Greedy Algorithm

1. Greedy Algorithms | Set 1 (Activity Selection Problem)

Greedy is an algorithmic paradigm that builds up a solution piece by piece, always choosing the next piece that offers the most obvious and immediate benefit. Greedy algorithms are used for optimization problems. An optimization problem can be solved using Greedy if the problem has the following property: *At every step, we can make a choice that looks best at the moment, and we get the optimal solution of the complete problem*.
If a Greedy Algorithm can solve a problem, then it generally becomes the best method to solve that problem as the Greedy algorithms are in general more efficient than other techniques like Dynamic Programming. But Greedy algorithms cannot always be applied. For example, Fractional Knapsack problem (See this) can be solved using Greedy, but 0-1 Knapsack cannot be solved using Greedy.

Following are some standard algorithms that are Greedy algorithms.
1) Kruskal's Minimum Spanning Tree (MST): In Kruskal's algorithm, we create a MST by picking edges one by one. The Greedy Choice is to pick the smallest weight edge that doesn't cause a cycle in the MST constructed so far.
2) Prim's Minimum Spanning Tree: In Prim's algorithm also, we create a MST by picking edges one by one. We maintain two sets: set of the vertices already included in MST and the set of the vertices not yet included. The Greedy Choice is to pick the smallest weight edge that connects the two sets.
3) Dijkstra's Shortest Path: The Dijkstra's algorithm is very similar to Prim's algorithm. The shortest path tree is built up, edge by edge. We maintain two sets: set of the vertices already included in the tree and the set of the vertices not yet included. The Greedy Choice is to pick the edge that connects the two sets and is on the smallest weight path from source to the set that contains not yet included vertices.
4) Huffman Coding: Huffman Coding is a loss-less compression technique. It assigns variable length bit codes to different characters. The Greedy Choice is to assign least bit length code to the most frequent character.

The greedy algorithms are sometimes also used to get an approximation for Hard optimization problems. For example, Traveling Salesman Problem is a NP Hard problem. A Greedy choice for this problem is to pick the nearest unvisited city from the current city at every step. This solutions doesn't always produce the best optimal solution, but can be used to get an approximate optimal solution.

Let us consider the Activity Selection problem as our first example of Greedy algorithms. Following is the problem statement.
You are given n activities with their start and finish times. Select the maximum number of activities that can be performed by a single person, assuming that a person can only

work on a single activity at a time.
Example:

```
Consider the following 6 activities.
     start[]  =  {1, 3, 0, 5, 8, 5};
     finish[] =  {2, 4, 6, 7, 9, 9};
The maximum set of activities that can be executed
by a single person is {0, 1, 3, 4}
```

The greedy choice is to always pick the next activity whose finish time is least among the remaining activities and the start time is more than or equal to the finish time of previously selected activity. We can sort the activities according to their finishing time so that we always consider the next activity as minimum finishing time activity.

1) Sort the activities according to their finishing time
2) Select the first activity from the sorted array and print it.
3) Do following for remaining activities in the sorted array.
…….a) If the start time of this activity is greater than the finish time of previously selected activity then select this activity and print it.

In the following C implementation, it is assumed that the activities are already sorted according to their finish time.

```c
#include<stdio.h>

// Prints a maximum set of activities that can be done by a single
// person, one at a time.
//  n   -->  Total number of activities
//  s[] -->  An array that contains start time of all activities
//  f[] -->  An array that contains finish time of all activities
void printMaxActivities(int s[], int f[], int n)
{
    int i, j;

    printf ("Following activities are selected \n");

    // The first activity always gets selected
    i = 0;
    printf("%d ", i);

    // Consider rest of the activities
    for (j = 1; j < n; j++)
    {
      // If this activity has start time greater than or equal to the
      // time of previously selected activity, then select it
      if (s[j] >= f[i])
      {
          printf ("%d ", j);
          i = j;
      }
    }
}

// driver program to test above function
int main()
{
    int s[] =  {1, 3, 0, 5, 8, 5};
    int f[] =  {2, 4, 6, 7, 9, 9};
    int n = sizeof(s)/sizeof(s[0]);
    printMaxActivities(s, f, n);
    getchar();
    return 0;
}
```

Output:

```
Following activities are selected
0 1 3 4
```

How does Greedy Choice work for Activities sorted according to finish time?
Let the give set of activities be S = {1, 2, 3, ..n} and activities be sorted by finish time. The greedy choice is to always pick activity 1. How come the activity 1 always provides one of the optimal solutions. We can prove it by showing that if there is another solution B with first activity other than 1, then there is also a solution A of same size with activity 1 as first activity. Let the first activity selected by B be k, then there always exist A = {B – {k}} U {1}.(Note that the activities in B are independent and k has smallest finishing time among all. Since k is not 1, finish(k) >= finish(1)).

References:
Introduction to Algorithms by Thomas H. Cormen, Charles E. Leiserson, Ronald L.

Rivest, Clifford Stein
Algorithms by S. Dasgupta, C.H. Papadimitriou, and U.V. Vazirani
http://en.wikipedia.org/wiki/Greedy_algorithm

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

2. Greedy Algorithms | Set 2 (Kruskal's Minimum Spanning Tree Algorithm)

What is Minimum Spanning Tree?
Given a connected and undirected graph, a *spanning tree* of that graph is a subgraph that is a tree and connects all the vertices together. A single graph can have many different spanning trees. A *minimum spanning tree (MST)* or minimum weight spanning tree for a weighted, connected and undirected graph is a spanning tree with weight less than or equal to the weight of every other spanning tree. The weight of a spanning tree is the sum of weights given to each edge of the spanning tree.

How many edges does a minimum spanning tree has?
A minimum spanning tree has (V – 1) edges where V is the number of vertices in the given graph.

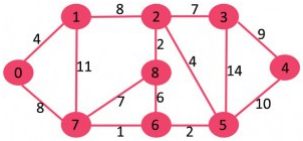
What are the applications of Minimum Spanning Tree?
See this for applications of MST.

Below are the steps for finding MST using Kruskal's algorithm

```
1. Sort all the edges in non-decreasing order of their weight.

2. Pick the smallest edge. Check if it forms a cycle with the spanning tree
formed so far. If cycle is not formed, include this edge. Else, discard it.

3. Repeat step#2 until there are (V-1) edges in the spanning tree.
```

The step#2 uses Union-Find algorithm to detect cycle. So we recommend to read following post as a prerequisite.
Union-Find Algorithm | Set 1 (Detect Cycle in a Graph)
Union-Find Algorithm | Set 2 (Union By Rank and Path Compression)

The algorithm is a Greedy Algorithm. The Greedy Choice is to pick the smallest weight edge that does not cause a cycle in the MST constructed so far. Let us understand it with an example: Consider the below input graph.

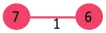


The graph contains 9 vertices and 14 edges. So, the minimum spanning tree formed will be having (9 – 1) = 8 edges.

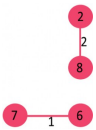
```
After sorting:
Weight   Src    Dest
1          7      6
2          8      2
2          6      5
4          0      1
4          2      5
6          8      6
7          2      3
7          7      8
8          0      7
8          1      2
9          3      4
10         5      4
11         1      7
14         3      5
```

Now pick all edges one by one from sorted list of edges

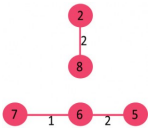
1. *Pick edge 7-6:* No cycle is formed, include it.



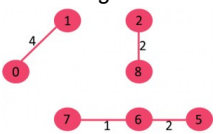
2. *Pick edge 8-2:* No cycle is formed, include it.



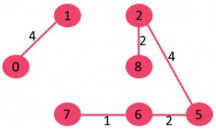
3. *Pick edge 6-5:* No cycle is formed, include it.



4. *Pick edge 0-1:* No cycle is formed, include it.

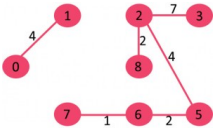


5. *Pick edge 2-5:* No cycle is formed, include it.



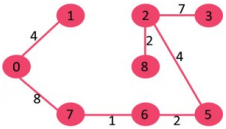
6. *Pick edge 8-6:* Since including this edge results in cycle, discard it.

7. *Pick edge 2-3:* No cycle is formed, include it.



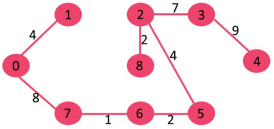
8. *Pick edge 7-8:* Since including this edge results in cycle, discard it.

9. *Pick edge 0-7:* No cycle is formed, include it.



10. *Pick edge 1-2:* Since including this edge results in cycle, discard it.

11. *Pick edge 3-4:* No cycle is formed, include it.



Since the number of edges included equals (V – 1), the algorithm stops here.

```
// Kruskal's algortihm to find Minimum Spanning Tree of a given connec
// undirected and weighted graph
#include <stdio.h>
#include <stdlib.h>
#include <string.h>

// a structure to represent a weighted edge in graph
struct Edge
{
    int src, dest, weight;
};

// a structure to represent a connected, undirected and weighted graph
struct Graph
{
    // V-> Number of vertices, E-> Number of edges
    int V, E;

    // graph is represented as an array of edges. Since the graph is
    // undirected, the edge from src to dest is also edge from dest
    // to src. Both are counted as 1 edge here.
    struct Edge* edge;
```

```
};

// Creates a graph with V vertices and E edges
struct Graph* createGraph(int V, int E)
{
    struct Graph* graph = (struct Graph*) malloc( sizeof(struct Graph)
    graph->V = V;
    graph->E = E;

    graph->edge = (struct Edge*) malloc( graph->E * sizeof( struct Edg

    return graph;
}

// A structure to represent a subset for union-find
struct subset
{
    int parent;
    int rank;
};

// A utility function to find set of an element i
// (uses path compression technique)
int find(struct subset subsets[], int i)
{
    // find root and make root as parent of i (path compression)
    if (subsets[i].parent != i)
        subsets[i].parent = find(subsets, subsets[i].parent);

    return subsets[i].parent;
}

// A function that does union of two sets of x and y
// (uses union by rank)
void Union(struct subset subsets[], int x, int y)
{
    int xroot = find(subsets, x);
    int yroot = find(subsets, y);

    // Attach smaller rank tree under root of high rank tree
    // (Union by Rank)
    if (subsets[xroot].rank < subsets[yroot].rank)
        subsets[xroot].parent = yroot;
    else if (subsets[xroot].rank > subsets[yroot].rank)
        subsets[yroot].parent = xroot;

    // If ranks are same, then make one as root and increment
    // its rank by one
    else
    {
        subsets[yroot].parent = xroot;
        subsets[xroot].rank++;
    }
}

// Compare two edges according to their weights.
// Used in qsort() for sorting an array of edges
int myComp(const void* a, const void* b)
{
    struct Edge* a1 = (struct Edge*)a;
    struct Edge* b1 = (struct Edge*)b;
    return a1->weight > b1->weight;
}
```

```
}

// The main function to construct MST using Kruskal's algorithm
void KruskalMST(struct Graph* graph)
{
    int V = graph->V;
    struct Edge result[V];  // Tnis will store the resultant MST
    int e = 0;  // An index variable, used for result[]
    int i = 0;  // An index variable, used for sorted edges

    // Step 1:  Sort all the edges in non-decreasing order of their we
    // If we are not allowed to change the given graph, we can create
    // array of edges
    qsort(graph->edge, graph->E, sizeof(graph->edge[0]), myComp);

    // Allocate memory for creating V ssubsets
    struct subset *subsets =
        (struct subset*) malloc( V * sizeof(struct subset) );

    // Create V subsets with single elements
    for (int v = 0; v < V; ++v)
    {
        subsets[v].parent = v;
        subsets[v].rank = 0;
    }

    // Number of edges to be taken is equal to V-1
    while (e < V - 1)
    {
        // Step 2: Pick the smallest edge. And increment the index
        // for next iteration
        struct Edge next_edge = graph->edge[i++];

        int x = find(subsets, next_edge.src);
        int y = find(subsets, next_edge.dest);

        // If including this edge does't cause cycle, include it
        // in result and increment the index of result for next edge
        if (x != y)
        {
            result[e++] = next_edge;
            Union(subsets, x, y);
        }
        // Else discard the next_edge
    }

    // print the contents of result[] to display the built MST
    printf("Following are the edges in the constructed MST\n");
    for (i = 0; i < e; ++i)
        printf("%d -- %d == %d\n", result[i].src, result[i].dest,
                                                 result[i].weight);
    return;
}

// Driver program to test above functions
int main()
{
    /* Let us create following weighted graph
             10
        0--------1
        |  \     |
       6|   5\   |15
        |      \ |
```

```
        |       \ |
        2--------3
            4       */
    int V = 4;  // Number of vertices in graph
    int E = 5;  // Number of edges in graph
    struct Graph* graph = createGraph(V, E);


    // add edge 0-1
    graph->edge[0].src = 0;
    graph->edge[0].dest = 1;
    graph->edge[0].weight = 10;

    // add edge 0-2
    graph->edge[1].src = 0;
    graph->edge[1].dest = 2;
    graph->edge[1].weight = 6;

    // add edge 0-3
    graph->edge[2].src = 0;
    graph->edge[2].dest = 3;
    graph->edge[2].weight = 5;

    // add edge 1-3
    graph->edge[3].src = 1;
    graph->edge[3].dest = 3;
    graph->edge[3].weight = 15;

    // add edge 2-3
    graph->edge[4].src = 2;
    graph->edge[4].dest = 3;
    graph->edge[4].weight = 4;

    KruskalMST(graph);

    return 0;
}
```

```
Following are the edges in the constructed MST
2 -- 3 == 4
0 -- 3 == 5
0 -- 1 == 10
```

Time Complexity: O(ElogE) or O(ElogV). Sorting of edges takes O(ELogE) time. After sorting, we iterate through all edges and apply find-union algorithm. The find and union operations can take atmost O(LogV) time. So overall complexity is O(ELogE + ELogV) time. The value of E can be atmost V^2, so O(LogV) are O(LogE) same. Therefore, overall time complexity is O(ElogE) or O(ElogV)

References:
http://www.ics.uci.edu/~eppstein/161/960206.html
http://en.wikipedia.org/wiki/Minimum_spanning_tree

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

3. Greedy Algorithms | Set 3 (Huffman Coding)

Huffman coding is a lossless data compression algorithm. The idea is to assign variable-legth codes to input characters, lengths of the assigned codes are based on the frequencies of corresponding characters. The most frequent character gets the smallest code and the least frequent character gets the largest code.
The variable-length codes assigned to input characters are Prefix Codes, means the codes (bit sequences) are assigned in such a way that the code assigned to one character is not prefix of code assigned to any other character. This is how Huffman Coding makes sure that there is no ambiguity when decoding the generated bit stream.
Let us understand prefix codes with a counter example. Let there be four characters a, b, c and d, and their corresponding variable length codes be 00, 01, 0 and 1. This coding leads to ambiguity because code assigned to c is prefix of codes assigned to a and b. If the compressed bit stream is 0001, the de-compressed output may be “cccd” or “ccb” or “acd” or “ab”.

