

# Principles of Programming Languages - Homework 8

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## 1 Problem 1

(a)

(i): The expression is well typed. It will evaluate to a type number.

(ii): The expression is not well typed. You cannot add a Number to a Boolean. This happens on Line 2. This is dictated by the TypeArith rule.

(iii): The expression is well typed. It will evaluate to a type number  $\Rightarrow$  number.

(iv): The expression is not well typed. Line 4 doesn't follow the TypeEqual rule. Line 4 tries to test equality of two functions.

(v): The expression is not well typed. The anonymous function does not define a return type. This is dictated by the TypeFunctionRec rule. However, if  $x > y$  then this will return a Number, if  $x < y$  then it will never return (infinite loop).

(b)

(i)

$t_1$ : number  $\Rightarrow$  number

$t_2$ : number  $\Rightarrow$  number

$t_3$ : number

$f$ : (number  $\Rightarrow$  number)  $\Rightarrow$  (number  $\Rightarrow$  number)

(ii) Kind of like the opposite of (i). Since  $t_3$  can be anything, which implies that  $t_1$  can be anything, which implies that  $t_2$  can be anything.

$t_1 : t_3$ : Any

$t_2 : t_3$ : Any

$t_3$ : Any

Let's take an example of  $t_3$  being Boolean. Then  $f$ : (boolean  $\Rightarrow$  boolean)  $\Rightarrow$  (boolean  $\Rightarrow$  boolean). This is a universal rule to this program.

(iii) There exist no concrete types for the mission parameter types such that the given program is well-typed according to the rules. For it to be well-typed  $t_2$  has to be both a Boolean and a Number as it needs to satisfy both  $f(g)(\text{true})$  and  $f(h)(1)$ . This isn't possible so it isn't possible for it to be well-typed.