Who Pays? Inefficiencies Arising from Pressure in Joint Liability Lending Microfinance Programs

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"... top-down repayment pressure can lead to forms of borrower discipline which are unnecessarily exclusionary, and which can contradict the broader (social) aims of solidarity group lending"

— Montgomery

"... despite the promise of microcredit as a development tool aimed at helping borrowers escape poverty, there was a general consensus among high-level staff that credit in and of itself was not sufficient to achieve Fundacion Paraguaya's mission to alleviate poverty."

— Caroline E. Schuster

Abstract

This paper presents a game theoretic model of Joint Liability Lending (JLL) microfinance programs with endogenous peer pressure to repay. In addition, this paper describes a role for institutional pressure applied by microfinance institutions. We find that this model helps better explain two important empirical findings in the literature. Firstly, observed repayment rates in not for profit microfinance programs are very high. Secondly, the welfare implications of these programs (as evidenced by RCTs) are small. A sequential game is analyzed where the MFI's interest rate, the projects selected by the group members and the subsequent peer pressure and repayment decision are endogenized. I also characterize the solutions and analyze the outcomes computed over a large range of parameter values.

The most striking intuition generated by the model is that when (risk-averse) participants can choose between low risk-low reward and high risk-high reward investments, and the MFI prefers to set low interest rates, the resulting equilibrium boasts inefficiently high repayment rates. This leads to an inefficient transfer of the burden of risk bearing onto the participants who respond by inefficiently choosing low risk-low reward investments. Thus, counter to the main purpose of these programs of poverty alleviation, this implies that growth generating investments (high risk-high reward) are left under funded in equilibrium. Thus, the model provides a more satisfactory explanation of some of the empirical findings in this literature.

1 Introduction

Joint Liability Lending (JLL) is a common practice in microfinance programs where participating households arrange into groups to receive a joint liability loan. In such a program, the obligation of repaying the loan falls collectively on the group and not on individual members. If the group fails to repay, the microfinance institution refuses to offer the members of the group a loan thereafter. Thus, if some recipients of the loan are unable to meet their repayment obligations, other participants are incentivized to cover for them. This paper focuses on microfinance institutions (MFIs) that operate as non-profits (or NGOs) and not on commercial microfinance programs. Grameen MFI, BRAC and the early days of SKS are all examples of microfinance programs that operate as non-profits.

The main aim of this paper is to understand the role of top down pressure in encouraging repayment and project (investment) selection in microfinance programs. Top down pressure captures the two types of pressure to repay thought to be in action in these programs: pressure to repay from the MFI, and endogenous peer pressure by group members. In this paper, peer pressure is a negative pressure applied by a participant of the program to incentivize her fellow group member to meet its share of repayment. As a concept, this is often brought up in case studies (see leading head-quote for an example) and accepted by economists but loosely considered within theoretical models. When considered, it is often collected under the broad umbrella of social collateral and assumed exogenous to the model. In this paper, I seek to endogenize this. Pressure from the MFI to encourage repayment on the other hand is a direct result of the relationship between the MFI and its costumers. At least in its early days, MFIs spent resources on developing relationships with costumers in villages. MFIs often leveraged these relationships to discourage strategic defaults. Like the idea of peer pressure, this relationship building aspect of microfinance though recognized, fails to get much attention in theoretical models of microfinance.

The role of top-down pressure is particularly interesting in light of two important empirical findings of JLL microfinance. Firstly, repayment rates are predominantly high with most instances of microfinance resulting in very high repayment rates (between 89% and 98%) and some select examples (Andhra Pradesh) where repayment rates plummeted to around 5% when the program was commercialized (operate *for* profit). Secondly, evidence from randomized control trials suggests an insignificant average effect of these programs on a variety of welfare indicators such as access to nutrition, health and sanitation facilities. Within the purview of the existing models, these empirical findings seem at odds with each other. This is because the existing literature rationalizes the high repayment rates through an efficient allocation of resources. The lack of welfare improvements generated seems to suggest that even an efficient alleviation of credit constraints does not help these local economies grow. This is at odds with the law of diminishing returns to capital, a staple feature of concave production technologies.

The model I present here allows households to apply (costly) *peer pressure* to facilitate *good repayment behavior*. Here, good repayment is a norm that specifies how much each member of a group repays in different states of the world while peer pressure denotes the threat of severing a social tie across which households share resources in a mutually beneficial way. The model also details the strategic choices of the MFI. I argue based on written comments and statements from MFIs that they invest in building relationships with their clients. This relationship allows them some control over the repayment decisions of the members. In my model, I suggest that the relationships enable MFIs to *select* pressure and repayment equilibrium in the event of multiple equilibrium. Finally, I am also able to describe and solve the social planner's problem in this environment, allowing me to perform welfare analysis.

The most striking intuition generated by the model is that when (risk-averse) participants can choose between low risk-low reward and high risk-high reward investments, and the MFI prefers to set low interest rates, the resulting equilibrium boasts inefficiently high repayment rates. This leads to an inefficient transfer of the burden of risk bearing onto the participants who respond by inefficiently choosing low risk-low reward investments. This is detrimental to the economy since it implies that growth generating investments (high risk-high reward) are left under funded in equilibrium. Thus, this model provides an alternate explanation of the empirical findings that is not at odds with the law of diminishing returns.

2 Literature

Formal analysis of microfinance begins with Stiglitz (1990) and Varian (1990), which talks about the role of group lending in mitigating the moral hazard problem. This has forged a literature that explores the role of joint liability in harnessing (exogenous) peer monitoring to solve problems of adverse selection and moral hazard often associated in providing loans without collateral. Examples of this include Ghatak (1999) and Ghatak, Guinnane (1999). Much of this work suggests that the social collateral possessed by members of the community effectively replaces the need for financial collateral in MFIs. Much of this literature sets up economically rational agents that always (exogenously) exert their influence on peers to ensure repayments who in kind repay when it is rational to do so. Besley, Coate (1995) is a notable exception which shows how group lending could sometimes lead to lower repayment rates than individual lending when repayment decisions are strategically chosen. This paper follows suit and goes one step further to endogenize the peer influence and successive repayment decision as stages of a sequential game.

An empirical literature on microfinance explores the welfare effects of microfinance loans on consumers. Karlan, Zinman (2011) show positive effects on employment and start-ups in microfinance programs in South Africa while Banerjee, Duflo, Glennerster, Kinnan (2015) find no effect of welfare measures (like consumption, health, access to sanitation, etc) apart from at the top 5% of the income distribution. Finally, Crepon, Devoto, Duflo, Pariente (2015) suggests positive effect on business profits but no increase in consumption (a useful benchmark for welfare, particularly in poor communities). There is also a literature on repayment rates in joint liability programs. For example, Morduch (1999b) finds that repayment rates are usually between 89% and 93%, while Cull, Demirgüç-Kunt, Morduch (2009) also confirm high repayment rates $\sim 98\%$. This is also indicative of some heterogeneity in the repayment rates. Finally, Ahlin, Townsend (2007) uses network data to relate the repayment rates to the strength of local sanctions.

More recently, many microfinance lenders have switched away from group lending spurring further research on the matter. Gine, Karlan (2014) conduct an RCT to demonstrate the repayment rates do not fall when groups are chosen randomly instead of being self selected. Ghatak (2016) compares the welfare implications of individual and joint liability lending programs in the light of the recent shift away from joint liability lending. Ghatak (2018) explores sustainability of MFIs when they are commercialized.

3 Context

Micro-finance programs are typically targeted towards sections of the society with low income and hence large credit constraints. As a consequence, participants in micro-finance programs often have little to no collateral to offer MFIs in exchange for loans. In the absence of any collateral, the MFI fearing the low repayment rate (a consequence of insufficient incentives to repay under limited liability) set interest rates prohibitively high. Joint liability lending is a popular mechanism used by micro-finance lenders to incentivize repayments among participants. Since participating households must organize into groups and continued access to loans is contingent on repayment by *all* group members, micro-finance programs effectively encourage participants to sanction each other to induce repayment. Thus, these programs are able to leverage informal ties between group members to force repayment in the absence of formal incentives to repay. In what follows, I specify the economic narrative that is formalized in section 4.

