + arr[i];
        if (sum > right_sum)
          right_sum = sum;
    }

    // Return sum of elements on left and right of mid
    return left_sum + right_sum;
}

// Returns sum of maxium sum subarray in aa[l..h]
int maxSubArraySum(int arr[], int l, int h)
{
   // Base Case: Only one element
   if (l == h)
     return arr[l];
```

```
  // Find middle point
  int m = (l + h)/2;

  /* Return maximum of following three possible cases
     a) Maximum subarray sum in left half
     b) Maximum subarray sum in right half
     c) Maximum subarray sum such that the subarray crosses the midpoint */
  return max(maxSubArraySum(arr, l, m),
             maxSubArraySum(arr, m+1, h),
             maxCrossingSum(arr, l, m, h));
}

/*Driver program to test maxSubArraySum*/
int main()
{
  int arr[] = {2, 3, 4, 5, 7};
  int n = sizeof(arr)/sizeof(arr[0]);
  int max_sum = maxSubArraySum(arr, 0, n-1);
  printf("Maximum contiguous sum is %d\n", max_sum);
  getchar();
  return 0;
}
```

**Time Complexity:** maxSubArraySum() is a recursive method and time complexity can be expressed as following recurrence relation.
T(n) = 2T(n/2) + $\Theta(n)$
The above recurrence is similar to Merge Sort and can be solved either using Recurrence Tree method or Master method. It falls in case II of Master Method and solution of the recurrence is $\Theta(nLogn)$.

**The Kadaneâ€™s Algorithm** for this problem takes O(n) time. Therefore the Kadaneâ€™s algorithm is better than the Divide and Conquer approach, but this problem can be considered as a good example to show power of Divide and Conquer. The above simple approach where we divide the array in two halves, reduces the time complexity from O(n^2) to O(nLogn).

**References:**
Introduction to Algorithms by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

Counting sort is a sorting technique based on keys between a specific range. It works by counting the number of objects having distinct key values (kind of hashing). Then doing some arithmetic to calculate the position of each object in the output sequence.

Let us understand it with the help of an example.

```
For simplicity, consider the data in the range 0 to 9.
Input data: 1, 4, 1, 2, 7, 5, 2
 1) Take a count array to store the count of each unique object.
 Index:    0  1  2  3  4  5  6  7  8  9
 Count:    0  2  2  0   1  1  0  1  0  0

 2) Modify the count array such that each element at each index
 stores the sum of previous counts.
 Index:    0  1  2  3  4  5  6  7  8  9
 Count:    0  2  4  4  5  6  6  7  7  7

The modified count array indicates the position of each object in
the output sequence.

 3) Output each object from the input sequence followed by
 decreasing its count by 1.
 Process the input data: 1, 4, 1, 2, 7, 5, 2. Position of 1 is 2.
 Put data 1 at index 2 in output. Decrease count by 1 to place
 next data 1 at an index 1 smaller than this index.
```

Following is C implementation of counting sort.

```
// C Program for counting sort
#include <stdio.h>
#include <string.h>
#define RANGE 255

// The main function that sort the given string str in alphabatical order
void countSort(char *str)
{
   // The output character array that will have sorted str
 char output[strlen(str)];

 // Create a count array to store count of inidividul characters and
 // initialize count array as 0
 int count[RANGE + 1], i;
```

```
 memset(count, 0, sizeof(count));

   // Store count of each character
 for(i = 0; str[i]; ++i)
  ++count[str[i]];

   // Change count[i] so that count[i] now contains actual position of
   // this character in output array
 for (i = 1; i <= RANGE; ++i)
  count[i] += count[i-1];

   // Build the output character array
 for (i = 0; str[i]; ++i)
 {
  output[count[str[i]]-1] = str[i];
  --count[str[i]];
 }

 // Copy the output array to str, so that str now
 // contains sorted characters
 for (i = 0; str[i]; ++i)
  str[i] = output[i];
}

// Driver program to test above function
int main()
{
 char str[] = "geeksforgeeks";//"applepp";

 countSort(str);

 printf("Sorted string is %s\n", str);
 return 0;
}
```

Output:

```
Sorted character array is eeeefggkkorss
```

**Time Complexity:** O(n+k) where n is the number of elements in input array and k is the range of input.
**Auxiliary Space:** O(n+k)

**Points to be noted:**
**1.** Counting sort is efficient if the range of input data is not significantly greater than

the number of objects to be sorted. Consider the situation where the input sequence is between range 1 to 10K and the data is 10, 5, 10K, 5K.
**2.** It is not a comparison based sorting. It running time complexity is O(n) with space proportional to the range of data.
**3.** It is often used as a sub-routine to another sorting algorithm like radix sort.
**4.** Counting sort uses a partial hashing to count the occurrence of the data object in O(1).
**5.** Counting sort can be extended to work for negative inputs also.

**Exercise:**
**1.** Modify above code to sort the input data in the range from M to N.
**2.** Modify above code to sort negative input data.
**3.** Is counting sort stable and online?
**4.** Thoughts on parallelizing the counting sort algorithm.

This article is compiled by Aashish Barnwal. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

94. Merge Overlapping Intervals

Given a set of time intervals in any order, merge all overlapping intervals into one and output the result which should have only mutually exclusive intervals. Let the intervals be represented as pairs of integers for simplicity.
For example, let the given set of intervals be {{1,3}, {2,4}, {5,7}, {6,8} }. The intervals {1,3} and {2,4} overlap with each other, so they should be merged and become {1, 4}. Similarly {5, 7} and {6, 8} should be merged and become {5, 8}

Write a function which produces the set of merged intervals for the given set of intervals.

A **simple approach** is to start from the first interval and compare it with all other intervals for overlapping, if it overlaps with any other interval, then remove the other interval from list and merge the other into the first interval. Repeat the same steps for remaining intervals after first. This approach cannot be implemented in better than O(n^2) time.

An **efficient approach** is to first sort the intervals according to starting time. Once we have the sorted intervals, we can combine all intervals in a linear traversal. The idea is, in sorted array of intervals, if interval[i] doesnâ€™t overlap with interval[i-1], then interval[i+1] cannot overlap with interval[i-1] because starting time of interval[i+1] must be greater than or equal to interval[i]. Following is the detailed step by step algorithm.

**1.** Sort the intervals based on increasing order of starting time.
**2.** Push the first interval on to a stack.
**3.** For each interval do the following
â€¦â€¦**a.** If the current interval does not overlap with the stack top, push it.
â€¦â€¦**b.** If the current interval overlaps with stack top and ending time of current interval is more than that of stack top, update stack top with the ending time of current interval.
**4.** At the end stack contains the merged intervals.

Below is a C++ implementation of the above approach.

```
// A C++ program for merging overlapping intervals
#include <iostream>
#include <vector>
#include <algorithm>
#include <stack>
using namespace std;

// An interval has start time and end time
struct Interval
{
    int start;
    int end;
};

// Compares two intervals according to their staring time.
// This is needed for sorting the intervals using library
// function std::sort(). See http://goo.gl/iGspV
bool compareInterval(Interval i1, Interval i2)
{  return (i1.start < i2.start)? true: false; }

// The main function that takes a set of intervals, merges
// overlapping intervals and prints the result
void mergeIntervals(vector<Interval>& intervals)
{
    // Test if the given set has at least one interval
    if (intervals.size() <= 0)
        return;

    // Create an empty stack of intervals
    stack<Interval> s;

    // sort the intervals based on start time
    sort(intervals.begin(), intervals.end(), compareInterval);
```

```
    // push the first interval to stack
    s.push(intervals[0]);

    // Start from the next interval and merge if necessary
    for (int i = 1 ; i < intervals.size(); i++)
    {
        // get interval from stack top
        Interval top = s.top();

        // if current interval is not overlapping with stack top,
        // push it to the stack
        if (top.end < intervals[i].start)
        {
            s.push( intervals[i] );
        }
        // Otherwise update the ending time of top if ending of current
        // interval is more
        else if (top.end < intervals[i].end)
        {
            top.end = intervals[i].end;
            s.pop();
            s.push(top);
        }
    }

    // Print contents of stack
    cout << "\n The Merged Intervals are: ";
    while (!s.empty())
    {
        Interval t = s.top();
        cout << "[" << t.start << "," << t.end << "]" << " ";
        s.pop();
    }

    return;
}

// Functions to run test cases
void TestCase1()
{
    // Create a set of intervals
    Interval intvls[] = { {6,8}, {1,9}, {2,4}, {4,7} };
    vector<Interval> intervals(intvls, intvls+4);
```

```
    // Merge overlapping inervals and print result
    mergeIntervals(intervals);
}
void TestCase2()
{
    // Create a set of intervals
    Interval intvls[] = { {6,8},{1,3},{2,4},{4,7} };
    vector<Interval> intervals(intvls, intvls+4);

    // Merge overlapping inervals and print result
    mergeIntervals(intervals);
}
void TestCase3()
{
    // Create a set of intervals
    Interval intvls[] = { {1,3},{7,9},{4,6},{10,13} };
    vector<Interval> intervals(intvls, intvls+4);

    // Merge overlapping inervals and print result
    mergeIntervals(intervals);
}

// Driver program to test above functions
int main()
{
    TestCase1();
    TestCase2();
    TestCase3();
    return 0;
}
```

Output:

```
 The Merged Intervals are: [1,9]
 The Merged Intervals are: [1,8]
 The Merged Intervals are: [10,13] [7,9] [4,6] [1,3]
```

Time complexity of the method is O(nLogn) which is for sorting. Once the array of intervals is sorted, merging takes linear time.

This article is compiled by Ravi Chandra Enaganti. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

95. Find the maximum repeating number in O(n) time and O(1) extra space

Given an array of size *n*, the array contains numbers in range from 0 to *k-1* where *k* is a positive integer and *k <= n.* Find the maximum repeating number in this array. For example, let *k* be 10 the given array be *arr[]* = {1, 2, 2, 2, 0, 2, 0, 2, 3, 8, 0, 9, 2, 3}, the maximum repeating number would be 2. Expected time complexity is *O(n)* and extra space allowed is *O(1)*. Modifications to array are allowed.

The **naive approach** is to run two loops, the outer loop picks an element one by one, the inner loop counts number of occurrences of the picked element. Finally return the element with maximum count. Time complexity of this approach is *O(n^2)*.

A **better approach** is to create a count array of size k and initialize all elements of *count[]* as 0. Iterate through all elements of input array, and for every element *arr[i]*, increment *count[arr[i]]*. Finally, iterate through *count[]* and return the index with maximum value. This approach takes O(n) time, but requires O(k) space.

Following is the ***O(n)*** **time and** ***O(1)*** **extra space** approach. Let us understand the approach with a simple example where *arr[]* = {2, 3, 3, 5, 3, 4, 1, 7}, *k* = 8, *n* = 8 (number of elements in arr[]).

**1)** Iterate though input array *arr[]*, for every element *arr[i]*, increment *arr[arr[i]%k]* by *k* (*arr[]* becomes {2, 11, 11, 29, 11, 12, 1, 15 })

**2)** Find the maximum value in the modified array (maximum value is 29). Index of the maximum value is the maximum repeating element (index of 29 is 3).

**3)** If we want to get the original array back, we can iterate through the array one more time and do *arr[i] = arr[i] % k* where *i* varies from 0 to *n-1*.

*How does the above algorithm work?* Since we use *arr[i]%k* as index and add value *k* at the index *arr[i]%k*, the index which is equal to maximum repeating element will have the maximum value in the end. Note that *k* is added maximum number of times at the index equal to maximum repeating element and all array elements are smaller than *k.*

Following is C++ implementation of the above algorithm.

```
#include<iostream>
using namespace std;

// Returns maximum repeating element in arr[0..n-1].
// The array elements are in range from 0 to k-1
int maxRepeating(int* arr, int n, int k)
```

```
{
    // Iterate though input array, for every element
    // arr[i], increment arr[arr[i]%k] by k
    for (int i = 0; i< n; i++)
        arr[arr[i]%k] += k;

    // Find index of the maximum repeating element
    int max = arr[0], result = 0;
    for (int i = 1; i < n; i++)
    {
        if (arr[i] > max)
        {
            max = arr[i];
            result = i;
        }
    }

    /* Uncomment this code to get the original array back
       for (int i = 0; i< n; i++)
          arr[i] = arr[i]%k; */

    // Return index of the maximum element
    return result;
}

// Driver program to test above function
int main()
{
    int arr[] = {2, 3, 3, 5, 3, 4, 1, 7};
    int n = sizeof(arr)/sizeof(arr[0]);
    int k = 8;

    cout << "The maximum repeating number is " <<
         maxRepeating(arr, n, k) << endl;

    return 0;
}
```

Output:

```
The maximum repeating number is 3
```

**Exercise:**

The above solution prints only one repeating element and doesnâ€™t work if we want to print all maximum repeating elements. For example, if the input array is {2, 3, 2, 3},

the above solution will print only 3. What if we need to print both of 2 and 3 as both of them occur maximum number of times. Write a O(n) time and O(1) extra space function that prints all maximum repeating elements. (Hint: We can use maximum quotient arr[i]/n instead of maximum value in step 2).

Note that the above solutions may cause overflow if adding k repeatedly makes the value more than INT_MAX.

This article is compiled by Ashish Anand and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

96. Stock Buy Sell to Maximize Profit

The cost of a stock on each day is given in an array, find the max profit that you can make by buying and selling in those days. For example, if the given array is {100, 180, 260, 310, 40, 535, 695}, the maximum profit can earned by buying on day 0, selling on day 3. Again buy on day 4 and sell on day 6. If the given array of prices is sorted in decreasing order, then profit cannot be earned at all.

If we are allowed to buy and sell only once, then we can use following algorithm. Maximum difference between two elements. Here we are allowed to buy and sell multiple times. Following is algorithm for this problem.
**1.** Find the local minima and store it as starting index. If not exists, return.
**2.** Find the local maxima. and store it as ending index. If we reach the end, set the end as ending index.
**3.** Update the solution (Increment count of buy sell pairs)
**4.** Repeat the above steps if end is not reached.

```
// Program to find best buying and selling days
#include <stdio.h>

// solution structure
struct Interval
{
    int buy;
    int sell;
};

// This function finds the buy sell schedule for maximum profit
void stockBuySell(int price[], int n)
{
```

```
// Prices must be given for at least two days
if (n == 1)
    return;

int count = 0; // count of solution pairs

// solution vector
Interval sol[n/2 + 1];

// Traverse through given price array
int i = 0;
while (i < n-1)
{
    // Find Local Minima. Note that the limit is (n-2) as we are
    // comparing present element to the next element.
    while ((i < n-1) && (price[i+1] <= price[i]))
        i++;

    // If we reached the end, break as no further solution possible
    if (i == n-1)
        break;

    // Store the index of minima
    sol[count].buy = i++;

    // Find Local Maxima.  Note that the limit is (n-1) as we are
    // comparing to previous element
    while ((i < n) && (price[i] >= price[i-1]))
        i++;

    // Store the index of maxima
    sol[count].sell = i-1;

    // Increment count of buy/sell pairs
    count++;
}

// print solution
if (count == 0)
    printf("There is no day when buying the stock will make profit\n");
else
{
   for (int i = 0; i < count; i++)
      printf("Buy on day: %d\t Sell on day: %d\n", sol[i].buy, sol[i].sell);
```

```
    }

    return;
}

// Driver program to test above functions
int main()
{
    // stock prices on consecutive days
    int price[] = {100, 180, 260, 310, 40, 535, 695};
    int n = sizeof(price)/sizeof(price[0]);

    // fucntion call
    stockBuySell(price, n);

    return 0;
}
```

Output:

```
Buy on day : 0   Sell on day: 3
Buy on day : 4   Sell on day: 6
```

**Time Complexity:** The outer loop runs till i becomes n-1. The inner two loops increment value of i in every iteration. So overall time complexity is O(n)

This article is compiled by Ashish Anand and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

97. Rearrange positive and negative numbers in O(n) time and O(1) extra space

An array contains both positive and negative numbers in random order. Rearrange the array elements so that positive and negative numbers are placed alternatively. Number of positive and negative numbers need not be equal. If there are more positive numbers they appear at the end of the array. If there are more negative numbers, they too appear in the end of the array.

For example, if the input array is [-1, 2, -3, 4, 5, 6, -7, 8, 9], then the output should be [9, -7, 8, -3, 5, -1, 2, 4, 6]

The solution is to first separate positive and negative numbers using partition process of QuickSort. In the partition process, consider 0 as value of pivot element so that all

negative numbers are placed before positive numbers. Once negative and positive numbers are separated, we start from the first negative number and first positive number, and swap every alternate negative number with next positive number.

```
// A C++ program to put positive numbers at even indexes (0, 2, 4,..)
// and negative numbers at odd indexes (1, 3, 5, ..)
#include <stdio.h>

// prototype for swap
void swap(int *a, int *b);

// The main function that rearranges elements of given array.  It puts
// positive elements at even indexes (0, 2, ..) and negative numbers at
// odd indexes (1, 3, ..).
void rearrange(int arr[], int n)
{
   // The following few lines are similar to partition process
   // of QuickSort.  The idea is to consider 0 as pivot and
   // divide the array around it.
   int i = -1;
   for (int j = 0; j < n; j++)
   {
      if (arr[j] < 0)
      {
         i++;
         swap(&arr[i], &arr[j]);
      }
   }

   // Now all positive numbers are at end and negative numbers at
   // the beginning of array. Initialize indexes for starting point
   // of positive and negative numbers to be swapped
   int pos = i+1, neg = 0;

   // Increment the negative index by 2 and positive index by 1, i.e.,
   // swap every alternate negative number with next positive number
   while (pos < n && neg < pos && arr[neg] < 0)
   {
      swap(&arr[neg], &arr[pos]);
      pos++;
      neg += 2;
   }
}
```

```
// A utility function to swap two elements
void swap(int *a, int *b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// A utility function to print an array
void printArray(int arr[], int n)
{
    for (int i = 0; i < n; i++)
        printf("%4d ", arr[i]);
}

// Driver program to test above functions
int main()
{
    int arr[] = {-1, 2, -3, 4, 5, 6, -7, 8, 9};
    int n = sizeof(arr)/sizeof(arr[0]);
    rearrange(arr, n);
    printArray(arr, n);
    return 0;
}
```

Output:

```
   4   -3    5   -1    6   -7    2    8    9
```

**Time Complexity:** O(n) where n is number of elements in given array.
**Auxiliary Space:** O(1)

Note that the partition process changes relative order of elements.

This article is compiled by **Abhay Rathi**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.
If you like GeeksforGeeks and would like to contribute, you can also write an article and mail your article to contribute@geeksforgeeks.org. See your article appearing on the GeeksforGeeks main page and help other Geeks.

Â  Â

Given an array of integers, sort the array according to frequency of elements. For example, if the input array is {2, 3, 2, 4, 5, 12, 2, 3, 3, 3, 12}, then modify the array to {3, 3, 3, 3, 2, 2, 2, 12, 12, 4, 5}.
In the previous post, we have discussed all methods for sorting according to frequency. In this post, method 2 is discussed in detail and C++ implementation for the same is provided.

Following is detailed algorithm.
**1)** Create a BST and while creating BST maintain the count i,e frequency of each coming element in same BST. This step may take O(nLogn) time if a self balancing BST is used.
**2)** Do Inorder traversal of BST and store every element and count of each element in an auxiliary array. Let us call the auxiliary array as â€˜count[]â€™. Note that every element of this array is element and frequency pair. This step takes O(n) time.
**3)** Sort â€˜count[]â€™ according to frequency of the elements. This step takes O(nLohn) time if a O(nLogn) sorting algorithm is used.
**4)** Traverse through the sorted array â€˜count[]â€™. For each element x, print it â€˜freqâ€™ times where â€˜freqâ€™ is frequency of x. This step takes O(n) time.

Overall time complexity of the algorithm can be minimum O(nLogn) if we use a O(nLogn) sorting algorithm and use a self balancing BST with O(Logn) insert operation.

Following is C++ implementation of the above algorithm.