See this for applications of Huffman Coding.

There are mainly two major parts in Huffman Coding
1) Build a Huffman Tree from input characters.
2) Traverse the Huffman Tree and assign codes to characters.

Steps to build Huffman Tree
Input is array of unique characters along with their frequency of occurrences and output is Huffman Tree.

1. Create a leaf node for each unique character and build a min heap of all leaf nodes (Min Heap is used as a priority queue. The value of frequency field is used to compare two nodes in min heap. Initially, the least frequent character is at root)

2. Extract two nodes with the minimum frequency from the min heap.

3. Create a new internal node with frequency equal to the sum of the two nodes frequencies. Make the first extracted node as its left child and the other extracted node as its right child. Add this node to the min heap.

4. Repeat steps#2 and #3 until the heap contains only one node. The remaining node is the root node and the tree is complete.

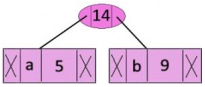
Let us understand the algorithm with an example:

```
character   Frequency
    a         5
```

```
    b           9
    c           12
    d           13
    e           16
    f           45
```

Step 1. Build a min heap that contains 6 nodes where each node represents root of a tree with single node.

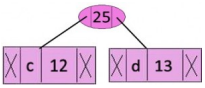
Step 2 Extract two minimum frequency nodes from min heap. Add a new internal node with frequency 5 + 9 = 14.



Now min heap contains 5 nodes where 4 nodes are roots of trees with single element each, and one heap node is root of tree with 3 elements

```
character           Frequency
       c               12
       d               13
 Internal Node         14
       e               16
       f                45
```

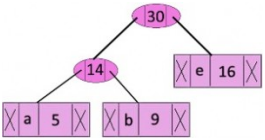
Step 3: Extract two minimum frequency nodes from heap. Add a new internal node with frequency 12 + 13 = 25



Now min heap contains 4 nodes where 2 nodes are roots of trees with single element each, and two heap nodes are root of tree with more than one nodes.

```
character           Frequency
Internal Node          14
       e               16
Internal Node          25
       f               45
```

Step 4: Extract two minimum frequency nodes. Add a new internal node with frequency 14 + 16 = 30

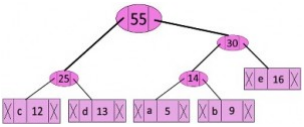


Now min heap contains 3 nodes.

```
character          Frequency
Internal Node         25
```

```
Internal Node          30
      f                45
```

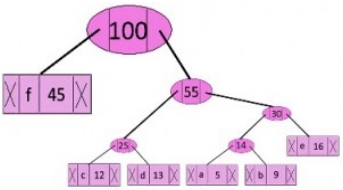
Step 5: Extract two minimum frequency nodes. Add a new internal node with frequency 25 + 30 = 55



Now min heap contains 2 nodes.

```
character      Frequency
       f          45
Internal Node     55
```

Step 6: Extract two minimum frequency nodes. Add a new internal node with frequency 45 + 55 = 100



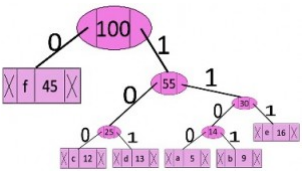
Now min heap contains only one node.

```
character       Frequency
Internal Node     100
```

Since the heap contains only one node, the algorithm stops here.

Steps to print codes from Huffman Tree:

Traverse the tree formed starting from the root. Maintain an auxiliary array. While moving to the left child, write 0 to the array. While moving to the right child, write 1 to the array. Print the array when a leaf node is encountered.



The codes are as follows:

```
character    code-word
    f           0
    c           100
    d           101
    a           1100
```

```
    b          1101
    e          111
```

```c
// C program for Huffman Coding
#include <stdio.h>
#include <stdlib.h>

// This constant can be avoided by explicitly calculating height of Hu
#define MAX_TREE_HT 100

// A Huffman tree node
struct MinHeapNode
{
    char data;  // One of the input characters
    unsigned freq;  // Frequency of the character
    struct MinHeapNode *left, *right; // Left and right child of this
};

// A Min Heap:  Collection of min heap (or Hufmman tree) nodes
struct MinHeap
{
    unsigned size;    // Current size of min heap
    unsigned capacity;   // capacity of min heap
    struct MinHeapNode **array;  // Attay of minheap node pointers
};

// A utility function allocate a new min heap node with given characte
// and frequency of the character
struct MinHeapNode* newNode(char data, unsigned freq)
{
    struct MinHeapNode* temp =
          (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    temp->left = temp->right = NULL;
    temp->data = data;
    temp->freq = freq;
    return temp;
}

// A utility function to create a min heap of given capacity
struct MinHeap* createMinHeap(unsigned capacity)
{
    struct MinHeap* minHeap =
         (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->size = 0;  // current size is 0
    minHeap->capacity = capacity;
    minHeap->array =
     (struct MinHeapNode**)malloc(minHeap->capacity * sizeof(struct Mi
    return minHeap;
}

// A utility function to swap two min heap nodes
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}

// The standard minHeapify function.
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest = idx;
```

```
    int left = 2 * idx + 1;
    int right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->array[left]->freq < minHeap->array[smallest]->freq)
      smallest = left;

    if (right < minHeap->size &&
        minHeap->array[right]->freq < minHeap->array[smallest]->freq)
      smallest = right;

    if (smallest != idx)
    {
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx
        minHeapify(minHeap, smallest);
    }
}

// A utility function to check if size of heap is 1 or not
int isSizeOne(struct MinHeap* minHeap)
{
    return (minHeap->size == 1);
}

// A standard function to extract minimum value node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    struct MinHeapNode* temp = minHeap->array[0];
    minHeap->array[0] = minHeap->array[minHeap->size - 1];
    --minHeap->size;
    minHeapify(minHeap, 0);
    return temp;
}

// A utility function to insert a new node to Min Heap
void insertMinHeap(struct MinHeap* minHeap, struct MinHeapNode* minHea
{
    ++minHeap->size;
    int i = minHeap->size - 1;
    while (i && minHeapNode->freq < minHeap->array[(i - 1)/2]->freq)
    {
        minHeap->array[i] = minHeap->array[(i - 1)/2];
        i = (i - 1)/2;
    }
    minHeap->array[i] = minHeapNode;
}

// A standard funvtion to build min heap
void buildMinHeap(struct MinHeap* minHeap)
{
    int n = minHeap->size - 1;
    int i;
    for (i = (n - 1) / 2; i >= 0; --i)
        minHeapify(minHeap, i);
}

// A utility function to print an array of size n
void printArr(int arr[], int n)
{
    int i;
    for (i = 0; i < n; ++i)
        printf("%d", arr[i]);
```

```
    printf("\n");
}

// Utility function to check if this node is leaf
int isLeaf(struct MinHeapNode* root)
{
    return !(root->left) && !(root->right) ;
}

// Creates a min heap of capacity equal to size and inserts all charac
// data[] in min heap. Initially size of min heap is equal to capacity
struct MinHeap* createAndBuildMinHeap(char data[], int freq[], int siz
{
    struct MinHeap* minHeap = createMinHeap(size);
    for (int i = 0; i < size; ++i)
        minHeap->array[i] = newNode(data[i], freq[i]);
    minHeap->size = size;
    buildMinHeap(minHeap);
    return minHeap;
}

// The main function that builds Huffman tree
struct MinHeapNode* buildHuffmanTree(char data[], int freq[], int size
{
    struct MinHeapNode *left, *right, *top;

    // Step 1: Create a min heap of capacity equal to size.  Initially
    // modes equal to size.
    struct MinHeap* minHeap = createAndBuildMinHeap(data, freq, size);

    // Iterate while size of heap doesn't become 1
    while (!isSizeOne(minHeap))
    {
        // Step 2: Extract the two minimum freq items from min heap
        left = extractMin(minHeap);
        right = extractMin(minHeap);

        // Step 3:  Create a new internal node with frequency equal to
        // sum of the two nodes frequencies. Make the two extracted no
        // left and right children of this new node. Add this node to
        // '$' is a special value for internal nodes, not used
        top = newNode('$', left->freq + right->freq);
        top->left = left;
        top->right = right;
        insertMinHeap(minHeap, top);
    }

    // Step 4: The remaining node is the root node and the tree is com
    return extractMin(minHeap);
}

// Prints huffman codes from the root of Huffman Tree.  It uses arr[]
// store codes
void printCodes(struct MinHeapNode* root, int arr[], int top)
{
    // Assign 0 to left edge and recur
    if (root->left)
    {
        arr[top] = 0;
        printCodes(root->left, arr, top + 1);
    }

    // Assign 1 to right edge and recur
```

```
    // Assign 1 to right edge and recur
    if (root->right)
    {
        arr[top] = 1;
        printCodes(root->right, arr, top + 1);
    }

    // If this is a leaf node, then it contains one of the input
    // characters, print the character and its code from arr[]
    if (isLeaf(root))
    {
        printf("%c: ", root->data);
        printArr(arr, top);
    }
}

// The main function that builds a Huffman Tree and print codes by tra
// the built Huffman Tree
void HuffmanCodes(char data[], int freq[], int size)
{
   //  Construct Huffman Tree
   struct MinHeapNode* root = buildHuffmanTree(data, freq, size);

   // Print Huffman codes using the Huffman tree built above
   int arr[MAX_TREE_HT], top = 0;
   printCodes(root, arr, top);
}

// Driver program to test above functions
int main()
{
    char arr[] = {'a', 'b', 'c', 'd', 'e', 'f'};
    int freq[] = {5, 9, 12, 13, 16, 45};
    int size = sizeof(arr)/sizeof(arr[0]);
    HuffmanCodes(arr, freq, size);
    return 0;
}
```

```
f: 0
c: 100
d: 101
a: 1100
b: 1101
e: 111
```

Time complexity: O(nlogn) where n is the number of unique characters. If there are n nodes, extractMin() is called 2*(n – 1) times. extractMin() takes O(logn) time as it calles minHeapify(). So, overall complexity is O(nlogn).

If the input array is sorted, there exists a linear time algorithm. We will soon be discussing in our next post.

Reference:

http://en.wikipedia.org/wiki/Huffman_coding

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

4. Greedy Algorithms | Set 4 (Efficient Huffman Coding for Sorted Input)

We recommend to read following post as a prerequisite for this.

Greedy Algorithms | Set 3 (Huffman Coding)

Time complexity of the algorithm discussed in above post is O(nLogn). If we know that the given array is sorted (by non-decreasing order of frequency), we can generate Huffman codes in O(n) time. Following is a O(n) algorithm for sorted input.

1. Create two empty queues.

2. Create a leaf node for each unique character and Enqueue it to the first queue in non-decreasing order of frequency. Initially second queue is empty.

3. Dequeue two nodes with the minimum frequency by examining the front of both queues. Repeat following steps two times
…..**a)** If second queue is empty, dequeue from first queue.
…..**b)** If first queue is empty, dequeue from second queue.
…..**c)** Else, compare the front of two queues and dequeue the minimum.

4. Create a new internal node with frequency equal to the sum of the two nodes frequencies. Make the first Dequeued node as its left child and the second Dequeued node as right child. Enqueue this node to second queue.