As is traditional within this literature, I model the canonical group with two participating households. Participants are aware that if the group does not repay, the MFI refuses to offer the group (or any of its members) a loan thereafter. On the other hand, if the group has successfully repaid, participants within the group have the option of borrowing again in the future from the same program.

Consider that a $1 loan being made available to each participating household. Consistent with the literature, participating households are assumed to invest this loan in a known *investment technology* which I model as a simple lottery. In later sections, I allow participants to choose between different investment technologies. I assume throughout this paper that the realizations of the investment technology across participants are independent events. Thus, realizations of these investments will be purely exogenous in this section. Let us denote these investment realizations as states λ. Further, I assume that in the tightly knit communities where such programs are made available, investment outcomes of all participating households are common knowledge. Each participant values continued access to the program. Let $\phi(r)$ denote the temporally discounted option value of having access to this source of credit, where r is the interest rate at which the loan is made available.

In the model I propose that applying social pressure involves two components:

- an *exogenous* and *á priori* known to all *social repayment norm* that prescribes what each group member pays.
- a mechanism by which participants can *pressure* their fellow group members to follow the social repayment norm.

The *social repayment norm* prescribes for every realization of investment outcomes λ, the fraction of the total group loan that each participant is required to pay in that state $\tilde{x}(\lambda)$. A simplifying assumption made is that this social repayment structure is a consensus reached by the society that is exogenous to this model. Although it is indeed a consensus, observe that given a realization of λ, participants need not find it optimal to pay the suggested sum. Thus, either participant may still choose not to follow this social repayment norm. Given the implications that one participant's choice has on the other participant, participants are allowed to exert pressure on each other. To my knowledge, this is the first paper to endogenize the decision to apply pressure. This pressure might incentivize each other to follow the social repayment norm. The story I am trying to capture here is of applying pressure by refusing to exchange goods or services. In an agrarian economy, different households may own different shared [1] tools of farming such as ploughs, bullocks, shovels or carts as well as consumption goods like kerosene and other rations. Since groups are self-selected by participants, group members engage in such sharing of resources, i.e. share a *social tie*. When a household exerts *peer pressure* on another, it may prohibit the non-cooperating participant from sharing tools, which effectively places a cost on the non-repaying participant that now

[1] The term 'shared' here is not being used to suggest shared ownership. The reader may think of this as freely lent resources.

has to make alternative arrangements for ploughs and kerosene. In exerting this pressure, participants applying the pressure must entail a cost since the household applying the pressure may now have to purchases a lockable shed to store its shovels/carts in. Additionally, participants applying the pressure must also be prepared for retaliation in kind by the other participant. Viewed in this light, Joint Liability Loans are merely formal arrangements that bootstraps from the pre-existing informal arrangements within communities to facilitate high repayments.

In practice, microfinance programs (at least in its early days) were built with an emphasis on the relationship between clients and the MFI. Bankers would spend time in developing these relations and often leveraged these relations to discourage strategic non-repayment of loans. This ensured that the MFIs were able to keep interest rates low which was thought to help make these loans more widely accessible to the most credit constrained of borrowers.

4 Game played within microfinance groups

Let A and B denote the two households participating in a microfinance group. The groups form such that members of the group share a social tie. This paper considers the homogenous setting where each household is endowed with wealth level w. Each participating household receives a \$1 investment at gross interest rate r and chooses to invest their loan in an (independent across households) personal investment technology which is modeled as a lottery with probability law:

$$\text{return} = \begin{cases} R, \text{ with probability } \mu \\ 0, \text{ with probability } 1-\mu \end{cases}$$

where $0 < \mu < 1$ and $R > 0$. I suppose that the investment technology of participants is public knowledge. In a later section, I allow for households to choose from two different investment technologies.

The investment outcomes that are realized are denoted by vector $\lambda = (\lambda_i)_{i\in\{A,B\}} \in \{0,1\}^2$ where $\lambda_i = 1$ denotes return of R while $\lambda_i = 0$ denotes return of 0. Once this has been realized (which is assumed to be perfectly observed by all households in the group), then each household faces a repayment decision. In this paper, I model the repayment decision as a choice of either paying the *social repayment norm* suggested amount, $\tilde{x}_i(\lambda)$ or not. The *social repayment norm* is simply an exogenous, ex-ante known and agreed upon social norm which suggests how much each household in a group would contribute towards the repayment of the loan taking into account the realizations of investment outcomes. I suppose this particular exogenous norm:

$$\tilde{x}_i(\lambda) = \begin{cases} \theta, \text{ if } \lambda_i = 0 \\ 1 + (1 - \lambda_j) \cdot (1 - \theta), \text{ if } \lambda_i = 1 \end{cases}$$

where $\theta \in (0, 0.5]$ is the share of loan a household whose investment has failed is expected to repay and $j = \{A, B\} \setminus i$.

This current specification of the norm suggests that each failed node ($\lambda_i = 0$) pays a fraction θ of what it owes. A successful node ($\lambda_i = 1$) on the other hand would pick up the slack if its group member has failed in addition to paying its own dues. Consequently, one observes that if both participants fail in their investments, even when all participants follow the norm, full repayment does not result.

As discussed above, the salient feature of joint liability lending programs is that future loans are conditional on group repayment. More formally, if the group as a whole successfully repays, households may continue to borrow from this lender. Denote by $\phi(r)$ the net present value of this credit line. Thus, the utility for household i in state λ as a function of households' repayment decisions, $d^\lambda = (d_i^\lambda)_{i \in \{A,B\}}$: for all $i \in \{A, B\}$,

$$U_i(d^\lambda; \lambda) = u(w + R_i \cdot \lambda_i - r \cdot \tilde{x}_i(\lambda) \cdot d_i^\lambda) + \phi(r) \cdot 1\Big\{d_A^\lambda \cdot \tilde{x}_A(\lambda) + d_B^\lambda \cdot \tilde{x}_B(\lambda) = 2\Big\}$$

where $u(.)$ is a known felicity function that is increasing and concave. In the illustrations to follow, I suppose that this felicity function is a member of the CARA family.

As evident from the equation above, each household no longer decides how much it will contribute towards repayment but only whether or not it would follow the social repayment norm. Hence, failing to follow the norm corresponds to paying *nothing*.

A novel feature in this model allows households to endogenously exert peer pressure on their fellow group members to ensure repayment of the group loan. I refer to the decision to apply peer pressure as *stage 1* and the consequent repayment decision as *stage 2*. Denote the pressure participant i exerts by δ_i. The decision to apply pressure is binary. i.e. $\delta_i \in$

$\{0,1\}$, $\forall i \in \{A,B\}$. So, in *stage 1*, households simultaneously choose whether or not to pressure their fellow group members having observed the investment realization λ. The decision to apply pressure is associated with a padlock cost (eg. cost of buying a padlock or the emotional cost of issuing a stern warning to a friend) which is denoted by η. Further, applying pressure is also associated with a contingent cost (γ) of a damaged or severed social tie in the event that the fellow group member fails to comply with the social repayment norm. This second cost encodes the reciprocal non-sharing of resources discussed in the previous section. Since γ is a cost arising from a severed or damaged social tie, it is only natural that both: the household applying the pressure as well as the household on which pressure is applied and fails to repay - face this cost. It should be noted here that I assume a full commitment punishment mechanism where the decision to apply pressure in stage 1 corresponds to households also committing to punish the group member if it deviates from the norm. One can explicitly write out this decision to follow through on the threat of punishment as the third stage. Call this stage the *sharing* game and suppose the stage game payoffs are as described below:

Table 1: 2x2 Matrix: Normal form representation of sharing stage game

		HH 2	
		Share	Don't Share
HH 1	Share	$0,0$	w,z
	Don't Share	z,w	$-\gamma,-\gamma$

This game has two equilibria (Share, Share) and (Don't Share, Don't Share) such that the former Pareto dominates the latter. To be sure, assumptions are required on x, y, z to make it so. Specifically:

$$0 > z > -\gamma > w$$

This assumption is not without loss of generality. One way to justify this would be by *fairness minded preferences*. That is, households prefer *to do unto others as others do unto them*. The *threat* of punishment can now be thought of as committing to playing the Pareto dominated equilibrium (Don't Share, Don't Share). This is essentially a hidden third stage of the game played between members of the group.

