Principles of Programming Languages - Homework 5

Abhi Agarwal

1 Problem 1

(a)

(i) $$\dfrac{\dfrac{}{\{x\to3,y\to-2\}\vdash3\Downarrow3}\,EvalVal \quad \dfrac{x\in dom(\{x\to3,y\to-2\})}{\{x\to3,y\to-2\}\vdash x\Downarrow3}\,EvalVar}{\{x\to3,y\to-2\}\vdash x*3\Downarrow9}\,EvalTimes \quad \dfrac{}{(\{x\to3,y\to-2\})\vdash2\Downarrow2}\,EvalVal}{\{x\to3,y\to-2\}\vdash3*x+2\Downarrow11}\,EvalPlus$$

(ii) $$\dfrac{\dfrac{\dfrac{}{\{x\to3,y\to-2\}\vdash2\Downarrow2}\,EvalVal \quad \dfrac{y\in dom(\{x\to3,y\to-2\})}{\{x\to3,y\to-2\}\vdash y\Downarrow-2}\,EvalVar}{\{x\to3,y\to-2\}\vdash2+y\Downarrow0}\,EvalPlus}{\{x\to3,y\to-2\}\vdash constb=0\Downarrow0}\,EvalConstDecl\ false{=}toBool(0) \quad \dfrac{y\in dom(\{x\to3,y\to-2\})}{\{x\to3,y\to-2\}\vdash y\Downarrow-2}\,EvalVar}{\{x\to3,y\to-2\}\vdash b?x{:}y\Downarrow-2}\,EvalIfThen$$

(b)

(i) $$\dfrac{\dfrac{}{\{\varnothing\}\vdash1\&\&5\to5}\,toBool(1){=}true\ DoAndTrue \quad v{=}toNum(3){+}toNum(5)}{\{\varnothing\}\vdash3+(1\&\&5)\to v}\,DoPlus$$

$3 + \underline{(1 \;\&\&\; 5)} \xrightarrow{a} \underline{3 + (1 \;\&\&\; 5)} \xrightarrow{b} 3 + 5 \xrightarrow{c} 8$
a: SearchConstDecl2, DoAndTrue
b: SearchConstDecl3, DoPlus
c: DoConstDecl

(ii) $$\dfrac{\dfrac{\dfrac{}{\{\varnothing\}\vdash2+1\to3}\,v_0{=}toNum(2){+}toNum(1)\ DoPlus \quad \dfrac{}{\{v_0{=}3\}\vdash const\ x{=}3}\,DoConstDecl \quad \dfrac{v_1{=}toNum(x){*}toNum(0)}{\{v_0{=}3,x{=}3\}\vdash x*0\to0}\,DoTimes}{\{v_0{=}3,x{=}3\}\vdash x{+}x\to x{+}x}\,DoIfElse\ v_2{=}toNum(3){+}toNum(3)}{\{v_0{=}3,x{=}3,v_1{=}0\}\vdash x{+}x\to v_2}\,DoPlu$$

const x = $\underline{2 + 1}$; x * 0 ? x : x + x $\xrightarrow{a}$ const x = 3; $\underline{\text{x}}$ * 0 ? x : x + x $\xrightarrow{c}$ const x = 3; $\underline{0}$? x : x + x $\xrightarrow{d}$ const x = 3; $\underline{\text{x + x}}$ $\xrightarrow{e}$ 6

a: SearchConstDecl1, DoPlus
b: SearchConstDecl2, DoVar
c: SearchIf
d: SearchConstDecl3, DoPlus
e: DoConstDecl

(c)

SearchPlus1 $\frac{env \vdash e_2 \to e_2'}{env \vdash e_1 + e_2 \to e_1 + e_2'}$
SearchPlus2 $\frac{env \vdash e_1 \to e_1'}{env \vdash e_1 + v_2 \to e_1' + v_2}$
DoPlus $\frac{v = toNum(v_1) + toNum(v_2)}{env \vdash v_1 + v_2 = v}$
Order for right-to-left evaluation: SearchPlus1, SearchPlus2, DoPlus

(d)

Big-step SOS $\frac{env \vdash e_1 \to v_1 \;\; env \vdash e_2 \to v_2 \;\; v = v_2}{env \vdash e_1, e_2 \Downarrow v}$
Small-step SOS
SearchComma1 $\frac{env \vdash e_1 \to e_1'}{env \vdash e_1, e_2 \to e_1', e_2}$
SearchComma2 $\frac{env \vdash e_2' \to e_2'}{env \vdash v_1, e_2 \to v_1, e_2'}$
DoReturnValue $\frac{v = v_2}{env \vdash v_1, v_2 = v}$
Order for right-to-left evaluation: SearchComma1, SearchComma2, DoReturnValue

(e)

(i) This program evaluates to 5. During evaluation, the EvalVar is applied three times as follows:

The first application is the using occurrence of y in the definition of the function f on line 3. In this case, y was bound to the variable 3 in the call to f on line 4.

The second application is the occurrence of x in the definition of the function g on line 2. x is bound to the value 2 in the constant declaration of 2 on line 1.

The third application is for the occurrence of y in the definition of the function g on line 2. This occurrence of y was bound to the value 3 in the call to g on line 3.

(ii) This program evaluates to 6. During evaluation, the EvalVar rule is applied four times as follows:

The first application is the using occurrence of y in the definition of function f on line 3. In this case, y was bound to the value 3 in the call to f on line 4.

The second application is the using occurrence of x in the definition of the function g on line 2. This occurrence was bound to the value 3 in the call to g on line 3.

The third application is the using occurrence of y in the definition of the function g on line 2. This occurrence was bound to the value that will be passed into the function by the function call in line 3. The function call to g returns a function, and that function is called on line 3 using the value 3. So the occurrence was bound to the value 3 in the call to the anonymous function on line 3.

The fourth application is the using occurrence of y in the definition of function f on line 3. This occurrence was bound to the value 3 in the call to the anonymous function that is returned by calling g on line 3.

2 Problem 2

(a)

$$e_1 = (3 * y) + 4$$

(b)

$$e_1 = (x * y) + 4$$

(c)

$e_2 = const\, y = y;\, 3 + y$

(d)

$e_2 = const\, y = 3; x + y$

(e)

(f)