5. Repeat steps#3 and #4 until there is more than one node in the queues. The remaining node is the root node and the tree is complete.

```
// C Program for Efficient Huffman Coding for Sorted input
#include <stdio.h>
#include <stdlib.h>

// This constant can be avoided by explicitly calculating height of Hu
#define MAX_TREE_HT 100

// A node of huffman tree
struct QueueNode
{
    char data;
    unsigned freq;
    struct QueueNode *left, *right;
};

// Structure for Queue: collection of Huffman Tree nodes (or QueueNode
```

```
struct Queue
{
    int front, rear;
    int capacity;
    struct QueueNode **array;
};

// A utility function to create a new Queuenode
struct QueueNode* newNode(char data, unsigned freq)
{
    struct QueueNode* temp =
       (struct QueueNode*) malloc(sizeof(struct QueueNode));
    temp->left = temp->right = NULL;
    temp->data = data;
    temp->freq = freq;
    return temp;
}

// A utility function to create a Queue of given capacity
struct Queue* createQueue(int capacity)
{
    struct Queue* queue = (struct Queue*) malloc(sizeof(struct Queue))
    queue->front = queue->rear = -1;
    queue->capacity = capacity;
    queue->array =
      (struct QueueNode**) malloc(queue->capacity * sizeof(struct Queu
    return queue;
}

// A utility function to check if size of given queue is 1
int isSizeOne(struct Queue* queue)
{
    return queue->front == queue->rear && queue->front != -1;
}

// A utility function to check if given queue is empty
int isEmpty(struct Queue* queue)
{
    return queue->front == -1;
}

// A utility function to check if given queue is full
int isFull(struct Queue* queue)
{
    return queue->rear == queue->capacity - 1;
}

// A utility function to add an item to queue
void enQueue(struct Queue* queue, struct QueueNode* item)
{
    if (isFull(queue))
        return;
    queue->array[++queue->rear] = item;
    if (queue->front == -1)
        ++queue->front;
}

// A utility function to remove an item from queue
struct QueueNode* deQueue(struct Queue* queue)
{
    if (isEmpty(queue))
        return NULL;
```

```
    struct QueueNode* temp = queue->array[queue->front];
    if (queue->front == queue->rear)  // If there is only one item in
        queue->front = queue->rear = -1;
    else
        ++queue->front;
    return temp;
}

// A utility function to get from of queue
struct QueueNode* getFront(struct Queue* queue)
{
    if (isEmpty(queue))
        return NULL;
    return queue->array[queue->front];
}

/* A function to get minimum item from two queues */
struct QueueNode* findMin(struct Queue* firstQueue, struct Queue* seco
{
    // Step 3.a: If second queue is empty, dequeue from first queue
    if (isEmpty(firstQueue))
        return deQueue(secondQueue);

    // Step 3.b: If first queue is empty, dequeue from second queue
    if (isEmpty(secondQueue))
        return deQueue(firstQueue);

    // Step 3.c:  Else, compare the front of two queues and dequeue mi
    if (getFront(firstQueue)->freq < getFront(secondQueue)->freq)
        return deQueue(firstQueue);

    return deQueue(secondQueue);
}

// Utility function to check if this node is leaf
int isLeaf(struct QueueNode* root)
{
    return !(root->left) && !(root->right) ;
}

// A utility function to print an array of size n
void printArr(int arr[], int n)
{
    int i;
    for (i = 0; i < n; ++i)
        printf("%d", arr[i]);
    printf("\n");
}

// The main function that builds Huffman tree
struct QueueNode* buildHuffmanTree(char data[], int freq[], int size)
{
    struct QueueNode *left, *right, *top;

    // Step 1: Create two empty queues
    struct Queue* firstQueue  = createQueue(size);
    struct Queue* secondQueue = createQueue(size);

    // Step 2:Create a leaf node for each unique character and Enqueue
    // the first queue in non-decreasing order of frequency. Initially
    // queue is empty
    for (int i = 0; i < size; ++i)
        enQueue(firstQueue, newNode(data[i], freq[i]));
```

```
        enQueue(firstQueue, newNode(data[i], freq[i]));

    // Run while Queues contain more than one node. Finally, first que
    // be empty and second queue will contain only one node
    while (!(isEmpty(firstQueue) && isSizeOne(secondQueue)))
    {
        // Step 3: Dequeue two nodes with the minimum frequency by exa
        // the front of both queues
        left = findMin(firstQueue, secondQueue);
        right = findMin(firstQueue, secondQueue);

        // Step 4: Create a new internal node with frequency equal to
        // of the two nodes frequencies. Enqueue this node to second q
        top = newNode('$' , left->freq + right->freq);
        top->left = left;
        top->right = right;
        enQueue(secondQueue, top);
    }

    return deQueue(secondQueue);
}

// Prints huffman codes from the root of Huffman Tree.  It uses arr[]
// store codes
void printCodes(struct QueueNode* root, int arr[], int top)
{
    // Assign 0 to left edge and recur
    if (root->left)
    {
        arr[top] = 0;
        printCodes(root->left, arr, top + 1);
    }

    // Assign 1 to right edge and recur
    if (root->right)
    {
        arr[top] = 1;
        printCodes(root->right, arr, top + 1);
    }

    // If this is a leaf node, then it contains one of the input
    // characters, print the character and its code from arr[]
    if (isLeaf(root))
    {
        printf("%c: ", root->data);
        printArr(arr, top);
    }
}

// The main function that builds a Huffman Tree and print codes by tra
// the built Huffman Tree
void HuffmanCodes(char data[], int freq[], int size)
{
   //  Construct Huffman Tree
   struct QueueNode* root = buildHuffmanTree(data, freq, size);

   // Print Huffman codes using the Huffman tree built above
   int arr[MAX_TREE_HT], top = 0;
   printCodes(root, arr, top);
}

// Driver program to test above functions
int main()
```

```
{
    char arr[] = {'a', 'b', 'c', 'd', 'e', 'f'};
    int freq[] = {5, 9, 12, 13, 16, 45};
    int size = sizeof(arr)/sizeof(arr[0]);
    HuffmanCodes(arr, freq, size);
    return 0;
}
```

Output:

```
f: 0
c: 100
d: 101
a: 1100
b: 1101
e: 111
```

Time complexity: O(n)

If the input is not sorted, it need to be sorted first before it can be processed by the above algorithm. Sorting can be done using heap-sort or merge-sort both of which run in Theta(nlogn). So, the overall time complexity becomes O(nlogn) for unsorted input.

Reference:
http://en.wikipedia.org/wiki/Huffman_coding

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

5. Greedy Algorithms | Set 5 (Prim's Minimum Spanning Tree (MST))

We have discussed Kruskal's algorithm for Minimum Spanning Tree. Like Kruskal's algorithm, Prim's algorithm is also a Greedy algorithm. It starts with an empty spanning tree. The idea is to maintain two sets of vertices. The first set contains the vertices already included in the MST, the other set contains the vertices not yet included. At every step, it considers all the edges that connect the two sets, and picks the minimum weight edge from these edges. After picking the edge, it moves the other endpoint of the edge to the set containing MST.
A group of edges that connects two set of vertices in a graph is called cut in graph theory. *So, at every step of Prim's algorithm, we find a cut (of two sets, one contains the vertices already included in MST and other contains rest of the verices), pick the minimum weight edge from the cut and include this vertex to MST Set (the set that contains already included vertices).*

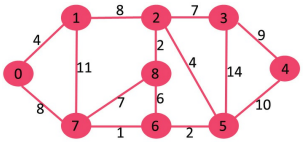
How does Prim's Algorithm Work? The idea behind Prim's algorithm is simple, a spanning tree means all vertices must be connected. So the two disjoint subsets (discussed above) of vertices must be connected to make a *Spanning* Tree. And they must be connected with the minimum weight edge to make it a *Minimum* Spanning Tree.

Algorithm

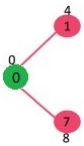
1) Create a set *mstSet* that keeps track of vertices already included in MST.
2) Assign a key value to all vertices in the input graph. Initialize all key values as INFINITE. Assign key value as 0 for the first vertex so that it is picked first.
3) While mstSet doesn't include all vertices
….**a)** Pick a vertex *u* which is not there in *mstSet* and has minimum key value.
….**b)** Include *u* to mstSet.
….**c)** Update key value of all adjacent vertices of *u*. To update the key values, iterate through all adjacent vertices. For every adjacent vertex *v*, if weight of edge *u-v* is less than the previous key value of *v*, update the key value as weight of *u-v*

The idea of using key values is to pick the minimum weight edge from cut. The key values are used only for vertices which are not yet included in MST, the key value for these vertices indicate the minimum weight edges connecting them to the set of vertices included in MST.

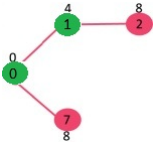
Let us understand with the following example:



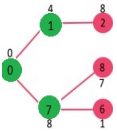
The set *mstSet* is initially empty and keys assigned to vertices are {0, INF, INF, INF, INF, INF, INF, INF} where INF indicates infinite. Now pick the vertex with minimum key value. The vertex 0 is picked, include it in *mstSet*. So *mstSet* becomes {0}. After including to *mstSet*, update key values of adjacent vertices. Adjacent vertices of 0 are 1 and 7. The key values of 1 and 7 are updated as 4 and 8. Following subgraph shows vertices and their key values, only the vertices with finite key values are shown. The vertices included in MST are shown in green color.



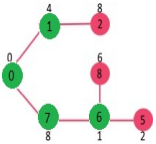
Pick the vertex with minimum key value and not already included in MST (not in mstSET). The vertex 1 is picked and added to mstSet. So mstSet now becomes {0, 1}. Update the key values of adjacent vertices of 1. The key value of vertex 2 becomes 8.



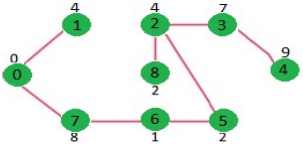
Pick the vertex with minimum key value and not already included in MST (not in mstSET). We can either pick vertex 7 or vertex 2, let vertex 7 is picked. So mstSet now becomes {0, 1, 7}. Update the key values of adjacent vertices of 7. The key value of vertex 6 and 8 becomes finite (7 and 1 respectively).



Pick the vertex with minimum key value and not already included in MST (not in mstSET). Vertex 6 is picked. So mstSet now becomes {0, 1, 7, 6}. Update the key values of adjacent vertices of 6. The key value of vertex 5 and 8 are updated.



We repeat the above steps until *mstSet* includes all vertices of given graph. Finally, we get the following graph.



How to implement the above algorithm?

We use a boolean array mstSet[] to represent the set of vertices included in MST. If a value mstSet[v] is true, then vertex v is included in MST, otherwise not. Array key[] is used to store key values of all vertices. Another array parent[] to store indexes of parent nodes in MST. The parent array is the output array which is used to show the constructed MST.

```
// A C / C++ program for Prim's Minimum Spanning Tree (MST) algorithm.
// The program is for adjacency matrix representation of the graph

#include <stdio.h>
#include <limits.h>

// Number of vertices in the graph
#define V 5

// A utility function to find the vertex with minimum key value, from
// the set of vertices not yet included in MST
int minKey(int key[], bool mstSet[])
```

```
int minKey(int key[], bool mstSet[])
{
   // Initialize min value
   int min = INT_MAX, min_index;

   for (int v = 0; v < V; v++)
     if (mstSet[v] == false && key[v] < min)
         min = key[v], min_index = v;

   return min_index;
}

// A utility function to print the constructed MST stored in parent[]
int printMST(int parent[], int n, int graph[V][V])
{
   printf("Edge   Weight\n");
   for (int i = 1; i < V; i++)
      printf("%d - %d    %d \n", parent[i], i, graph[i][parent[i]]);
}

// Function to construct and print MST for a graph represented using a
// matrix representation
void primMST(int graph[V][V])
{
     int parent[V]; // Array to store constructed MST
     int key[V];   // Key values used to pick minimum weight edge in c
     bool mstSet[V];  // To represent set of vertices not yet included

     // Initialize all keys as INFINITE
     for (int i = 0; i < V; i++)
        key[i] = INT_MAX, mstSet[i] = false;

     // Always include first 1st vertex in MST.
     key[0] = 0;     // Make key 0 so that this vertex is picked as fi
     parent[0] = -1; // First node is always root of MST

     // The MST will have V vertices
     for (int count = 0; count < V-1; count++)
     {
        // Pick thd minimum key vertex from the set of vertices
        // not yet included in MST
        int u = minKey(key, mstSet);

        // Add the picked vertex to the MST Set
        mstSet[u] = true;

        // Update key value and parent index of the adjacent vertices
        // the picked vertex. Consider only those vertices which are n
        // included in MST
        for (int v = 0; v < V; v++)

           // graph[u][v] is non zero only for adjacent vertices of m
           // mstSet[v] is false for vertices not yet included in MST
           // Update the key only if graph[u][v] is smaller than key[v
          if (graph[u][v] && mstSet[v] == false && graph[u][v] <  key[
             parent[v]  = u, key[v] = graph[u][v];
     }

     // print the constructed MST
     printMST(parent, V, graph);
}
```

```
// driver program to test above function
int main()
{
   /* Let us create the following graph
          2    3
      (0)--(1)--(2)
       |   / \   |
      6| 8/   \5 |7
       | /     \ |
      (3)-------(4)
            9          */
   int graph[V][V] = {{0, 2, 0, 6, 0},
                      {2, 0, 3, 8, 5},
                      {0, 3, 0, 0, 7},
                      {6, 8, 0, 0, 9},
                      {0, 5, 7, 9, 0},
                     };

    // Print the solution
    primMST(graph);

    return 0;
}
```

Output:

```
Edge   Weight
0 - 1    2
1 - 2    3
0 - 3    6
1 - 4    5
```

Time Complexity of the above program is O(V^2). If the input graph is represented using adjacency list, then the time complexity of Prim's algorithm can be reduced to O(E log V) with the help of binary heap. We will soon be discussing O(E Log V) algorithm as a separate post.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

6. Greedy Algorithms | Set 6 (Prim's MST for Adjacency List Representation)

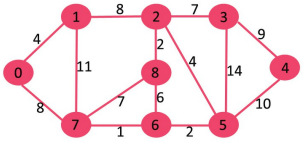
We recommend to read following two posts as a prerequisite of this post.

1. Greedy Algorithms | Set 5 (Prim's Minimum Spanning Tree (MST))
2. Graph and its representations

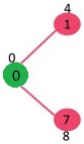
We have discussed Prim's algorithm and its implementation for adjacency matrix representation of graphs. The time complexity for the matrix representation is O(V^2). In this post, O(ELogV) algorithm for adjacency list representation is discussed.
As discussed in the previous post, in Prim's algorithm, two sets are maintained, one set contains list of vertices already included in MST, other set contains vertices not yet included. With adjacency list representation, all vertices of a graph can be traversed in O(V+E) time using BFS. The idea is to traverse all vertices of graph using BFS and use a Min Heap to store the vertices not yet included in MST. Min Heap is used as a priority queue to get the minimum weight edge from the cut. Min Heap is used as time complexity of operations like extracting minimum element and decreasing key value is O(LogV) in Min Heap.

Following are the detailed steps.
1) Create a Min Heap of size V where V is the number of vertices in the given graph. Every node of min heap contains vertex number and key value of the vertex.
2) Initialize Min Heap with first vertex as root (the key value assigned to first vertex is 0). The key value assigned to all other vertices is INF (infinite).
3) While Min Heap is not empty, do following
…..**a)** Extract the min value node from Min Heap. Let the extracted vertex be u.
…..**b)** For every adjacent vertex v of u, check if v is in Min Heap (not yet included in MST). If v is in Min Heap and its key value is more than weight of u-v, then update the key value of v as weight of u-v.