```
// Implementation of above algorithm in C++.
#include <iostream>
#include <stdlib.h>
using namespace std;

/* A BST node has data, freq, left and right pointers */
struct BSTNode
{
    struct BSTNode *left;
    int data;
    int freq;
    struct BSTNode *right;
};

// A structure to store data and its frequency
struct dataFreq
{
    int data;
    int freq;
```

```
};

/* Function for qsort() implementation. Compare frequencies to
   sort the array according to decreasing order of frequency */
int compare(const void *a, const void *b)
{
    return ( (*(const dataFreq*)b).freq - (*(const dataFreq*)a).freq );
}

/* Helper function that allocates a new node with the given data,
   frequency as 1 and NULL left and right  pointers.*/
BSTNode* newNode(int data)
{
    struct BSTNode* node = new BSTNode;
    node->data = data;
    node->left = NULL;
    node->right = NULL;
    node->freq = 1;
    return (node);
}

// A utility function to insert a given key to BST. If element
// is already present, then increases frequency
BSTNode *insert(BSTNode *root, int data)
{
    if (root == NULL)
        return newNode(data);
    if (data == root->data) // If already present
        root->freq += 1;
    else if (data < root->data)
        root->left = insert(root->left, data);
    else
        root->right = insert(root->right, data);
    return root;
}

// Function to copy elements and their frequencies to count[].
void store(BSTNode *root, dataFreq count[], int *index)
{
    // Base Case
    if (root == NULL) return;

    // Recur for left substree
    store(root->left, count, index);
```

```
    // Store item from root and increment index
    count[(*index)].freq = root->freq;
    count[(*index)].data = root->data;
    (*index)++;

    // Recur for right subtree
    store(root->right, count, index);
}

// The main function that takes an input array as an argument
// and sorts the array items according to frequency
void sortByFrequency(int arr[], int n)
{
    // Create an empty BST and insert all array items in BST
    struct BSTNode *root = NULL;
    for (int i = 0; i < n; ++i)
        root = insert(root, arr[i]);

    // Create an auxiliary array 'count[]' to store data and
    // frequency pairs. The maximum size of this array would
    // be n when all elements are different
    dataFreq count[n];
    int index = 0;
    store(root, count, &index);

    // Sort the count[] array according to frequency (or count)
    qsort(count, index, sizeof(count[0]), compare);

    // Finally, traverse the sorted count[] array and copy the
    // i'th item 'freq' times to original array 'arr[]'
    int j = 0;
    for (int i = 0; i < index; i++)
    {
        for (int freq = count[i].freq; freq > 0; freq--)
            arr[j++] = count[i].data;
    }
}

// A utility function to print an array of size n
void printArray(int arr[], int n)
{
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";
```

```
    cout << endl;
}

/* Driver program to test above functions */
int main()
{
    int arr[] = {2, 3, 2, 4, 5, 12, 2, 3, 3, 3, 12};
    int n = sizeof(arr)/sizeof(arr[0]);
    sortByFrequency(arr, n);
    printArray(arr, n);
    return 0;
}
```

Output:

```
3 3 3 3 2 2 2 12 12 5 4
```

**Exercise:**
The above implementation doesnâ€™t guarantee original order of elements with same frequency (for example, 4 comes before 5 in input, but 4 comes after 5 in output). Extend the implementation to maintain original order. For example, if two elements have same frequency then print the one which came 1st in input array.

This article is compiled by **Chandra Prakash**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

Given an array of integers, sort the array according to frequency of elements. For example, if the input array is {2, 3, 2, 4, 5, 12, 2, 3, 3, 3, 12}, then modify the array to {3, 3, 3, 3, 2, 2, 2, 12, 12, 4, 5}.
In the previous post, we have discussed all methods for sorting according to frequency. In this post, method 2 is discussed in detail and C++ implementation for the same is provided.

Following is detailed algorithm.
**1)** Create a BST and while creating BST maintain the count i,e frequency of each coming element in same BST. This step may take O(nLogn) time if a self balancing BST is used.
**2)** Do Inorder traversal of BST and store every element and count of each element in an auxiliary array. Let us call the auxiliary array as â€˜count[]â€™. Note that every element of this array is element and frequency pair. This step takes O(n) time.
**3)** Sort â€˜count[]â€™ according to frequency of the elements. This step takes

O(nLohn) time if a O(nLogn) sorting algorithm is used.
**4)** Traverse through the sorted array â€˜count[]â€™. For each element x, print it â€˜freqâ€™ times where â€˜freqâ€™ is frequency of x. This step takes O(n) time.

Overall time complexity of the algorithm can be minimum O(nLogn) if we use a O(nLogn) sorting algorithm and use a self balancing BST with O(Logn) insert operation.

Following is C++ implementation of the above algorithm.

```
// Implementation of above algorithm in C++.
#include <iostream>
#include <stdlib.h>
using namespace std;

/* A BST node has data, freq, left and right pointers */
struct BSTNode
{
    struct BSTNode *left;
    int data;
    int freq;
    struct BSTNode *right;
};

// A structure to store data and its frequency
struct dataFreq
{
    int data;
    int freq;
};

/* Function for qsort() implementation. Compare frequencies to
   sort the array according to decreasing order of frequency */
int compare(const void *a, const void *b)
{
    return ( (*(const dataFreq*)b).freq - (*(const dataFreq*)a).freq );
}

/* Helper function that allocates a new node with the given data,
   frequency as 1 and NULL left and right  pointers.*/
BSTNode* newNode(int data)
{
    struct BSTNode* node = new BSTNode;
    node->data = data;
    node->left = NULL;
```

```
    node->right = NULL;
    node->freq = 1;
    return (node);
}

// A utility function to insert a given key to BST. If element
// is already present, then increases frequency
BSTNode *insert(BSTNode *root, int data)
{
    if (root == NULL)
        return newNode(data);
    if (data == root->data) // If already present
        root->freq += 1;
    else if (data < root->data)
        root->left = insert(root->left, data);
    else
        root->right = insert(root->right, data);
    return root;
}

// Function to copy elements and their frequencies to count[].
void store(BSTNode *root, dataFreq count[], int *index)
{
    // Base Case
    if (root == NULL) return;

    // Recur for left substree
    store(root->left, count, index);

    // Store item from root and increment index
    count[(*index)].freq = root->freq;
    count[(*index)].data = root->data;
    (*index)++;

    // Recur for right subtree
    store(root->right, count, index);
}

// The main function that takes an input array as an argument
// and sorts the array items according to frequency
void sortByFrequency(int arr[], int n)
{
    // Create an empty BST and insert all array items in BST
    struct BSTNode *root = NULL;
```

```
    for (int i = 0; i < n; ++i)
        root = insert(root, arr[i]);

    // Create an auxiliary array 'count[]' to store data and
    // frequency pairs. The maximum size of this array would
    // be n when all elements are different
    dataFreq count[n];
    int index = 0;
    store(root, count, &index);

    // Sort the count[] array according to frequency (or count)
    qsort(count, index, sizeof(count[0]), compare);

    // Finally, traverse the sorted count[] array and copy the
    // i'th item 'freq' times to original array 'arr[]'
    int j = 0;
    for (int i = 0; i < index; i++)
    {
        for (int freq = count[i].freq; freq > 0; freq--)
            arr[j++] = count[i].data;
    }
}

// A utility function to print an array of size n
void printArray(int arr[], int n)
{
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";
    cout << endl;
}

/* Driver program to test above functions */
int main()
{
    int arr[] = {2, 3, 2, 4, 5, 12, 2, 3, 3, 3, 12};
    int n = sizeof(arr)/sizeof(arr[0]);
    sortByFrequency(arr, n);
    printArray(arr, n);
    return 0;
}
```

Output:

```
3 3 3 3 2 2 2 12 12 5 4
```

**Exercise:**
The above implementation doesnâ€™t guarantee original order of elements with same frequency (for example, 4 comes before 5 in input, but 4 comes after 5 in output). Extend the implementation to maintain original order. For example, if two elements have same frequency then print the one which came 1st in input array.

This article is compiled by **Chandra Prakash**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

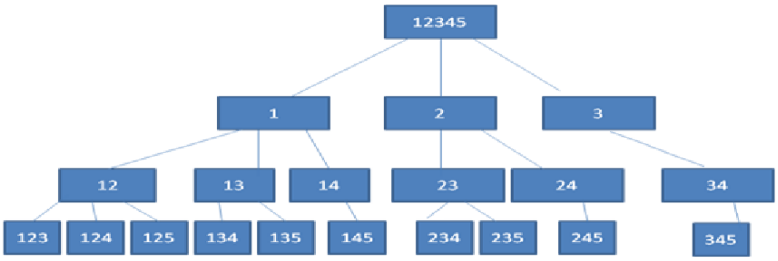
100. Print all possible combinations of r elements in a given array of size n

Given an array of size n, generate and print all possible combinations of r elements in array. For example, if input array is {1, 2, 3, 4} and r is 2, then output should be {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4} and {3, 4}.

Following are two methods to do this.

**Method 1 (Fix Elements and Recur)**
We create a temporary array â€˜data[]â€™ which stores all outputs one by one. The idea is to start from first index (index = 0) in data[], one by one fix elements at this index and recur for remaining indexes. Let the input array be {1, 2, 3, 4, 5} and r be 3. We first fix 1 at index 0 in data[], then recur for remaining indexes, then we fix 2 at index 0 and recur. Finally, we fix 3 and recur for remaining indexes. When number of elements in data[] becomes equal to r (size of a combination), we print data[].

Following diagram shows recursion tree for same input.




Following is C++ implementation of above approach.

```
// Program to print all combination of size r in an array of size n
#include <stdio.h>
void combinationUtil(int arr[], int data[], int start, int end, int index, int r);

// The main function that prints all combinations of size r
// in arr[] of size n. This function mainly uses combinationUtil()
```

```
void printCombination(int arr[], int n, int r)
{
    // A temporary array to store all combination one by one
    int data[r];

    // Print all combination using temprary array 'data[]'
    combinationUtil(arr, data, 0, n-1, 0, r);
}

/* arr[]  ---> Input Array
   data[] ---> Temporary array to store current combination
   start & end ---> Staring and Ending indexes in arr[]
   index  ---> Current index in data[]
   r ---> Size of a combination to be printed */
void combinationUtil(int arr[], int data[], int start, int end, int index, int r)
{
    // Current combination is ready to be printed, print it
    if (index == r)
    {
        for (int j=0; j<r; j++)
            printf("%d ", data[j]);
        printf("\n");
        return;
    }

    // replace index with all possible elements. The condition
    // "end-i+1 >= r-index" makes sure that including one element
    // at index will make a combination with remaining elements
    // at remaining positions
    for (int i=start; i<=end && end-i+1 >= r-index; i++)
    {
        data[index] = arr[i];
        combinationUtil(arr, data, i+1, end, index+1, r);
    }
}

// Driver program to test above functions
int main()
{
    int arr[] = {1, 2, 3, 4, 5};
    int r = 3;
    int n = sizeof(arr)/sizeof(arr[0]);
    printCombination(arr, n, r);
}
```

Output:

```
1 2 3
1 2 4
1 2 5
1 3 4
1 3 5
1 4 5
2 3 4
2 3 5
2 4 5
3 4 5
```

*How to handle duplicates?*
Note that the above method doesnâ€™t handle duplicates. For example, if input array is {1, 2, 1} and r is 2, then the program prints {1, 2} and {2, 1} as two different combinations. We can avoid duplicates by adding following two additional things to above code.
1) Add code to sort the array before calling combinationUtil() in printCombination()
2) Add following lines at the end of for loop in combinationUtil()

```
    // Since the elements are sorted, all occurrences of an element
    // must be together
    while (arr[i] == arr[i+1])
        i++;
```

See **this** for an implementation that handles duplicates.

**Method 2 (Include and Exclude every element)**
Like the above method, We create a temporary array data[]. The idea here is similar to Subset Sum Problem. We one by one consider every element of input array, and recur for two cases:

1) The element is included in current combination (We put the element in data[] and increment next available index in data[])
2) The element is excluded in current combination (We do not put the element and do not change index)

When number of elements in data[] become equal to r (size of a combination), we print it.

This method is mainly based on Pascalâ€™s Identity, i.e. $\mathbf{n_{c_r} = n\text{-}1_{c_r} + n\text{-}1_{c_{r-1}}}$

Following is C++ implementation of method 2.

```
// Program to print all combination of size r in an array of size n
```

```
#include<stdio.h>
void combinationUtil(int arr[],int n,int r,int index,int data[],int i);

// The main function that prints all combinations of size r
// in arr[] of size n. This function mainly uses combinationUtil()
void printCombination(int arr[], int n, int r)
{
    // A temporary array to store all combination one by one
    int data[r];

    // Print all combination using temprary array 'data[]'
    combinationUtil(arr, n, r, 0, data, 0);
}

/* arr[]  ---> Input Array
   n      ---> Size of input array
   r      ---> Size of a combination to be printed
   index  ---> Current index in data[]
   data[] ---> Temporary array to store current combination
   i      ---> index of current element in arr[]     */
void combinationUtil(int arr[], int n, int r, int index, int data[], int i)
{
    // Current cobination is ready, print it
    if (index == r)
    {
        for (int j=0; j<r; j++)
            printf("%d ",data[j]);
        printf("\n");
        return;
    }

    // When no more elements are there to put in data[]
    if (i >= n)
        return;

    // current is included, put next at next location
    data[index] = arr[i];
    combinationUtil(arr, n, r, index+1, data, i+1);

    // current is excluded, replace it with next (Note that
    // i+1 is passed, but index is not changed)
    combinationUtil(arr, n, r, index, data, i+1);
}
```

```
// Driver program to test above functions
int main()
{
    int arr[] = {1, 2, 3, 4, 5};
    int r = 3;
    int n = sizeof(arr)/sizeof(arr[0]);
    printCombination(arr, n, r);
    return 0;
}
```

Output:

```
1 2 3
1 2 4
1 2 5
1 3 4
1 3 5
1 4 5
2 3 4
2 3 5
2 4 5
3 4 5
```

*How to handle duplicates in method 2?*
Like method 1, we can following two things to handle duplicates.
1) Add code to sort the array before calling combinationUtil() in printCombination()
2) Add following lines between two recursive calls of combinationUtil() in combinationUtil()

```
    // Since the elements are sorted, all occurrences of an element
    // must be together
    while (arr[i] == arr[i+1])
        i++;
```

See **this** for an implementation that handles duplicates.

This article is contributed by **Bateesh**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

101. Given an array of of size n and a number k, find all elements that appear more than n/k times

Given an array of size n, find all elements in array that appear more than n/k times. For example, if the input arrays is {3, 1, 2, 2, 1, 2, 3, 3} and k is 4, then the output

should be [2, 3]. Note that size of array is 8 (or n = 8), so we need to find all elements that appear more than 2 (or 8/4) times. There are two elements that appear more than two times, 2 and 3.

A **simple method** is to pick all elements one by one. For every picked element, count its occurrences by traversing the array, if count becomes more than n/k, then print the element. Time Complexity of this method would be $O(n^2)$.

A better solution is to **use sorting**. First, sort all elements using a O(nLogn) algorithm. Once the array is sorted, we can find all required elements in a linear scan of array. So overall time complexity of this method is O(nLogn) + O(n) which is O(nLogn).

Following is an interesting **O(nk) solution:**
We can solve the above problem in O(nk) time using O(k-1) extra space. Note that there can never be more than k-1 elements in output (Why?). There are mainly three steps in this algorithm.

**1)** Create a temporary array of size (k-1) to store elements and their counts (The output elements are going to be among these k-1 elements). Following is structure of temporary array elements.

```
struct eleCount {
   int element;
   int count;
};
struct eleCount temp[];
```

This step takes O(k) time.

**2)** Traverse through the input array and update temp[] (add/remove an element or increase/decrease count) for every traversed element. The array temp[] stores potential (k-1) candidates at every step. This step takes O(nk) time.

**3)** Iterate through final (k-1) potential candidates (stored in temp[]). or every element, check if it actually has count more than n/k. This step takes O(nk) time.

The main step is step 2, how to maintain (k-1) potential candidates at every point? The steps used in step 2 are like famous game: Tetris. We treat each number as a piece in Tetris, which falls down in our temporary array temp[]. Our task is to try to keep the same number stacked on the same column (count in temporary array is incremented).

```
Consider k = 4, n = 9
Given array: 3 1 2 2 2 1 4 3 3


i = 0
        3 _ _
temp[] has one element, 3 with count 1
```

```
i = 1
        3 1 _
temp[] has two elements, 3 and 1 with
counts 1 and 1 respectively

i = 2
        3 1 2
temp[] has three elements, 3, 1 and 2 with
counts as 1, 1 and 1 respectively.

i = 3
        - - 2
        3 1 2
temp[] has three elements, 3, 1 and 2 with
counts as 1, 1 and 2 respectively.

i = 4
        - - 2
        - - 2
        3 1 2
temp[] has three elements, 3, 1 and 2 with
counts as 1, 1 and 3 respectively.

i = 5
        - - 2
        - 1 2
        3 1 2
temp[] has three elements, 3, 1 and 2 with
counts as 1, 2 and 3 respectively.
```

Now the question arises, what to do when temp[] is full and we see a new element â€“ we remove the bottom row from stacks of elements, i.e., we decrease count of every element by 1 in temp[]. We ignore the current element.

```
i = 6
        - - 2
        - 1 2
temp[] has two elements, 1 and 2 with
counts as 1 and 2 respectively.

i = 7
         - 2
        3 1 2
```

```
temp[] has three elements, 3, 1 and 2 with
counts as 1, 1 and 2 respectively.

i = 8
        3 - 2
        3 1 2
temp[] has three elements, 3, 1 and 2 with
counts as 2, 1 and 2 respectively.
```

Finally, we have at most k-1 numbers in temp[]. The elements in temp are {3, 1, 2}. Note that the counts in temp[] are useless now, the counts were needed only in step 2. Now we need to check whether the actual counts of elements in temp[] are more than n/k (9/4) or not. The elements 3 and 2 have counts more than 9/4. So we print 3 and 2.

Note that the algorithm doesnâ€™t miss any output element. There can be two possibilities, many occurrences are together or spread across the array. If occurrences are together, then count will be high and wonâ€™t become 0. If occurrences are spread, then the element would come again in temp[]. Following is C++ implementation of above algorithm.

```
// A C++ program to print elements with count more than n/k
#include<iostream>
using namespace std;

// A structure to store an element and its current count
struct eleCount
{
   int e;  // Element
   int c;  // Count
};

// Prints elements with more than n/k occurrences in arr[] of
// size n. If there are no such elements, then it prints nothing.
void moreThanNdK(int arr[], int n, int k)
{
   // k must be greater than 1 to get some output
   if (k < 2)
      return;

   /* Step 1: Create a temporary array (contains element
      and count) of size k-1. Initialize count of all
      elements as 0 */
   struct eleCount temp[k-1];
   for (int i=0; i<k-1; i++)
       temp[i].c = 0;
```

```
/* Step 2: Process all elements of input array */
for (int i = 0; i < n; i++)
{
    int j;

    /* If arr[i] is already present in
       the element count array, then increment its count */
    for (j=0; j<k-1; j++)
    {
        if (temp[j].e == arr[i])
        {
            temp[j].c += 1;
            break;
        }
    }

    /* If arr[i] is not present in temp[] */
    if (j == k-1)
    {
        int l;

        /* If there is position available in temp[], then place
          arr[i] in the first available position and set count as 1*/
        for (l=0; l<k-1; l++)
        {
            if (temp[l].c == 0)
            {
                temp[l].e = arr[i];
                temp[l].c = 1;
                break;
            }
        }

        /* If all the position in the temp[] are filled, then
           decrease count of every element by 1 */
        if (l == k-1)
            for (l=0; l<k; l++)
                temp[l].c -= 1;
    }
}

/*Step 3: Check actual counts of potential candidates in temp[]*/
for (int i=0; i<k-1; i++)
```

```
    {
        // Calculate actual count of elements
        int ac = 0;  // actual count
        for (int j=0; j<n; j++)
            if (arr[j] == temp[i].e)
                ac++;

        // If actual count is more than n/k, then print it
        if (ac > n/k)
            cout << "Number:" << temp[i].e
                 << " Count:" << ac << endl;
    }
}

/* Driver program to test above function */
int main()
{
    cout << "First Test\n";
    int arr1[] = {4, 5, 6, 7, 8, 4, 4};
    int size = sizeof(arr1)/sizeof(arr1[0]);
    int k = 3;
    moreThanNdK(arr1, size, k);

    cout << "\nSecond Test\n";
    int arr2[] = {4, 2, 2, 7};
    size = sizeof(arr2)/sizeof(arr2[0]);
    k = 3;
    moreThanNdK(arr2, size, k);

    cout << "\nThird Test\n";
    int arr3[] = {2, 7, 2};
    size = sizeof(arr3)/sizeof(arr3[0]);
    k = 2;
    moreThanNdK(arr3, size, k);

    cout << "\nFourth Test\n";
    int arr4[] = {2, 3, 3, 2};
    size = sizeof(arr4)/sizeof(arr4[0]);
    k = 3;
    moreThanNdK(arr4, size, k);

    return 0;
}
```

Output:

```
First Test
Number:4 Count:3

Second Test
Number:2 Count:2

Third Test
Number:2 Count:2

Fourth Test
Number:2 Count:2
Number:3 Count:2
```

Time Complexity: O(nk)
Auxiliary Space: O(k)

Generally asked variations of this problem are, find all elements that appear n/3 times or n/4 times in O(n) time complexity and O(1) extra space.

**Hashing** can also be an efficient solution. With a good hash function, we can solve the above problem in O(n) time on average. Extra space required hashing would be higher than O(k). Also, hashing cannot be used to solve above variations with O(1) extra space.

**Exercise:**
The above problem can be solved in O(nLogk) time with the help of more appropriate data structures than array for auxiliary storage of k-1 elements. Suggest a O(nLogk) approach.

This article is contributed by **Kushagra Jaiswal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

102. Unbounded Binary Search Example (Find the point where a monotonically increasing function becomes positive first time)

Given a function â€˜int f(unsigned int x)â€™ which takes a **non-negative integer** â€˜xâ€™ as input and returns an **integer** as output. The function is monotonically increasing with respect to value of x, i.e., the value of f(x+1) is greater than f(x) for every input x. Find the value â€˜nâ€™ where f() becomes positive for the first time. Since f() is monotonically increasing, values of f(n+1), f(n+2),â€¦ must be positive and

values of f(n-2), f(n-3), .. must be negative.
Find n in O(logn) time, you may assume that f(x) can be evaluated in O(1) time for any input x.

A **simple solution** is to start from i equals to 0 and one by one calculate value of f(i) for 1, 2, 3, 4 .. etc until we find a positive f(i). This works, but takes O(n) time.

**Can we apply Binary Search to find n in O(Logn) time?** We canâ€™t directly apply Binary Search as we donâ€™t have an upper limit or high index. The idea is to do repeated doubling until we find a positive value, i.e., check values of f() for following values until f(i) becomes positive.

