

Principles of Programming Languages - Homework 5

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1 Problem 1

(a)

$$\begin{aligned}
 \text{(i)} \quad & \frac{\frac{\frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 3 \Downarrow 3}{\{x \rightarrow 3, y \rightarrow -2\} \vdash x * 3 \Downarrow 9} \text{EvalVal} \quad \frac{\frac{x \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash x \Downarrow 3} \text{EvalVar}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 3 * x + 2 \Downarrow 11} \text{EvalTimes} \quad \frac{\frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2} \text{EvalVal}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2} \text{EvalPlus}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 3 * x + 2 \Downarrow 11} \text{EvalPlus} \\
 \text{(ii)} \quad & \frac{\frac{\frac{\frac{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 \Downarrow 2}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 + y \Downarrow 0} \text{EvalVal} \quad \frac{\frac{y \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVar}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash 2 + y \Downarrow 0} \text{EvalPlus}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash \text{const } b = 0 \Downarrow 0} \text{EvalConstDecl} \quad \frac{\text{false} = \text{toBool}(0)}{\{x \rightarrow 3, y \rightarrow -2\} \vdash \text{const } b = 0 \Downarrow 0} \text{EvalConstDecl} \quad \frac{\frac{y \in \text{dom}(\{x \rightarrow 3, y \rightarrow -2\})}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVar}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash y \Downarrow -2} \text{EvalVar}}{\{x \rightarrow 3, y \rightarrow -2\} \vdash b ? x : y \Downarrow -2} \text{EvalIfThen}
 \end{aligned}$$

(b)

$$\text{(i)} \quad \frac{\frac{\text{toBool}(1) = \text{true}}{\{\emptyset\} \vdash 1 \&\& 5 \rightarrow 5} \text{DoAndTrue} \quad v = \text{toNum}(3) + \text{toNum}(5)}{\{\emptyset\} \vdash 3 + (1 \&\& 5) \rightarrow v} \text{DoPlus}$$

$$3 + (1 \&\& 5) \xrightarrow{a} 3 + (1 \&\& 5) \xrightarrow{b} 3 + 5 \xrightarrow{c} 8$$

a : SearchConstDecl2, DoAndTrue

b : SearchConstDecl3, DoPlus

c : DoConstDecl

$$\text{(ii)} \quad \frac{\frac{v_0 = \text{toNum}(2) + \text{toNum}(1)}{\{\emptyset\} \vdash 2 + 1 \rightarrow 3} \text{DoPlus} \quad \frac{\text{DoConstDecl} \quad \frac{v_1 = \text{toNum}(x) * \text{toNum}(0)}{\{v_0 = 3, x = 3\} \vdash x * 0 \rightarrow 0} \text{DoTimes} \quad \frac{\text{toBool}(v_1) = \text{false}}{\{v_0 = 3, x = 3, v_1 = 0\} \vdash ?x : x + x \rightarrow x + x} \text{DoIfElse} \quad v_2 = \text{toNum}(3) + \text{toNum}(3)}{\{v_0 = 3, x = 3, v_1 = 0\} \vdash x + x \rightarrow v_2} \text{DoPlus}$$

$\text{const } x = \underline{2} + 1; x * 0 ? x : x + x \xrightarrow{a} \text{const } x = 3; \underline{x} * 0 ? x : x + x \xrightarrow{c} \text{const } x = 3; \underline{0} ? x : x + x \xrightarrow{d} \text{const } x =$
 $3; \underline{x + x} \xrightarrow{e} 6$

a: SearchConstDecl1, DoPlus

b: SearchConstDecl2, DoVar

c: SearchIf

d: SearchConstDecl3, DoPlus

e: DoConstDecl

(c)

(d)

(e)

2 Problem 2

(a)

$$e_1 = (3 * y) + 4$$

(b)

$$e_1 = (x * y) + 4$$

(c)

$$e_2 = \text{const } y = y; 3 + y$$

(d)

$$e_2 = \text{const } y = 2; x + y$$

(e)

$$e_3 = \text{const } x = (\text{function}(y)(x(y))); x(y(2))$$

(f)

$$e_3 = \text{const } x = (\text{function}(y)((y(x))(y))); x(y)$$