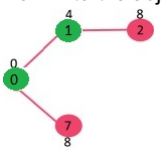
Let us understand the above algorithm with the following example:



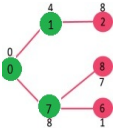
Initially, key value of first vertex is 0 and INF (infinite) for all other vertices. So vertex 0 is extracted from Min Heap and key values of vertices adjacent to 0 (1 and 7) are updated. Min Heap contains all vertices except vertex 0.
The vertices in green color are the vertices included in MST.



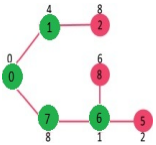
Since key value of vertex 1 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 1 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 1 to the adjacent). Min Heap contains all vertices except vertex 0 and 1.



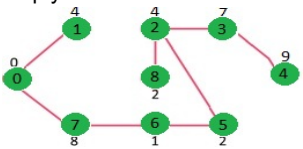
Since key value of vertex 7 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 7 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 7 to the adjacent). Min Heap contains all vertices except vertex 0, 1 and 7.



Since key value of vertex 6 is minimum among all nodes in Min Heap, it is extracted from Min Heap and key values of vertices adjacent to 6 are updated (Key is updated if the a vertex is not in Min Heap and previous key value is greater than the weight of edge from 6 to the adjacent). Min Heap contains all vertices except vertex 0, 1, 7 and 6.



The above steps are repeated for rest of the nodes in Min Heap till Min Heap becomes empty



```
// C / C++ program for Prim's MST for adjacency list representation of

#include <stdio.h>
#include <stdlib.h>
#include <limits.h>

// A structure to represent a node in adjacency list
struct AdjListNode
{
    int dest;
    int weight;
    struct AdjListNode* next;
};

// A structure to represent an adjacency liat
struct AdjList
{
    struct AdjListNode *head;  // pointer to head node of list
};

// A structure to represent a graph. A graph is an array of adjacency
// Size of array will be V (number of vertices in graph)
struct Graph
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest, int weight)
```

```
{
    struct AdjListNode* newNode =
            (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->weight = weight;
    newNode->next = NULL;
    return newNode;
}

// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph = (struct Graph*) malloc(sizeof(struct Graph))
    graph->V = V;

    // Create an array of adjacency lists.  Size of array will be V
    graph->array = (struct AdjList*) malloc(V * sizeof(struct AdjList)

     // Initialize each adjacency list as empty by making head as NULL
    for (int i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}

// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest, int weight)
{
    // Add an edge from src to dest.  A new node is added to the adjac
    // list of src.  The node is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest, weight);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;

    // Since graph is undirected, add an edge from dest to src also
    newNode = newAdjListNode(src, weight);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}

// Structure to represent a min heap node
struct MinHeapNode
{
    int  v;
    int key;
};

// Structure to represent a min heap
struct MinHeap
{
    int size;      // Number of heap nodes present currently
    int capacity;  // Capacity of min heap
    int *pos;     // This is needed for decreaseKey()
    struct MinHeapNode **array;
};

// A utility function to create a new Min Heap Node
struct MinHeapNode* newMinHeapNode(int v, int key)
{
    struct MinHeapNode* minHeapNode =
           (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    minHeapNode->v = v;
    minHeapNode->key = key;
```

```
    minHeapNode->key = key;
    return minHeapNode;
}

// A utilit function to create a Min Heap
struct MinHeap* createMinHeap(int capacity)
{
    struct MinHeap* minHeap =
         (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->pos = (int *)malloc(capacity * sizeof(int));
    minHeap->size = 0;
    minHeap->capacity = capacity;
    minHeap->array =
         (struct MinHeapNode**) malloc(capacity * sizeof(struct MinHea
    return minHeap;
}

// A utility function to swap two nodes of min heap. Needed for min he
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}

// A standard function to heapify at given idx
// This function also updates position of nodes when they are swapped.
// Position is needed for decreaseKey()
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest, left, right;
    smallest = idx;
    left = 2 * idx + 1;
    right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->array[left]->key < minHeap->array[smallest]->key )
      smallest = left;

    if (right < minHeap->size &&
        minHeap->array[right]->key < minHeap->array[smallest]->key )
      smallest = right;

    if (smallest != idx)
    {
        // The nodes to be swapped in min heap
        MinHeapNode *smallestNode = minHeap->array[smallest];
        MinHeapNode *idxNode = minHeap->array[idx];

        // Swap positions
        minHeap->pos[smallestNode->v] = idx;
        minHeap->pos[idxNode->v] = smallest;

        // Swap nodes
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx

        minHeapify(minHeap, smallest);
    }
}

// A utility function to check if the given minHeap is ampty or not
int isEmpty(struct MinHeap* minHeap)
{
```

```
    return minHeap->size == 0;
}

// Standard function to extract minimum node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    if (isEmpty(minHeap))
        return NULL;

    // Store the root node
    struct MinHeapNode* root = minHeap->array[0];

    // Replace root node with last node
    struct MinHeapNode* lastNode = minHeap->array[minHeap->size - 1];
    minHeap->array[0] = lastNode;

    // Update position of last node
    minHeap->pos[root->v] = minHeap->size-1;
    minHeap->pos[lastNode->v] = 0;

    // Reduce heap size and heapify root
    --minHeap->size;
    minHeapify(minHeap, 0);

    return root;
}

// Function to decreasy key value of a given vertex v. This function
// uses pos[] of min heap to get the current index of node in min heap
void decreaseKey(struct MinHeap* minHeap, int v, int key)
{
    // Get the index of v in  heap array
    int i = minHeap->pos[v];

    // Get the node and update its key value
    minHeap->array[i]->key = key;

    // Travel up while the complete tree is not hepified.
    // This is a O(Logn) loop
    while (i && minHeap->array[i]->key < minHeap->array[(i - 1) / 2]->
    {
        // Swap this node with its parent
        minHeap->pos[minHeap->array[i]->v] = (i-1)/2;
        minHeap->pos[minHeap->array[(i-1)/2]->v] = i;
        swapMinHeapNode(&minHeap->array[i],  &minHeap->array[(i - 1) /

        // move to parent index
        i = (i - 1) / 2;
    }
}

// A utility function to check if a given vertex
// 'v' is in min heap or not
bool isInMinHeap(struct MinHeap *minHeap, int v)
{
   if (minHeap->pos[v] < minHeap->size)
     return true;
   return false;
}

// A utility function used to print the constructed MST
void printArr(int arr[], int n)
```

```
{
    for (int i = 1; i < n; ++i)
        printf("%d - %d\n", arr[i], i);
}

// The main function that constructs Minimum Spanning Tree (MST)
// using Prim's algorithm
void PrimMST(struct Graph* graph)
{
    int V = graph->V;// Get the number of vertices in graph
    int parent[V];   // Array to store constructed MST
    int key[V];      // Key values used to pick minimum weight edge in

    // minHeap represents set E
    struct MinHeap* minHeap = createMinHeap(V);

    // Initialize min heap with all vertices. Key value of
    // all vertices (except 0th vertex) is initially infinite
    for (int v = 1; v < V; ++v)
    {
        parent[v] = -1;
        key[v] = INT_MAX;
        minHeap->array[v] = newMinHeapNode(v, key[v]);
        minHeap->pos[v] = v;
    }

    // Make key value of 0th vertex as 0 so that it
    // is extracted first
    key[0] = 0;
    minHeap->array[0] = newMinHeapNode(0, key[0]);
    minHeap->pos[0]   = 0;

    // Initially size of min heap is equal to V
    minHeap->size = V;

    // In the followin loop, min heap contains all nodes
    // not yet added to MST.
    while (!isEmpty(minHeap))
    {
        // Extract the vertex with minimum key value
        struct MinHeapNode* minHeapNode = extractMin(minHeap);
        int u = minHeapNode->v; // Store the extracted vertex number

        // Traverse through all adjacent vertices of u (the extracted
        // vertex) and update their key values
        struct AdjListNode* pCrawl = graph->array[u].head;
        while (pCrawl != NULL)
        {
            int v = pCrawl->dest;

            // If v is not yet included in MST and weight of u-v is
            // less than key value of v, then update key value and
            // parent of v
            if (isInMinHeap(minHeap, v) && pCrawl->weight < key[v])
            {
                key[v] = pCrawl->weight;
                parent[v] = u;
                decreaseKey(minHeap, v, key[v]);
            }
            pCrawl = pCrawl->next;
        }
    }
```

```
    // print edges of MST
    printArr(parent, V);
}

// Driver program to test above functions
int main()
{
    // Let us create the graph given in above fugure
    int V = 9;
    struct Graph* graph = createGraph(V);
    addEdge(graph, 0, 1, 4);
    addEdge(graph, 0, 7, 8);
    addEdge(graph, 1, 2, 8);
    addEdge(graph, 1, 7, 11);
    addEdge(graph, 2, 3, 7);
    addEdge(graph, 2, 8, 2);
    addEdge(graph, 2, 5, 4);
    addEdge(graph, 3, 4, 9);
    addEdge(graph, 3, 5, 14);
    addEdge(graph, 4, 5, 10);
    addEdge(graph, 5, 6, 2);
    addEdge(graph, 6, 7, 1);
    addEdge(graph, 6, 8, 6);
    addEdge(graph, 7, 8, 7);

    PrimMST(graph);

    return 0;
}
```

Output:

```
0 - 1
5 - 2
2 - 3
3 - 4
6 - 5
7 - 6
0 - 7
2 - 8
```

Time Complexity: The time complexity of the above code/algorithm looks O(V^2) as there are two nested while loops. If we take a closer look, we can observe that the statements in inner loop are executed O(V+E) times (similar to BFS). The inner loop has decreaseKey() operation which takes O(LogV) time. So overall time complexity is O(E+V)*O(LogV) which is O((E+V)*LogV) = O(ELogV) (For a connected graph, V = O(E))

References:

Introduction to Algorithms by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L.

http://en.wikipedia.org/wiki/Prim's_algorithm

This article is compiled by Aashish Barnwal and reviewed by GeeksforGeeks team. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

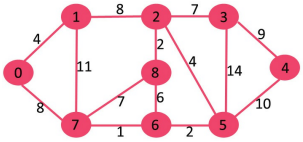
7. Greedy Algorithms | Set 7 (Dijkstra's shortest path algorithm)

Given a graph and a source vertex in graph, find shortest paths from source to all vertices in the given graph.

Dijkstra's algorithm is very similar to Prim's algorithm for minimum spanning tree. Like Prim's MST, we generate a *SPT (shortest path tree)* with given source as root. We maintain two sets, one set contains vertices included in shortest path tree, other set includes vertices not yet included in shortest path tree. At every step of the algorithm, we find a vertex which is in the other set (set of not yet included) and has minimum distance from source.

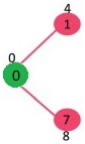
Below are the detailed steps used in Dijkstra's algorithm to find the shortest path from a single source vertex to all other vertices in the given graph.
Algorithm
1) Create a set *sptSet* (shortest path tree set) that keeps track of vertices included in shortest path tree, i.e., whose minimum distance from source is calculated and finalized. Initially, this set is empty.
2) Assign a distance value to all vertices in the input graph. Initialize all distance values as INFINITE. Assign distance value as 0 for the source vertex so that it is picked first.
3) While *sptSet* doesn't include all vertices
….**a)** Pick a vertex u which is not there in *sptSet*and has minimum distance value.
….**b)** Include u to *sptSet*.
….**c)** Update distance value of all adjacent vertices of u. To update the distance values, iterate through all adjacent vertices. For every adjacent vertex v, if sum of distance value of u (from source) and weight of edge u-v, is less than the distance value of v, then update the distance value of v.

Let us understand with the following example:

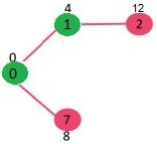


The set *sptSet*is initially empty and distances assigned to vertices are {0, INF, INF, INF, INF, INF, INF, INF} where INF indicates infinite. Now pick the vertex with minimum distance value. The vertex 0 is picked, include it in *sptSet*. So *sptSet* becomes {0}. After

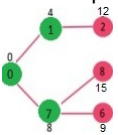
including 0 to *sptSet*, update distance values of its adjacent vertices. Adjacent vertices of 0 are 1 and 7. The distance values of 1 and 7 are updated as 4 and 8. Following subgraph shows vertices and their distance values, only the vertices with finite distance values are shown. The vertices included in SPT are shown in green color.



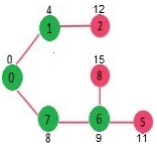
Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). The vertex 1 is picked and added to sptSet. So sptSet now becomes {0, 1}. Update the distance values of adjacent vertices of 1. The distance value of vertex 2 becomes 12.



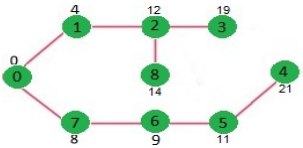
Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). Vertex 7 is picked. So sptSet now becomes {0, 1, 7}. Update the distance values of adjacent vertices of 7. The distance value of vertex 6 and 8 becomes finite (15 and 9 respectively).



Pick the vertex with minimum distance value and not already included in SPT (not in sptSET). Vertex 6 is picked. So sptSet now becomes {0, 1, 7, 6}. Update the distance values of adjacent vertices of 6. The distance value of vertex 5 and 8 are updated.



We repeat the above steps until *sptSet* doesn't include all vertices of given graph. Finally, we get the following Shortest Path Tree (SPT).



How to implement the above algorithm?