```
f(0)
f(1)
f(2)
f(4)
f(8)
f(16)
f(32)
....
....
f(high)
```

Let 'high' be the value of i when f() becomes positive for first time.

Can we apply Binary Search to find n after finding â€˜highâ€™? We can apply Binary Search now, we can use â€˜high/2â€² as low and â€˜highâ€™ as high indexes in binary search. The result n must lie between â€˜high/2â€² and â€˜highâ€™.

Number of steps for finding â€˜highâ€™ is O(Logn). So we can find â€˜highâ€™ in O(Logn) time. What about time taken by Binary Search between high/2 and high? The value of â€˜highâ€™ must be less than 2*n. The number of elements between high/2 and high must be O(n). Therefore, time complexity of Binary Search is O(Logn) and overall time complexity is 2*O(Logn) which is O(Logn).

```
#include <stdio.h>
int binarySearch(int low, int high); // prototype

// Let's take an example function as f(x) = x^2 - 10*x - 20
// Note that f(x) can be any monotonocally increasing function
int f(int x) { return (x*x - 10*x - 20); }

// Returns the value x where above function f() becomes positive
// first time.
int findFirstPositive()
{
    // When first value itself is positive
```

```
    if (f(0) > 0)
        return 0;

    // Find 'high' for binary search by repeated doubling
    int i = 1;
    while (f(i) <= 0)
        i = i*2;

    //  Call binary search
    return binarySearch(i/2, i);
}

// Searches first positive value of f(i) where low <= i <= high
int binarySearch(int low, int high)
{
    if (high >= low)
    {
        int mid = low + (high - low)/2; /* mid = (low + high)/2 */

        // If f(mid) is greater than 0 and one of the following two
        // conditions is true:
        // a) mid is equal to low
        // b) f(mid-1) is negative
        if (f(mid) > 0 && (mid == low || f(mid-1) <= 0))
            return mid;

        // If f(mid) is smaller than or equal to 0
        if (f(mid) <= 0)
            return binarySearch((mid + 1), high);
        else // f(mid) > 0
            return binarySearch(low, (mid -1));
    }

    /* Return -1 if there is no positive value in given range */
    return -1;
}

/* Driver program to check above functions */
int main()
{
    printf("The value n where f() becomes positive first is %d",
           findFirstPositive());
    return 0;
}
```

Output:

```
The value n where f() becomes positive first is 12
```

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

103. Find the Increasing subsequence of length three with maximum product

Given a sequence of non-negative integers, find the subsequence of length 3 having maximum product with the numbers of the subsequence being in ascending order.

Examples:

```
Input:
arr[] = {6, 7, 8, 1, 2, 3, 9, 10}
Output:
8 9 10

Input:
arr[] = {1, 5, 10, 8, 9}
Output: 5 8 9
```

Since we want to find the maximum product, we need to find following two things for every element in the given sequence:
**LSL:** The largest smaller element on left of given element
**LGR:** The largest greater element on right of given element.

Once we find LSL and LGR for an element, we can find the product of element with its LSL and LGR (if they both exist). We calculate this product for every element and return maximum of all products.

A **simple method** is to use nested loops. The outer loop traverses every element in sequence. Inside the outer loop, run two inner loops (one after other) to find LSL and LGR of current element. Time complexity of this method is $O(n^2)$.

We can do this **in O(nLogn) time**. For simplicity, let us first create two arrays LSL[] and LGR[] of size n each where n is number of elements in input array arr[]. The main task is to fill two arrays LSL[] and LGR[]. Once we have these two arrays filled, all we need to find maximum product LSL[i]*arr[i]*LGR[i] where 0 < i < n-1 (Note that LSL[i] doesn't exist for i = 0 and LGR[i] doesn't exist for i = n-1).

We can **fill LSL[]** in O(nLogn) time. The idea is to use a Balanced Binary Search Tree like AVL. We start with empty AVL tree, insert the leftmost element in it. Then we traverse the input array starting from the second element to second last element. For every element currently being traversed, we find the floor of it in AVL tree. If floor exists, we store the floor in LSL[], otherwise we store NIL. After storing the floor, we insert the current element in the AVL tree.

We can **fill LGR[]** in O(n) time. The idea is similar to this post. We traverse from right side and keep track of the largest element. If the largest element is greater than current element, we store it in LGR[], otherwise we store NIL.

Finally, we run a O(n) loop and **return maximum of LSL[i]*arr[i]*LGR[i]**

Overall complexity of this approach is O(nLogn) + O(n) + O(n) which is O(nLogn). Auxiliary space required is O(n). Note that we can avoid space required for LSL, we can find and use LSL values in final loop.

This article is contributed by **Amit Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

104. Find the minimum element in a sorted and rotated array

A sorted array is rotated at some unknown point, find the minimum element in it.

Following solution assumes that all elements are distinct.

Examples

```
Input: {5, 6, 1, 2, 3, 4}
Output: 1

Input: {1, 2, 3, 4}
Output: 1

Input: {2, 1}
Output: 1
```

A simple solution is to traverse the complete array and find minimum. This solution requires $\Theta(n)$ time.
We can do it in O(Logn) using Binary Search. If we take a closer look at above examples, we can easily figure out following pattern: The minimum element is the only element whose previous element is greater than it. If there is no such element, then there is no rotation and first element is the minimum element. Therefore, we do binary

search for an element which is smaller than the previous element. We strongly recommend you to try it yourself before seeing the following C implementation.

```
// C program to find minimum element in a sorted and rotated array
#include <stdio.h>

int findMin(int arr[], int low, int high)
{
    // This condition is needed to handle the case when array is not
    // rotated at all
    if (high < low)  return arr[0];

    // If there is only one element left
    if (high == low) return arr[low];

    // Find mid
    int mid = low + (high - low)/2; /*(low + high)/2;*/

    // Check if element (mid+1) is minimum element. Consider
    // the cases like {3, 4, 5, 1, 2}
    if (mid < high && arr[mid+1] < arr[mid])
      return arr[mid+1];

    // Check if mid itself is minimum element
    if (mid > low && arr[mid] < arr[mid - 1])
      return arr[mid];

    // Decide whether we need to go to left half or right half
    if (arr[high] > arr[mid])
      return findMin(arr, low, mid-1);
    return findMin(arr, mid+1, high);
}

// Driver program to test above functions
int main()
{
    int arr1[] =  {5, 6, 1, 2, 3, 4};
    int n1 = sizeof(arr1)/sizeof(arr1[0]);
    printf("The minimum element is %d\n", findMin(arr1, 0, n1-1));

    int arr2[] =  {1, 2, 3, 4};
    int n2 = sizeof(arr2)/sizeof(arr2[0]);
    printf("The minimum element is %d\n", findMin(arr2, 0, n2-1));
```

```
    int arr3[] =  {1};
    int n3 = sizeof(arr3)/sizeof(arr3[0]);
    printf("The minimum element is %d\n", findMin(arr3, 0, n3-1));

    int arr4[] =  {1, 2};
    int n4 = sizeof(arr4)/sizeof(arr4[0]);
    printf("The minimum element is %d\n", findMin(arr4, 0, n4-1));

    int arr5[] =  {2, 1};
    int n5 = sizeof(arr5)/sizeof(arr5[0]);
    printf("The minimum element is %d\n", findMin(arr5, 0, n5-1));

    int arr6[] =  {5, 6, 7, 1, 2, 3, 4};
    int n6 = sizeof(arr6)/sizeof(arr6[0]);
    printf("The minimum element is %d\n", findMin(arr6, 0, n6-1));

    int arr7[] =  {1, 2, 3, 4, 5, 6, 7};
    int n7 = sizeof(arr7)/sizeof(arr7[0]);
    printf("The minimum element is %d\n", findMin(arr7, 0, n7-1));

    int arr8[] =  {2, 3, 4, 5, 6, 7, 8, 1};
    int n8 = sizeof(arr8)/sizeof(arr8[0]);
    printf("The minimum element is %d\n", findMin(arr8, 0, n8-1));

    int arr9[] =  {3, 4, 5, 1, 2};
    int n9 = sizeof(arr9)/sizeof(arr9[0]);
    printf("The minimum element is %d\n", findMin(arr9, 0, n9-1));

    return 0;
}
```

Output:

```
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
The minimum element is 1
```

**How to handle duplicates?**

It turned out that duplicates canâ€™t be handled in O(Logn) time in all cases. Thanks

to Amit Jain for inputs. The special cases that cause problems are like {2, 2, 2, 2, 2, 2, 2, 2, 0, 1, 1, 2} and {2, 2, 2, 0, 2, 2, 2, 2, 2, 2, 2, 2}. It doesnâ€™t look possible to go to left half or right half by doing constant number of comparisons at the middle. Following is an implementation that handles duplicates. It may become O(n) in worst case though.

```
// C program to find minimum element in a sorted and rotated array
#include <stdio.h>

int min(int x, int y) { return (x < y)? x :y; }

// The function that handles duplicates.  It can be O(n) in worst case.
int findMin(int arr[], int low, int high)
{
    // This condition is needed to handle the case when array is not
    // rotated at all
    if (high < low)  return arr[0];

    // If there is only one element left
    if (high == low) return arr[low];

    // Find mid
    int mid = low + (high - low)/2; /*(low + high)/2;*/

    // Check if element (mid+1) is minimum element. Consider
    // the cases like {1, 1, 0, 1}
    if (mid < high && arr[mid+1] < arr[mid])
      return arr[mid+1];

    // This case causes O(n) time
    if (arr[low] == arr[mid] && arr[high] == arr[mid])
       return min(findMin(arr, low, mid-1), findMin(arr, mid+1, high));

    // Check if mid itself is minimum element
    if (mid > low && arr[mid] < arr[mid - 1])
      return arr[mid];

    // Decide whether we need to go to left half or right half
    if (arr[high] > arr[mid])
      return findMin(arr, low, mid-1);
    return findMin(arr, mid+1, high);
}

// Driver program to test above functions
```

```
int main()
{
   int arr1[] =  {5, 6, 1, 2, 3, 4};
   int n1 = sizeof(arr1)/sizeof(arr1[0]);
   printf("The minimum element is %d\n", findMin(arr1, 0, n1-1));

   int arr2[] =  {1, 1, 0, 1};
   int n2 = sizeof(arr2)/sizeof(arr2[0]);
   printf("The minimum element is %d\n", findMin(arr2, 0, n2-1));

   int arr3[] =  {1, 1, 2, 2, 3};
   int n3 = sizeof(arr3)/sizeof(arr3[0]);
   printf("The minimum element is %d\n", findMin(arr3, 0, n3-1));

   int arr4[] =  {3, 3, 3, 4, 4, 4, 4, 5, 3, 3};
   int n4 = sizeof(arr4)/sizeof(arr4[0]);
   printf("The minimum element is %d\n", findMin(arr4, 0, n4-1));

   int arr5[] =  {2, 2, 2, 2, 2, 2, 2, 2, 0, 1, 1, 2};
   int n5 = sizeof(arr5)/sizeof(arr5[0]);
   printf("The minimum element is %d\n", findMin(arr5, 0, n5-1));

   int arr6[] =  {2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1};
   int n6 = sizeof(arr6)/sizeof(arr6[0]);
   printf("The minimum element is %d\n", findMin(arr6, 0, n6-1));

   int arr7[] =  {2, 2, 2, 0, 2, 2, 2, 2, 2, 2, 2, 2};
   int n7 = sizeof(arr7)/sizeof(arr7[0]);
   printf("The minimum element is %d\n", findMin(arr7, 0, n7-1));

   return 0;
}
```

Output:

```
The minimum element is 1
The minimum element is 0
The minimum element is 1
The minimum element is 3
The minimum element is 0
The minimum element is 1
The minimum element is 0
```

This article is contributed by **Abhay Rathi**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â Â

105. Stable Marriage Problem

Given N men and N women, where each person has ranked all members of the opposite sex in order of preference, marry the men and women together such that there are no two people of opposite sex who would both rather have each other than their current partners. If there are no such people, all the marriages are â€œstableâ€ (Source Wiki).

Consider the following example.

Let there be two men m1 and m2 and two women w1 and w2.
Let m1â€˜s list of preferences be {w1, w2}
Let m2â€˜s list of preferences be {w1, w2}
Let w1â€˜s list of preferences be {m1, m2}
Let w2â€˜s list of preferences be {m1, m2}

The matching { {m1, w2}, {w1, m2} } is not stable because m1 and w1 would prefer each other over their assigned partners. The matching {m1, w1} and {m2, w2} is stable because there are no two people of opposite sex that would prefer each other over their assigned partners.

It is always possible to form stable marriages from lists of preferences (See references for proof). Following is Galeâ€“Shapley algorithm to find a stable matching:
The idea is to iterate through all free men while there is any free man available. Every free man goes to all women in his preference list according to the order. For every woman he goes to, he checks if the woman is free, if yes, they both become engaged. If the woman is not free, then the woman chooses either says no to him or dumps her current engagement according to her preference list. So an engagement done once can be broken if a woman gets better option.
Following is complete algorithm from Wiki

```
Initialize all men and women to free
while there exist a free man m who still has a woman w to propose to
{
   w = m's highest ranked such woman to whom he has not yet proposed
   if w is free
     (m, w) become engaged
   else some pair (m', w) already exists
     if w prefers m to m'
        (m, w) become engaged
         m' becomes free
     else
```

```
        (m', w) remain engaged
}
```

**Input & Output:** Input is a 2D matrix of size (2*N)*N where N is number of women or men. Rows from 0 to N-1 represent preference lists of men and rows from N to 2*N â€“ 1 represent preference lists of women. So men are numbered from 0 to N-1 and women are numbered from N to 2*N â€“ 1. The output is list of married pairs.

Following is C++ implementation of the above algorithm.

```
// C++ program for stable marriage problem
#include <iostream>
#include <string.h>
#include <stdio.h>
using namespace std;

// Number of Men or Women
#define N  4

// This function returns true if woman 'w' prefers man 'm1' over man 'm'
bool wPrefersM1OverM(int prefer[2*N][N], int w, int m, int m1)
{
   // Check if w prefers m over her current engagment m1
   for (int i = 0; i < N; i++)
   {
      // If m1 comes before m in lisr of w, then w prefers her
      // cirrent engagement, don't do anything
      if (prefer[w][i] == m1)
          return true;

      // If m cmes before m1 in w's list, then free her current
      // engagement and engage her with m
      if (prefer[w][i] == m)
         return false;
   }
}

// Prints stable matching for N boys and N girls. Boys are numbered as 0 to
// N-1. Girls are numbereed as N to 2N-1.
void stableMarriage(int prefer[2*N][N])
{
   // Stores partner of women. This is our output array that
   // stores paing information.  The value of wPartner[i]
   // indicates the partner assigned to woman N+i.  Note that
   // the woman numbers between N and 2*N-1. The value -1
```

```
// indicates that (N+i)'th woman is free
int wPartner[N];

// An array to store availability of men.  If mFree[i] is
// false, then man 'i' is free, otherwise engaged.
bool mFree[N];

// Initialize all men and women as free
memset(wPartner, -1, sizeof(wPartner));
memset(mFree, false, sizeof(mFree));
int freeCount = N;

// While there are free men
while (freeCount > 0)
{
    // Pick the first free man (we could pick any)
    int m;
    for (m = 0; m < N; m++)
        if (mFree[m] == false)
            break;

    // One by one go to all women according to m's preferences.
    // Here m is the picked free man
    for (int i = 0; i < N && mFree[m] == false; i++)
    {
        int w = prefer[m][i];

        // The woman of preference is free, w and m become
        // partners (Note that the partnership maybe changed
        // later). So we can say they are engaged not married
        if (wPartner[w-N] == -1)
        {
            wPartner[w-N] = m;
            mFree[m] = true;
            freeCount--;
        }

        else  // If w is not free
        {
            // Find current engagement of w
            int m1 = wPartner[w-N];

            // If w prefers m over her current engagement m1,
            // then break the engagement between w and m1 and
```

```
            // engage m with w.
            if (wPrefersM1OverM(prefer, w, m, m1) == false)
            {
                wPartner[w-N] = m;
                mFree[m] = true;
                mFree[m1] = false;
            }
          } // End of Else
       } // End of the for loop that goes to all women in m's list
    } // End of main while loop


    // Print the solution
    cout << "Woman   Man" << endl;
    for (int i = 0; i < N; i++)
       cout << " " << i+N << "\t" << wPartner[i] << endl;
}

// Driver program to test above functions
int main()
{
    int prefer[2*N][N] = { {7, 5, 6, 4},
        {5, 4, 6, 7},
        {4, 5, 6, 7},
        {4, 5, 6, 7},
        {0, 1, 2, 3},
        {0, 1, 2, 3},
        {0, 1, 2, 3},
        {0, 1, 2, 3},
    };
    stableMarriage(prefer);

    return 0;
}
```

Output:

```
Woman   Man
 4      2
 5      1
 6      3
 7      0
```

**References:**

http://www.csee.wvu.edu/~ksmani/courses/fa01/random/lecnotes/lecture5.pdf

http://www.youtube.com/watch?v=5RSMLgy06Ew#t=11m4s

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

106. Merge k sorted arrays | Set 1

Given k sorted arrays of size n each, merge them and print the sorted output.

Example:

**Input:**
k = 3, n = 4
arr[][] = { {1, 3, 5, 7},
{2, 4, 6, 8},
{0, 9, 10, 11}} ;

**Output:** 0 1 2 3 4 5 6 7 8 9 10 11

A simple solution is to create an output array of size n*k and one by one copy all arrays to it. Finally, sort the output array using any O(nLogn) sorting algorithm. This approach takes O(nkLognk) time.

We can merge arrays in O(nk*Logk) time using Min Heap. Following is detailed algorithm.

**1.** Create an output array of size n*k.
**2.** Create a min heap of size k and insert 1st element in all the arrays into a the heap
**3.** Repeat following steps n*k times.
Â Â Â Â **a)** Get minimum element from heap (minimum is always at root) and store it in output array.
Â Â Â Â **b)** Replace heap root with next element from the array from which the element is extracted. If the array doesnâ€™t have any more elements, then replace root with infinite. After replacing the root, heapify the tree.

Following is C++ implementation of the above algorithm.