I now tie this together with the stage 1 decision of exerting peer pressure and the stage 2 decision of repayment. In stage 2, having observed investment outcomes λ, and stage 1 pressures δ, households now choose whether or not to pay the *social repayment norm* $\tilde{x}$ suggested amount.

The resulting utility to member $i \in \{A, B\}$ in state λ given pressure vector δ^λ and repayment vector d^λ:

$$U_i(\delta^\lambda, d^\lambda; \lambda) = u\big(w + R\lambda_i - r d_i^\lambda \tilde{x}_i(\lambda)\big) + \phi(r) \cdot 1\Big\{d_A^\lambda \cdot \tilde{x}_A(\lambda) + d_B^\lambda \cdot \tilde{x}_B(\lambda) = 2\Big\}$$
$$- \gamma \max\big\{\delta_i^\lambda(1 - d_j^\lambda), \delta_j(1 - d_i^\lambda)\big\} - \eta\delta_i^\lambda$$

The third term accounts for the loss of utility to household i arising from a severed social tie with household j in the even that either i applies pressure and j breaks norm or j applies pressure and i breaks norm.

4.1 Equilibrium Concept

Definition 1 (Repayment play)**.** *A* repayment play *for a given realization of investment outcomes, $\lambda \in \{0, 1\}^2$, is a (pure) strategy profile listing the repayment decision of each household in each sub-game that may result after stage 1 (pressure). This will be denoted by $d^\lambda(.) : \{0, 1\}^2 \to \{0, 1\}^2$, the superscript is ignored when convenient to enhance readability.*

Definition 2 (Strategy profile)**.** *A (pure) strategy profile in this game lists for each realization of investment outcomes, $\lambda \in \{0, 1\}^2$, a profile of pressure decisions, $\delta = (\delta_A^\lambda, \delta_B^\lambda) \in \{0, 1\}^2$, and a consequent* repayment play *that determines repayment decisions in every sub-game that may result after the pressure stage, $d^\lambda(.) : \{0, 1\}^2 \to \{0, 1\}^2$. σ denotes a strategy profile $\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda\big)$.*

From the above section, the payoff to every participant associated with each strategy profile is defined to be the expected utility to the participant:

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall\lambda\big) = \sum_\lambda \mathbb{P}(\lambda) \times U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big)$$

where

$$\begin{aligned} U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) = {} & u\big(w + R\lambda_i - r d_i^\lambda(\delta_A^\lambda, \delta_B^\lambda)\tilde{x}_i(\lambda)\big) \\ & + \phi(r) 1\Big\{ \sum_{j \in \{A,B\}} d_j^\lambda(\delta_A^\lambda, \delta_B^\lambda)\tilde{x}_j(\lambda) = M \Big\} \\ & - \gamma \max\Big\{ \delta_A^\lambda\big(1 - d_B^\lambda(\delta_A^\lambda, \delta_B^\lambda)\big), \delta_B^\lambda\big(1 - d_A^\lambda(\delta_A^\lambda, \delta_B^\lambda)\big) \Big\} \\ & - \eta \delta_i^\lambda \end{aligned}$$

Definition 3 (Sub-game perfect equilibrium). *A strategy profile,* $\big\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall \lambda \in \{0,1\}^2\big\}$ *is a sub-game perfect equilibrium if the following conditions hold:*

1. $\forall \lambda,\ i \in \{A, B\}$, *given* $\delta \in \{0,1\}^2$*:*

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) \geq U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^{\lambda\prime}(\delta_A^\lambda, \delta_B^\lambda); \lambda\big)$$

   for any alternate repayment play: $d^{\lambda\prime} : \{0,1\}^2 \to \{0,1\}^2$

2. $\forall \lambda,\ i \in \{A, B\}$, *given* $d^\lambda(.)$*:*

$$U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) \geq U_i\big(\delta_A^{\lambda\prime}, \delta_B^{\lambda\prime}, d^\lambda(\delta_A^{\lambda\prime}, \delta_B^{\lambda\prime}); \lambda\big)$$

Notice that the above restrictions need to hold for each realization of λ. In fact, owing to the simple and symmetric nature of the uncertainty in this context, a strategy $\big\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall \lambda \in \{0,1\}^2\big\}$ is a sub-game perfect equilibrium if and only if, for every realization of λ, the strategy $\big\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \big\}$ is sub-game perfect in the sub-game beginning at realization of investment outcomes λ. This simplifies the analysis of equilibria in this game.

Based on the structure set up, I now discuss an intuitive result that arises in any sub-game perfect equilibrium.

Proposition 4 (No pressure wasted). *Suppose there is a sub-game perfect equilibrium* $\sigma = \big\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall \lambda \in \{0,1\}^2\big\}$ *where for some* λ, $\delta^\lambda \neq (0,0)$, *then it must be that* $d_A^\lambda(\delta^\lambda)\tilde{x}_A(\lambda) + d_B^\lambda(\delta^\lambda)\tilde{x}_B(\lambda) = 2$ *i.e. full repayment occurs.*

It basically states that if any household applies pressure in an equilibrium, then full repayment results in than equilibrium. This arises from the strictly positive cost of applying pressure, $\eta > 0$.

Proof: Suppose to the contrary that the repayment is not full, then, $\sum_j d^\lambda(\delta^\lambda)_j \tilde{x}_j(\lambda) < 2$. Thus, both households in the group are penalized and are unable to participate in the program thereafter. Let i denote a household such that $\delta_i^\lambda = 1$. Exploring the profitability of a one shot deviation for node i.

Consider the utility to household i of applying pressure ($j = \{A, B\} \setminus i$):

$$U_i(\text{ applying pressure }, \sigma; \lambda) = u(w + R\lambda_i - r\tilde{x}_i(\lambda)d_i^\lambda) - \eta \\ - \gamma \max\left\{(1 - d_j^\lambda), \delta_j^\lambda(1 - d_i^\lambda)\right\}$$

Consider now its utility of not applying pressure:

$$U_i(\text{ no pressure }, \sigma; \lambda) = u(w + R\lambda_i - r\tilde{x}_i(\lambda)d_i^\lambda) \\ - \gamma\delta_j^\lambda(1 - d_i^\lambda)$$

Given $\eta > 0$, one can see that

$$U_i(\text{ no pressure }, \sigma; \lambda) > U_i(\text{ applying pressure }, \sigma; \lambda)$$

Hence there exists a profitable one-shot deviation for i implying that σ cannot be a sub-game perfect equilibrium so long as $\mathbb{P}(\lambda) > 0$. ■

Remark: This proof emplys the *one shot deviation principle*: a popular result that states that a strategy profile for finite extensive form game is SPE iff there exists no profitable deviation for every sub-game and for every player.

4.2 Further restriction on action set

Within the analysis, owing to the large number of possible sub-game equilibria[2], I propose additional restrictions on participant behavior that are arguably very reasonable. Formally:

[2] This is a consequence of the pressure stage since may combinations of pressure δ in the second stage can support a particular profile of repayments in the third stage.

