

# Principles of Programming Languages - Homework 5

Abhi Agarwal

## 1 Problem 1

(a)

$$\begin{aligned}
 \text{(i)} \quad & \frac{\frac{\frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 3 \Downarrow 3}{\{x \rightarrow 3, y \rightarrow -2\} \vdash x \Downarrow 3} \text{EvalVal} \quad \frac{x \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash x \Downarrow 3} \text{EvalVar} \quad 9=3*3}{\{x \rightarrow 3, y \rightarrow -2\} \vdash x*3 \Downarrow 9} \text{EvalTimes} \quad \frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 11=9+2} \text{EvalVal} \quad 11=9+2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 3*x+2 \Downarrow 11} \text{EvalPlus} \\
 \text{(ii)} \quad & \frac{\frac{\frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVal} \quad \frac{y \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVar} \quad 0=2+-2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2+y \Downarrow 0} \text{EvalPlus} \quad \frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash \text{const } b=0 \Downarrow 0}{\{x \rightarrow 3, y \rightarrow -2\} \vdash \text{false}=\text{toBool}(0)} \text{EvalConstDecl} \quad \frac{y \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVar}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash b?x:y \Downarrow -2} \text{EvalIfThen}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \text{(i)} \quad & 3 + (1 \ \&\& \ 5) \xrightarrow{a} 3 + 5 \xrightarrow{b} 8 \xrightarrow{c} 8 \\
 & a: \text{SearchBop1, DoAndTrue} \\
 & b: \text{SearchBop2, DoPlus} \\
 & c: \text{DoConstDecl}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \text{const } x = \underline{2} + \underline{1}; x * 0 ? x : x + x \xrightarrow{a} \text{const } x = 3; \underline{x} * 0 ? x : x + x \xrightarrow{c} \text{const } x = 3; \underline{0} ? x : x + x \xrightarrow{d} \text{const } x \\
 & = 3; \underline{3} + \underline{3} \xrightarrow{e} 6 \\
 & a: \text{SearchConstDecl1, DoPlus} \\
 & b: \text{SearchConstDecl2, DoVar}
 \end{aligned}$$

*c*: SearchIf, DoTimes  
*d*: SearchConstDecl1, DoPlus  
*e*: DoConstDecl

(c)

SearchPlus1  $\frac{env \vdash e_2 \rightarrow e'_2}{env \vdash e_1 + e_2 \rightarrow e_1 + e'_2}$   
 SearchPlus2  $\frac{env \vdash e_1 \rightarrow e'_1}{env \vdash e_1 + v_2 \rightarrow e'_1 + v_2}$   
 DoPlus  $\frac{v = toNum(v_1) + toNum(v_2)}{env \vdash v_1 + v_2 = v}$   
 Order for right-to-left evaluation: SearchPlus1, SearchPlus2, DoPlus

(d)

Big-step SOS  $\frac{env \vdash e_1 \rightarrow v_1 \quad env \vdash e_2 \rightarrow v_2 \quad v = v_2}{env \vdash e_1, e_2 \Downarrow v}$   
 Small-step SOS  
 SearchComma1  $\frac{env \vdash e_1 \rightarrow e'_1}{env \vdash e_1, e_2 \rightarrow e'_1, e_2}$   
 SearchComma2  $\frac{env \vdash e'_2 \rightarrow e'_2}{env \vdash v_1, e_2 \rightarrow v_1, e'_2}$   
 DoReturnValue  $\frac{v = v_2}{env \vdash v_1, v_2 = v}$   
 Order for right-to-left evaluation: SearchComma1, SearchComma2, DoReturnValue

(e)

(i) This program evaluates to 5. During evaluation, the EvalVar is applied three times as follows:

The first application is the using occurrence of *y* in the definition of the function *f* on line 3. In this case, *y* was bound to the variable 3 in the call to *f* on line 4.

The second application is the occurrence of *x* in the definition of the function *g* on line 2. *x* is bound to the value 2 in the constant declaration of 2 on line 1.

The third application is for the occurrence of  $y$  in the definition of the function  $g$  on line 2. This occurrence of  $y$  was bound to the value 3 in the call to  $g$  on line 3.

(ii) This program evaluates to 6. During evaluation, the EvalVar rule is applied four times as follows:

The first application is the using occurrence of  $y$  in the definition of function  $f$  on line 3. In this case,  $y$  was bound to the value 3 in the call to  $f$  on line 4.

The second application is the using occurrence of  $x$  in the definition of the function  $g$  on line 2. This occurrence was bound to the value 3 in the call to  $g$  on line 3.

The third application is the using occurrence of  $y$  in the definition of the function  $g$  on line 2. This occurrence was bound to the value that will be passed into the function by the function call in line 3. The function call to  $g$  returns a function, and that function is called on line 3 using the value 3. So the occurrence was bound to the value 3 in the call to the anonymous function on line 3.

The fourth application is the using occurrence of  $y$  in the definition of function  $f$  on line 3. This occurrence was bound to the value 3 in the call to the anonymous function that is returned by calling  $g$  on line 3.

## 2 Problem 2

(a)

$$e_1 = (3 * y) + 4$$

(b)

$$e_1 = (x * y) + 4$$

(c)

$$e_2 = \text{const } y = y; 3 + y$$

(d)

$$e_2 = \text{const } y = 3; x + y$$

(e)

(f)