```
// C++ program to merge k sorted arrays of size n each.
#include<iostream>
#include<limits.h>
using namespace std;

#define n 4
```

```
// A min heap node
struct MinHeapNode
{
    int element; // The element to be stored
    int i; // index of the array from which the element is taken
    int j; // index of the next element to be picked from array
};

// Prototype of a utility function to swap two min heap nodes
void swap(MinHeapNode *x, MinHeapNode *y);

// A class for Min Heap
class MinHeap
{
    MinHeapNode *harr; // pointer to array of elements in heap
    int heap_size; // size of min heap
public:
    // Constructor: creates a min heap of given size
    MinHeap(MinHeapNode a[], int size);

    // to heapify a subtree with root at given index
    void MinHeapify(int );

    // to get index of left child of node at index i
    int left(int i) { return (2*i + 1); }

    // to get index of right child of node at index i
    int right(int i) { return (2*i + 2); }

    // to get the root
    MinHeapNode getMin() { return harr[0]; }

    // to replace root with new node x and heapify() new root
    void replaceMin(MinHeapNode x) { harr[0] = x;  MinHeapify(0); }
};

// This function takes an array of arrays as an argument and
// All arrays are assumed to be sorted. It merges them together
// and prints the final sorted output.
int *mergeKArrays(int arr[][n], int k)
{
    int *output = new int[n*k];  // To store output array
```

```
    // Create a min heap with k heap nodes.  Every heap node
    // has first element of an array
    MinHeapNode *harr = new MinHeapNode[k];
    for (int i = 0; i < k; i++)
    {
        harr[i].element = arr[i][0]; // Store the first element
        harr[i].i = i;  // index of array
        harr[i].j = 1;  // Index of next element to be stored from array
    }
    MinHeap hp(harr, k); // Create the heap

    // Now one by one get the minimum element from min
    // heap and replace it with next element of its array
    for (int count = 0; count < n*k; count++)
    {
        // Get the minimum element and store it in output
        MinHeapNode root = hp.getMin();
        output[count] = root.element;

        // Find the next elelement that will replace current
        // root of heap. The next element belongs to same
        // array as the current root.
        if (root.j < n)
        {
            root.element = arr[root.i][root.j];
            root.j += 1;
        }
        // If root was the last element of its array
        else root.element =  INT_MAX; //INT_MAX is for infinite

        // Replace root with next element of array
        hp.replaceMin(root);
    }

    return output;
}

// FOLLOWING ARE IMPLEMENTATIONS OF STANDARD MIN HEAP METHODS
// FROM CORMEN BOOK
// Constructor: Builds a heap from a given array a[] of given size
MinHeap::MinHeap(MinHeapNode a[], int size)
{
    heap_size = size;
    harr = a;  // store address of array
```

```
    int i = (heap_size - 1)/2;
    while (i >= 0)
    {
        MinHeapify(i);
        i--;
    }
}

// A recursive method to heapify a subtree with root at given index
// This method assumes that the subtrees are already heapified
void MinHeap::MinHeapify(int i)
{
    int l = left(i);
    int r = right(i);
    int smallest = i;
    if (l < heap_size && harr[l].element < harr[i].element)
        smallest = l;
    if (r < heap_size && harr[r].element < harr[smallest].element)
        smallest = r;
    if (smallest != i)
    {
        swap(&harr[i], &harr[smallest]);
        MinHeapify(smallest);
    }
}

// A utility function to swap two elements
void swap(MinHeapNode *x, MinHeapNode *y)
{
    MinHeapNode temp = *x;  *x = *y;  *y = temp;
}

// A utility function to print array elements
void printArray(int arr[], int size)
{
   for (int i=0; i < size; i++)
       cout << arr[i] << " ";
}

// Driver program to test above functions
int main()
{
    // Change n at the top to change number of elements
    // in an array
```

```
    int arr[][n] =  {{2, 6, 12, 34},
                     {1, 9, 20, 1000},
                     {23, 34, 90, 2000}};
    int k = sizeof(arr)/sizeof(arr[0]);

    int *output = mergeKArrays(arr, k);

    cout << "Merged array is " << endl;
    printArray(output, n*k);

    return 0;
}
```

Output:

```
Merged array is
1 2 6 9 12 20 23 34 34 90 1000 2000
```

**Time Complexity:** The main step is 3rd step, the loop runs n*k times. In every iteration of loop, we call heapify which takes O(Logk) time. Therefore, the time complexity is O(nk Logk).

There are other interesting methods to merge k sorted arrays in O(nkLogk), we will sonn be discussing them as separate posts.

Thanks to vignesh for suggesting this problem and initial solution. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

107.  Radix Sort

The lower bound for Comparison based sorting algorithm (Merge Sort, Heap Sort, Quick-Sort .. etc) is $\Omega(nLogn)$, i.e., they cannot do better than nLogn.

Counting sort is a linear tine sorting algorithm that sort in O(n+k) time when elements are in range from 1 to k.

***What if the elements are in range from 1 to $n^2$?***

We canâ€™t use counting sort because counting sort will take O($n^2$) which is worse than comparison based sorting algorithms. Can we sort such an array in linear time? Radix Sort is the answer. The idea of Radix Sort is to do digit by digit sort starting from least significant digit to most significant digit. Radix sort uses counting sort as a subroutine to sort.

***The Radix Sort Algorithm***

**1)** Do following for each digit i where i varies from least significant digit to the most significant digit.

â€¦â€¦â€¦.**a)** Sort input array using counting sort (or any stable sort) according to the iâ€™th digit.

**Example:**

Original, unsorted list:

170, 45, 75, 90, 802, 24, 2, 66

Sorting by least significant digit (1s place) gives: [*Notice that we keep 802 before 2, because 802 occurred before 2 in the original list, and similarly for pairs 170 & 90 and 45 & 75.]

170, 90, 802,Â 2, 24, 45, 75, 66

Sorting by next digit (10s place) gives: [*Notice that 802 again comes before 2 as 802 comes before 2 in the previous list.]

802, 2,Â 24,Â 45,Â 66, 170,Â 75,Â 90

Sorting by most significant digit (100s place) gives:

2, 24, 45, 66, 75, 90,Â 170,Â 802

***What is the running time of Radix Sort?***

Let there be d digits in input integers. Radix Sort takes O(d*(n+b)) time where b is the base for representing numbers, for example, for decimal system, b is 10. What is the value of d? If k is the maximum possible value, then d would be $O(\log_b(k))$. So overall time complexity is $O((n+b) * \log_b(k))$. Which looks more than the time complexity of comparison based sorting algorithms for a large k. Let us first limit k. Let k <= $n^c$ where c is a constant. In that case, the complexity becomes $O(n \log_b(n))$. But it still doesnâ€™t beat comparison based sorting algorithms.

What if we make value of b larger?. What should be the value of b to make the time complexity linear? If we set b as n, we get the time complexity as O(n). In other words, we can sort an array of integers with range from 1 to $n^c$ if the numbers are represented in base n (or every digit takes $\log_2(n)$ bits).

***Is Radix Sort preferable to Comparison based sorting algorithms like Quick-Sort?***

If we have $\log_2(n)$ bits for every digit, the running time of Radix appears to be better than Quick Sort for a wide range of input numbers. The constant factors hidden in asymptotic notation are higher for Radix Sort and Quick-Sort uses hardware caches more effectively. Also, Radix sort uses counting sort as a subroutine and counting sort takes extra space to sort numbers.

**Implementation of Radix Sort**

Following is a simple C++ implementation of Radix Sort. For simplicity, the value of d is assumed to be 10. We recommend you to see Counting Sort for details of countSort() function in below code.

```
// C++ implementation of Radix Sort
#include<iostream>
using namespace std;

// A utility function to get maximum value in arr[]
int getMax(int arr[], int n)
{
    int mx = arr[0];
    for (int i = 1; i < n; i++)
        if (arr[i] > mx)
            mx = arr[i];
    return mx;
}

// A function to do counting sort of arr[] according to
// the digit represented by exp.
void countSort(int arr[], int n, int exp)
{
    int output[n]; // output array
    int i, count[10] = {0};

    // Store count of occurrences in count[]
    for (i = 0; i < n; i++)
        count[ (arr[i]/exp)%10 ]++;

    // Change count[i] so that count[i] now contains actual position of
    // this digit in output[]
    for (i = 1; i < 10; i++)
        count[i] += count[i - 1];

    // Build the output array
    for (i = n - 1; i >= 0; i--)
    {
        output[count[ (arr[i]/exp)%10 ] - 1] = arr[i];
        count[ (arr[i]/exp)%10 ]--;
    }

    // Copy the output array to arr[], so that arr[] now
    // contains sorted numbers according to curent digit
```

```
    for (i = 0; i < n; i++)
        arr[i] = output[i];
}

// The main function to that sorts arr[] of size n using Radix Sort
void radixsort(int arr[], int n)
{
    // Find the maximum number to know number of digits
    int m = getMax(arr, n);

    // Do counting sort for every digit. Note that instead of passing digit
    // number, exp is passed. exp is 10^i where i is current digit number
    for (int exp = 1; m/exp > 0; exp *= 10)
        countSort(arr, n, exp);
}

// A utility function to print an array
void print(int arr[], int n)
{
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";
}

// Driver program to test above functions
int main()
{
    int arr[] = {170, 45, 75, 90, 802, 24, 2, 66};
    int n = sizeof(arr)/sizeof(arr[0]);
    radixsort(arr, n);
    print(arr, n);
    return 0;
}
```

Output:

```
2 24 45 66 75 90 170 802
```

**References:**


http://en.wikipedia.org/wiki/Radix_sort
http://alg12.wikischolars.columbia.edu/file/view/RADIX.pdf
MIT Video Lecture
Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest


Please write comments if you find anything incorrect, or you want to share more

information about the topic discussed above

Â Â

108. Move all zeroes to end of array

Given an array of random numbers, Push all the zeroâ€™s of a given array to the end of the array. For example, if the given arrays is {1, 9, 8, 4, 0, 0, 2, 7, 0, 6, 0}, it should be changed to {1, 9, 8, 4, 2, 7, 6, 0, 0, 0, 0}. The order of all other elements should be same. Expected time complexity is O(n) and extra space is O(1).

There can be many ways to solve this problem. Following is a simple and interesting way to solve this problem.
Traverse the given array â€˜arrâ€™ from left to right. While traversing, maintain count of non-zero elements in array. Let the count be â€˜countâ€™. For every non-zero element arr[i], put the element at â€˜arr[count]â€™ and increment â€˜countâ€™. After complete traversal, all non-zero elements have already been shifted to front end and â€˜countâ€™ is set as index of first 0. Now all we need to do is that run a loop which makes all elements zero from â€˜countâ€™ till end of the array.

Below is C++ implementation of the above approach.

```
// A C++ program to move all zeroes at the end of array
#include <iostream>
using namespace std;

// Function which pushes all zeros to end of an array.
void pushZerosToEnd(int arr[], int n)
{
    int count = 0;  // Count of non-zero elements

    // Traverse the array. If element encountered is non-zero, then
    // replace the element at index 'count' with this element
    for (int i = 0; i < n; i++)
        if (arr[i] != 0)
            arr[count++] = arr[i]; // here count is incremented

    // Now all non-zero elements have been shifted to front and 'count' is
    // set as index of first 0. Make all elements 0 from count to end.
    while (count < n)
        arr[count++] = 0;
}

// Driver program to test above function
```

```
int main()
{
    int arr[] = {1, 9, 8, 4, 0, 0, 2, 7, 0, 6, 0, 9};
    int n = sizeof(arr) / sizeof(arr[0]);
    pushZerosToEnd(arr, n);
    cout << "Array after pushing all zeros to end of array :\n";
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";
    return 0;
}
```

Output:

```
Array after pushing all zeros to end of array :
1 9 8 4 2 7 6 9 0 0 0 0
```

**Time Complexity:** O(n) where n is number of elements in input array.

**Auxiliary Space:** O(1)

This article is contributed by **Chandra Prakash**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

109. Find number of pairs such that x^y > y^x

Given two arrays X[] and Y[] of positive integers, find number of pairs such that **x^y > y^x** where x is an element from X[] and y is an element from Y[].

Examples:

```
Input: X[] = {2, 1, 6}, Y = {1, 5}
Output: 3
// There are total 3 pairs where pow(x, y) is greater than pow(y, x)
// Pairs are (2, 1), (2, 5) and (6, 1)


Input: X[] = {10, 19, 18}, Y[] = {11, 15, 9};
Output: 2
// There are total 2 pairs where pow(x, y) is greater than pow(y, x)
// Pairs are (10, 11) and (10, 15)
```

The **brute force solution** is to consider each element of X[] and Y[], and check

whether the given condition satisfies or not. Time Complexity of this solution is **O(m*n)** where m and n are sizes of given arrays.

Following is C++ code based on brute force solution.

```
int countPairsBruteForce(int X[], int Y[], int m, int n)
{
    int ans = 0;
    for (int i = 0; i < m; i++)
        for (int j = 0; j < n; j++)
            if (pow(X[i], Y[j]) > pow(Y[j], X[i]))
                ans++;
    return ans;
}
```

**Efficient Solution:**
The problem can be solved in **O(nLogn + mLogn)** time. The trick here is, if y > x then x^y > y^x with some exceptions. Following are simple steps based on this trick.

**1)** Sort array Y[].
**2)** For every x in X[], find the index idx of smallest number greater than x (also called ceil of x) in Y[] using binary search or we can use the inbuilt function upper_bound() in algorithm library.
**3)** All the numbers after idx satisfy the relation so just add (n-idx) to the count.

**Base Cases and Exceptions:**
Following are exceptions for x from X[] and y from Y[]
If x = 0, then the count of pairs for this x is 0.
If x = 1, then the count of pairs for this x is equal to count of 0s in Y[].
The following cases must be handled separately as they donâ€™t follow the general rule that x smaller than y means x^y is greater than y^x.
a) x = 2, y = 3 or 4
b) x = 3, y = 2
Note that the case where x = 4 and y = 2 is not there

Following diagram shows all exceptions in tabular form. The value 1 indicates that the corresponding (x, y) form a valid pair.

| X \ Y | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 1 |
| 4 | 1 | 1 | 0 | 0 | 0 |

Following is C++ implementation. In the following implementation, we pre-process the

Y array and count 0, 1, 2, 3 and 4 in it, so that we can handle all exceptions in constant time. The array NoOfY[] is used to store the counts.

```
#include<iostream>
#include<algorithm>
using namespace std;

// This function return count of pairs with x as one element
// of the pair. It mainly looks for all values in Y[] where
// x ^ Y[i] > Y[i] ^ x
int count(int x, int Y[], int n, int NoOfY[])
{
    // If x is 0, then there cannot be any value in Y such that
    // x^Y[i] > Y[i]^x
    if (x == 0) return 0;

    // If x is 1, then the number of pais is equal to number of
    // zeroes in Y[]
    if (x == 1) return NoOfY[0];

    // Find number of elements in Y[] with values greater than x
    // upper_bound() gets address of first greater element in Y[0..n-1]
    int* idx = upper_bound(Y, Y + n, x);
    int ans = (Y + n) - idx;

    // If we have reached here, then x must be greater than 1,
    // increase number of pairs for y=0 and y=1
    ans += (NoOfY[0] + NoOfY[1]);

    // Decrease number of pairs for x=2 and (y=4 or y=3)
    if (x == 2)  ans -= (NoOfY[3] + NoOfY[4]);

    // Increase number of pairs for x=3 and y=2
    if (x == 3)  ans += NoOfY[2];

    return ans;
}

// The main function that returns count of pairs (x, y) such that
// x belongs to X[], y belongs to Y[] and x^y > y^x
int countPairs(int X[], int Y[], int m, int n)
{
    // To store counts of 0, 1, 2, 3 and 4 in array Y
    int NoOfY[5] = {0};
```

```
    for (int i = 0; i < n; i++)
        if (Y[i] < 5)
            NoOfY[Y[i]]++;

    // Sort Y[] so that we can do binary search in it
    sort(Y, Y + n);

    int total_pairs = 0; // Initialize result

    // Take every element of X and count pairs with it
    for (int i=0; i<m; i++)
        total_pairs += count(X[i], Y, n, NoOfY);

    return total_pairs;
}

// Driver program to test above functions
int main()
{
    int X[] = {2, 1, 6};
    int Y[] = {1, 5};

    int m = sizeof(X)/sizeof(X[0]);
    int n = sizeof(Y)/sizeof(Y[0]);

    cout << "Total pairs = " << countPairs(X, Y, m, n);

    return 0;
}
```

Output:

```
Total pairs = 3
```

**Time Complexity :** Let m and n be the sizes of arrays X[] and Y[] respectively. The sort step takes O(nLogn) time. Then every element of X[] is searched in Y[] using binary search. This step takes O(mLogn) time. Overall time complexity is O(nLogn + mLogn).

This article is contributed by **Shubham Mittal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

Given two arrays X[] and Y[] of positive integers, find number of pairs such that **x^y > y^x** where x is an element from X[] and y is an element from Y[].

Examples:

```
Input: X[] = {2, 1, 6}, Y = {1, 5}
Output: 3
// There are total 3 pairs where pow(x, y) is greater than pow(y, x)
// Pairs are (2, 1), (2, 5) and (6, 1)



Input: X[] = {10, 19, 18}, Y[] = {11, 15, 9};
Output: 2
// There are total 2 pairs where pow(x, y) is greater than pow(y, x)
// Pairs are (10, 11) and (10, 15)
```

The **brute force solution** is to consider each element of X[] and Y[], and check whether the given condition satisfies or not. Time Complexity of this solution is **O(m*n)** where m and n are sizes of given arrays.

Following is C++ code based on brute force solution.

```
int countPairsBruteForce(int X[], int Y[], int m, int n)
{
   int ans = 0;
   for (int i = 0; i < m; i++)
      for (int j = 0; j < n; j++)
         if (pow(X[i], Y[j]) > pow(Y[j], X[i]))
            ans++;
   return ans;
}
```

**Efficient Solution:**
The problem can be solved in **O(nLogn + mLogn)** time. The trick here is, if y > x then x^y > y^x with some exceptions. Following are simple steps based on this trick.

**1)** Sort array Y[].
**2)** For every x in X[], find the index idx of smallest number greater than x (also called ceil of x) in Y[] using binary search or we can use the inbuilt function upper_bound() in algorithm library.
**3)** All the numbers after idx satisfy the relation so just add (n-idx) to the count.

**Base Cases and Exceptions:**
Following are exceptions for x from X[] and y from Y[]
If x = 0, then the count of pairs for this x is 0.

If x = 1, then the count of pairs for this x is equal to count of 0s in Y[].
The following cases must be handled separately as they donâ€™t follow the general rule that x smaller than y means x^y is greater than y^x.
a) x = 2, y = 3 or 4
b) x = 3, y = 2
Note that the case where x = 4 and y = 2 is not there

Following diagram shows all exceptions in tabular form. The value 1 indicates that the corresponding (x, y) form a valid pair.

| X \ Y | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 1 |
| 4 | 1 | 1 | 0 | 0 | 0 |

Following is C++ implementation. In the following implementation, we pre-process the Y array and count 0, 1, 2, 3 and 4 in it, so that we can handle all exceptions in constant time. The array NoOfY[] is used to store the counts.

```
#include<iostream>
#include<algorithm>
using namespace std;

// This function return count of pairs with x as one element
// of the pair. It mainly looks for all values in Y[] where
// x ^ Y[i] > Y[i] ^ x
int count(int x, int Y[], int n, int NoOfY[])
{
    // If x is 0, then there cannot be any value in Y such that
    // x^Y[i] > Y[i]^x
    if (x == 0) return 0;

    // If x is 1, then the number of pais is equal to number of
    // zeroes in Y[]
    if (x == 1) return NoOfY[0];

    // Find number of elements in Y[] with values greater than x
    // upper_bound() gets address of first greater element in Y[0..n-1]
    int* idx = upper_bound(Y, Y + n, x);
    int ans = (Y + n) - idx;

    // If we have reached here, then x must be greater than 1,
    // increase number of pairs for y=0 and y=1
```

```
    ans += (NoOfY[0] + NoOfY[1]);

    // Decrease number of pairs for x=2 and (y=4 or y=3)
    if (x == 2)  ans -= (NoOfY[3] + NoOfY[4]);

    // Increase number of pairs for x=3 and y=2
    if (x == 3)  ans += NoOfY[2];

    return ans;
}

// The main function that returns count of pairs (x, y) such that
// x belongs to X[], y belongs to Y[] and x^y > y^x
int countPairs(int X[], int Y[], int m, int n)
{
    // To store counts of 0, 1, 2, 3 and 4 in array Y
    int NoOfY[5] = {0};
    for (int i = 0; i < n; i++)
        if (Y[i] < 5)
            NoOfY[Y[i]]++;

    // Sort Y[] so that we can do binary search in it
    sort(Y, Y + n);

    int total_pairs = 0; // Initialize result

    // Take every element of X and count pairs with it
    for (int i=0; i<m; i++)
        total_pairs += count(X[i], Y, n, NoOfY);

    return total_pairs;
}

// Driver program to test above functions
int main()
{
    int X[] = {2, 1, 6};
    int Y[] = {1, 5};

    int m = sizeof(X)/sizeof(X[0]);
    int n = sizeof(Y)/sizeof(Y[0]);

    cout << "Total pairs = " << countPairs(X, Y, m, n);
```

```
    return 0;
}
```

Output:

```
Total pairs = 3
```

**Time Complexity :** Let m and n be the sizes of arrays X[] and Y[] respectively. The sort step takes O(nLogn) time. Then every element of X[] is searched in Y[] using binary search. This step takes O(mLogn) time. Overall time complexity is O(nLogn + mLogn).

This article is contributed by **Shubham Mittal**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Â  Â

111. Count all possible paths from top left to bottom right of a mXn matrix

The problem is to count all the possible paths from top left to bottom right of a mXn matrix with the constraints thatÂ ***from each cell you can either move only to right or down***

We have discussed a solution to print all possible paths, counting all paths is easier. Let NumberOfPaths(m, n) be the count of paths to reach row number m and column number n in the matrix, NumberOfPaths(m, n) can be recursively written as following.