Assumption 5 (Monotone repayment response to peer pressure).
In any sub-game perfect equilibrium $\sigma = \left\{\delta_A^\lambda, \delta_B^\lambda, d^\lambda(.) : \forall \lambda \in \{0,1\}^2\right\}$, $\forall \lambda \neq (0,0)$, *if* $\delta^\lambda > \delta^{\lambda\prime}$ *(greater in the vector sense) in any sub-game perfect equilibrium, then* $d^\lambda(\delta) \geq d^\lambda(\delta^{\lambda\prime})$ *in that equilibrium.*
Additionally, when $\lambda \neq (0,0)$*, the only equilibrium play is* $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$

The above assumption rules out what I consider strange repayment plays in equilibrium. Essentially, the first part rules out repayment play where when additional household applied pressure, at least one household that would have otherwise applied pressure doesn't. Consider the following behavior that would be consistent with sub-game perfection driven entirely by the complementarity of repayments:

In state $\lambda \neq (0,0)$:

$$\left\{\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (1,1), & \text{if } \delta = (0,0) \\ (0,0), & \text{if } \delta \neq (0,0) \end{cases} \right\}$$

Although this is optimal, as long as

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

While the model would regard this behavior as rational, I would like to rule out this kind of behavior. That is, if the participants select an equilibrium with full repayment without pressure, then the addition of pressure should not cause them to select an equilibrium with lower repayment.

The second part of the assumption basically states that in the event that the investments of both participants fail, the *unique* equilibrium play that would result is that neither households applies any pressure and neither pays its share of the norm. This assumption is essentially without loss of generality. First of all, note that when $\lambda = (0,0)$, $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$ is trivially a best response since even full compliance with the norm doesn't lead to full repayment, thus from the contrapositive of the above lemma, neither participant has any incentive to apply pressure. Further note that in every equilibrium in the sub-game starting at this state, there will be no repayment and no pressure on the equilibrium path.

Proposition 6. *When $\lambda = (0,0)$ one can assume $\{\delta_A^\lambda = \delta_B^\lambda = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\}$ is the only sub-game perfect equilibrium* without any loss of generality

Proof: When $\lambda = (0,0)$, notice that $\tilde{x}_A(\lambda) = \tilde{x}_B(\lambda) = 0$, i.e. the social repayment norm does not yield full repayment. In the parlance of the model, $\phi(r) \cdot 1\big\{d_A\tilde{x}_A(\lambda) + d_B\tilde{x}_B(\lambda) = 2\big\} = 0$. Thus, there is no incentive for households to apply any costly pressure $(\eta > 0)$ to force other households to follow the social repayment norm. An application of the contrapositive of the *proposition 4* yields that in any sub-game perfect equilibrium, no pressure is applied, the repayment and punishments observed are identical. ■

Denote sub-game perfect equilibria that satisfy assumption 5 by *admissible equilibria.*

4.2.1 Additional assumptions

Assumption 7 (CARA felicity function)**.** *Suppose that the felicity function for individuals is given by this constant absolute risk aversion function*

$$u(c) = \frac{1 - e^{-\alpha \cdot c}}{\alpha}$$

Definition 8 (Maximum equilibrium repayment rate $\bar{\pi}$)**.** *denotes the maximum repayment rate sustained in an equilibrium of the above described game. Let Σ denote the set of all equilibrium strategy profiles of the game.*

For any strategy profile $\sigma \in \Sigma$, denote the repayment rate implied by that strategy as $\pi(\sigma)$, i.e.

$$\pi(\sigma) = \sum_\lambda \mathbb{P}(\lambda) \times 1\big\{d_A^\lambda(\delta_A^\lambda, \delta_B^\lambda) + d_B^\lambda(\delta_A^\lambda, \delta_B^\lambda) = 2\big\}$$

Then,

$$\bar{\pi} = \max_{\sigma\in\Sigma} \pi(\sigma)$$

Assumption 9 (Setting γ and η)**.** *Set:*

$$\gamma = \frac{1}{2} \times u(w)$$

$$\eta = \epsilon \times \gamma$$

Assumption 10 (Efficient Threats)**.**

$$\gamma > \eta > 0$$

4.3 Setting $\phi(r)$

Recall that $\phi(r)$ denotes the net benefit to having access to microfinance loans. Denote by $\phi(r)_1$ the continuation value with microfinance and by $\phi(r)_0$ the continuation value without microfinance. Here $\beta \in (0,1)$ is the discount factor. Then, given the lottery, this system is characterized by the three equations:

$$\begin{aligned}\phi(r)_1 =\beta\Big\{&\mu^2\big[u(w+R-r)+\phi(r)_1\big]+\mu(1-\mu)\big[u(w+R-(2-\theta)r)+\phi(r)_1\big]\\&+(1-\mu)\mu\big[u(w-\theta r)+\phi(r)_1\big]+(1-\mu)^2\big[u(w)+\phi(r)_0\big]\Big\}\end{aligned} \tag{1}$$

$$\phi(r)_0 = \beta\Big[u(w)+\phi(r)_0\Big] \tag{2}$$

$$\phi(r) = \phi(r)_1 - \phi(r)_0 \tag{3}$$

Notice that the net benefit of having access to the microfinance loan is only available when the full repayment occurs, i.e. in every state except for $\lambda = (0,0)$.

The solution to this system is attained at:

$$\phi(r)_0 = \frac{\beta u(w)}{1-\beta} \tag{4}$$

$$\phi(r)_1 = \frac{\beta EU(norm) + \beta\phi(r)_0(1-\mu)^2}{1-\beta\mu(2-\mu)} \tag{5}$$

where

$$EU(norm) = \mu^2\big[u(w+R-r)\big]+\mu(1-\mu)\big[u(w+R-(2-\theta)r)\big]+(1-\mu)\mu\big[u(w-\theta r)\big]+(1-\mu)^2\big[u(w)\big] \tag{6}$$

and hence

$$\phi(r) = \frac{\beta EU(norm) + (\beta-1)\phi(r)_0}{1-\beta\mu(2-\mu)} \tag{7}$$

4.4 Characterization of Admissible equilibria

There are 4 states (investment outcome realizations) in this model: both succeed, both fail, A succeeds while B fails and vice versa. Further, in each state that there are 256 distinct *repayment plays* that agents may choose. The following lemma recognizes that only a small subset of *repayment plays* are in fact sub-game perfect. In the two person context, out of the 256 candidate *repayment plays*, only 36 can be sustained as a part of a sub-game perfect equilibrium, thereby reducing the complexity involved in computing the equilibrium. This is formalized in the lemma below. Since assumption 5, dictates the play that will arise when $\lambda = (0,0)$, the following results apply for all other states, i.e. $\lambda \neq (0,0)$:

Proposition 11 (Game implied restrictions)**.** *Given the game described above, for all* continuation profiles *supported in any sub-game perfect equilibrium in sub-games with* $\lambda \neq (0,0)$*:*

1. $d^{\lambda}(0,0) \in \{(0,0),\ (1,1)\}$
2. $d^{\lambda}(1,0) \neq (1,0)$ *likewise* $d^{\lambda}(0,1) \neq (0,1)$
3. $d^{\lambda}(1,1) \in \{(0,0),\ (1,1)\}$

Intuition: The first assertion in this lemma states that in any optimal continuation profile, partial repayment cannot occur when neither household applies any pressure, i.e. either both households repay or both do not. This is a direct consequence of the strategic complementarity in repayments. Notice that in the event of partial repayment, neither household will be allowed to borrow from the microfinance institution thereafter. Thus, the household that is paying is only loosing money (be repaying now) and now getting any additional utility from continued access to this resource. So, in the event that this household is not under pressure to comply with the repayment norm, it must simply choose not to repay. The second and third assertions are a little more subtle. Collectively, they suggests that the act of applying pressure will not induce the household applying pressure to repay without also inducing the other household to repay. This is because a household applying pressure faces the same (if not worse) incentives to repaying its share when it chooses to apply pressure. The worse incentives to repay come from the max $\{\delta_A^{\lambda}(1-d_B^{\lambda}), \delta_B^{\lambda}(1-d_A^{\lambda})\}$ term which essentially means that when household A applies pressure, and B still won't repay, pressure from B will be inconsequential since the link will severed regardless of household A's repayment decision.