We use a boolean array sptSet[] to represent the set of vertices included in SPT. If a value sptSet[v] is true, then vertex v is included in SPT, otherwise not. Array dist[] is used to store shortest distance values of all vertices.

```
// A C / C++ program for Dijkstra's single source shortest path algori
// The program is for adjacency matrix representation of the graph

#include <stdio.h>
#include <limits.h>

// Number of vertices in the graph
#define V 9

// A utility function to find the vertex with minimum distance value,
// the set of vertices not yet included in shortest path tree
int minDistance(int dist[], bool sptSet[])
{
   // Initialize min value
   int min = INT_MAX, min_index;

   for (int v = 0; v < V; v++)
     if (sptSet[v] == false && dist[v] <= min)
         min = dist[v], min_index = v;

   return min_index;
}

// A utility function to print the constructed distance array
int printSolution(int dist[], int n)
{
   printf("Vertex   Distance from Source\n");
   for (int i = 0; i < V; i++)
      printf("%d \t\t %d\n", i, dist[i]);
}

// Funtion that implements Dijkstra's single source shortest path algo
// for a graph represented using adjacency matrix representation
void dijkstra(int graph[V][V], int src)
{
     int dist[V];     // The output array.  dist[i] will hold the shor
                      // distance from src to i

     bool sptSet[V]; // sptSet[i] will true if vertex i is included in
                     // path tree or shortest distance from src to i i

     // Initialize all distances as INFINITE and stpSet[] as false
     for (int i = 0; i < V; i++)
        dist[i] = INT_MAX, sptSet[i] = false;

     // Distance of source vertex from itself is always 0
     dist[src] = 0;

     // Find shortest path for all vertices
     for (int count = 0; count < V-1; count++)
     {
       // Pick the minimum distance vertex from the set of vertices no
       // yet processed. u is always equal to src in first iteration.
       int u = minDistance(dist, sptSet);

       // Mark the picked vertex as processed
```

```
        sptSet[u] = true;

        // Update dist value of the adjacent vertices of the picked ver
        for (int v = 0; v < V; v++)

          // Update dist[v] only if is not in sptSet, there is an edge
          // u to v, and total weight of path from src to  v through u
          // smaller than current value of dist[v]
          if (!sptSet[v] && graph[u][v] && dist[u] != INT_MAX
                                        && dist[u]+graph[u][v] < dist[v
             dist[v] = dist[u] + graph[u][v];
      }

      // print the constructed distance array
      printSolution(dist, V);
}

// driver program to test above function
int main()
{
   /* Let us create the example graph discussed above */
   int graph[V][V] = {{0, 4, 0, 0, 0, 0, 0, 8, 0},
                      {4, 0, 8, 0, 0, 0, 0, 11, 0},
                      {0, 8, 0, 7, 0, 4, 0, 0, 2},
                      {0, 0, 7, 0, 9, 14, 0, 0, 0},
                      {0, 0, 0, 9, 0, 10, 0, 0, 0},
                      {0, 0, 4, 0, 10, 0, 2, 0, 0},
                      {0, 0, 0, 14, 0, 2, 0, 1, 6},
                      {8, 11, 0, 0, 0, 0, 1, 0, 7},
                      {0, 0, 2, 0, 0, 0, 6, 7, 0}
                     };

    dijkstra(graph, 0);

    return 0;
}
```

Output:

```
Vertex   Distance from Source
0                0
1                4
2                12
3                19
4                21
5                11
6                9
7                8
8                14
```

Notes:

1) The code calculates shortest distance, but doesn't calculate the path information. We can create a parent array, update the parent array when distance is updated (like prim's implementation) and use it show the shortest path from source to different vertices.

2) The code is for undirected graph, same dijekstra function can be used for directed graphs also.

3) The code finds shortest distances from source to all vertices. If we are interested only in shortest distance from source to a single target, we can break the for loop when the picked minimum distance vertex is equal to target (Step 3.a of algorithm).

4) Time Complexity of the implementation is O(V^2). If the input graph is represented using adjacency list, it can be reduced to O(E log V) with the help of binary heap. We will soon be discussing O(E Log V) algorithm as a separate post.

5) Dijkstra's algorithm doesn't work for graphs with negative weight edges. For graphs with negative weight edges, Bellman–Ford algorithm can be used, we will soon be discussing it as a separate post.
Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

8. Greedy Algorithms | Set 8 (Dijkstra's Algorithm for Adjacency List Representation)

We recommend to read following two posts as a prerequisite of this post.

1. Greedy Algorithms | Set 7 (Dijkstra's shortest path algorithm)
2. Graph and its representations

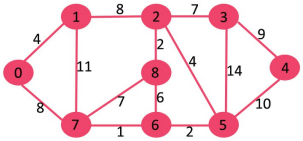
We have discussed Dijkstra's algorithm and its implementation for adjacency matrix representation of graphs. The time complexity for the matrix representation is O(V^2). In this post, O(ELogV) algorithm for adjacency list representation is discussed.

As discussed in the previous post, in Dijkstra's algorithm, two sets are maintained, one set contains list of vertices already included in SPT (Shortest Path Tree), other set contains vertices not yet included. With adjacency list representation, all vertices of a graph can be traversed in O(V+E) time using BFS. The idea is to traverse all vertices of graph using BFS and use a Min Heap to store the vertices not yet included in SPT (or the vertices for which shortest distance is not finalized yet). Min Heap is used as a priority queue to get the minimum distance vertex from set of not yet included vertices. Time complexity of operations like extract-min and decrease-key value is O(LogV) for Min Heap.

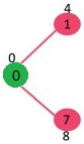
Following are the detailed steps.
1) Create a Min Heap of size V where V is the number of vertices in the given graph. Every node of min heap contains vertex number and distance value of the vertex.
2) Initialize Min Heap with source vertex as root (the distance value assigned to source

vertex is 0). The distance value assigned to all other vertices is INF (infinite).
3) While Min Heap is not empty, do following
…..**a)** Extract the vertex with minimum distance value node from Min Heap. Let the extracted vertex be u.
…..**b)** For every adjacent vertex v of u, check if v is in Min Heap. If v is in Min Heap and distance value is more than weight of u-v plus distance value of u, then update the distance value of v.

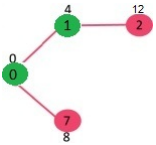
Let us understand with the following example. Let the given source vertex be 0



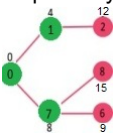
Initially, distance value of source vertex is 0 and INF (infinite) for all other vertices. So source vertex is extracted from Min Heap and distance values of vertices adjacent to 0 (1 and 7) are updated. Min Heap contains all vertices except vertex 0.
The vertices in green color are the vertices for which minimum distances are finalized and are not in Min Heap



Since distance value of vertex 1 is minimum among all nodes in Min Heap, it is extracted from Min Heap and distance values of vertices adjacent to 1 are updated (distance is updated if the a vertex is not in Min Heap and distance through 1 is shorter than the previous distance). Min Heap contains all vertices except vertex 0 and 1.

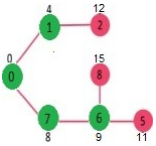


Pick the vertex with minimum distance value from min heap. Vertex 7 is picked. So min heap now contains all vertices except 0, 1 and 7. Update the distance values of adjacent vertices of 7. The distance value of vertex 6 and 8 becomes finite (15 and 9 respectively).

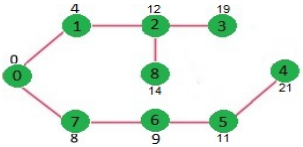


Pick the vertex with minimum distance from min heap. Vertex 6 is picked. So min heap now contains all vertices except 0, 1, 7 and 6. Update the distance values of adjacent

vertices of 6. The distance value of vertex 5 and 8 are updated.



Above steps are repeated till min heap doesn't become empty. Finally, we get the following shortest path tree.




```c
// C / C++ program for Dijkstra's shortest path algorithm for adjacenc
// list representation of graph

#include <stdio.h>
#include <stdlib.h>
#include <limits.h>

// A structure to represent a node in adjacency list
struct AdjListNode
{
    int dest;
    int weight;
    struct AdjListNode* next;
};

// A structure to represent an adjacency liat
struct AdjList
{
    struct AdjListNode *head;  // pointer to head node of list
};

// A structure to represent a graph. A graph is an array of adjacency
// Size of array will be V (number of vertices in graph)
struct Graph
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest, int weight)
{
    struct AdjListNode* newNode =
            (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->weight = weight;
    newNode->next = NULL;
    return newNode;
}

// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
```

```
{
    struct Graph* graph = (struct Graph*) malloc(sizeof(struct Graph))
    graph->V = V;

    // Create an array of adjacency lists.  Size of array will be V
    graph->array = (struct AdjList*) malloc(V * sizeof(struct AdjList)

     // Initialize each adjacency list as empty by making head as NULL
    for (int i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}

// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest, int weight)
{
    // Add an edge from src to dest.  A new node is added to the adjac
    // list of src.  The node is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest, weight);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;

    // Since graph is undirected, add an edge from dest to src also
    newNode = newAdjListNode(src, weight);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}

// Structure to represent a min heap node
struct MinHeapNode
{
    int  v;
    int dist;
};

// Structure to represent a min heap
struct MinHeap
{
    int size;      // Number of heap nodes present currently
    int capacity;  // Capacity of min heap
    int *pos;     // This is needed for decreaseKey()
    struct MinHeapNode **array;
};

// A utility function to create a new Min Heap Node
struct MinHeapNode* newMinHeapNode(int v, int dist)
{
    struct MinHeapNode* minHeapNode =
           (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    minHeapNode->v = v;
    minHeapNode->dist = dist;
    return minHeapNode;
}

// A utility function to create a Min Heap
struct MinHeap* createMinHeap(int capacity)
{
    struct MinHeap* minHeap =
         (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->pos = (int *)malloc(capacity * sizeof(int));
    minHeap->size = 0;
    minHeap->capacity = capacity;
```

```
    minHeap->array =
         (struct MinHeapNode**) malloc(capacity * sizeof(struct MinHea
    return minHeap;
}

// A utility function to swap two nodes of min heap. Needed for min he
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}

// A standard function to heapify at given idx
// This function also updates position of nodes when they are swapped.
// Position is needed for decreaseKey()
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest, left, right;
    smallest = idx;
    left = 2 * idx + 1;
    right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->array[left]->dist < minHeap->array[smallest]->dist )
      smallest = left;

    if (right < minHeap->size &&
        minHeap->array[right]->dist < minHeap->array[smallest]->dist )
      smallest = right;

    if (smallest != idx)
    {
        // The nodes to be swapped in min heap
        MinHeapNode *smallestNode = minHeap->array[smallest];
        MinHeapNode *idxNode = minHeap->array[idx];

        // Swap positions
        minHeap->pos[smallestNode->v] = idx;
        minHeap->pos[idxNode->v] = smallest;

        // Swap nodes
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx

        minHeapify(minHeap, smallest);
    }
}

// A utility function to check if the given minHeap is ampty or not
int isEmpty(struct MinHeap* minHeap)
{
    return minHeap->size == 0;
}

// Standard function to extract minimum node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    if (isEmpty(minHeap))
        return NULL;

    // Store the root node
    struct MinHeapNode* root = minHeap->array[0];
```

```
    // Replace root node with last node
    struct MinHeapNode* lastNode = minHeap->array[minHeap->size - 1];
    minHeap->array[0] = lastNode;

    // Update position of last node
    minHeap->pos[root->v] = minHeap->size-1;
    minHeap->pos[lastNode->v] = 0;

    // Reduce heap size and heapify root
    --minHeap->size;
    minHeapify(minHeap, 0);

    return root;
}

// Function to decreasy dist value of a given vertex v. This function
// uses pos[] of min heap to get the current index of node in min heap
void decreaseKey(struct MinHeap* minHeap, int v, int dist)
{
    // Get the index of v in  heap array
    int i = minHeap->pos[v];

    // Get the node and update its dist value
    minHeap->array[i]->dist = dist;

    // Travel up while the complete tree is not hepified.
    // This is a O(Logn) loop
    while (i && minHeap->array[i]->dist < minHeap->array[(i - 1) / 2]-
    {
        // Swap this node with its parent
        minHeap->pos[minHeap->array[i]->v] = (i-1)/2;
        minHeap->pos[minHeap->array[(i-1)/2]->v] = i;
        swapMinHeapNode(&minHeap->array[i],  &minHeap->array[(i - 1) /

        // move to parent index
        i = (i - 1) / 2;
    }
}

// A utility function to check if a given vertex
// 'v' is in min heap or not
bool isInMinHeap(struct MinHeap *minHeap, int v)
{
   if (minHeap->pos[v] < minHeap->size)
     return true;
   return false;
}

// A utility function used to print the solution
void printArr(int dist[], int n)
{
    printf("Vertex   Distance from Source\n");
    for (int i = 0; i < n; ++i)
        printf("%d \t\t %d\n", i, dist[i]);
}

// The main function that calulates distances of shortest paths from s
// vertices. It is a O(ELogV) function
void dijkstra(struct Graph* graph, int src)
{
    int V = graph->V;// Get the number of vertices in graph
    int dist[V];      // dist values used to pick minimum weight edge
```

```
    int dist[V];      // dist values used to pick minimum weight edge

    // minHeap represents set E
    struct MinHeap* minHeap = createMinHeap(V);

    // Initialize min heap with all vertices. dist value of all vertic
    for (int v = 0; v < V; ++v)
    {
        dist[v] = INT_MAX;
        minHeap->array[v] = newMinHeapNode(v, dist[v]);
        minHeap->pos[v] = v;
    }

    // Make dist value of src vertex as 0 so that it is extracted firs
    minHeap->array[src] = newMinHeapNode(src, dist[src]);
    minHeap->pos[src]   = src;
    dist[src] = 0;
    decreaseKey(minHeap, src, dist[src]);

    // Initially size of min heap is equal to V
    minHeap->size = V;

    // In the followin loop, min heap contains all nodes
    // whose shortest distance is not yet finalized.
    while (!isEmpty(minHeap))
    {
        // Extract the vertex with minimum distance value
        struct MinHeapNode* minHeapNode = extractMin(minHeap);
        int u = minHeapNode->v; // Store the extracted vertex number

        // Traverse through all adjacent vertices of u (the extracted
        // vertex) and update their distance values
        struct AdjListNode* pCrawl = graph->array[u].head;
        while (pCrawl != NULL)
        {
            int v = pCrawl->dest;

            // If shortest distance to v is not finalized yet, and dis
            // through u is less than its previously calculated distan
            if (isInMinHeap(minHeap, v) && dist[u] != INT_MAX &&
                                          pCrawl->weight + dist[u] < d
            {
                dist[v] = dist[u] + pCrawl->weight;

                // update distance value in min heap also
                decreaseKey(minHeap, v, dist[v]);
            }
            pCrawl = pCrawl->next;
        }
    }

    // print the calculated shortest distances
    printArr(dist, V);
}


// Driver program to test above functions
int main()
{
    // create the graph given in above fugure
    int V = 9;
    struct Graph* graph = createGraph(V);
    addEdge(graph, 0, 1, 4);
```

```
    addEdge(graph, 0, 7, 8);
    addEdge(graph, 1, 2, 8);
    addEdge(graph, 1, 7, 11);
    addEdge(graph, 2, 3, 7);
    addEdge(graph, 2, 8, 2);
    addEdge(graph, 2, 5, 4);
    addEdge(graph, 3, 4, 9);
    addEdge(graph, 3, 5, 14);
    addEdge(graph, 4, 5, 10);
    addEdge(graph, 5, 6, 2);
    addEdge(graph, 6, 7, 1);
    addEdge(graph, 6, 8, 6);
    addEdge(graph, 7, 8, 7);

    dijkstra(graph, 0);

    return 0;
}
```