```
#include <iostream>
using namespace std;

// Returns count of possible paths to reach cell at row number m and column
// number n from the topmost leftmost cell (cell at 1, 1)
int  numberOfPaths(int m, int n)
{
  // If either given row number is first or given column number is first
  if (m == 1 || n == 1)
      return 1;

  // If diagonal movements are allowed then the last addition
  // is required.
  return  numberOfPaths(m-1, n) + numberOfPaths(m, n-1);
        // + numberOfPaths(m-1,n-1);
}
```

```
int main()
{
 cout << numberOfPaths(3, 3);
 return 0;
}
```

Output:

```
6
```

The time complexity of above recursive solution is exponential. There are many overlapping subproblems. We can draw a recursion tree for numberOfPaths(3, 3) and see many overlapping subproblems. The recursion tree would be similar to Recursion tree for Longest Common Subsequence problem.
So this problem has both properties (seeÂ thisÂ andÂ this) of a dynamic programming problem. Like other typicalÂ Dynamic Programming(DP) problems, recomputations of same subproblems can be avoided by constructing a temporary array count[][] in bottom up manner using the above recursive formula.

```
#include <iostream>
using namespace std;

// Returns count of possible paths to reach cell at row number m and column
// number n from the topmost leftmost cell (cell at 1, 1)
int numberOfPaths(int m, int n)
{
    // Create a 2D table to store results of subproblems
    int count[m][n];

    // Count of paths to reach any cell in first column is 1
    for (int i = 0; i < m; i++)
        count[i][0] = 1;

    // Count of paths to reach any cell in first column is 1
    for (int j = 0; j < n; j++)
        count[0][j] = 1;

    // Calculate count of paths for other cells in bottom-up manner using
    // the recursive solution
    for (int i = 1; i < m; i++)
    {
        for (int j = 1; j < n; j++)

            // By uncommenting the last part the code calculatest he total
            // possible paths if the diagonal Movements are allowed
```

```
            count[i][j] = count[i-1][j] + count[i][j-1]; //+ count[i-1][j-1];

        }
        return count[m-1][n-1];
}

// Driver program to test above functions
int main()
{
    cout << numberOfPaths(3, 3);
    return 0;
}
```

Output:

```
6
```

Time complexity of the above dynamic programming solution is O(mn).

Note the count can also be calculated using the formula (m-1 + n-1)!/(m-1)!(n-1)! as mentioned in the comments of this article.

This article is contributed by **Hariprasad NG**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

112. Suffix Array | Set 1 (Introduction)

We strongly recommend to read following post on suffix trees as a pre-requisite for this post.

Pattern Searching | Set 8 (Suffix Tree Introduction)

***A suffix array is a sorted array of all suffixes of a given string***. The definition is similar to Suffix Tree which is compressed trie of all suffixes of the given text. Any suffix tree based algorithm can be replaced with an algorithm that uses a suffix array enhanced with additional information and solves the same problem in the same time complexity (Source Wiki).
A suffix array can be constructed from Suffix tree by doing a DFS traversal of the suffix tree. In fact Suffix array and suffix tree both can be constructed from each other in linear time.
Advantages of suffix arrays over suffix trees include improved space requirements, simpler linear time construction algorithms (e.g., compared to Ukkonenâ€™s

algorithm) and improved cache locality (Source: Wiki)

### *Example:*

```
Let the given string be "banana".

0 banana                          5 a
1 anana     Sort the Suffixes     3 ana
2 nana      ----------------->    1 anana
3 ana       alphabetically        0 banana
4 na                              4 na
5 a                               2 nana

So the suffix array for "banana" is {5, 3, 1, 0, 4, 2}
```

### *Naive method to build Suffix Array*

A simple method to construct suffix array is to make an array of all suffixes and then sort the array. Following is implementation of simple method.

```
// Naive algorithm for building suffix array of a given text
#include <iostream>
#include <cstring>
#include <algorithm>
using namespace std;

// Structure to store information of a suffix
struct suffix
{
    int index;
    char *suff;
};

// A comparison function used by sort() to compare two suffixes
int cmp(struct suffix a, struct suffix b)
{
    return strcmp(a.suff, b.suff) < 0? 1 : 0;
}

// This is the main function that takes a string 'txt' of size n as an
// argument, builds and return the suffix array for the given string
int *buildSuffixArray(char *txt, int n)
{
    // A structure to store suffixes and their indexes
    struct suffix suffixes[n];
```

```
    // Store suffixes and their indexes in an array of structures.
    // The structure is needed to sort the suffixes alphabatically
    // and maintain their old indexes while sorting
    for (int i = 0; i < n; i++)
    {
        suffixes[i].index = i;
        suffixes[i].suff = (txt+i);
    }

    // Sort the suffixes using the comparison function
    // defined above.
    sort(suffixes, suffixes+n, cmp);

    // Store indexes of all sorted suffixes in the suffix array
    int *suffixArr = new int[n];
    for (int i = 0; i < n; i++)
        suffixArr[i] = suffixes[i].index;

    // Return the suffix array
    return  suffixArr;
}

// A utility function to print an array of given size
void printArr(int arr[], int n)
{
    for(int i = 0; i < n; i++)
        cout << arr[i] << " ";
    cout << endl;
}

// Driver program to test above functions
int main()
{
    char txt[] = "banana";
    int n = strlen(txt);
    int *suffixArr = buildSuffixArray(txt,  n);
    cout << "Following is suffix array for " << txt << endl;
    printArr(suffixArr, n);
    return 0;
}
```

Output:

```
Following is suffix array for banana
```

The time complexity of above method to build suffix array is $O(n^2Logn)$ if we consider a O(nLogn) algorithm used for sorting. The sorting step itself takes $O(n^2Logn)$ time as every comparison is a comparison of two strings and the comparison takes O(n) time. There are many efficient algorithms to build suffix array. We will soon be covering them as separate posts.

***Search a pattern using the built Suffix Array***

To search a pattern in a text, we preprocess the text and build a suffix array of the text. Since we have a sorted array of all suffixes, Binary Search can be used to search. Following is the search function. Note that the function doesnâ€™t report all occurrences of pattern, it only report one of them.

```
// This code only contains search() and main. To make it a complete running
// above code or see http://ideone.com/1Io9eN

// A suffix array based search function to search a given pattern
// 'pat' in given text 'txt' using suffix array suffArr[]
void search(char *pat, char *txt, int *suffArr, int n)
{
    int m = strlen(pat);  // get length of pattern, needed for strncmp()

    // Do simple binary search for the pat in txt using the
    // built suffix array
    int l = 0, r = n-1;  // Initilize left and right indexes
    while (l <= r)
    {
        // See if 'pat' is prefix of middle suffix in suffix array
        int mid = l + (r - l)/2;
        int res = strncmp(pat, txt+suffArr[mid], m);

        // If match found at the middle, print it and return
        if (res == 0)
        {
            cout << "Pattern found at index " << suffArr[mid];
            return;
        }

        // Move to left half if pattern is alphabtically less than
        // the mid suffix
        if (res < 0) r = mid - 1;

        // Otherwise move to right half
```

```
        else l = mid + 1;
    }

    // We reach here if return statement in loop is not executed
    cout << "Pattern not found";
}

// Driver program to test above function
int main()
{
    char txt[] = "banana";  // text
    char pat[] = "nan";   // pattern to be searched in text

    // Build suffix array
    int n = strlen(txt);
    int *suffArr = buildSuffixArray(txt, n);

    // search pat in txt using the built suffix array
    search(pat, txt, suffArr, n);

    return 0;
}
```

Output:

```
Pattern found at index 2
```

The time complexity of the above search function is O(mLogn). There are more efficient algorithms to search pattern once the suffix array is built. In fact there is a O(m) suffix array based algorithm to search a pattern. We will soon be discussing efficient algorithm for search.

***Applications of Suffix Array***

Suffix array is an extremely useful data structure, it can be used for a wide range of problems. Following are some famous problems where Suffix array can be used.

1) Pattern Searching

2) Finding the longest repeated substring

3) Finding the longest common substring

4) Finding the longest palindrome in a string

See this for more problems where Suffix arrays can be used.

This post is a simple introduction. There is a lot to cover in Suffix arrays. We have discussed a O(nLogn) algorithm for Suffix Array construction here. We will soon be discussing more efficient suffix array algorithms.

**References:**
http://www.stanford.edu/class/cs97si/suffix-array.pdf
http://en.wikipedia.org/wiki/Suffix_array

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

We strongly recommend to read following post on suffix trees as a pre-requisite for this post.

Pattern Searching | Set 8 (Suffix Tree Introduction)

***A suffix array is a sorted array of all suffixes of a given string***. The definition is similar to Suffix Tree which is compressed trie of all suffixes of the given text. Any suffix tree based algorithm can be replaced with an algorithm that uses a suffix array enhanced with additional information and solves the same problem in the same time complexity (Source Wiki).
A suffix array can be constructed from Suffix tree by doing a DFS traversal of the suffix tree. In fact Suffix array and suffix tree both can be constructed from each other in linear time.
Advantages of suffix arrays over suffix trees include improved space requirements, simpler linear time construction algorithms (e.g., compared to Ukkonenâ€™s algorithm) and improved cache locality (Source: Wiki)

***Example:***

```
Let the given string be "banana".

0 banana                          5 a
1 anana     Sort the Suffixes     3 ana
2 nana      ----------------->    1 anana
3 ana       alphabetically        0 banana
4 na                              4 na
5 a                               2 nana

So the suffix array for "banana" is {5, 3, 1, 0, 4, 2}
```

***Naive method to build Suffix Array***
A simple method to construct suffix array is to make an array of all suffixes and then sort the array. Following is implementation of simple method.

```
// Naive algorithm for building suffix array of a given text
#include <iostream>
```

```
#include <cstring>
#include <algorithm>
using namespace std;

// Structure to store information of a suffix
struct suffix
{
    int index;
    char *suff;
};

// A comparison function used by sort() to compare two suffixes
int cmp(struct suffix a, struct suffix b)
{
    return strcmp(a.suff, b.suff) < 0? 1 : 0;
}

// This is the main function that takes a string 'txt' of size n as an
// argument, builds and return the suffix array for the given string
int *buildSuffixArray(char *txt, int n)
{
    // A structure to store suffixes and their indexes
    struct suffix suffixes[n];

    // Store suffixes and their indexes in an array of structures.
    // The structure is needed to sort the suffixes alphabatically
    // and maintain their old indexes while sorting
    for (int i = 0; i < n; i++)
    {
        suffixes[i].index = i;
        suffixes[i].suff = (txt+i);
    }

    // Sort the suffixes using the comparison function
    // defined above.
    sort(suffixes, suffixes+n, cmp);

    // Store indexes of all sorted suffixes in the suffix array
    int *suffixArr = new int[n];
    for (int i = 0; i < n; i++)
        suffixArr[i] = suffixes[i].index;

    // Return the suffix array
    return  suffixArr;
```

```
}

// A utility function to print an array of given size
void printArr(int arr[], int n)
{
    for(int i = 0; i < n; i++)
        cout << arr[i] << " ";
    cout << endl;
}

// Driver program to test above functions
int main()
{
    char txt[] = "banana";
    int n = strlen(txt);
    int *suffixArr = buildSuffixArray(txt,  n);
    cout << "Following is suffix array for " << txt << endl;
    printArr(suffixArr, n);
    return 0;
}
```

Output:

```
Following is suffix array for banana
5 3 1 0 4 2
```

The time complexity of above method to build suffix array is $O(n^2Logn)$ if we consider a O(nLogn) algorithm used for sorting. The sorting step itself takes $O(n^2Logn)$ time as every comparison is a comparison of two strings and the comparison takes O(n) time. There are many efficient algorithms to build suffix array. We will soon be covering them as separate posts.

***Search a pattern using the built Suffix Array***

To search a pattern in a text, we preprocess the text and build a suffix array of the text. Since we have a sorted array of all suffixes, Binary Search can be used to search. Following is the search function. Note that the function doesnâ€™t report all occurrences of pattern, it only report one of them.

```
// This code only contains search() and main. To make it a complete running
// above code or see http://ideone.com/1Io9eN

// A suffix array based search function to search a given pattern
// 'pat' in given text 'txt' using suffix array suffArr[]
void search(char *pat, char *txt, int *suffArr, int n)
{
```

```
    int m = strlen(pat);  // get length of pattern, needed for strncmp()

    // Do simple binary search for the pat in txt using the
    // built suffix array
    int l = 0, r = n-1;  // Initilize left and right indexes
    while (l <= r)
    {
        // See if 'pat' is prefix of middle suffix in suffix array
        int mid = l + (r - l)/2;
        int res = strncmp(pat, txt+suffArr[mid], m);

        // If match found at the middle, print it and return
        if (res == 0)
        {
            cout << "Pattern found at index " << suffArr[mid];
            return;
        }

        // Move to left half if pattern is alphabtically less than
        // the mid suffix
        if (res < 0) r = mid - 1;

        // Otherwise move to right half
        else l = mid + 1;
    }

    // We reach here if return statement in loop is not executed
    cout << "Pattern not found";
}

// Driver program to test above function
int main()
{
    char txt[] = "banana";  // text
    char pat[] = "nan";   // pattern to be searched in text

    // Build suffix array
    int n = strlen(txt);
    int *suffArr = buildSuffixArray(txt, n);

    // search pat in txt using the built suffix array
    search(pat, txt, suffArr, n);

    return 0;
```

```
}
```

Output:

```
Pattern found at index 2
```

The time complexity of the above search function is O(mLogn). There are more efficient algorithms to search pattern once the suffix array is built. In fact there is a O(m) suffix array based algorithm to search a pattern. We will soon be discussing efficient algorithm for search.

***Applications of Suffix Array***

Suffix array is an extremely useful data structure, it can be used for a wide range of problems. Following are some famous problems where Suffix array can be used.

1) Pattern Searching
2) Finding the longest repeated substring
3) Finding the longest common substring
4) Finding the longest palindrome in a string

See this for more problems where Suffix arrays can be used.

This post is a simple introduction. There is a lot to cover in Suffix arrays. We have discussed a O(nLogn) algorithm for Suffix Array construction here. We will soon be discussing more efficient suffix array algorithms.

**References:**

http://www.stanford.edu/class/cs97si/suffix-array.pdf
http://en.wikipedia.org/wiki/Suffix_array

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

114. Sort n numbers in range from 0 to n^2 – 1 in linear time

Given an array of numbers of size n. It is also given that the array elements are in range from 0 to $n^2$ â€“ 1. Sort the given array in linear time.

```
Examples:
Since there are 5 elements, the elements can be from 0 to 24.
Input: arr[] = {0, 23, 14, 12, 9}
Output: arr[] = {0, 9, 12, 14, 23}

Since there are 3 elements, the elements can be from 0 to 8.
Input: arr[] = {7, 0, 2}
```

```
Output: arr[] = {0, 2, 7}
```

***We strongly recommend to minimize the browser and try this yourself first.***

**Solution:** If we use Counting Sort, it would take O(n^2) time as the given range is of size n^2. Using any comparison based sorting like Merge Sort, Heap Sort, .. etc would take O(nLogn) time.
Now question arises how to do this in 0(n)? Firstly, is it possible? Can we use data given in question? n numbers in range from 0 to $n^2$ â€“ 1?
The idea is to use Radix Sort. Following is standard Radix Sort algorithm.

```
1) Do following for each digit i where i varies from least
   significant digit to the most significant digit.
â€¦â€¦â€¦..a) Sort input array using counting sort (or any stable
        sort) according to the iâ€™th digit
```

Let there be d digits in input integers. Radix Sort takes O(d*(n+b)) time where b is the base for representing numbers, for example, for decimal system, b is 10. Since $n^2$-1 is the maximum possible value, the value of d would be $O(\log_b(n))$. So overall time complexity is $O((n+b)*\log_b(n))$. Which looks more than the time complexity of comparison based sorting algorithms for a large k. The idea is to change base b. If we set b as n, the value of $O(\log_b(n))$ becomes O(1) and overall time complexity becomes O(n).

```
arr[] = {0, 10, 13, 12, 7}

Let us consider the elements in base 5. For example 13 in
base 5 is 23, and 7 in base 5 is 12.
arr[] = {00(0), 20(10), 23(13), 22(12), 12(7)}

After first iteration (Sorting according to the last digit in
base 5),  we get.
arr[] = {00(0), 20(10), 12(7), 22(12), 23(13)}

After second iteration, we get
arr[] = {00(0), 12(7), 20(10), 22(12), 23(13)}
```

Following is C++ implementation to sort an array of size n where elements are in range from 0 to $n^2$ â€“ 1.

```
#include<iostream>
using namespace std;

// A function to do counting sort of arr[] according to
// the digit represented by exp.
```

```
int countSort(int arr[], int n, int exp)
{

    int output[n]; // output array
    int i, count[n] ;
    for (int i=0; i < n; i++)
       count[i] = 0;

    // Store count of occurrences in count[]
    for (i = 0; i < n; i++)
        count[ (arr[i]/exp)%n ]++;

    // Change count[i] so that count[i] now contains actual
    // position of this digit in output[]
    for (i = 1; i < n; i++)
        count[i] += count[i - 1];

    // Build the output array
    for (i = n - 1; i >= 0; i--)
    {
        output[count[ (arr[i]/exp)%n] - 1] = arr[i];
        count[(arr[i]/exp)%n]--;
    }

    // Copy the output array to arr[], so that arr[] now
    // contains sorted numbers according to curent digit
    for (i = 0; i < n; i++)
        arr[i] = output[i];
}


// The main function to that sorts arr[] of size n using Radix Sort
void sort(int arr[], int n)
{
    // Do counting sort for first digit in base n. Note that
    // instead of passing digit number, exp (n^0 = 0) is passed.
    countSort(arr, n, 1);

    // Do counting sort for second digit in base n. Note that
    // instead of passing digit number, exp (n^1 = n) is passed.
    countSort(arr, n, n);
}

// A utility function to print an array
```

```
void printArr(int arr[], int n)
{
    for (int i = 0; i < n; i++)
        cout << arr[i] << " ";
}

// Driver program to test above functions
int main()
{
    // Since array size is 7, elements should be from 0 to 48
    int arr[] = {40, 12, 45, 32, 33, 1, 22};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << "Given array is \n";
    printArr(arr, n);

    sort(arr, n);

    cout << "\nSorted array is \n";
    printArr(arr, n);
    return 0;
}
```

Output:

```
Given array is
40 12 45 32 33 1 22
Sorted array is
1 12 22 32 33 40 45
```

**How to sort if range is from 1 to $n^2$?**

If range is from 1 to n $n^2$, the above process can not be directly applied, it must be changed. Consider n = 100 and range from 1 to 10000. Since the base is 100, a digit must be from 0 to 99 and there should be 2 digits in the numbers. But the number 10000 has more than 2 digits. So to sort numbers in a range from 1 to $n^2$, we can use following process.

1) Subtract all numbers by 1.

2) Since the range is now 0 to $n^2$, do counting sort twice as done in the above implementation.

3) After the elements are sorted, add 1 to all numbers to obtain the original numbers.

**How to sort if range is from 0 to n^3 -1?**

Since there can be 3 digits in base n, we need to call counting sort 3 times.

This article is contributed by **Bateesh**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

115. Count all possible groups of size 2 or 3 that have sum as multiple of 3

Given an unsorted integer (positive values only) array of size â€˜nâ€™, we can form a group of two or three, the group should be such that the sum of all elements in that group is a multiple of 3. Count all possible number of groups that can be generated in this way.

```
Input: arr[] = {3, 6, 7, 2, 9}
Output: 8
// Groups are {3,6}, {3,9}, {9,6}, {7,2}, {3,6,9},
//            {3,7,2}, {7,2,6}, {7,2,9}



Input: arr[] = {2, 1, 3, 4}
Output: 4
// Groups are {2,1}, {2,4}, {2,1,3}, {2,4,3}
```

***We strongly recommend to minimize the browser and try this yourself first.***

The idea is to see remainder of every element when divided by 3. A set of elements can form a group only if sun of their remainders is multiple of 3. Since the task is to enumerate groups, we count all elements with different remainders.