Proof:

1. Suppose to the contrary that $d^{\lambda}(0,0) = (1,0)$ in some state of the world λ. Looking at the payoffs to household A implied by the game.

   When household A repays:

   $$U_A(\text{repay}) = u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) \cdot 0$$

   When household A deviates by not repaying:

   $$U_A(\text{deviate}) = u\big(w + R\lambda_A\big) + \phi(r) \cdot 0 - \gamma \cdot 0$$

   Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. This yields a contradiction in every λ. By a symmetric argument, one can establish a contradiction for $d^{\lambda}(0,0) = (0,1)$.

   So, this establish that for all $\lambda \neq (0,0)$, $d^{\lambda}(0,0) \in \{(0,0),\ (1,1)\}$.

   This reduces the number of candidate sub game optimal repayment plays to 128.

2. Suppose in some state $\lambda \neq (0,0)$ where households play a sub-game perfect repayment play with $d^{\lambda}(.)$. Now, consider the event where household A applies pressure; notice that for household A, utility of following social repayment norm is

   $$U_A(\text{repay}) = u\big(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) - \gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) - \gamma$$

   while the utility of deviating is

   $$U_A(\text{deviate}) = u\big(w + R\lambda_A\big) - \gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w + R\lambda_A\big) - \gamma$$

   Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. Thus, $d^{\lambda}(1,0) = (1,0)$ in not optimal.

   By a symmetric argument, one can establish a contradiction for $d^{\lambda}(0,1) = (0,1)$ is sub optimal in any $\lambda \neq (0,0)$.

   This further reduces the number of candidate sub game optimal repayment plays to 64.

3. Suppose in some state $\lambda \neq (0,0)$ where households play a sub-game perfect equilibrium with repayment play $d^\lambda(1,1) = (1,0)$. Now, consider the event where household A applies pressure, notice that for household A, utility of following social repayment norm is

$$U_A(\text{repay}) = u\big(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) - \gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) - \gamma$$

while the utility of deviating is

$$U_A(\text{deviate}) = u\big(w+R\lambda_A\big) - \gamma \max\{\delta_A(1-d_B), \delta_B(1-d_A)\} = u\big(w+R\lambda_A\big) - \gamma$$

Here, $U_A(\text{repay}) < U_A(\text{deviate})$ since the repayment norm specified $x_A(\lambda) > 0$ in any $\lambda \neq (0,0)$ and $r > 1$. Thus, as long as at least one household succeeds, household A would strictly prefer not repaying. Thus, $d^\lambda(1,1) = (1,0)$ in not optimal.

By a symmetric argument, one can establish a contradiction for $d^\lambda(1,1) = (0,1)$ is sub optimal in any $\lambda \neq (0,0)$.

This further reduces the number of candidate sub game optimal repayment plays to 36.

■

Proposition 12 (Restrictions)**.** *Under assumption 5, for all $\lambda \neq (0,0)$, the number of optimal repayment plays reduces from 36 to 11.*

This can be verified by running through all the 36 options and checking if they satisfy assumption 5. In the interest of brevity, this is omitted form the paper. However, I provide a list of the 11 sub-game perfect consistent *repayment plays* that satisfy the assumption.

- When $\lambda \neq (0,0)$:

  1. $$d^\lambda(\delta) = (0,0), \text{ for all } \delta$$

  2. $$d^\lambda(\delta) = (1,1), \text{ for all } \delta$$

  3. $$d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$$

4.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$$

5.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B < 2 \\ (1,1), \text{ if } \delta_A + \delta_B = 2 \end{cases}$$

6.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0 \\ (0,1), \text{ if } \delta_A = 1, \ \delta_B = 0 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$

7.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1, \ \delta_B = 1 \end{cases}$$

8.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1, \ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$$

9.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \ \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \ \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$$

10.
$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B = 0 \\ (1,1), \text{ if } \delta_A + \delta_B \geq 1 \end{cases}$$

11.

$$d^{\lambda}(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$

- When $\lambda = (0,0)$, restrict attention to $d^{\lambda}(\delta) = (0,0),\ \forall \delta$.

Notice that restriction made in the sub-game with $\lambda = (0,0)$ is shown to be without loss in generality in proposition 11. For sub-games with $\lambda \neq (0,0)$, any of the 11 possible *repayment plays* listed above could be chosen. Consequently, there are a large number of possible sub-gam equilibrium profiles. In what follows, I solve for the 'on equilibrium path' actions implied by the equilibria by backward-induction.

4.4.1 Stage 2 best responses

Now, restricting attention to these 11 repayment plays, consider the set of sub-game perfect equilibria that emerge and how that varies over the parameter space. Given the assumption that deals with optimal play in sub-games with $\lambda = (0,0)$, the rest of the analysis of the equilibrium will focus on dealing with sub-games where $\lambda \neq (0,0)$. For households $i \in \{A, B\}$ and $j \in \{A, B\} - i$, let $d_i(\delta_i, \delta_j, d_j)$ denote the best response by household i given δ_i, δ_j and d_j which also depends on λ but has been suppressed in notation to enhance readability:

1. $d_i(\delta_j = d_j = 0) =?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) - \gamma\delta_i - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma\delta_i - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \quad &< \quad u\big(w + R\lambda_i\big) \end{aligned}$$

since $r\tilde{x}_i(\lambda) > 0$ for all $\lambda \neq (0,0)$. Thus,

$$d_i(\delta_j = d_j = 0) = 0, \text{ for all parameter values}$$

2. $d_i(\delta_j = 0,\ d_j = 1) = ?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) \end{aligned}$$

Thus,

$$d_i(\delta_j = 0,\ d_j = 1) = \begin{cases} 1, \ \text{if } \phi(r) \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, \ \text{if } \phi(r) < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$$

3. $d_i(\delta_j = 1,\ d_j = 0) = ?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) - \gamma\delta_i - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma - \eta\delta_i \\ (1 - \delta_i)\gamma \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{aligned}$$

This depends on whether or not node i is applying pressure and thus can be split into two cases:

(a) $d_i(\delta_i = 0,\ \delta_j = 1,\ d_j = 0) = \begin{cases} 1, \ \text{if } \gamma \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, \ \text{if } \gamma < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$

(b) $d_i(\delta_i = 1,\ \delta_j = 1,\ d_j = 0) = 0,$ for all parameter values

4. $d_i(\delta_j = 1,\ d_j = 1) = ?$

$$\begin{aligned} d_i = 1 \quad &\underline{\text{vs}} \quad d_i = 0 \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) - \eta\delta_i \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma - \eta\delta_i \\ u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - \gamma \\ \gamma + \phi(r) \quad &\underline{\text{vs}} \quad u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{aligned}$$

Thus,

$$d_i(\delta_j = 1,\ d_j = 1) = \begin{cases} 1, \ \text{if } \gamma + \phi(r) \geq u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \\ 0, \ \text{if } \gamma + \phi(r) < u\big(w + R\lambda_i\big) - u\big(w + R\lambda_i - r\tilde{x}_i(\lambda)\big) \end{cases}$$

This is now used to find parametric ranges where each of the 11 repayment plays are supported for different realizations of $\lambda \neq (0,0)$.