Output:

```
Vertex   Distance from Source
0                0
1                4
2                12
3                19
4                21
5                11
6                9
7                8
8                14
```

Time Complexity: The time complexity of the above code/algorithm looks O(V^2) as there are two nested while loops. If we take a closer look, we can observe that the statements in inner loop are executed O(V+E) times (similar to BFS). The inner loop has decreaseKey() operation which takes O(LogV) time. So overall time complexity is O(E+V)*O(LogV) which is O((E+V)*LogV) = O(ELogV)
Note that the above code uses Binary Heap for Priority Queue implementation. Time complexity can be reduced to O(E + VLogV) using Fibonacci Heap. The reason is, Fibonacci Heap takes O(1) time for decrease-key operation while Binary Heap takes O(Logn) time.

Notes:
1) The code calculates shortest distance, but doesn't calculate the path information. We can create a parent array, update the parent array when distance is updated (like prim's implementation) and use it show the shortest path from source to different vertices.

2) The code is for undirected graph, same dijekstra function can be used for directed graphs also.

3) The code finds shortest distances from source to all vertices. If we are interested only

in shortest distance from source to a single target, we can break the for loop when the picked minimum distance vertex is equal to target (Step 3.a of algorithm).

4) Dijkstra's algorithm doesn't work for graphs with negative weight edges. For graphs with negative weight edges, Bellman–Ford algorithm can be used, we will soon be discussing it as a separate post.

References:
Introduction to Algorithms by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L.
Algorithms by Sanjoy Dasgupta, Christos Papadimitriou, Umesh Vazirani

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

9. Graph Coloring | Set 2 (Greedy Algorithm)

We introduced graph coloring and applications in previous post. As discussed in the previous post, graph coloring is widely used. Unfortunately, there is no efficient algorithm available for coloring a graph with minimum number of colors as the problem is a known NP Complete problem. There are approximate algorithms to solve the problem though. Following is the basic Greedy Algorithm to assign colors. It doesn't guarantee to use minimum colors, but it guarantees an upper bound on the number of colors. The basic algorithm never uses more than d+1 colors where d is the maximum degree of a vertex in the given graph.

Basic Greedy Coloring Algorithm:

```
1. Color first vertex with first color.
2. Do following for remaining V-1 vertices.
          a) Consider the currently picked vertex and color it with the
             lowest numbered color that has not been used on any previously
             colored vertices adjacent to it. If all previously used colors
             appear on vertices adjacent to v, assign a new color to it.
```

Following is C++ implementation of the above Greedy Algorithm.

```
// A C++ program to implement greedy algorithm for graph coloring
#include <iostream>
#include <list>
using namespace std;

// A class that represents an undirected graph
class Graph
{
    int V;    // No. of vertices
```

```
    list<int> *adj;    // A dynamic array of adjacency lists
public:
    // Constructor and destructor
    Graph(int V)   { this->V = V; adj = new list<int>[V]; }
    ~Graph()       { delete [] adj; }

    // function to add an edge to graph
    void addEdge(int v, int w);

    // Prints greedy coloring of the vertices
    void greedyColoring();
};

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w);
    adj[w].push_back(v);  // Note: the graph is undirected
}
```

```
// Assigns colors (starting from 0) to all vertices and prints
// the assignment of colors
void Graph::greedyColoring()
{
    int result[V];

    // Assign the first color to first vertex
    result[0]  = 0;

    // Initialize remaining V-1 vertices as unassigned
    for (int u = 1; u < V; u++)
        result[u] = -1;  // no color is assigned to u

    // A temporary array to store the available colors. True
    // value of available[cr] would mean that the color cr is
    // assigned to one of its adjacent vertices
    bool available[V];
    for (int cr = 0; cr < V; cr++)
        available[cr] = false;

    // Assign colors to remaining V-1 vertices
    for (int u = 1; u < V; u++)
    {
        // Process all adjacent vertices and flag their colors
        // as unavailable
        list<int>::iterator i;
        for (i = adj[u].begin(); i != adj[u].end(); ++i)
            if (result[*i] != -1)
                available[result[*i]] = true;

        // Find the first available color
        int cr;
        for (cr = 0; cr < V; cr++)
            if (available[cr] == false)
                break;

        result[u] = cr; // Assign the found color

        // Reset the values back to false for the next iteration
        for (i = adj[u].begin(); i != adj[u].end(); ++i)
            if (result[*i] != -1)
                available[result[*i]] = false;
    }
```

```
    // print the result
    for (int u = 0; u < V; u++)
        cout << "Vertex " << u << " --->  Color "
             << result[u] << endl;
}

// Driver program to test above function
int main()
{
    Graph g1(5);
    g1.addEdge(0, 1);
    g1.addEdge(0, 2);
    g1.addEdge(1, 2);
    g1.addEdge(1, 3);
    g1.addEdge(2, 3);
    g1.addEdge(3, 4);
    cout << "Coloring of Graph 1 \n";
    g1.greedyColoring();

    Graph g2(5);
    g2.addEdge(0, 1);
    g2.addEdge(0, 2);
    g2.addEdge(1, 2);
    g2.addEdge(1, 4);
    g2.addEdge(2, 4);
    g2.addEdge(4, 3);
    cout << "\nColoring of Graph 2 \n";
    g2.greedyColoring();

    return 0;
}
```

Output:

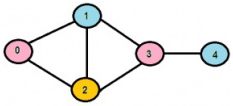
```
Coloring of Graph 1
Vertex 0 --->  Color 0
Vertex 1 --->  Color 1
Vertex 2 --->  Color 2
Vertex 3 --->  Color 0
Vertex 4 --->  Color 1

Coloring of Graph 2
Vertex 0 --->  Color 0
Vertex 1 --->  Color 1
Vertex 2 --->  Color 2
Vertex 3 --->  Color 0
Vertex 4 --->  Color 3
```

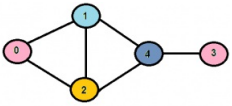
Time Complexity: O(V^2 + E) in worst case.

Analysis of Basic Algorithm

The above algorithm doesn't always use minimum number of colors. Also, the number of colors used sometime depend on the order in which vertices are processed. For example, consider the following two graphs. Note that in graph on right side, vertices 3

and 4 are swapped. If we consider the vertices 0, 1, 2, 3, 4 in left graph, we can color the graph using 3 colors. But if we consider the vertices 0, 1, 2, 3, 4 in right graph, we need 4 colors.





So the order in which the vertices are picked is important. Many people have suggested different ways to find an ordering that work better than the basic algorithm on average. The most common is Welsh–Powell Algorithm which considers vertices in descending order of degrees.

How does the basic algorithm guarantee an upper bound of d+1?
Here d is the maximum degree in the given graph. Since d is maximum degree, a vertex cannot be attached to more than d vertices. When we color a vertex, at most d colors could have already been used by its adjacent. To color this vertex, we need to pick the smallest numbered color that is not used by the adjacent vertices. If colors are numbered like 1, 2, …., then the value of such smallest number must be between 1 to d+1 (Note that d numbers are already picked by adjacent vertices).
This can also be proved using induction. See this video lecture for proof.

We will soon be discussing some interesting facts about chromatic number and graph coloring.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

10. Rearrange a string so that all same characters become d distance away

Given a string and a positive integer d. Some characters may be repeated in the given string. Rearrange characters of the given string such that the same characters become d distance away from each other. Note that there can be many possible rearrangements, the output should be one of the possible rearrangements. If no such arrangement is possible, that should also be reported.
Expected time complexity is O(n) where n is length of input string.

```
Examples:
Input:  "abb", d = 2
```

```
Output: "bab"

Input:  "aacbbc", d = 3
Output: "abcabc"

Input: "geeksforgeeks", d = 3
Output: egkegkesfesor

Input:  "aaa",  d = 2
Output: Cannot be rearranged
```

We strongly recommend to minimize the browser and try this yourself first.
Hint: Alphabet size may be assumed as constant (256) and extra space may be used.

Solution: The idea is to count frequencies of all characters and consider the most frequent character first and place all occurrences of it as close as possible. After the most frequent character is placed, repeat the same process for remaining characters.

1) Let the given string be str and size of string be n

2) Traverse str, store all characters and their frequencies in a Max Heap MH. The value of frequency decides the order in MH, i.e., the most frequent character is at the root of MH.

3) Make all characters of str as '\0'.

4) Do following while MH is not empty.
…**a)** Extract the Most frequent character. Let the extracted character be x and its frequency be f.
…**b)** Find the first available position in str, i.e., find the first '\0' in str.
…**c)** Let the first position be p. Fill x at p, p+d,.. p+(f-1)d

Following is C++ implementation of above algorithm.

```
// Rearrange a string so that all same characters become at least d
// distance away
#include <iostream>
#include <cstring>
#include <cstdlib>
#define MAX 256
using namespace std;

// A structure to store a character 'c' and its frequency 'f'
// in input string
struct charFreq {
    char c;
    int f;
};

// A utility function to swap two charFreq items.
void swap(charFreq *x, charFreq *y) {
    charFreq z = *x;
    *x = *y;
```

```
    *y = z;
}

// A utility function to maxheapify the node freq[i] of a heap
// stored in freq[]
void maxHeapify(charFreq freq[], int i, int heap_size)
{
    int l = i*2 + 1;
    int r = i*2 + 2;
    int largest = i;
    if (l < heap_size && freq[l].f > freq[i].f)
        largest = l;
    if (r < heap_size && freq[r].f > freq[largest].f)
        largest = r;
    if (largest != i)
    {
        swap(&freq[i], &freq[largest]);
        maxHeapify(freq, largest, heap_size);
    }
}

// A utility function to convert the array freq[] to a max heap
void buildHeap(charFreq freq[], int n)
{
    int i = (n - 1)/2;
    while (i >= 0)
    {
        maxHeapify(freq, i, n);
        i--;
    }
}

// A utility function to remove the max item or root from max heap
charFreq extractMax(charFreq freq[], int heap_size)
{
    charFreq root = freq[0];
    if (heap_size > 1)
    {
        freq[0] = freq[heap_size-1];
        maxHeapify(freq, 0, heap_size-1);
    }
    return root;
}

// The main function that rearranges input string 'str' such that
// two same characters become d distance away
void rearrange(char str[], int d)
{
    // Find length of input string
    int n = strlen(str);

    // Create an array to store all characters and their
    // frequencies in str[]
    charFreq freq[MAX] = {{0, 0}};

    int m = 0; // To store count of distinct characters in str[]

    // Traverse the input string and store frequencies of all
    // characters in freq[] array.
    for (int i = 0; i < n; i++)
    {
        char x = str[i];
```

```
        // If this character has occurred first time, increment m
        if (freq[x].c == 0)
            freq[x].c = x, m++;

        (freq[x].f)++;
        str[i] = '\0';  // This change is used later
    }

    // Build a max heap of all characters
    buildHeap(freq, MAX);

    // Now one by one extract all distinct characters from max heap
    // and put them back in str[] with the d distance constraint
    for (int i = 0; i < m; i++)
    {
        charFreq x = extractMax(freq, MAX-i);

        // Find the first available position in str[]
        int p = i;
        while (str[p] != '\0')
            p++;

        // Fill x.c at p, p+d, p+2d, .. p+(f-1)d
        for (int k = 0; k < x.f; k++)
        {
            // If the index goes beyond size, then string cannot
            // be rearranged.
            if (p + d*k >= n)
            {
                cout << "Cannot be rearranged";
                exit(0);
            }
            str[p + d*k] = x.c;
        }
    }
}

// Driver program to test above functions
int main()
{
    char str[] = "aabbcc";
    rearrange(str, 3);
    cout << str;
}
```

Output:

```
abcabc
```

Algorithmic Paradigm: Greedy Algorithm

Time Complexity: Time complexity of above implementation is O(n + mLog(MAX)). Here n is the length of str, m is count of distinct characters in str[] and MAX is maximum possible different characters. MAX is typically 256 (a constant) and m is smaller than MAX. So the time complexity can be considered as O(n).

More Analysis:
The above code can be optimized to store only m characters in heap, we have kept it

this way to keep the code simple. So the time complexity can be improved to O(n + mLogm). It doesn't much matter through as MAX is a constant.

Also, the above algorithm can be implemented using a O(mLogm) sorting algorithm. The first steps of above algorithm remain same. Instead of building a heap, we can sort the freq[] array in non-increasing order of frequencies and then consider all characters one by one from sorted array.