```
1. Hash all elements in a count array based on remainder, i.e,
   for all elements a[i], do c[a[i]%3]++;
2. Now c[0] contains the number of elements which when divided
   by 3 leave remainder 0 and similarly c[1] for remainder 1
   and c[2] for 2.
3. Now for group of 2, we have 2 possibilities
   a. 2 elements of remainder 0 group. Such possibilities are
      c[0]*(c[0]-1)/2
   b. 1 element of remainder 1 and 1 from remainder 2 group
      Such groups are c[1]*c[2].
4. Now for group of 3,we have 4 possibilities
   a. 3 elements from remainder group 0.
      No. of such groups are c[0]C3
   b. 3 elements from remainder group 1.
      No. of such groups are c[1]C3
   c. 3 elements from remainder group 2.
      No. of such groups are c[2]C3
   d. 1 element from each of 3 groups.
```

No. of such groups are c[0]*c[1]*c[2].

5. Add all the groups in steps 3 and 4 to obtain the result.

```
#include<stdio.h>

// Returns count of all possible groups that can be formed from elements
// of a[].
int findgroups(int arr[], int n)
{
    // Create an array C[3] to store counts of elements with remainder
    // 0, 1 and 2.  c[i] would store count of elements with remainder i
    int c[3] = {0}, i;

    int res = 0; // To store the result

    // Count elements with remainder 0, 1 and 2
    for (i=0; i<n; i++)
        c[arr[i]%3]++;

    // Case 3.a: Count groups of size 2 from 0 remainder elements
    res += ((c[0]*(c[0]-1))>>1);

    // Case 3.b: Count groups of size 2 with one element with 1
    // remainder and other with 2 remainder
    res += c[1] * c[2];

    // Case 4.a: Count groups of size 3 with all 0 remainder elements
    res += (c[0] * (c[0]-1) * (c[0]-2))/6;

    // Case 4.b: Count groups of size 3 with all 1 remainder elements
    res += (c[1] * (c[1]-1) * (c[1]-2))/6;

    // Case 4.c: Count groups of size 3 with all 2 remainder elements
    res += ((c[2]*(c[2]-1)*(c[2]-2))/6);

    // Case 4.c: Count groups of size 3 with different remainders
    res += c[0]*c[1]*c[2];

    // Return total count stored in res
    return res;
}

// Driver program to test above functions
int main()
```

```
{
   int arr[] = {3, 6, 7, 2, 9};
   int n = sizeof(arr)/sizeof(arr[0]);
   printf("Required number of groups are %d\n", findgroups(arr,n));
   return 0;
}
```

Output:

```
Required number of groups are 8
```

Time Complexity: O(n)
Auxiliary Space: O(1)

This article is contributed by Amit Jain. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

116. Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)

Given two square matrices A and B of size n x n each, find their multiplication matrix.

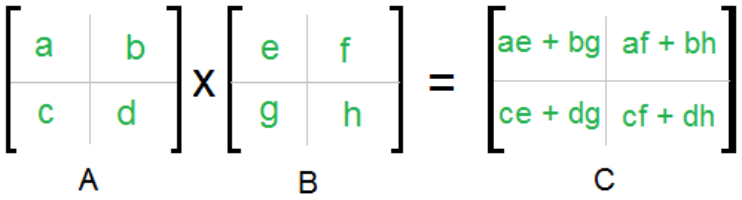
***Naive Method***
Following is a simple way to multiply two matrices.

```
void multiply(int A[][N], int B[][N], int C[][N])
{
   for (int i = 0; i < N; i++)
   {
      for (int j = 0; j < N; j++)
      {
         C[i][j] = 0;
         for (int k = 0; k < N; k++)
         {
            C[i][j] += A[i][k]*B[k][j];
         }
      }
   }
}
```

Time Complexity of above method is $O(N^3)$.

***Divide and Conquer***
Following is simple Divide and Conquer method to multiply two square matrices.

1) Divide matrices A and B in 4 sub-matrices of size N/2 x N/2 as shown in the below diagram.
2) Calculate following values recursively. ae + bg, af + bh, ce + dg and cf + dh.




A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2

In the above method, we do 8 multiplications for matrices of size N/2 x N/2 and 4 additions. Addition of two matrices takes $O(N^2)$ time. So the time complexity can be written as

$$T(N) = 8T(N/2) + O(N^2)$$

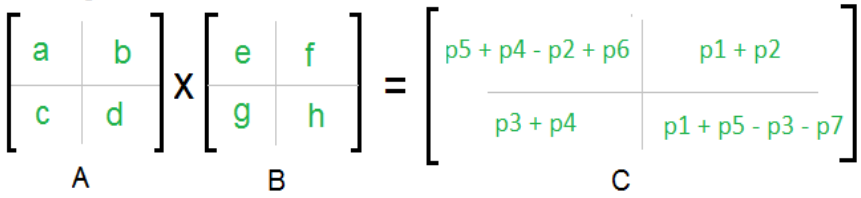
From Master's Theorem, time complexity of above method is $O(N^3)$
which is unfortunately same as the above naive method.

***Simple Divide and Conquer also leads to $O(N^3)$, can there be a better way?***

In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of **Strassenâ€™s method** is to reduce the number of recursive calls to 7. Strassenâ€™s method is similar to above simple divide and conquer method in the sense that this method also divide matrices to sub-matrices of size N/2 x N/2 as shown in the above diagram, but in Strassenâ€™s method, the four sub-matrices of result are calculated using following formulae.

**p1 = a(f - h)** **p2 = (a + b)h**
**p3 = (c + d)e** **p4 = d(g - e)**
**p5 = (a + d)(e + h)** **p6 = (b - d)(g + h)**
**p7 = (a - c)(e + f)**

The A x B can be calculated using above seven multiplications.
Following are values of four sub-matrices of result C




A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2
p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2

**Time Complexity of Strassenâ€™s Method**

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

$T(N) = 7T(N/2) + O(N^2)$

From Master's Theorem, time complexity of above method is
$O(N^{Log7})$ which is approximately $O(N^{2.8074})$

Generally Strassenâ€™s Method is not preferred for practical applications for following reasons.
1) The constants used in Strassenâ€™s method are high and for a typical application Naive method works better.
2) For Sparse matrices, there are better methods especially designed for them.
3) The submatrices in recursion take extra space.
4) Because of the limited precision of computer arithmetic on noninteger values, larger errors accumulate in Strassenâ€™s algorithm than in Naive Method (Source: CLRS Book)

**References:**
Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest
https://www.youtube.com/watch?v=LOLebQ8nKHA
https://www.youtube.com/watch?v=QXY4RskLQcI

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

117. Find if there is a subarray with 0 sum

Given an array of positive and negative numbers, find if there is a subarray with 0 sum.

Examples:

Input: {4, 2, -3, 1, 6}
Output: true
There is a subarray with zero sum from index 1 to 3.

Input: {4, 2, 0, 1, 6}
Output: true
There is a subarray with zero sum from index 2 to 2.

Input: {-3, 2, 3, 1, 6}
Output: false
There is no subarray with zero sum.

***We strongly recommend to minimize the browser and try this yourself first.***

A **simple solution** is to consider all subarrays one by one and check the sum of every subarray. We can run two loops: the outer loop picks a starting point i and the inner loop tries all subarrays starting from i (See this for implementation). Time complexity of this method is $O(n^2)$.

We can also **use hashing**. The idea is to iterate through the array and for every element arr[i], calculate sum of elements form 0 to i (this can simply be done as sum += arr[i]). If the current sum has been seen before, then there is a zero sum array. Hashing is used to store the sum values, so that we can quickly store sum and find out whether the current sum is seen before or not.

Following is Java implementation of the above approach.

```
// A Java program to find if there is a zero sum subarray
import java.util.HashMap;

class ZeroSumSubarray {

   // Returns true if arr[] has a subarray with sero sum
   static Boolean printZeroSumSubarray(int arr[])
   {
      // Creates an empty hashMap hM
      HashMap<Integer, Integer> hM = new HashMap<Integer, Integer>();

      // Initialize sum of elements
      int sum = 0;

      // Traverse through the given array
      for (int i = 0; i < arr.length; i++)
      {
         // Add current element to sum
         sum += arr[i];

         // Return true in following cases
         // a) Current element is 0
         // b) sum of elements from 0 to i is 0
         // c) sum is already present in hash map
         if (arr[i] == 0 || sum == 0 || hM.get(sum) != null)
            return true;

         // Add sum to hash map
         hM.put(sum, i);
      }
```

```
        // We reach here only when there is no subarray with 0 sum
        return false;
    }

    public static void main(String arg[])
    {
        int arr[] = {4, 2, -3, 1, 6};
        if (printZeroSumSubarray(arr))
            System.out.println("Found a subarray with 0 sum");
        else
            System.out.println("No Subarray with 0 sum");
    }
}
```

Output:

```
Found a subarray with 0 sum
```

Time Complexity of this solution can be considered as O(n) under the assumption that we have good hashing function that allows insertion and retrieval operations in O(1) time.

**Exercise:**
Extend the above program to print starting and ending indexes of all subarrays with 0 sum.

This article is contributed by **Chirag Gupta**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

118. Find the number of zeroes

Given an array of 1s and 0s which has all 1s first followed by all 0s. Find the number of 0s. Count the number of zeroes in the given array.

Examples:

```
Input: arr[] = {1, 1, 1, 1, 0, 0}
Output: 2

Input: arr[] = {1, 0, 0, 0, 0}
Output: 4
```

```
Input: arr[] = {0, 0, 0}
Output: 3

Input: arr[] = {1, 1, 1, 1}
Output: 0
```

***We strongly recommend to minimize the browser and try this yourself in time complexity better than O(n).***

A **simple solution** is to traverse the input array. As soon as we find a 0, we return n â€“ index of first 0. Here n is number of elements in input array. Time complexity of this solution would be O(n).

Since the input array is sorted, we can use **Binary Search** to find the first occurrence of 0. Once we have index of first element, we can return count as n â€“ index of first zero.

```
// A divide and conquer solution to find count of zeroes in an array
// where all 1s are present before all 0s
#include <stdio.h>

/* if 0 is present in arr[] then returns the index of FIRST occurrence
   of 0 in arr[low..high], otherwise returns -1 */
int firstZero(int arr[], int low, int high)
{
   if (high >= low)
   {
      // Check if mid element is first 0
      int mid = low + (high - low)/2;
      if (( mid == 0 || arr[mid-1] == 1) && arr[mid] == 0)
         return mid;

      if (arr[mid] == 1)  // If mid element is not 0
         return firstZero(arr, (mid + 1), high);
      else  // If mid element is 0, but not first 0
         return firstZero(arr, low, (mid -1));
   }
   return -1;
}

// A wrapper over recursive function firstZero()
int countOnes(int arr[], int n)
{
   // Find index of first zero in given array
```

```
    int first = firstZero(arr, 0, n-1);

    // If 0 is not present at all, return 0
    if (first == -1)
        return 0;

    return (n - first);
}

/* Driver program to check above functions */
int main()
{
    int arr[] =   {1, 1, 1, 0, 0, 0, 0, 0};
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Count of zeroes is %d", countOnes(arr, n));
    return 0;
}
```

Output:

```
Count of zeroes is 5
```

Time Complexity: O(Logn) where n is number of elements in arr[].

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

119.  Kth smallest element in a row-wise and column-wise sorted 2D array | Set 1

Given an n x n matrix, where every row and column is sorted in non-decreasing order. Find the kth smallest element in the given 2D array.

For example, consider the following 2D array.

```
      10, 20, 30, 40
      15, 25, 35, 45
      24, 29, 37, 48
      32, 33, 39, 50
The 3rd smallest element is 20 and 7th smallest element is 30
```

***We strongly recommend to minimize the browser and try this yourself first.***

The idea is to use min heap. Following are detailed step.

1) Build a min heap of elements from first row. A heap entry also stores row number and column number.
2) Do following k times.
â€¦a) Get minimum element (or root) from min heap.
â€¦b) Find row number and column number of the minimum element.
â€¦c) Replace root with the next element from same column and min-heapify the root.
3) Return the last extracted root.

Following is C++ implementation of above algorithm.

```
// kth largest element in a 2d array sorted row-wise and column-wise
#include<iostream>
#include<climits>
using namespace std;

// A structure to store an entry of heap.  The entry contains
// a value from 2D array, row and column numbers of the value
struct HeapNode {
    int val;  // value to be stored
    int r;    // Row number of value in 2D array
    int c;    // Column number of value in 2D array
};

// A utility function to swap two HeapNode items.
void swap(HeapNode *x, HeapNode *y) {
    HeapNode z = *x;
    *x = *y;
    *y = z;
}

// A utility function to minheapify the node harr[i] of a heap
// stored in harr[]
void minHeapify(HeapNode harr[], int i, int heap_size)
{
    int l = i*2 + 1;
    int r = i*2 + 2;
    int smallest = i;
    if (l < heap_size && harr[l].val < harr[i].val)
        smallest = l;
    if (r < heap_size && harr[r].val < harr[smallest].val)
        smallest = r;
    if (smallest != i)
    {
        swap(&harr[i], &harr[smallest]);
```

```
        minHeapify(harr, smallest, heap_size);
    }
}

// A utility function to convert harr[] to a max heap
void buildHeap(HeapNode harr[], int n)
{
    int i = (n - 1)/2;
    while (i >= 0)
    {
        minHeapify(harr, i, n);
        i--;
    }
}

// This function returns kth smallest element in a 2D array mat[][]
int kthSmallest(int mat[4][4], int n, int k)
{
    // k must be greater than 0 and smaller than n*n
    if (k <= 0 || k > n*n)
        return INT_MAX;

    // Create a min heap of elements from first row of 2D array
    HeapNode harr[n];
    for (int i = 0; i < n; i++)
        harr[i] =  {mat[0][i], 0, i};
    buildHeap(harr, n);

    HeapNode hr;
    for (int i = 0; i < k; i++)
    {
        // Get current heap root
        hr = harr[0];

        // Get next value from column of root's value. If the
        // value stored at root was last value in its column,
        // then assign INFINITE as next value
        int nextval = (hr.r < (n-1))? mat[hr.r + 1][hr.c]: INT_MAX;

        // Update heap root with next value
        harr[0] =  {nextval, (hr.r) + 1, hr.c};

        // Heapify root
        minHeapify(harr, 0, n);
```

```
    }

    // Return the value at last extracted root
    return hr.val;
}

// driver program to test above function
int main()
{
  int mat[4][4] = { {10, 20, 30, 40},
                    {15, 25, 35, 45},
                    {25, 29, 37, 48},
                    {32, 33, 39, 50},
                  };
  cout << "7th smallest element is " << kthSmallest(mat, 4, 7);
  return 0;
}
```

Output:

```
7th smallest element is 30
```

Time Complexity: The above solution involves following steps.
1) Build a min heap which takes O(n) time
2) Heapify k times which takes O(kLogn) time.
Therefore, overall time complexity is O(n + kLogn) time.

The above code can be optimized to build a heap of size k when k is smaller than n. In that case, the kth smallest element must be in first k rows and k columns.

We will soon be publishing more efficient algorithms for finding the kth smallest element.

This article is compiled by Ravi Gupta. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

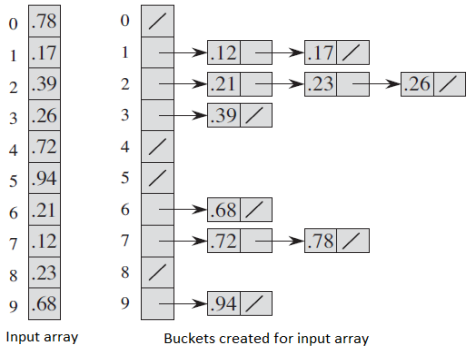
120.  Bucket Sort

Bucket sort is mainly useful when input is uniformly distributed over a range. For example, consider the following problem.
*Sort a large set of floating point numbers which are in range from 0.0 to 1.0 and are uniformly distributed across the range. How do we sort the numbers efficiently?*

A simple way is to apply a comparison based sorting algorithm. The lower bound for Comparison based sorting algorithm (Merge Sort, Heap Sort, Quick-Sort .. etc) is $\Omega(nLogn)$, i.e., they cannot do better than nLogn.
Can we sort the array in linear time? Counting sort can not be applied here as we use keys as index in counting sort. Here keys are floating point numbers.Â
The idea is to use bucket sort. Following is bucket algorithm.

**bucketSort(arr[], n)**
1) Create n empty buckets (Or lists).
2) Do following for every array element arr[i].
.....a) Insert arr[i] into bucket[n*array[i]]
3) Sort individual buckets using insertion sort.
4) Concatenate all sorted buckets.

Following diagram (taken from CLRS book) demonstrates working of bucket sort.




**Time Complexity:** If we assume that insertion in a bucket takes O(1) time then steps 1 and 2 of the above algorithm clearly take O(n) time. The O(1) is easily possible if we use a linked list to represent a bucket (In the following code, C++ vector is used for simplicity). Step 4 also takes O(n) time as there will be n items in all buckets.
The main step to analyze is step 3. This step also takes O(n) time on average if all numbers are uniformly distributed (please refer CLRS book for more details)

Following is C++ implementation of the above algorithm.

```
// C++ program to sort an array using bucket sort
#include <iostream>
#include <algorithm>
#include <vector>
using namespace std;

// Function to sort arr[] of size n using bucket sort
void bucketSort(float arr[], int n)
{
   // 1) Create n empty buckets
   vector<float> b[n];
```

```
    // 2) Put array elements in different buckets
    for (int i=0; i<n; i++)
    {
      int bi = n*arr[i]; // Index in bucket
      b[bi].push_back(arr[i]);
    }

    // 3) Sort individual buckets
    for (int i=0; i<n; i++)
      sort(b[i].begin(), b[i].end());

    // 4) Concatenate all buckets into arr[]
    int index = 0;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < b[i].size(); j++)
          arr[index++] = b[i][j];
}

/* Driver program to test above funtion */
int main()
{
    float arr[] = {0.897, 0.565, 0.656, 0.1234, 0.665, 0.3434};
    int n = sizeof(arr)/sizeof(arr[0]);
    bucketSort(arr, n);

    cout << "Sorted array is \n";
    for (int i=0; i<n; i++)
      cout << arr[i] << " ";
    return 0;
}
```

Output:

```
Sorted array is
0.1234 0.3434 0.565 0.656 0.665 0.897
```

**References:**

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest

http://en.wikipedia.org/wiki/Bucket_sort

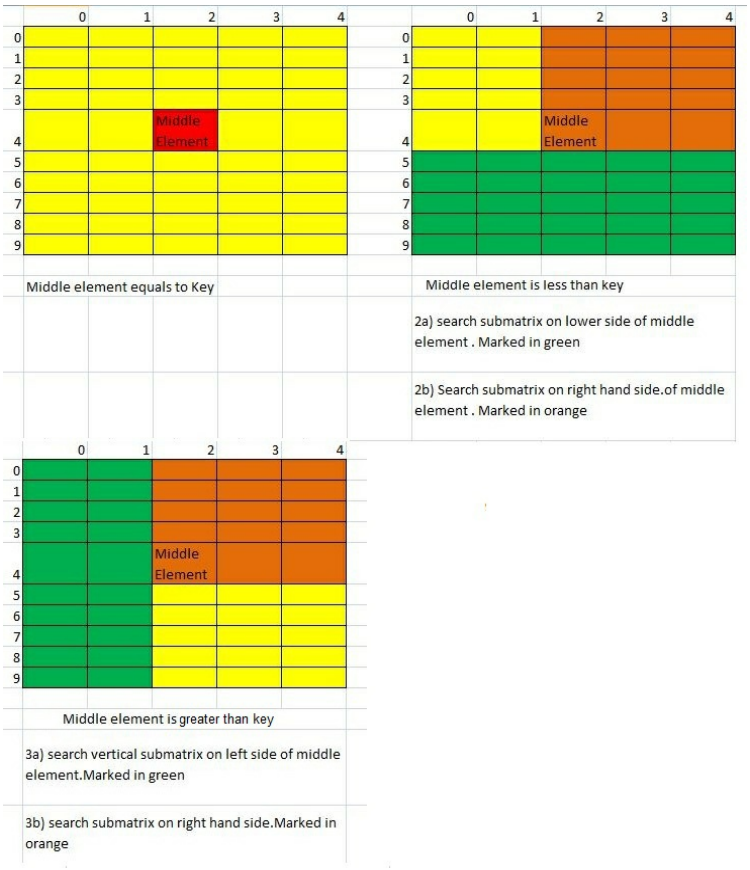
Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

121. Divide and Conquer | Set 6 (Search in a Row-wise and Column-wise Sorted 2D Array)

Given an n x n matrix, where every row and column is sorted in increasing order. Given a key, how to decide whether this key is in the matrix.
A linear time complexity is discussed in the previous post. This problem can also be a very good example for divide and conquer algorithms. Following is divide and conquer algorithm.

1) Find the middle element.
2) If middle element is same as key return.
3) If middle element is lesser than key then
â€¦.3a) search submatrix on lower side of middle element
â€¦.3b) Search submatrix on right hand side.of middle element
4) If middle element is greater than key then
â€¦.4a) search vertical submatrix on left side of middle element
â€¦.4b) search submatrix on right hand side.




Following Java implementation of above algorithm.

