Principles of Programming Languages - Homework 5

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1 Problem 1

(a)

(i) $$\dfrac{\dfrac{}{\{x\to 3,y\to -2\}\vdash 3\Downarrow 3}\,EvalVal \quad \dfrac{x\in dom(\{x\to 3,y\to -2\})}{\{x\to 3,y\to -2\}\vdash x\Downarrow 3}\,EvalVar \;\; 9=3*3}{\{x\to 3,y\to -2\}\vdash x*3\Downarrow 9}\,EvalTimes \quad \dfrac{}{(\{x\to 3,y\to -2\})\vdash 2\Downarrow 2}\,EvalVal \;\; 11=9+2}{\{x\to 3,y\to -2\}\vdash 3*x+2\Downarrow 11}\,EvalPlus$$

(ii) $$\dfrac{\dfrac{\dfrac{}{\{x\to 3,y\to -2\}\vdash 2\Downarrow 2}\,EvalVal \quad \dfrac{y\in dom(\{x\to 3,y\to -2\})}{\{x\to 3,y\to -2\}\vdash y\Downarrow -2}\,EvalVar \;\; 0=2+-2}{\{x\to 3,y\to -2\}\vdash 2+y\Downarrow 0}\,EvalPlus \quad env'=env[b\to 0] \quad \dfrac{\dfrac{b\in dom(\{x\to 3,y\to -2,b\to 0\})}{env'\vdash b\Downarrow 0}\,EvalVar \quad toBool(0)=false \quad \dfrac{y\in dom(\{x\to 3,y\to -2,b\to 0\})}{env'\vdash y\Downarrow -2}\,EvalVar}{\{x\to 3,y\to -2,b\to 0\}\vdash y\Downarrow -2}\,EvalIfElse}{\{x\to 3,y\to -2\}\vdash const\ b=2+y;b?x:y\Downarrow -2}\,EvalIfThen$$

(b)

(i) 3 + $\underline{(1 \;\&\&\; 5)}$ $\xrightarrow{a}$ $\underline{3 + 5}$ $\xrightarrow{b}$ 8
a: SearchBop2, DoAndTrue
b: DoPlus

(ii) const x = $\underline{2 + 1}$; x * 0 ? x : x + x $\xrightarrow{a}$ const x = 3; $\underline{\text{x}}$ * 0 ? x : x + x $\xrightarrow{b}$ const x = 3; $\underline{3 * 0}$? x : x + x $\xrightarrow{c}$ const x = 3; $\underline{0 \text{ ? x : x + x}}$ $\xrightarrow{d}$ const x = 3; $\underline{\text{x}}$ + x $\xrightarrow{e}$ const x = 3; 3 + $\underline{\text{x}}$ $\xrightarrow{f}$ const x = 3; 3 + 3 $\xrightarrow{g}$ const x = 3; $\underline{3 + 3}$ $\xrightarrow{h}$ 6
a: SearchConstDecl1, DoPlus
b: SearchConstDecl2, DoVar
c: SearchConstDecl2, DoTimes
d: SearchConstDecl2, DoIfFalse
e: SearchConstDecl2, DoVar
f: SearchConstDecl2, DoVar
g: SearchConstDecl1, DoPlus
h: DoConstDecl

(c)

SearchPlus1 $\dfrac{env\vdash e_2\to e_2'}{env\vdash e_1+e_2\to e_1+e_2'}$
SearchPlus2 $\dfrac{env\vdash e_1\to e_1'}{env\vdash e_1+v_2\to e_1'+v_2}$
DoPlus $\dfrac{v=toNum(v_1)+toNum(v_2)}{env\vdash v_1+v_2=v}$
Order for right-to-left evaluation: SearchPlus1, SearchPlus2, DoPlus

(d)

Big-step SOS $\frac{env \vdash e_1 \to v_1 \ env \vdash e_2 \to v_2 \ v = v_2}{env \vdash e_1, e_2 \Downarrow v}$
Small-step SOS
SearchSeq1 $\frac{env \vdash e_1 \to e_1'}{env \vdash e_1, e_2 \to e_1', e_2}$
SearchSeq2 $\frac{env \vdash e_2' \to e_2'}{env \vdash v_1, e_2 \to v_1, e_2'}$
DoReturnValue $\frac{}{env \vdash v_1, v_2 = v}$
Order for right-to-left evaluation: SearchSeq1, SearchSeq2, DoReturnValue

(e)

(i) This program evaluates to 5. During evaluation, the EvalVar is applied three times as follows:

The first application is the using occurrence of y in the definition of the function f on line 3. In this case, y was bound to the variable 3 in the call to f on line 4.

The second application is the occurrence of x in the definition of the function g on line 2. x is bound to the value 2 in the constant declaration of 2 on line 1.

The third application is for the occurrence of y in the definition of the function g on line 2. This occurrence of y was bound to the value 3 in the call to g on line 3.

(ii) This program evaluates to 6. During evaluation, the EvalVar rule is applied four times as follows:

The first application is the using occurrence of y in the definition of function f on line 3. In this case, y was bound to the value 3 in the call to f on line 4.

The second application is the using occurrence of x in the definition of the function g on line 2. This occurrence was bound to the value 3 in the call to g on line 3.

The third application is the using occurrence of y in the definition of the function g on line 2. This occurrence was bound to the value that will be passed into the function by the function call in line 3. The function call to g returns a function, and that function is called on line 3 using the value 3. So the occurrence was bound to the value 3 in the call to the anonymous function on line 3.

The fourth application is the using occurrence of y in the definition of function f on line 3. This occurrence was bound to the value 3 in the call to the anonymous function that is returned by calling g on line 3.

2 Problem 2

(a)

$e_1 = (3 * y) + 4$

(b)

$e_1 = (x * y) + 4$

(c)

$e_2 = const\, y = y;\ 3 + y$

(d)

$e_2 = const\, y = 3; x + y$

(e)

$e_3 = const\, x = (function(z)(x(z))); x(y(2))$

(f)

$e_3 = const\, x = (function(z)(y(x(z)))); x(y)$