1. $$d^{\lambda}(\delta) = (0,0), \text{ for all } \delta$$

   This is supported when:

   $$\gamma < u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
   $$\gamma < u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

2. $$d^{\lambda}(\delta) = (1,1), \text{ for all } \delta$$

   This is supported when:

   $$\phi(r) \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
   $$\phi(r) \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

3. $$d^{\lambda}(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$$

   This is supported when:

   $$\phi(r) + \gamma \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
   $$\phi(r) \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

4. $$d^{\lambda}(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}$$

   This is supported when:

   $$\phi(r) \geq u\big(w + R\lambda_A) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
   $$\phi(r) + \gamma \geq u\big(w + R\lambda_B) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

5.

$$d^{\lambda}(\delta)=\begin{cases}(0,0), \text{ if } \delta_A+\delta_B<2\\(1,1), \text{ if } \delta_A+\delta_B=2\end{cases}$$

This is supported when:

$$\phi(r)+\gamma \geq u\big(w+R\lambda_A)-u(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)>\gamma$$
$$\phi(r)+\gamma \geq u\big(w+R\lambda_B)-u(w+R\lambda_B-r\tilde{x}_B(\lambda)\big)>\gamma$$

6.

$$d^{\lambda}(\delta)=\begin{cases}(0,0), \text{ if } \delta_A=0\\(0,1), \text{ if } \delta_A=1,\ \delta_B=0\\(1,1), \text{ if } \delta_A=1, \delta_B=1\end{cases}$$

This is supported when:

$$\phi(r)+\gamma \geq u\big(w+R\lambda_A)-u(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)>\max\{\phi(r),\gamma\}$$
$$\gamma \geq u\big(w+R\lambda_B)-u(w+R\lambda_B-r\tilde{x}_B(\lambda)\big)$$

7.

$$d^{\lambda}(\delta)=\begin{cases}(0,0), \text{ if } \delta_B=0\\(1,0), \text{ if } \delta_B=1,\ \delta_A=0\\(1,1), \text{ if } \delta_A=1,\ \delta_B=1\end{cases}$$

This is supported when:

$$\gamma \geq u\big(w+R\lambda_A)-u(w+R\lambda_A-r\tilde{x}_A(\lambda)\big)$$
$$\phi(r)+\gamma \geq u\big(w+R\lambda_B)-u(w+R\lambda_B-r\tilde{x}_B(\lambda)\big)>\max\{\phi(r),\gamma\}$$

8.

$$d^{\lambda}(\delta)=\begin{cases}(0,0), \text{ if } \delta_A=0,\ \delta_B=0\\(1,0), \text{ if } \delta_B=1,\ \delta_A=0\\(1,1), \text{ if } \delta_A=1\end{cases}$$

This is supported when:

$$\min\{\phi(r),\gamma\} \geq u\big(w+R\lambda_A) - u(w+R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r)+\gamma \geq u\big(w+R\lambda_B) - u(w+R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

9.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0,\ \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), \text{ if } \delta_B = 1 \end{cases}$$

This is supported when:

$$\phi(r)+\gamma \geq u\big(w+R\lambda_A) - u(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\min\{\phi(r),\gamma\} \geq u\big(w+R\lambda_B) - u(w+R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

10.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B = 0 \\ (1,1), \text{ if } \delta_A + \delta_B \geq 1 \end{cases}$$

This is supported when:

$$\phi(r) \geq u\big(w+R\lambda_A) - u(w+R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w+R\lambda_B) - u(w+R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

11.

$$d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$$

This is supported when:

$$\gamma \geq u\big(w+R\lambda_A) - u(w+R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\gamma \geq u\big(w+R\lambda_B) - u(w+R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

Notice also that when $\lambda = (0,0)$, $d^\lambda(\delta) = (0,0)$, $\forall\delta$ is supported on all parameter values since the norm does not induce repayment.

4.4.2 Stage 1 best responses

The first stage best response depends on the chosen sub-game perfect repayment play. Here, the best responses under the 11 different repayment plays are explored.

1. If $d^\lambda(\delta) = (0,0),\ \forall\delta$ is the repayment play:

   For A:

   $$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
   $$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big) - \gamma\delta_B$$

   Thus, $\delta_A^* = 0$ and by a similar argument, $\delta_B^* = 0$.

   This results in the following equilibrium profile that may arise in sub-games with any realization of λ

   $$\big(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = (0,0),\ \forall\delta\big)$$

   and the required parametric restrictions are:

   - If $\lambda = (0,0)$, without further parametric restrictions
   - If $\lambda \neq (0,0)$, then the parametric restrictions are

   $$u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
   $$u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

2. If $d^\lambda(\delta) = (1,1),\ \forall\delta$ is the repayment play:

   For A:

   $$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
   $$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta < u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

   Thus, $\delta_A^* = 0$ and by a similar argument, $\delta_B^* = 0$.

   This results in sub-game perfect equilibrium profile

   $$\big(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = (1,1),\ \forall\delta\big)$$

and the required parametric restrictions are:

$\lambda \neq (0,0)$, further parametric restrictions are

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

3. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta < u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

Thus, $\delta_A^* = 0$ is dominant for A.

For B (given $\delta_A^* = 0$):

$$U_B(\text{apply pressure}) \underline{?}\, U_B(\text{no pressure})$$
$$\text{depend on whether } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ \geq u\big(w + R\lambda_B\big)$$
$$\text{or } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \ < u\big(w + R\lambda_B\big)$$

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \gamma$$

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\phi(r) - \eta, \gamma\}$$

4. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}$ is the repayment play:

A symmetric analysis to the above case yields that there are two equilibrium profiles in sub-games with $\lambda \neq (0,0)$:

- $\left(\delta_A^* = 1, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}\right)$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \gamma$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}\right)$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \max\{\phi(r) - \eta, \gamma\}$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

5. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B < 2 \\ (1,1), & \text{if } \delta_A + \delta_B = 2 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ \underline{?}\ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ \underline{?}\ u\big(w + R\lambda_A\big) - \gamma$$

Thus, $\delta_A^*(\delta_B) = \delta_B$ if

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

and $\delta_A^*(\delta_B) = 0$ if

$$\phi(r) + \gamma - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

For B, the analysis is symmetric:
$\delta_B^*(\delta_A) = \delta_A$ if

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

and $\delta_B^*(\delta_A) = 0$ if

$$\phi(r) + \gamma - \eta < u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

There are thus five equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B < 2 \\ (1,1), \text{ if } \delta_A + \delta_B = 2 \end{cases}\right)$ when

$$\phi(r) + \gamma \geq u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) - u\big(w + R\lambda_A\big) > \gamma$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) > \gamma$$

- $\left(\delta_A^* = \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A + \delta_B < 2 \\ (1,1), \text{ if } \delta_A + \delta_B = 2 \end{cases}\right)$ when

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) - u\big(w + R\lambda_A\big) > \gamma$$
$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) > \gamma$$

6. If $d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0 \\ (0,1), \text{ if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}$ is the repayment play:

By symmetric analysis to the following case, there exist two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $$\left(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right) \text{ when}$$

$$\begin{aligned} \phi(r) + \gamma &\geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \max\{\gamma, \phi(r) + \gamma - \eta\} \\ \gamma &\geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

- $$\left(\delta_A^* = \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right) \text{ when}$$

$$\begin{aligned} \phi(r) + \gamma - \eta &\geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \max\{\gamma, \phi(r)\} \\ \gamma &\geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \end{aligned}$$

7. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1,\ \delta_B = 1 \end{cases}$ is the repayment play:

   For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) < U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \gamma - \eta < u\big(w + R\lambda_A\big)$$

   (when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ > \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ > \ u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

   Thus, $\delta_A^*(\delta_B) = \delta_B$.