```
// Java program for implementation of divide and conquer algorithm
// to find a given key in a row-wise and column-wise sorted 2D array
class SearchInMatrix
{
    public static void main(String[] args)
```

```
{
    int[][] mat = new int[][] { {10, 20, 30, 40},
                                {15, 25, 35, 45},
                                {27, 29, 37, 48},
                                {32, 33, 39, 50}};
    int rowcount = 4,colCount=4,key=50;
    for (int i=0; i<rowcount; i++)
      for (int j=0; j<colCount; j++)
         search(mat, 0, rowcount-1, 0, colCount-1, mat[i][j]);
}

// A divide and conquer method to search a given key in mat[]
// in rows from fromRow to toRow and columns from fromCol to
// toCol
public static void search(int[][] mat, int fromRow, int toRow,
                          int fromCol, int toCol, int key)
{
    // Find middle and compare with middle
    int i = fromRow + (toRow-fromRow )/2;
    int j = fromCol + (toCol-fromCol )/2;
    if (mat[i][j] == key) // If key is present at middle
      System.out.println("Found "+ key + " at "+ i +
                         " " + j);
    else
    {
        // right-up quarter of matrix is searched in all cases.
        // Provided it is different from current call
        if (i!=toRow || j!=fromCol)
         search(mat,fromRow,i,j,toCol,key);

        // Special case for iteration with 1*2 matrix
        // mat[i][j] and mat[i][j+1] are only two elements.
        // So just check second element
        if (fromRow == toRow && fromCol + 1 == toCol)
          if (mat[fromRow][toCol] == key)
            System.out.println("Found "+ key+ " at "+
                               fromRow + " " + toCol);

        // If middle key is lesser then search lower horizontal
        // matrix and right hand side matrix
        if (mat[i][j] < key)
        {
            // search lower horizontal if such matrix exists
            if (i+1<=toRow)
```

```
            search(mat, i+1, toRow, fromCol, toCol, key);
        }

        // If middle key is greater then search left vertical
        // matrix and right hand side matrix
        else
        {
            // search left vertical if such matrix exists
            if (j-1>=fromCol)
              search(mat, fromRow, toRow, fromCol, j-1, key);
        }
      }
   }
}
```

**Time complexity:**
We are given a n*n matrix, the algorithm can be seen as recurring for 3 matrices of size n/2 x n/2. Following is recurrence for time complexity

```
T(n) = 3T(n/2) + O(1)
```

The solution of recurrence is $O(n^{1.58})$ using Master Method.
But the actual implementation calls for one submatrix of size n x n/2 or n/2 x n, and other submatrix of size n/2 x n/2.

This article is contributed by **Kaushik Lele**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

122. Remove minimum elements from either side such that 2*min becomes more than max

Given an unsorted array, trim the array such that twice of minimum is greater than maximum in the trimmed array. Elements should be removed either end of the array.

Number of removals should be minimum.

Examples:

```
arr[] = {4, 5, 100, 9, 10, 11, 12, 15, 200}
Output: 4
We need to remove 4 elements (4, 5, 100, 200)
so that 2*min becomes more than max.
```

```
arr[] = {4, 7, 5, 6}
Output: 0
We don't need to remove any element as
4*2 > 7 (Note that min = 4, max = 8)

arr[] = {20, 7, 5, 6}
Output: 1
We need to remove 20 so that 2*min becomes
more than max

arr[] = {20, 4, 1, 3}
Output: 3
We need to remove any three elements from ends
like 20, 4, 1 or 4, 1, 3 or 20, 3, 1 or 20, 4, 1
```

**Naive Solution:**

A naive solution is to try every possible case using recurrence. Following is the naive recursive algorithm. Note that the algorithm only returns minimum numbers of removals to be made, it doesnâ€™t print the trimmed array. It can be easily modified to print the trimmed array as well.

```
// Returns minimum number of removals to be made in
// arr[l..h]
minRemovals(int arr[], int l, int h)
1) Find min and max in arr[l..h]
2) If 2*min > max, then return 0.
3) Else return minimum of "minRemovals(arr, l+1, h) + 1"
   and "minRemovals(arr, l, h-1) + 1"
```

Following is C++ implementation of above algorithm.

```
#include <iostream>
using namespace std;

// A utility function to find minimum of two numbers
int min(int a, int b) {return (a < b)? a : b;}

// A utility function to find minimum in arr[l..h]
int min(int arr[], int l, int h)
{
   int mn = arr[l];
   for (int i=l+1; i<=h; i++)
      if (mn > arr[i])
        mn = arr[i];
```

```
    return mn;
}

// A utility function to find maximum in arr[l..h]
int max(int arr[], int l, int h)
{
    int mx = arr[l];
    for (int i=l+1; i<=h; i++)
       if (mx < arr[i])
         mx = arr[i];
    return mx;
}

// Returns the minimum number of removals from either end
// in arr[l..h] so that 2*min becomes greater than max.
int minRemovals(int arr[], int l, int h)
{
    // If there is 1 or less elements, return 0
    // For a single element, 2*min > max
    // (Assumption: All elements are positive in arr[])
    if (l >= h) return 0;

    // 1) Find minimum and maximum in arr[l..h]
    int mn = min(arr, l, h);
    int mx = max(arr, l, h);

    //If the property is followed, no removals needed
    if (2*mn > mx)
       return 0;

    // Otherwise remove a character from left end and recur,
    // then remove a character from right end and recur, take
    // the minimum of two is returned
    return min(minRemovals(arr, l+1, h),
               minRemovals(arr, l, h-1)) + 1;
}

// Driver program to test above functions
int main()
{
  int arr[] = {4, 5, 100, 9, 10, 11, 12, 15, 200};
  int n = sizeof(arr)/sizeof(arr[0]);
  cout << minRemovals(arr, 0, n-1);
  return 0;
```

```
}
```

Output:

```
4
```

Time complexity: Time complexity of the above function can be written as following

```
T(n) = 2T(n-1) + O(n)
```

An upper bound on solution of above recurrence would be $O(n \times 2^n)$.

**Dynamic Programming:**
The above recursive code exhibits many overlapping subproblems. For example minRemovals(arr, l+1, h-1) is evaluated twice. So Dynamic Programming is the choice to optimize the solution. Following is Dynamic Programming based solution.

```
#include <iostream>
using namespace std;

// A utility function to find minimum of two numbers
int min(int a, int b) {return (a < b)? a : b;}

// A utility function to find minimum in arr[l..h]
int min(int arr[], int l, int h)
{
   int mn = arr[l];
   for (int i=l+1; i<=h; i++)
     if (mn > arr[i])
       mn = arr[i];
   return mn;
}

// A utility function to find maximum in arr[l..h]
int max(int arr[], int l, int h)
{
   int mx = arr[l];
   for (int i=l+1; i<=h; i++)
     if (mx < arr[i])
       mx = arr[i];
   return mx;
}

// Returns the minimum number of removals from either end
// in arr[l..h] so that 2*min becomes greater than max.
int minRemovalsDP(int arr[], int n)
```

```
{
   // Create a table to store solutions of subproblems
   int table[n][n], gap, i, j, mn, mx;

   // Fill table using above recursive formula. Note that the table
   // is filled in diagonal fashion (similar to http://goo.gl/PQqoS),
   // from diagonal elements to table[0][n-1] which is the result.
   for (gap = 0; gap < n; ++gap)
   {
      for (i = 0, j = gap; j < n; ++i, ++j)
      {
         mn = min(arr, i, j);
         mx = max(arr, i, j);
         table[i][j] = (2*mn > mx)? 0: min(table[i][j-1]+1,
                                           table[i+1][j]+1);
      }
   }
   return table[0][n-1];
}

// Driver program to test above functions
int main()
{
  int arr[] = {20, 4, 1, 3};
  int n = sizeof(arr)/sizeof(arr[0]);
  cout << minRemovalsDP(arr, n);
  return 0;
}
```

Time Complexity: $O(n^3)$ where n is the number of elements in arr[].

Further Optimizations:
The above code can be optimized in many ways.
**1)** We can avoid calculation of min() and/or max() when min and/or max is/are not changed by removing corner elements.

**2)** We can pre-process the array and build segment tree in O(n) time. After the segment tree is built, we can query range minimum and maximum in O(Logn) time. The overall time complexity is reduced to $O(n^2Logn)$ time.

**A O(n^2) Solution**
The idea is to find the maximum sized subarray such that 2*min > max. We run two nested loops, the outer loop chooses a starting point and the inner loop chooses ending point for the current starting point. We keep track of longest subarray with the given property.

Following is C++ implementation of the above approach. Thanks to Richard Zhang for suggesting this solution.

```
// A O(n*n) solution to find the minimum of elements to
// be removed
#include <iostream>
#include <climits>
using namespace std;

// Returns the minimum number of removals from either end
// in arr[l..h] so that 2*min becomes greater than max.
int minRemovalsDP(int arr[], int n)
{
    // Initialize starting and ending indexes of the maximum
    // sized subarray with property 2*min > max
    int longest_start = -1, longest_end = 0;

    // Choose different elements as starting point
    for (int start=0; start<n; start++)
    {
        // Initialize min and max for the current start
        int min = INT_MAX, max = INT_MIN;

        // Choose different ending points for current start
        for (int end = start; end < n; end ++)
        {
            // Update min and max if necessary
            int val = arr[end];
            if (val < min) min = val;
            if (val > max) max = val;

            // If the property is violated, then no
            // point to continue for a bigger array
            if (2 * min <= max) break;

            // Update longest_start and longest_end if needed
            if (end - start > longest_end - longest_start ||
                longest_start == -1)
            {
                longest_start = start;
                longest_end = end;
            }
        }
    }
```

```
    // If not even a single element follow the property,
    // then return n
    if (longest_start == -1) return n;

    // Return the number of elements to be removed
    return (n - (longest_end - longest_start + 1));
}

// Driver program to test above functions
int main()
{
    int arr[] = {4, 5, 100, 9, 10, 11, 12, 15, 200};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << minRemovalsDP(arr, n);
    return 0;
}
```

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

123. Smallest subarray with sum greater than a given value

Given an array of integers and a number x, find the smallest subarray with sum greater than the given value.

```
Examples:
arr[] = {1, 4, 45, 6, 0, 19}
   x  =  51
Output: 3
Minimum length subarray is {4, 45, 6}

arr[] = {1, 10, 5, 2, 7}
   x  = 9
Output: 1
Minimum length subarray is {10}

arr[] = {1, 11, 100, 1, 0, 200, 3, 2, 1, 250}
    x = 280
Output: 4
Minimum length subarray is {100, 1, 0, 200}
```

A **simple solution** is to use two nested loops. The outer loop picks a starting element, the inner loop considers all elements (on right side of current start) as ending element. Whenever sum of elements between current start and end becomes more than the given number, update the result if current length is smaller than the smallest length so far.
Following is C++ implementation of simple approach.

```
# include <iostream>
using namespace std;

// Returns length of smallest subarray with sum greater than x.
// If there is no subarray with given sum, then returns n+1
int smallestSubWithSum(int arr[], int n, int x)
{
   //  Initilize length of smallest subarray as n+1
    int min_len = n + 1;

    // Pick every element as starting point
    for (int start=0; start<n; start++)
    {
        // Initialize sum starting with current start
        int curr_sum = arr[start];

        // If first element itself is greater
        if (curr_sum > x) return 1;

        // Try different ending points for curremt start
        for (int end=start+1; end<n; end++)
        {
            // add last element to current sum
            curr_sum += arr[end];

            // If sum becomes more than x and length of
            // this subarray is smaller than current smallest
            // length, update the smallest length (or result)
            if (curr_sum > x && (end - start + 1) < min_len)
               min_len = (end - start + 1);
        }
    }
    return min_len;
}

/* Driver program to test above function */
int main()
```

```
{
    int arr1[] = {1, 4, 45, 6, 10, 19};
    int x = 51;
    int n1 = sizeof(arr1)/sizeof(arr1[0]);
    cout << smallestSubWithSum(arr1, n1, x) << endl;

    int arr2[] = {1, 10, 5, 2, 7};
    int n2 = sizeof(arr2)/sizeof(arr2[0]);
    x  = 9;
    cout << smallestSubWithSum(arr2, n2, x) << endl;

    int arr3[] = {1, 11, 100, 1, 0, 200, 3, 2, 1, 250};
    int n3 = sizeof(arr3)/sizeof(arr3[0]);
    x  = 280;
    cout << smallestSubWithSum(arr3, n3, x) << endl;

    return 0;
}
```

Output:

```
3
1
4
```

Time Complexity: Time complexity of the above approach is clearly $O(n^2)$.

**Efficient Solution:** This problem can be solved in **O(n) time** using the idea used in this post. Thanks to Ankit and Nitin for suggesting this optimized solution.

```
// O(n) solution for finding smallest subarray with sum
// greater than x
#include <iostream>
using namespace std;

// Returns length of smallest subarray with sum greater than x.
// If there is no subarray with given sum, then returns n+1
int smallestSubWithSum(int arr[], int n, int x)
{
    // Initialize current sum and minimum length
    int curr_sum = 0, min_len = n+1;

    // Initialize starting and ending indexes
    int start = 0, end = 0;
    while (end < n)
```

```
    {
        // Keep adding array elements while current sum
        // is smaller than x
        while (curr_sum <= x && end < n)
            curr_sum += arr[end++];

        // If current sum becomes greater than x.
        while (curr_sum > x && start < n)
        {
            // Update minimum length if needed
            if (end - start < min_len)
                min_len = end - start;

            // remove starting elements
            curr_sum -= arr[start++];
        }
    }
    return min_len;
}

/* Driver program to test above function */
int main()
{
    int arr1[] = {1, 4, 45, 6, 10, 19};
    int x = 51;
    int n1 = sizeof(arr1)/sizeof(arr1[0]);
    cout << smallestSubWithSum(arr1, n1, x) << endl;

    int arr2[] = {1, 10, 5, 2, 7};
    int n2 = sizeof(arr2)/sizeof(arr2[0]);
    x  = 9;
    cout << smallestSubWithSum(arr2, n2, x) << endl;

    int arr3[] = {1, 11, 100, 1, 0, 200, 3, 2, 1, 250};
    int n3 = sizeof(arr3)/sizeof(arr3[0]);
    x  = 280;
    cout << smallestSubWithSum(arr3, n3, x);

    return 0;
}
```

Output:

```
3
```

```
1
4
```

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

124. Create a matrix with alternating rectangles of O and X

Write a code which inputs two numbers m and n and creates a matrix of size m x n (m rows and n columns) in which every elements is either X or 0. The Xs and 0s must be filled alternatively, the matrix should have outermost rectangle of Xs, then a rectangle of 0s, then a rectangle of Xs, and so on.

Examples:

```
Input: m = 3, n = 3
Output: Following matrix
X X X
X 0 X
X X X

Input: m = 4, n = 5
Output: Following matrix
X X X X X
X 0 0 0 X
X 0 0 0 X
X X X X X

Input:  m = 5, n = 5
Output: Following matrix
X X X X X
X 0 0 0 X
X 0 X 0 X
X 0 0 0 X
X X X X X

Input:  m = 6, n = 7
Output: Following matrix
X X X X X X X
X 0 0 0 0 0 X
X 0 X X X 0 X
X 0 X X X 0 X
```

```
X 0 0 0 0 0 X
X X X X X X X
```

***We strongly recommend to minimize the browser and try this yourself first.***

This question was asked in campus recruitment of Shreepartners Gurgaon. I followed the following approach.

**1)** Use the code for Printing Matrix in Spiral form.
**2)** Instead of printing the array, inserted the element â€˜Xâ€™ or â€˜0â€™ alternatively in the array.

Following is C implementation of the above approach.

```
#include <stdio.h>

// Function to print alternating rectangles of 0 and X
void fill0X(int m, int n)
{
    /*  k - starting row index
        m - ending row index
        l - starting column index
        n - ending column index
        i - iterator    */
    int i, k = 0, l = 0;

    // Store given number of rows and columns for later use
    int r = m, c = n;

    // A 2D array to store the output to be printed
    char a[m][n];
    char x = 'X'; // Iniitialize the character to be stoed in a[][]

    // Fill characters in a[][] in spiral form. Every iteration fills
    // one rectangle of either Xs or Os
    while (k < m && l < n)
    {
        /* Fill the first row from the remaining rows */
        for (i = l; i < n; ++i)
            a[k][i] = x;
        k++;

        /* Fill the last column from the remaining columns */
        for (i = k; i < m; ++i)
            a[i][n-1] = x;
```

```
        n--;

        /* Fill the last row from the remaining rows */
        if (k < m)
        {
            for (i = n-1; i >= l; --i)
                a[m-1][i] = x;
            m--;
        }

        /* Print the first column from the remaining columns */
        if (l < n)
        {
            for (i = m-1; i >= k; --i)
                a[i][l] = x;
            l++;
        }

        // Flip character for next iteration
        x = (x == '0')? 'X': '0';
    }

    // Print the filled matrix
    for (i = 0; i < r; i++)
    {
        for (int j = 0; j < c; j++)
            printf("%c ", a[i][j]);
        printf("\n");
    }
}

/* Driver program to test above functions */
int main()
{
    puts("Output for m = 5, n = 6");
    fill0X(5, 6);

    puts("\nOutput for m = 4, n = 4");
    fill0X(4, 4);

    puts("\nOutput for m = 3, n = 4");
    fill0X(3, 4);

    return 0;
```

```
}
```

Output:

```
Output for m = 5, n = 6
X X X X X X
X 0 0 0 0 X
X 0 X X 0 X
X 0 0 0 0 X
X X X X X X

Output for m = 4, n = 4
X X X X
X 0 0 X
X 0 0 X
X X X X

Output for m = 3, n = 4
X X X X
X 0 0 X
X X X X
```

Time Complexity: O(mn)
Auxiliary Space: O(mn)

Please suggest if someone has a better solution which is more efficient in terms of space and time.

This article is contributed by **Deepak Bisht**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

125. Find k closest elements to a given value

Given a sorted array arr[] and a value X, find the k closest elements to X in arr[].
Examples:

```
Input: K = 4, X = 35
      arr[] = {12, 16, 22, 30, 35, 39, 42,
               45, 48, 50, 53, 55, 56}
Output: 30 39 42 45
```

Note that if the element is present in array, then it should not be in output, only the

other closest elements are required.

In the following solutions, it is assumed that all elements of array are distinct.

A **simple solution** is to do linear search for k closest elements.
1) Start from the first element and search for the crossover point (The point before which elements are smaller than or equal to X and after which elements are greater). This step takes O(n) time.
2) Once we find the crossover point, we can compare elements on both sides of crossover point to print k closest elements. This step takes O(k) time.

The time complexity of the above solution is O(n).

An **Optimized Solution** is to find k elements in O(Logn + k) time. The idea is to use Binary Search to find the crossover point. Once we find index of crossover point, we can print k closest elements in O(k) time.

```
#include<stdio.h>

/* Function to find the cross over point (the point before
   which elements are smaller than or equal to x and after
   which greater than x)*/
int findCrossOver(int arr[], int low, int high, int x)
{
  // Base cases
  if (arr[high] <= x) // x is greater than all
    return high;
  if (arr[low] > x)  // x is smaller than all
    return low;

  // Find the middle point
  int mid = (low + high)/2;  /* low + (high - low)/2 */

  /* If x is same as middle element, then return mid */
  if (arr[mid] <= x && arr[mid+1] > x)
    return mid;

  /* If x is greater than arr[mid], then either arr[mid + 1]
    is ceiling of x or ceiling lies in arr[mid+1...high] */
  if(arr[mid] < x)
      return findCrossOver(arr, mid+1, high, x);

  return findCrossOver(arr, low, mid - 1, x);
}
```

```
// This function prints k closest elements to x in arr[].
// n is the number of elements in arr[]
void printKclosest(int arr[], int x, int k, int n)
{
    // Find the crossover point
    int l = findCrossOver(arr, 0, n-1, x); // le
    int r = l+1;   // Right index to search
    int count = 0; // To keep track of count of elements already printed

    // If x is present in arr[], then reduce left index
    // Assumption: all elements in arr[] are distinct
    if (arr[l] == x) l--;

    // Compare elements on left and right of crossover
    // point to find the k closest elements
    while (l >= 0 && r < n && count < k)
    {
        if (x - arr[l] < arr[r] - x)
            printf("%d ", arr[l--]);
        else
            printf("%d ", arr[r++]);
        count++;
    }

    // If there are no more elements on right side, then
    // print left elements
    while (count < k && l >= 0)
        printf("%d ", arr[l--]), count++;

    // If there are no more elements on left side, then
    // print right elements
    while (count < k && r < n)
        printf("%d ", arr[r++]), count++;
}

/* Driver program to check above functions */
int main()
{
   int arr[] ={12, 16, 22, 30, 35, 39, 42,
               45, 48, 50, 53, 55, 56};
   int n = sizeof(arr)/sizeof(arr[0]);
   int x = 35, k = 4;
   printKclosest(arr, x, 4, n);
   return 0;
```

```
}
```

Output:

```
39 30 42 45
```

The time complexity of this method is O(Logn + k).

**Exercise:** Extend the optimized solution to work for duplicates also, i.e., to work for arrays where elements donâ€™t have to be distinct.

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â Â

126. Count number of binary strings without consecutive 1's

Given a positive integer N, count all possible distinct binary strings of length N such that there are no consecutive 1â€™s.

Examples:

```
Input:  N = 2
Output: 3
// The 3 strings are 00, 01, 10

Input: N = 3
Output: 5
// The 5 strings are 000, 001, 010, 100, 101
```

This problem can be solved using Dynamic Programming. Let a[i] be the number of binary strings of length i which do not contain any two consecutive 1â€™s and which end in 0. Similarly, let b[i] be the number of such strings which end in 1. We can append either 0 or 1 to a string ending in 0, but we can only append 0 to a string ending in 1. This yields the recurrence relation:

```
a[i] = a[i - 1] + b[i - 1]
b[i] = a[i - 1]
```

The base cases of above recurrence are a[1] = b[1] = 1. The total number of strings of length i is just a[i] + b[i].

Following is C++ implementation of above solution. In the following implementation, indexes start from 0. So a[i] represents the number of binary strings for input length i+1. Similarly, b[i] represents binary strings for input length i+1.