We will soon be covering an extended version where same characters should be moved at least d distance away.

This article is contributed by **Himanshu Gupta**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

11. Connect n ropes with minimum cost

There are given n ropes of different lengths, we need to connect these ropes into one rope. The cost to connect two ropes is equal to sum of their lengths. We need to connect the ropes with minimum cost.

For example if we are given 4 ropes of lengths 4, 3, 2 and 6. We can connect the ropes in following ways.
1) First connect ropes of lengths 2 and 3. Now we have three ropes of lengths 4, 6 and 5.
2) Now connect ropes of lengths 4 and 5. Now we have two ropes of lengths 6 and 9.
3) Finally connect the two ropes and all ropes have connected.

Total cost for connecting all ropes is 5 + 9 + 15 = 29. This is the optimized cost for connecting ropes. Other ways of connecting ropes would always have same or more cost. For example, if we connect 4 and 6 first (we get three strings of 3, 2 and 10), then connect 10 and 3 (we get two strings of 13 and 2). Finally we connect 13 and 2. Total cost in this way is 10 + 13 + 15 = 38.

We strongly recommend to minimize the browser and try this yourself first.
If we observe the above problem closely, we can notice that the lengths of the ropes which are picked first are included more than once in total cost. Therefore, the idea is to connect smallest two ropes first and recur for remaining ropes. This approach is similar to Huffman Coding. We put smallest ropes down the tree so that they can be repeated multiple times rather than the longer ropes.

Following is complete algorithm for finding the minimum cost for connecting n ropes.
Let there be n ropes of lengths stored in an array len[0..n-1]

1) Create a min heap and insert all lengths into the min heap.
2) Do following while number of elements in min heap is not one.
……a) Extract the minimum and second minimum from min heap
……b) Add the above two extracted values and insert the added value to the min-heap.
3) Return the value of only left item in min heap.

Following is C++ implementation of above algorithm.

```
// C++ program for connecting n ropes with minimum cost
#include <iostream>
using namespace std;

// A Min Heap:  Collection of min heap nodes
struct MinHeap
{
    unsigned size;    // Current size of min heap
    unsigned capacity;   // capacity of min heap
    int *harr;  // Attay of minheap nodes
};

// A utility function to create a min heap of given capacity
struct MinHeap* createMinHeap(unsigned capacity)
{
    struct MinHeap* minHeap = new MinHeap;
    minHeap->size = 0;  // current size is 0
    minHeap->capacity = capacity;
    minHeap->harr = new int[capacity];
    return minHeap;
}

// A utility function to swap two min heap nodes
void swapMinHeapNode(int* a, int* b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// The standard minHeapify function.
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest = idx;
    int left = 2 * idx + 1;
    int right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->harr[left] < minHeap->harr[smallest])
      smallest = left;

    if (right < minHeap->size &&
        minHeap->harr[right] < minHeap->harr[smallest])
      smallest = right;

    if (smallest != idx)
    {
        swapMinHeapNode(&minHeap->harr[smallest], &minHeap->harr[idx])
        minHeapify(minHeap, smallest);
    }
}
```

```
// A utility function to check if size of heap is 1 or not
int isSizeOne(struct MinHeap* minHeap)
{
    return (minHeap->size == 1);
}

// A standard function to extract minimum value node from heap
int extractMin(struct MinHeap* minHeap)
{
    int temp = minHeap->harr[0];
    minHeap->harr[0] = minHeap->harr[minHeap->size - 1];
    --minHeap->size;
    minHeapify(minHeap, 0);
    return temp;
}

// A utility function to insert a new node to Min Heap
void insertMinHeap(struct MinHeap* minHeap, int val)
{
    ++minHeap->size;
    int i = minHeap->size - 1;
    while (i && (val < minHeap->harr[(i - 1)/2]))
    {
        minHeap->harr[i] = minHeap->harr[(i - 1)/2];
        i = (i - 1)/2;
    }
    minHeap->harr[i] = val;
}

// A standard funvtion to build min heap
void buildMinHeap(struct MinHeap* minHeap)
{
    int n = minHeap->size - 1;
    int i;
    for (i = (n - 1) / 2; i >= 0; --i)
        minHeapify(minHeap, i);
}

// Creates a min heap of capacity equal to size and inserts all values
// from len[] in it. Initially size of min heap is equal to capacity
struct MinHeap* createAndBuildMinHeap(int len[], int size)
{
    struct MinHeap* minHeap = createMinHeap(size);
    for (int i = 0; i < size; ++i)
        minHeap->harr[i] = len[i];
    minHeap->size = size;
    buildMinHeap(minHeap);
    return minHeap;
}

// The main function that returns the minimum cost to connect n ropes
// lengths stored in len[0..n-1]
int minCost(int len[], int n)
{
    int cost = 0;  // Initialize result

    // Create a min heap of capacity equal to n and put all ropes in it
    struct MinHeap* minHeap = createAndBuildMinHeap(len, n);

    // Iterate while size of heap doesn't become 1
    while (!isSizeOne(minHeap))
    {
        // Extract two minimum length ropes from min heap
```

```
        // Extract two minimum length ropes from min heap
        int min     = extractMin(minHeap);
        int sec_min = extractMin(minHeap);

        cost += (min + sec_min);  // Update total cost

        // Insert a new rope in min heap with length equal to sum
        // of two extracted minimum lengths
        insertMinHeap(minHeap, min+sec_min);
    }

    // Finally return total minimum cost for connecting all ropes
    return cost;
}

// Driver program to test above functions
int main()
{
    int len[] = {4, 3, 2, 6};
    int size = sizeof(len)/sizeof(len[0]);
    cout << "Total cost for connecting ropes is " << minCost(len, size
    return 0;
}
```

Output:

```
Total cost for connecting ropes is 29
```

Time Complexity: Time complexity of the algorithm is O(nLogn) assuming that we use a O(nLogn) sorting algorithm. Note that heap operations like insert and extract take O(Logn) time.

Algorithmic Paradigm: Greedy Algorithm

A simple implementation with STL in C++

Following is a simple implementation that uses priority_queue available in STL. Thanks to Pango89 for providing below code.

```
#include<iostream>
#include<queue>
using namespace std;

int minCost(int arr[], int n)
{
    // Create a priority queue ( http://www.cplusplus.com/reference/qu
    // By default 'less' is used which is for decreasing order
    // and 'greater' is used for increasing order
    priority_queue< int, vector<int>, greater<int> > pq(arr, arr+n);

    // Initialize result
    int res = 0;

    // While size of priority queue is more than 1
    while (pq.size() > 1)
    {
        // Extract shortest two ropes from pq
        int first = pq.top();
        pq.pop();
        int second = pq.top();
        pq.pop();

        // Connect the ropes: update result and
        // insert the new rope to pq
        res += first + second;
        pq.push(first + second);
    }

    return res;
}

// Driver program to test above function
int main()
{
    int len[] = {4, 3, 2, 6};
    int size = sizeof(len)/sizeof(len[0]);
    cout << "Total cost for connecting ropes is " << minCost(len, size
    return 0;
}
```

Output:

```
Total cost for connecting ropes is 29
```

This article is compiled by **Abhishek**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

12. Minimum Number of Platforms Required for a Railway/Bus Station

Given arrival and departure times of all trains that reach a railway station, find the

minimum number of platforms required for the railway station so that no train waits. We are given two arrays which represent arrival and departure times of trains that stop

Examples:

```
Input:  arr[]  = {9:00,  9:40, 9:50,  11:00, 15:00, 18:00}
        dep[]  = {9:10, 12:00, 11:20, 11:30, 19:00, 20:00}
Output: 3
There are at-most three trains at a time (time between 11:00 to 11:20)
```

We strongly recommend to minimize your browser and try this yourself first.
We need to find the maximum number of trains that are there on the given railway station at a time. A **Simple Solution** is to take every interval one by one and find the number of intervals that overlap with it. Keep track of maximum number of intervals that overlap with an interval. Finally return the maximum value. Time Complexity of this solution is $O(n^2)$.

We can solve the above problem **in O(nLogn) time**. The idea is to consider all evens in sorted order. Once we have all events in sorted order, we can trace the number of trains at any time keeping track of trains that have arrived, but not departed.

For example consider the above example.

```
    arr[]  = {9:00,  9:40, 9:50,  11:00, 15:00, 18:00}
    dep[]  = {9:10, 12:00, 11:20, 11:30, 19:00, 20:00}

All events sorted by time.
Total platforms at any time can be obtained by subtracting total
departures from total arrivals by that time.
 Time     Event Type     Total Platforms Needed at this Time
 9:00       Arrival                  1
 9:10       Departure                0
 9:40       Arrival                  1
 9:50       Arrival                  2
 11:00      Arrival                  3
 11:20      Departure                2
 11:30      Departure                1
 12:00      Departure                0
 15:00      Arrival                  1
 18:00      Arrival                  2
 19:00      Departure                1
 20:00      Departure                0

Minimum Platforms needed on railway station = Maximum platforms
                                              needed at any time
                                           = 3
```

Following is C++ implementation of above approach. Note that the implementation doesn't create a single sorted list of all events, rather it individually sorts arr[] and dep[] arrays, and then uses merge process of merge sort to process them together as a single sorted array.

```
// Program to find minimum number of platforms required on a railway s
#include<iostream>
#include<algorithm>
using namespace std;

// Returns minimum number of platforms reqquired
int findPlatform(int arr[], int dep[], int n)
{
   // Sort arrival and departure arrays
   sort(arr, arr+n);
   sort(dep, dep+n);

   // plat_needed indicates number of platforms needed at a time
   int plat_needed = 1, result = 1;
   int i = 1, j = 0;

   // Similar to merge in merge sort to process all events in sorted o
   while (i < n && j < n)
   {
      // If next event in sorted order is arrival, increment count of
      // platforms needed
      if (arr[i] < dep[j])
      {
          plat_needed++;
          i++;
          if (plat_needed > result)  // Update result if needed
              result = plat_needed;
      }
      else // Else decrement count of platforms needed
      {
          plat_needed--;
          j++;
      }
   }

   return result;
}

// Driver program to test methods of graph class
int main()
{
    int arr[] = {900, 940, 950, 1100, 1500, 1800};
    int dep[] = {910, 1200, 1120, 1130, 1900, 2000};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << "Minimum Number of Platforms Required = "
         << findPlatform(arr, dep, n);
    return 0;
}
```

Output:

```
Minimum Number of Platforms Required = 3
```

Algorithmic Paradigm: Dynamic Programming

Time Complexity: O(nLogn), assuming that a O(nLogn) sorting algorithm for sorting arr[] and dep[].

This article is contributed by **Shivam**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

13. Job Sequencing Problem | Set 1 (Greedy Algorithm)

Given an array of jobs where every job has a deadline and associated profit if the job is finished before the deadline. It is also given that every job takes single unit of time, so the minimum possible deadline for any job is 1. How to maximize total profit if only one job can be scheduled at a time.

Examples:

```
Input: Four Jobs with following deadlines and profits
  JobID    Deadline      Profit
    a         4            20
    b         1            10
    c         1            40
    d         1            30
Output: Following is maximum profit sequence of jobs
        c, a


Input:  Five Jobs with following deadlines and profits
   JobID     Deadline     Profit
     a          2           100
     b          1           19
     c          2           27
     d          1           25
     e          3           15
Output: Following is maximum profit sequence of jobs
        c, a, e
```

We strongly recommend to minimize your browser and try this yourself first.

A **Simple Solution** is to generate all subsets of given set of jobs and check individual subset for feasibility of jobs in that subset. Keep track of maximum profit among all feasible subsets. The time complexity of this solution is exponential.

This is a standard Greedy Algorithm problem. Following is algorithm.

```
1) Sort all jobs in decreasing order of profit.
2) Initialize the result sequence as first job in sorted jobs.
3) Do following for remaining n-1 jobs
.......a) If the current job can fit in the current result sequence
          without missing the deadline, add current job to the result.
          Else ignore the current job.
```

The Following is C++ implementation of above algorithm.

```
// Program to find the maximum profit job sequence from a given array
// of jobs with deadlines and profits
#include<iostream>
#include<algorithm>
using namespace std;

// A structure to represent a job
struct Job
{
   char id;      // Job Id
   int dead;    // Deadline of job
   int profit;  // Profit if job is over before or on deadline
};

// This function is used for sorting all jobs according to profit
bool comparison(Job a, Job b)
{
     return (a.profit > b.profit);
}

// Returns minimum number of platforms reqquired
void printJobScheduling(Job arr[], int n)
{
    // Sort all jobs according to decreasing order of prfit
    sort(arr, arr+n, comparison);

    int result[n]; // To store result (Sequence of jobs)
    bool slot[n];  // To keep track of free time slots

    // Initialize all slots to be free
    for (int i=0; i<n; i++)
        slot[i] = false;

    // Iterate through all given jobs
    for (int i=0; i<n; i++)
    {
       // Find a free slot for this job (Note that we start
       // from the last possible slot)
       for (int j=min(n, arr[i].dead)-1; j>=0; j--)
       {
          // Free slot found
          if (slot[j]==false)
          {
             result[j] = i;  // Add this job to result
             slot[j] = true; // Make this slot occupied
             break;
          }
       }
    }

    // Print the result
```

```
    for (int i=0; i<n; i++)
       if (slot[i])
         cout << arr[result[i]].id << " ";
}

// Driver program to test methods
int main()
{
    Job arr[5] = { {'a', 2, 100}, {'b', 1, 19}, {'c', 2, 27},
                   {'d', 1, 25}, {'e', 3, 15}};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << "Following is maximum profit sequence of jobs\n";
    printJobScheduling(arr, n);
    return 0;
}
```

Output:

```
Following is maximum profit sequence of jobs
c a e
```

Time Complexity of the above solution is $O(n^2)$. It can be optimized to almost O(n) by using union-find data structure. We will son be discussing the optimized solution.