```
// C++ program to count all distinct binary strings
// without two consecutive 1's
#include <iostream>
using namespace std;

int countStrings(int n)
{
    int a[n], b[n];
    a[0] = b[0] = 1;
    for (int i = 1; i < n; i++)
    {
        a[i] = a[i-1] + b[i-1];
        b[i] = a[i-1];
    }
    return a[n-1] + b[n-1];
}



// Driver program to test above functions
int main()
{
    cout << countStrings(3) << endl;
    return 0;
}
```

Output:

```
5
```

**Source:**
courses.csail.mit.edu/6.006/oldquizzes/solutions/q2-f2009-sol.pdf

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

127. Find next greater number with same set of digits

Given a number n, find the smallest number that has same set of digits as n and is greater than n. If x is the greatest possible number with its set of digits, then print â€œnot possibleâ€.

Examples:

For simplicity of implementation, we have considered input number as a string.

```
Input:  n = "218765"
Output: "251678"

Input:  n = "1234"
Output: "1243"

Input: n = "4321"
Output: "Not Possible"

Input: n = "534976"
Output: "536479"
```

**We strongly recommend to minimize the browser and try this yourself first.**

Following are few observations about the next greater number.
1) If all digits sorted in descending order, then output is always â€œNot Possibleâ€. For example, 4321.
2) If all digits are sorted in ascending order, then we need to swap last two digits. For example, 1234.
3) For other cases, we need to process the number from rightmost side (why? because we need to find the smallest of all greater numbers)

You can now try developing an algorithm yourself.

Following is the algorithm for finding the next greater number.
**I)** Traverse the given number from rightmost digit, keep traversing till you find a digit which is smaller than the previously traversed digit. For example, if the input number is â€œ534976â€, we stop at **4** because 4 is smaller than next digit 9. If we do not find such a digit, then output is â€œNot Possibleâ€.

**II)** Now search the right side of above found digit â€˜dâ€™ for the smallest digit greater than â€˜dâ€™. For â€œ53**4**976â€³, the right side of 4 contains â€œ976â€. The smallest digit greater than 4 is **6**.

**III)** Swap the above found two digits, we get 53**6**97**4** in above example.

**IV)** Now sort all digits from position next to â€˜dâ€™ to the end of number. The number that we get after sorting is the output. For above example, we sort digits in bold 536**974**. We get â€œ536**479**â€ which is the next greater number for input 534976.

Following is C++ implementation of above approach.

```
// C++ program to find the smallest number which greater than a given number
// and has same set of digits as given number
```

```
#include <iostream>
#include <cstring>
#include <algorithm>
using namespace std;

// Utility function to swap two digits
void swap(char *a, char *b)
{
    char temp = *a;
    *a = *b;
    *b = temp;
}

// Given a number as a char array number[], this function finds the
// next greater number.  It modifies the same array to store the result
void findNext(char number[], int n)
{
    int i, j;

    // I) Start from the right most digit and find the first digit that is
    // smaller than the digit next to it.
    for (i = n-1; i > 0; i--)
        if (number[i] > number[i-1])
            break;

    // If no such digit is found, then all digits are in descending order
    // means there cannot be a greater number with same set of digits
    if (i==0)
    {
        cout << "Next number is not possible";
        return;
    }

    // II) Find the smallest digit on right side of (i-1)'th digit that is
    // greater than number[i-1]
    int x = number[i-1], smallest = i;
    for (j = i+1; j < n; j++)
        if (number[j] > x && number[j] < number[smallest])
            smallest = j;

    // III) Swap the above found smallest digit with number[i-1]
    swap(&number[smallest], &number[i-1]);

    // IV) Sort the digits after (i-1) in ascending order
```

```
    sort(number + i, number + n);

    cout << "Next number with same set of digits is " << number;

    return;
}

// Driver program to test above function
int main()
{
    char digits[] = "534976";
    int n = strlen(digits);
    findNext(digits, n);
    return 0;
}
```

Output:

```
Next number with same set of digits is 536479
```

The above implementation can be optimized in following ways.
1) We can use binary search in step II instead of linear search.
2) In step IV, instead of doing simple sort, we can apply some clever technique to do it in linear time. Hint: We know that all digits are linearly sorted in reverse order except one digit which was swapped.

With above optimizations, we can say that the time complexity of this method is O(n).

This article is contributed by **Rahul Jain**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

128.  Maximum Sum Path in Two Arrays

Given two sorted arrays such the arrays may have some common elements. Find the sum of the maximum sum path to reach from beginning of any array to end of any of the two arrays. We can switch from one array to another array only at common elements.

Expected time complexity is O(m+n) where m is the number of elements in ar1[] and n is the number of elements in ar2[].

Examples:

```
Input:  ar1[] = {2, 3, 7, 10, 12}, ar2[] = {1, 5, 7, 8}
```

Output: 35
35 is sum of 1 + 5 + **7** + 10 + 12.
We start from first element of arr2 which is 1, then we
move to 5, then 7.  From 7, we switch to ar1 (7 is common)
and traverse 10 and 12.

Input:  ar1[] = {10, 12}, ar2 = {5, 7, 9}
Output: 22
22 is sum of 10 and 12.
Since there is no common element, we need to take all
elements from the array with more sum.

Input:  ar1[] = {2, 3, 7, 10, 12, 15, 30, 34}
       ar2[] = {1, 5, 7, 8, 10, 15, 16, 19}
Output: 122
122 is sum of 1, 5, **7**, 8, **10**, 12, **15**, 30, 34

**We strongly recommend to minimize the browser and try this yourself first.**

The idea is to do something similar to merge process of merge sort. We need to calculate sums of elements between all common points for both arrays. Whenever we see a common point, we compare the two sums and add the maximum of two to the result. Following are detailed steps.

1) Initialize result as 0. Also initialize two variables sum1 and sum2 as 0. Here sum1 and sum2 are used to store sum of element in ar1[] and ar2[] respectively. These sums are between two common points.

2) Now run a loop to traverse elements of both arrays. While traversing compare current elements of ar1[] and ar2[].

Â Â Â Â 2.a) If current element of ar1[] is smaller than current element of ar2[], then update sum1, else if current element of ar2[] is smaller, then update sum2.

Â Â Â Â 2.b) If current element of ar1[] and ar2[] are same, then take the maximum of sum1 and sum2 and add it to the result. Also add the common element to the result.

Following is C++ implementation of above approach.

```
#include<iostream>
using namespace std;

// Utility function to find maximum of two integers
int max(int x, int y) { return (x > y)? x : y; }

// This function returns the sum of elements on maximum path
```

```
// from beginning to end
int maxPathSum(int ar1[], int ar2[], int m, int n)
{
    // initialize indexes for ar1[] and ar2[]
    int i = 0, j = 0;

    // Initialize result and current sum through ar1[] and ar2[].
    int  result = 0, sum1 = 0, sum2 = 0;

    // Below 3 loops are similar to merge in merge sort
    while (i < m && j < n)
    {
        // Add elements of ar1[] to sum1
        if (ar1[i] < ar2[j])
            sum1 += ar1[i++];

        // Add elements of ar2[] to sum2
        else if (ar1[i] > ar2[j])
            sum2 += ar2[j++];

        else  // we reached a common point
        {
            // Take the maximum of two sums and add to result
            result += max(sum1, sum2);

            // Update sum1 and sum2 for elements after this
            // intersection point
            sum1 = 0, sum2 = 0;

            // Keep updating result while there are more common
            // elements
            while (i < m &&  j < n && ar1[i] == ar2[j])
            {
                result = result + ar1[i++];
                j++;
            }
        }
    }

    // Add remaining elements of ar1[]
    while (i < m)
        sum1  +=  ar1[i++];

    // Add remaining elements of ar2[]
```

```
    while (j < n)
        sum2 +=  ar2[j++];

    // Add maximum of two sums of remaining elements
    result +=  max(sum1, sum2);

    return result;
}

// Driver program to test above function
int main()
{
    int ar1[]  = {2, 3, 7, 10, 12, 15, 30, 34};
    int ar2[] =  {1, 5, 7, 8, 10, 15, 16, 19};
    int m = sizeof(ar1)/sizeof(ar1[0]);
    int n = sizeof(ar2)/sizeof(ar2[0]);
    cout << maxPathSum(ar1, ar2, m, n);
    return 0;
}
```

Output:

```
122
```

Time complexity: In every iteration of while loops, we process an element from either of the two arrays. There are total m + n elements. Therefore, time complexity is O(m+n).

This article is contributed by **Piyush Gupta**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

129.  Search in an almost sorted array

Given an array which is sorted, but after sorting some elements are moved to either of the adjacent positions, i.e., arr[i] may be present at arr[i+1] or arr[i-1]. Write an efficient function to search an element in this array. Basically the element arr[i] can only be swapped with either arr[i+1] or arr[i-1].

For example consider the array {2, 3, 10, 4, 40}, 4 is moved to next position and 10 is moved to previous position.

Example:

```
Input: arr[] =  {10, 3, 40, 20, 50, 80, 70}, key = 40
Output: 2
Output is index of 40 in given array

Input: arr[] =  {10, 3, 40, 20, 50, 80, 70}, key = 90
Output: -1
-1 is returned to indicate element is not present
```

A simple solution is to linearly search the given key in given array. Time complexity of this solution is O(n). We cab modify binary search to do it in O(Logn) time.

The idea is to compare the key with middle 3 elements, if present then return the index. If not present, then compare the key with middle element to decide whether to go in left half or right half. Comparing with middle element is enough as all the elements after mid+2 must be greater than element mid and all elements before mid-2 must be smaller than mid element.

Following is C++ implementation of this approach.

```
// C++ program to find an element in an almost sorted array
#include <stdio.h>

// A recursive binary search based function. It returns index of x in
// given array arr[l..r] is present, otherwise -1
int binarySearch(int arr[], int l, int r, int x)
{
  if (r >= l)
  {
      int mid = l + (r - l)/2;

      // If the element is present at one of the middle 3 positions
      if (arr[mid] == x)  return mid;
      if (mid > l && arr[mid-1] == x) return (mid - 1);
      if (mid < r && arr[mid+1] == x) return (mid + 1);

      // If element is smaller than mid, then it can only be present
      // in left subarray
      if (arr[mid] > x) return binarySearch(arr, l, mid-2, x);

      // Else the element can only be present in right subarray
      return binarySearch(arr, mid+2, r, x);
  }

  // We reach here when element is not present in array
  return -1;
```

```
}

// Driver program to test above function
int main(void)
{
   int arr[] = {3, 2, 10, 4, 40};
   int n = sizeof(arr)/ sizeof(arr[0]);
   int x = 4;
   int result = binarySearch(arr, 0, n-1, x);
   (result == -1)? printf("Element is not present in array")
                 : printf("Element is present at index %d", result);
   return 0;
}
```

Output:

```
Element is present at index 3
```

Time complexity of the above function is O(Logn).

This article is contributed by **Abhishek**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

130. Sort an array according to the order defined by another array

Given two arrays A1[] and A2[], sort A1 in such a way that the relative order among the elements will be same as those are in A2. For the elements not present in A2, append them at last in sorted order.

```
Input: A1[] = {2, 1, 2, 5, 7, 1, 9, 3, 6, 8, 8}
       A2[] = {2, 1, 8, 3}
Output: A1[] = {2, 2, 1, 1, 8, 8, 3, 5, 6, 7, 9}
```

The code should handle all cases like number of elements in A2[] may be more or less compared to A1[]. A2[] may have some elements which may not be there in A1[] and vice versa is also possible.

Source: Amazon Interview | Set 110 (On-Campus)

**We strongly recommend to minimize the browser and try this yourself first.**

**Method 1 (Using Sorting and Binary Search)**
Let size of A1[] be m and size of A2[] be n.
1) Create a temporary array temp of size m and copy contents of A1[] to it.

2) Create another array visited[] and initialize all entries in it as false. visited[] is used to mark those elements in temp[] which are copied to A1[].
3) Sort temp[]
4) Initialize the output index ind as 0.
5) Do following for every element of A2[i] in A2[]
â€¦..a) Binary search for all occurrences of A2[i] in temp[], if present then copy all occurrences to A1[ind] and increment ind. Also mark the copied elements visited[]
6) Copy all unvisited elements from temp[] to A1[].

Time complexity: The steps 1 and 2 require O(m) time. Step 3 requires O(mLogm) time. Step 5 requires O(nLogm) time. Therefore overall time complexity is O(m + nLogm).

Thanks to vivek for suggesting this method. Following is C++ implementation of above algorithm.

```
// A C++ program to sort an array according to the order defined
// by another array
#include <iostream>
#include <algorithm>
using namespace std;

/* A Binary Search based function to find index of FIRST occurrence
  of x in arr[].  If x is not present, then it returns -1 */
int first(int arr[], int low, int high, int x, int n)
{
    if (high >= low)
    {
        int mid = low + (high-low)/2;  /* (low + high)/2; */
        if ((mid == 0 || x > arr[mid-1]) && arr[mid] == x)
            return mid;
        if (x > arr[mid])
            return first(arr, (mid + 1), high, x, n);
        return first(arr, low, (mid -1), x, n);
    }
    return -1;
}

// Sort A1[0..m-1] according to the order defined by A2[0..n-1].
void sortAccording(int A1[], int A2[], int m, int n)
{
    // The temp array is used to store a copy of A1[] and visited[]
    // is used mark the visited elements in temp[].
    int temp[m], visited[m];
    for (int i=0; i<m; i++)
```

```
    {
        temp[i] = A1[i];
        visited[i] = 0;
    }

    // Sort elements in temp
    sort(temp, temp + m);

    int ind = 0;  // for index of output which is sorted A1[]

    // Consider all elements of A2[], find them in temp[]
    // and copy to A1[] in order.
    for (int i=0; i<n; i++)
    {
        // Find index of the first occurrence of A2[i] in temp
        int f = first(temp, 0, m-1, A2[i], m);

        // If not present, no need to proceed
        if (f == -1) continue;

        // Copy all occurrences of A2[i] to A1[]
        for (int j = f; (j<m && temp[j]==A2[i]); j++)
        {
            A1[ind++] = temp[j];
            visited[j] = 1;
        }
    }

    // Now copy all items of temp[] which are not present in A2[]
    for (int i=0; i<m; i++)
        if (visited[i] == 0)
            A1[ind++] = temp[i];
}

// Utility function to print an array
void printArray(int arr[], int n)
{
    for (int i=0; i<n; i++)
        cout << arr[i] << " ";
    cout << endl;
}

// Driver program to test above function.
int main()
```

```
{
    int A1[] = {2, 1, 2, 5, 7, 1, 9, 3, 6, 8, 8};
    int A2[] = {2, 1, 8, 3};
    int m = sizeof(A1)/sizeof(A1[0]);
    int n = sizeof(A2)/sizeof(A2[0]);
    cout << "Sorted array is \n";
    sortAccording(A1, A2, m, n);
    printArray(A1, m);
    return 0;
}
```

Output:

```
Sorted array is
2 2 1 1 8 8 3 5 6 7 9
```

**Method 2 (Using Self-Balancing Binary Search Tree)**
We can also use a self balancing BST like AVL Tree, Red Black Tree, etc. Following are detailed steps.
1) Create a self balancing BST of all elements in A1[]. In every node of BST, also keep track of count of occurrences of the key and a bool field visited which is initialized as false for all nodes.
2) Initialize the output index ind as 0.
3) Do following for every element of A2[i] in A2[]
â€¦..a) Search for A2[i] in the BST, if present then copy all occurrences to A1[ind] and increment ind. Also mark the copied elements visited in the BST node.
4) Do an inorder traversal of BST and copy all unvisited keys to A1[].

Time Complexity of this method is same as the previous method. Note that in a self balancing Binary Search Tree, all operations require logm time.

**Method 3 (Use Hashing)**
1. Loop through A1[], store the count of every number in a HashMap (key: number, value: count of number) .
2. Loop through A2[], check if it is present in HashMap, if so, put in output array that many times and remove the number from HashMap.
3. Sort the rest of the numbers present in HashMap and put in output array.

Thanks to Anurag Sigh for suggesting this method.

The steps 1 and 2 on average take O(m+n) time under the assumption that we have a good hashing function that takes O(1) time for insertion and search on average. The third step takes O(pLogp) time where p is the number of elements remained after considering elements of A2[].

**Method 4 (By Writing a Customized Compare Method)**

We can also customize compare method of a sorting algorithm to solve the above problem. For example qsort() in C allows us to pass our own customized compare method.
1. If num1 and num2 both are in A2 then number with lower index in A2 will be treated smaller than other.
2. If only one of num1 or num2 present in A2, then that number will be treated smaller than the other which doesnâ€™t present in A2.
3. If both are not in A2, then natural ordering will be taken.

Time complexity of this method is O(mnLogm) if we use a O(nLogn) time complexity sorting algorithm. We can improve time complexity to O(mLogm) by using a Hashing instead of doing linear search.

Following is C implementation of this method.

```
// A C++ program to sort an array according to the order defined
// by another array
#include <stdio.h>
#include <stdlib.h>

// A2 is made global here so that it can be accesed by compareByA2()
// The syntax of qsort() allows only two parameters to compareByA2()
int A2[5];
int size = 5; // size of A2[]

int search(int key)
{
 int i=0, idx = 0;
 for (i=0; i<size; i++)
  if (A2[i] == key)
   return i;
 return -1;
}

// A custom comapre method to compare elements of A1[] according
// to the order defined by A2[].
int compareByA2(const void * a, const void * b)
{
 int idx1 = search(*(int*)a);
 int idx2 = search(*(int*)b);
 if (idx1 != -1 && idx2 != -1)
  return idx1 - idx2;
 else if(idx1 != -1)
  return -1;
 else if(idx2 != -1)
```

```
 return 1;
 else
  return ( *(int*)a - *(int*)b );
}

// This method mainly uses qsort to sort A1[] according to A2[]
void sortA1ByA2(int A1[], int size1)
{
 qsort(A1, size1, sizeof (int), compareByA2);
}

// Driver program to test above function
int main(int argc, char *argv[])
{
 int A1[] = {2, 1, 2, 5, 7, 1, 9, 3, 6, 8, 8, 7, 5, 6, 9, 7, 5};

      //A2[] = {2, 1, 8, 3, 4};
 A2[0] = 2;
 A2[1] = 1;
 A2[2] = 8;
 A2[3] = 3;
 A2[4] = 4;
 int size1 = sizeof(A1)/sizeof(A1[0]);

 sortA1ByA2(A1, size1);

 printf("Sorted Array is ");
 int i;
 for (i=0; i<size1; i++)
  printf("%d ", A1[i]);
 return 0;
}
```

Output:

```
Sorted Array is 2 2 1 1 8 8 3 5 5 5 6 6 7 7 7 9 9
```

This method is based on comments by readers (Xinuo Chen, Pranay Doshi and javakurious) and compiled by Anurag Singh.

This article is compiled by **Piyush**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

Â  Â

**Warning**: file_get_contents(http://www.geeksforgeeks.org/rearrange-array-alternating-positive-negative-items-o1-extra-space/): failed to open stream: Connection timed out in **/opt/lampp/htdocs/geeksforgeeks/sample/code/simple_html_dom.php** on line **75**

**Fatal error**: Call to a member function find() on a non-object in **/opt/lampp/htdocs/geeksforgeeks/sample/code/sample.php** on line **213**