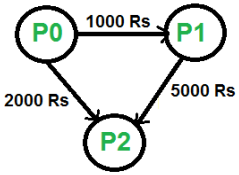
Sources:
http://ocw.mit.edu/courses/civil-and-environmental-engineering/1-204-computer-algorithms-in-systems-engineering-spring-2010/lecture-notes/MIT1_204S10_lec10.pdf

This article is contributed by **Shubham**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

14. Minimize Cash Flow among a given set of friends who have borrowed money from each other

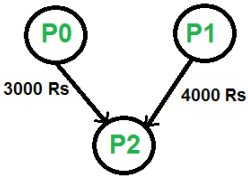
Given a number of friends who have to give or take some amount of money from one another. Design an algorithm by which the total cash flow among all the friends is minimized.

Example:
Following diagram shows input debts to be settled.



P0 has to pay 1000 Rs to P1
P0 also has to pay 2000 Rs to P2
P1 has to pay 5000 Rs to P2.

Above debts can be settled in following optimized way



P1 pays 4000 Rs to P2
P0 pays 3000 Rs to P2

We strongly recommend to minimize your browser and try this yourself first.
The idea is to use Greedy algorithm where at every step, settle all amounts of one person and recur for remaining n-1 persons.
How to pick the first person? To pick the first person, calculate the net amount for every person where net amount is obtained by subtracting all debts (amounts to pay) from all credits (amounts to be paid). Once net amount for every person is evaluated, find two persons with maximum and minimum net amounts. These two persons are the most creditors and debtors. The person with minimum of two is our first person to be settled and removed from list. Let the minimum of two amounts be x. We pay 'x' amount from the maximum debtor to maximum creditor and settle one person. If x is equal to the maximum debit, then maximum debtor is settled, else maximum creditor is settled.

The following is detailed algorithm.

Do following for every person Pi where i is from 0 to n-1.
1) Compute the net amount for every person. The net amount for person 'i' can be computed be subtracting sum of all debts from sum of all credits.

2) Find the two persons that are maximum creditor and maximum debtor. Let the maximum amount to be credited maximum creditor be maxCredit and maximum amount to be debited from maximum debtor be maxDebit. Let the maximum debtor be Pd and maximum creditor be Pc.

3) Find the minimum of maxDebit and maxCredit. Let minimum of two be x. Debit 'x' from Pd and credit this amount to Pc

4) If x is equal to maxCredit, then remove Pc from set of persons and recur for remaining (n-1) persons.

5) If x is equal to maxDebit, then remove Pd from set of persons and recur for remaining

(n-1) persons.

Thanks to Balaji S for suggesting this method in a comment here.

The following is C++ implementation of above algorithm.

```
// C++ program to fin maximum cash flow among a set of persons
#include<iostream>
using namespace std;

// Number of persons (or vertices in the graph)
#define N 3

// A utility function that returns index of minimum value in arr[]
int getMin(int arr[])
{
    int minInd = 0;
    for (int i=1; i<N; i++)
        if (arr[i] < arr[minInd])
            minInd = i;
    return minInd;
}

// A utility function that returns index of maximum value in arr[]
int getMax(int arr[])
{
    int maxInd = 0;
    for (int i=1; i<N; i++)
        if (arr[i] > arr[maxInd])
            maxInd = i;
    return maxInd;
}

// A utility function to return minimum of 2 values
int minOf2(int x, int y)
{
    return (x<y)? x: y;
}

// amount[p] indicates the net amount to be credited/debited
// to/from person 'p'
// If amount[p] is positive, then i'th person will amount[i]
// If amount[p] is negative, then i'th person will give  -amount[i]
void minCashFlowRec(int amount[])
{
    // Find the indexes of minimum and maximum values in amount[]
    // amount[mxCredit] indicates the maximum amount to be given
    //                  (or credited) to any person .
    // And amount[mxDebit] indicates the maximum amount to be taken
    //                  (or debited) from any person.
    // So if there is a positive value in amount[], then there must
    // be a negative value
    int mxCredit = getMax(amount), mxDebit = getMin(amount);

    // If both amounts are 0, then all amounts are settled
    if (amount[mxCredit] == 0 && amount[mxDebit] == 0)
        return;

    // Find the minimum of two amounts
    int min = minOf2(-amount[mxDebit], amount[mxCredit]);
    amount[mxCredit] -= min;
```

```
    amount[mxDebit] += min;

    // If minimum is the maximum amount to be
    cout << "Person " << mxDebit << " pays " << min
         << " to " << "Person " << mxCredit << endl;

    // Recur for the amount array.  Note that it is guaranteed that
    // the recursion would terminate as either amount[mxCredit]
    // or  amount[mxDebit] becomes 0
    minCashFlowRec(amount);
}

// Given a set of persons as graph[] where graph[i][j] indicates
// the amount that person i needs to pay person j, this function
// finds and prints the minimum cash flow to settle all debts.
void minCashFlow(int graph[][N])
{
    // Create an array amount[], initialize all value in it as 0.
    int amount[N] = {0};

    // Calculate the net amount to be paid to person 'p', and
    // stores it in amount[p]. The value of amount[p] can be
    // calculated by subtracting debts of 'p' from credits of 'p'
    for (int p=0; p<N; p++)
       for (int i=0; i<N; i++)
          amount[p] += (graph[i][p] -  graph[p][i]);

    minCashFlowRec(amount);
}

// Driver program to test above function
int main()
{
    // graph[i][j] indicates the amount that person i needs to
    // pay person j
    int graph[N][N] = { {0, 1000, 2000},
                        {0, 0, 5000},
                        {0, 0, 0},};

    // Print the solution
    minCashFlow(graph);
    return 0;
}
```

Output:

```
Person 1 pays 4000 to Person 2
Person 0 pays 3000 to Person 2
```

Algorithmic Paradigm: Greedy

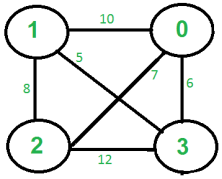
Time Complexity: $O(N^2)$ where N is the number of persons.

This article is contributed by **Gaurav**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

15. K Centers Problem | Set 1 (Greedy Approximate Algorithm)

Given n cities and distances between every pair of cities, select k cities to place warehouses (or ATMs) such that the maximum distance of a city to a warehouse (or ATM) is minimized.

For example consider the following four cities, 0, 1, 2 and 3 and distances between them, how do place 2 ATMs among these 4 cities so that the maximum distance of a city to an ATM is minimized.



k = 2

The two ATMs should be placed in cities 2 and 3. The maximum distance of a city from an ATM becomes 6 in this optimal placement (We can not get the maximum distance less than 7)

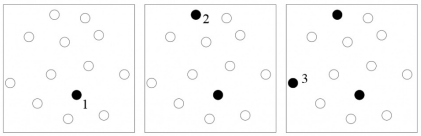
There is no polynomial time solution available for this problem as the problem is a known NP-Hard problem. There is a polynomial time Greedy approximate algorithm, the greedy algorithm provides a solution which is never worse that twice the optimal solution. The greedy solution works only if the distances between cities follow Triangular Inequality (Distance between two points is always smaller than sum of distances through a third point).

The 2-Approximate Greedy Algorithm:
1) Choose the first center arbitrarily.

2) Choose remaining k-1 centers using the following criteria.
Let c1, c2, c3, … ci be the already chosen centers. Choose (i+1)'th center by picking the city which is farthest from already selected centers, i.e, the point p which has following value as maximum

Min[dist(p, c1), dist(p, c2), dist(p, c3), …. dist(p, ci)]

The following diagram taken from here illustrates above algorithm.



Example (k = 3 in the above shown Graph)
a) Let the first arbitrarily picked vertex be 0.

b) The next vertex is 1 because 1 is the farthest vertex from 0.

c) Remaining cities are 2 and 3. Calculate their distances from already selected centers

(0 and 1). The greedy algorithm basically calculates following values.

Minimum of all distanced from 2 to already considered centers
Min[dist(2, 0), dist(2, 1)] = Min[7, 8] = 7

Minimum of all distanced from 3 to already considered centers
Min[dist(3, 0), dist(3, 1)] = Min[6, 5] = 5

After computing the above values, the city 2 is picked as the value corresponding to 2 is maximum.

Note that the greedy algorithm doesn't give best solution for k = 2 as this is just an approximate algorithm with bound as twice of optimal.

Proof that the above greedy algorithm is 2 approximate.
Let OPT be the maximum distance of a city from a center in the Optimal solution. We need to show that the maximum distance obtained from Greedy algorithm is 2*OPT.

The proof can be done using contradiction.

a) Assume that the distance from the furthest point to all centers is > 2·OPT.

b) This means that distances between all centers are also > 2·OPT.

c) We have k + 1 points with distances > 2·OPT between every pair.

d) Each point has a center of the optimal solution with distance ? OPT to it.

e) There exists a pair of points with the same center X in the optimal solution (pigeonhole principle: k optimal centers, k+1 points)

f) The distance between them is at most 2·OPT (triangle inequality) which is a contradiction.

Source:
http://algo2.iti.kit.edu/vanstee/courses/kcenter.pdf

This article is contributed by **Harshit**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

16. Set Cover Problem | Set 1 (Greedy Approximate Algorithm)

Given a universe U of n elements, a collection of subsets of U say S = {S_1, S_2…,S_m} where every subset S_i has an associated cost. Find a minimum cost subcollection of S that covers all elements of U.

Example:

```
   U = {1,2,3,4,5}
   S = {S1,S2,S3}

   S1 = {4,1,3},    Cost(S1) = 5
   S2 = {2,5},      Cost(S2) = 10
   S3 = {1,4,3,2},  Cost(S3) = 3

Output: Minimum cost of set cover is 13 and
        set cover is {S2, S3}

There are two possible set covers {S1, S2} with cost 15
and {S2, S3} with cost 13.
```

Why is it useful?
It was one of Karp's NP-complete problems, shown to be so in 1972. Other applications: edge covering, vertex cover
Interesting example: IBM finds computer viruses (wikipedia)
Elements- 5000 known viruses
Sets- 9000 substrings of 20 or more consecutive bytes from viruses, not found in 'good' code.
A set cover of 180 was found. It suffices to search for these 180 substrings to verify the existence of known computer viruses.

Another example: Consider General Motors needs to buy a certain amount of varied supplies and there are suppliers that offer various deals for different combinations of materials (Supplier A: 2 tons of steel + 500 tiles for $x; Supplier B: 1 ton of steel + 2000 tiles for $y; etc.). You could use set covering to find the best way to get all the materials while minimizing cost
Source: http://math.mit.edu/~goemans/18434S06/setcover-tamara.pdf

Set Cover is NP-Hard:
There is no polynomial time solution available for this problem as the problem is a known NP-Hard problem. There is a polynomial time Greedy approximate algorithm, the greedy algorithm provides a Logn approximate algorithm.

2-Approximate Greedy Algorithm:
Let U be the universe of elements, {S_1, S_2, … S_m} be collection of subsets of U and Cost(S_1), C(S_2), … Cost(S_m) be costs of subsets.

```
1) Let I represents set of elements included so far.  Initialize I = {}

2) Do following while I is not same as U.
    a) Find the set Si in {S1, S2, ... Sm} whose cost effectiveness is
```

```
        smallest, i.e., the ratio of cost C(Si) and number of newly added
        elements is minimum.
        Basically we pick the set for which following value is minimum.
            Cost(Si) / |Si - I|
    b) Add elements of above picked Si to I, i.e.,  I = I U Si
```

Example:
Let us consider the above example to understand Greedy Algorithm.

First Iteration:
I = {}

The per new element cost for S_1 = Cost(S_1)/|S_1 – I| = 5/3

The per new element cost for S_2 = Cost(S_2)/|S_2 – I| = 10/2

The per new element cost for S_3 = Cost(S_3)/|S_3 – I| = 3/4

Since S_3 has minimum value S_3 is added, I becomes {1,4,3,2}.

Second Iteration:
I = {1,4,3,2}

The per new element cost for S_1 = Cost(S_1)/|S_1 – I| = 5/0
Note that S_1 doesn't add any new element to I.

The per new element cost for S_2 = Cost(S_2)/|S_2 – I| = 10/1
Note that S_2 adds only 5 to I.

The greedy algorithm provides the optimal solution for above example, but it may not provide optimal solution all the time. Consider the following example.

```
S1 = {1, 2}
S2 = {2, 3, 4, 5}
S3 = {6, 7, 8, 9, 10, 11, 12, 13}
S4 = {1, 3, 5, 7, 9, 11, 13}
S5 = {2, 4, 6, 8, 10, 12, 13}

Let the cost of every set be same.

The greedy algorithm produces result as {S3, S2, S1}

The optimal solution is {S4, S5}
```

Proof that the above greedy algorithm is Logn approximate.
Let OPT be the cost of optimal solution. Say (k-1) elements are covered before an

iteration of above greedy algorithm. The cost of the k'th element <= OPT / (n-k+1) (Note that cost of an element is evaluated by cost of its set divided by number of elements added by its set). How did we get this result?
Since k'th element is not covered yet, there is a S_i that has not been covered before the current step of greedy algorithm and it is there in OPT. Since greedy algorithm picks the most cost effective S_i, per element cost in the picked set in Greedy must be smaller than OPT divided by remaining elements. Therefore cost of k'th element <= OPT/|U-I| (Note that U-I is set of not yet covered elements in Greedy Algorithm). The value of |U-I| is n - (k-1) which is n-k+1.

```
Cost of Greedy Algorithm = Sum of costs of n elements
                         [putting k = 1, 2..n in above formula]
                          <= (OPT/1 + OPT/2 + ... + OPT/n)
                          <= OPT(1 + 1/2 + ..... 1/n)
                         [Since 1 + 1/2 + .. 1/n ≈ Log n]
                          <= OPT * Logn
```

Source:
http://math.mit.edu/~goemans/18434S06/setcover-tamara.pdf

This article is contributed by **Harshit**. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.