   For B (when $\delta_A = 0$):

$$U_B(\text{apply pressure}) < U_B(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_B\big) - \eta < u\big(w + R\lambda_B\big)$$

(when $\delta_A = 1$):

$$U_B(\text{apply pressure}) \underline{?} U_B(\text{no pressure})$$

$$\text{since } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta \underline{?} u\big(w + R\lambda_B\big) - \gamma$$

Thus, $\delta_B^*(\delta_A) = \delta_A$ when

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big)$$

and $\delta_B^*(\delta_A) = 0$ when

$$u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) - u\big(w + R\lambda_B\big) > \phi(r) + \gamma - \eta$$

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $$\left(\delta_A^* = \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right) \text{ when}$$

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\gamma, \phi(r) + \gamma - \eta\}$$

- $$\left(\delta_A^* = \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1,\ \delta_B = 1 \end{cases}\right) \text{ when}$$

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

$$\phi(r) + \gamma - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \max\{\gamma, \phi(r)\}$$

8. If $d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0,\ \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0,\ \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \underline{?} U_A(\text{no pressure})$$

$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \underline{?} u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$\begin{aligned} U_A(\text{apply pressure}) &> U_A(\text{no pressure}) \\ \text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta &> u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \end{aligned}$$

Thus, $\delta_A^*(\delta_B) = \delta_B$ when

$$\phi(r) - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

and $\delta_A^*(\delta_B) = 1$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

For B (when $\delta_A = 0$):

$$U_B(\text{apply pressure}) < U_B(\text{no pressure})$$

$$\text{since } u\big(w + R\lambda_B\big) - \eta < u\big(w + R\lambda_B\big)$$

(when $\delta_B = 1$):

$$\begin{aligned} U_B(\text{apply pressure}) &< U_B(\text{no pressure}) \\ \text{since } u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) - \eta &< u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) + \phi(r) \end{aligned}$$

Thus, $\delta_B^*(\delta_A) = 0$

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0,\ \delta_B = 0 \\ (1,0), \text{ if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), \text{ if } \delta_A = 1 \end{cases} \right)$ when

$$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r) - \eta$$

$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

- $$\left(\delta^*_A = 1, \delta^*_B = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0,\ \delta_B = 0 \\ (1,0), & \text{if } \delta_B = 1,\ \delta_A = 0 \\ (1,1), & \text{if } \delta_A = 1 \end{cases}\right) \text{ when}$$

$$\min\{\phi(r) - \eta,\ \gamma\} \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) + \gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

9. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0,\ \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}$ is the repayment play:

   By symmetric analysis to the above case, the equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

   - $$\left(\delta^*_A = 0, \delta^*_B = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0,\ \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right) \text{ when}$$

   $$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
   $$\min\{\phi(r), \gamma\} \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r) - \eta$$

   - $$\left(\delta^*_A = 0, \delta^*_B = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0,\ \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1,\ \delta_B = 0 \\ (1,1), & \text{if } \delta_B = 1 \end{cases}\right) \text{ when}$$

   $$\phi(r) + \gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
   $$\min\{\phi(r) - \eta,\ \gamma\} \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

10. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}$ is the repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \;\underline{?}\; U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \;\underline{?}\; u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \;<\; U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \;<\; u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r)$$

Thus, $\delta_A^*(\delta_B) = 1 - \delta_B$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

and $\delta_A^*(\delta_B) = 0$ when

$$\phi(r) - \eta < u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

By symmetry, for B, $\delta_B^*(\delta_A) = 1 - \delta_A$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

and $\delta_B^*(\delta_A) = 0$ when

$$\phi(r) - \eta < u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

There are thus three equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\Bigg(\delta_A^* = 1, \delta_B^* = 0;\; d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}$ when

$$\phi(r) - \eta \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

- $\Bigg(\delta_A^* = 0, \delta_B^* = 1;\; d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$
$$\phi(r) - \eta \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big)$$

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A + \delta_B = 0 \\ (1,1), & \text{if } \delta_A + \delta_B \geq 1 \end{cases}\right.$ when

$$\phi(r) \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r) - \eta$$
$$\phi(r) \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r) - \eta$$

11. If $d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0, \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1, \delta_B = 0 \\ (1,0), & \text{if } \delta_A = 0, \delta_B = 1 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}$ is the chosen repayment play:

For A (when $\delta_B = 0$):

$$U_A(\text{apply pressure}) \leq U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A\big) - \eta \leq u\big(w + R\lambda_A\big)$$

(when $\delta_B = 1$):

$$U_A(\text{apply pressure}) \ > \ U_A(\text{no pressure})$$
$$\text{since } u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) + \phi(r) - \eta \ > \ u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big)$$

if and only if $\phi(r) > \eta$.

Thus, $\delta_A^*(\delta_B) = \delta_B$.

By symmetry, $\delta_B^*(\delta_A) = \delta_A$.

There are thus two equilibrium profiles in sub-games with $\lambda \neq (0,0)$ that may emerge:

- $\left(\delta_A^* = 0, \delta_B^* = 0;\ d^\lambda(\delta) = \begin{cases} (0,0), & \text{if } \delta_A = 0, \delta_B = 0 \\ (0,1), & \text{if } \delta_A = 1, \delta_B = 0 \\ (1,0), & \text{if } \delta_A = 0, \delta_B = 1 \\ (1,1), & \text{if } \delta_A = 1, \delta_B = 1 \end{cases}\right)$ when

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$

- $\left(\delta_A^* = 1, \delta_B^* = 1;\ d^\lambda(\delta) = \begin{cases} (0,0), \text{ if } \delta_A = 0, \delta_B = 0 \\ (0,1), \text{ if } \delta_A = 1, \delta_B = 0 \\ (1,0), \text{ if } \delta_A = 0, \delta_B = 1 \\ (1,1), \text{ if } \delta_A = 1, \delta_B = 1 \end{cases}\right)$ when

$$\gamma \geq u\big(w + R\lambda_A\big) - u\big(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) > \phi(r)$$
$$\gamma \geq u\big(w + R\lambda_B\big) - u\big(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) > \phi(r)$$
$$\phi(r) \geq \eta$$

Remark: households are not restricted to commit to a repayment play before the state has been realized.

4.4.3 On equilibrium path action profiles

The analysis is conducted for sub-games starting at investment outcome realization λ:

For ease of notation, define:

$$u\big(w + R\lambda_A\big) - u(w + R\lambda_A - r\tilde{x}_A(\lambda)\big) \equiv X_A(\lambda)$$
$$u\big(w + R\lambda_B\big) - u(w + R\lambda_B - r\tilde{x}_B(\lambda)\big) \equiv X_B(\lambda)$$

1. $\delta_A = \delta_B = 0;\ d_A = d_B = 0$

   For all parametric values when $\lambda = (0,0)$

   When $\lambda \neq (0,0)$:

   - $X_A(\lambda) > \gamma$ and $X_B(\lambda) > \gamma$
   - $\phi(r) + \gamma \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda) > \max\{\gamma, \phi(r) - \eta\}$
   - $\phi(r) \geq X_A(\lambda) > \max\{\gamma, \phi(r) - \eta\}$ and $\phi(r) + \gamma \geq X_B(\lambda)$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma \geq X_B(\lambda) > \gamma$
   - $\gamma \geq X_A(\lambda)$ and $\phi(r) + \gamma \geq X_B(\lambda) > \max\{\gamma, \phi(r) + \gamma - \eta\}$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \max\{\gamma, \phi(r) + \gamma - \eta\}$ and $\gamma \geq X_B(\lambda)$
   - $\phi(r) \geq X_A(\lambda) > \phi(r) - \eta$ and $\phi(r) \geq X_B(\lambda) > \phi(r) - \eta$

- $\gamma \geq X_A(\lambda) > \phi(r)$ and $\gamma \geq X_B(\lambda) > \phi(r)$
- $\min\{\phi(r), \gamma\} \geq X_A(\lambda) > \phi(r) - \eta$ and $\phi(r) + \gamma \geq X_B(\lambda) > \phi(r)$
- $\phi(r) + \gamma \geq X_A(\lambda) > \phi(r)$ and $\min\{\phi(r), \gamma\} \geq X_B(\lambda) > \phi(r) - \eta$

2. $\delta_A = \delta_B = 0;\ d_A = d_B = 1$

   When $\lambda \neq (0, 0)$:

   - $\phi(r) \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda)$

3. $\delta_A = 1, \delta_B = 0;\ d_A = d_B = 1$ When $\lambda \neq (0, 0)$:

   - $\phi(r) - \eta \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma \geq X_B(\lambda)$
   - $\min\{\phi(r) - \eta,\ \gamma\} \geq X_A(\lambda)$ and $\phi(r) + \gamma \geq X_B(\lambda) > \phi(r)$
   - $\phi(r) - \eta \geq X_A(\lambda)$ and $\phi(r) \geq X_B(\lambda)$

4. $\delta_A = 0, \delta_B = 1;\ d_A = d_B = 1$ When $\lambda \neq (0, 0)$:

   - $\phi(r) + \gamma \geq X_A(\lambda)$ and $\phi(r) - \eta \geq X_B(\lambda) > \gamma$
   - $\phi(r) + \gamma \geq X_A(\lambda) > \phi(r)$ and $\min\{\phi(r) - \eta,\ \gamma\} \geq X_B(\lambda)$
   - $\phi(r) \geq X_A(\lambda)$ and $\phi(r) - \eta \geq X_B(\lambda)$

5. $\delta_A = \delta_B = 1;\ d_A = d_B = 1$ When $\lambda \neq (0, 0)$:

   - $\phi(r) + \gamma - \eta \geq X_A(\lambda) > \gamma$ and $\phi(r) + \gamma - \eta \geq X_B(\lambda) > \gamma$
   - $\gamma \geq X_A(\lambda)$ and $\phi(r) + \gamma - \eta \geq X_B(\lambda) > \max\{\phi(r), \gamma\}$
   - $\phi(r) + \gamma - \eta \geq X_A(\lambda) > \max\{\phi(r), \gamma\}$ and $\gamma \geq X_B(\lambda)$
   - $\gamma \geq X_A(\lambda) > \phi(r)$ and $\gamma \geq X_B(\lambda) > \phi(r)$ and $\phi(r) \geq \eta$

4.5 Existence of admissible equilibrium

The assumptions made above simply the analysis by effectively reducing the number of sub-game perfect repayment plays that one needs to consider in computing the sub-game perfect equilibria. I now make an additional assumption to guarantee the existence of sub game perfect equilibrium.

Assumption 13 (Condition for non-existence)**.** *Both*

$$\max\{\phi(r)+\gamma, \gamma\} < u\big(w+R\lambda_i\big) - u\big(w+R\lambda_i - r\tilde{x}_i(\lambda)\big)$$

and

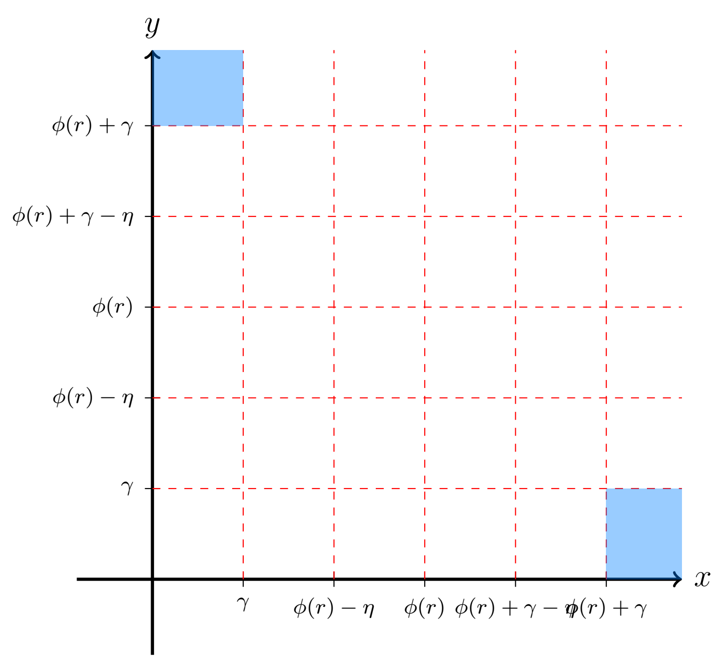
$$\gamma > u\big(w+R\lambda_j\big) - u\big(w+R\lambda_j - r\tilde{x}_j(\lambda)\big)$$

cannot hold simultaneously for any λ.

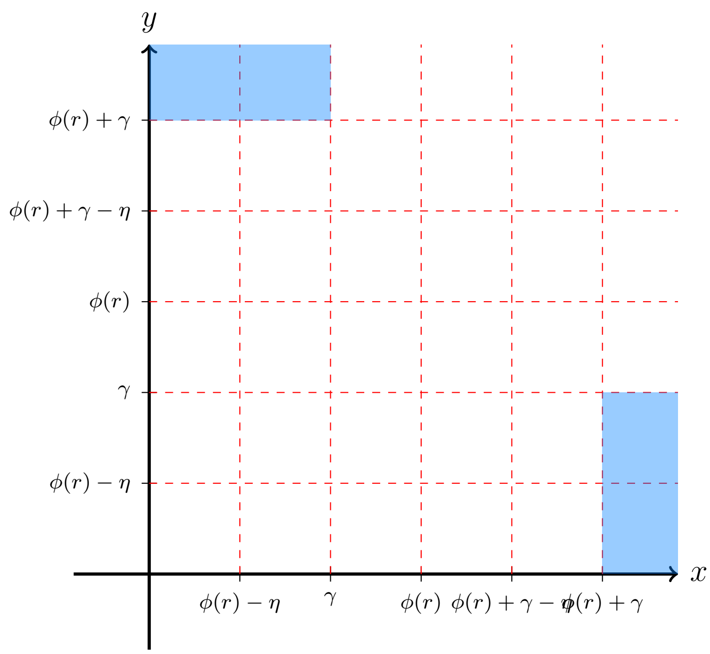
Lemma 14. *An admissible equilibrium exists if and only if assumption 13 fails.*

While this can be shown analytically, it is easies to demonstrate this equivalence in the following graphs. The five graphs below consider the four possible ways in which the parameters may be ordered. Note that the x axis represents X_A and y axis represents X_B. The blue shaded regions are regions of no equilibrium. These are obtained from the incentive compatibility constraints above.

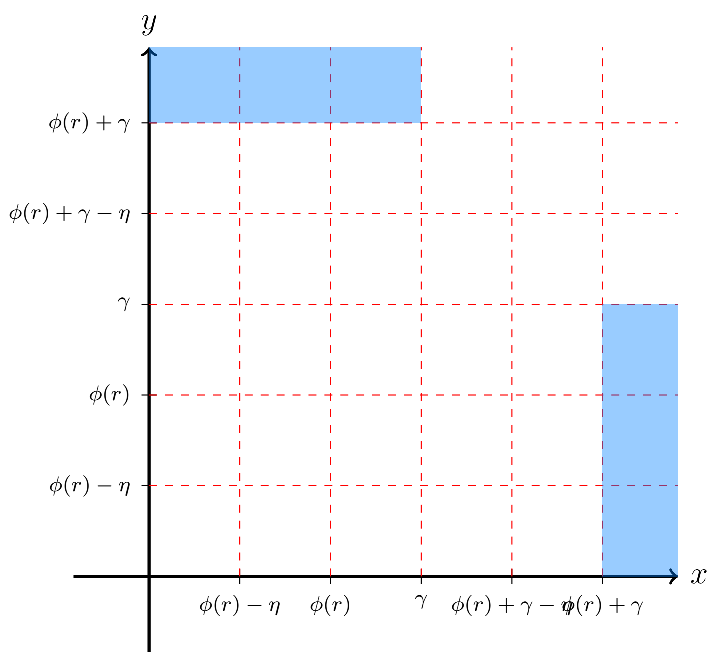
1. Figure 1: $\phi(r)+\gamma > \phi(r)+\gamma-\eta > \phi(r) > \phi(r)-\eta > \gamma$



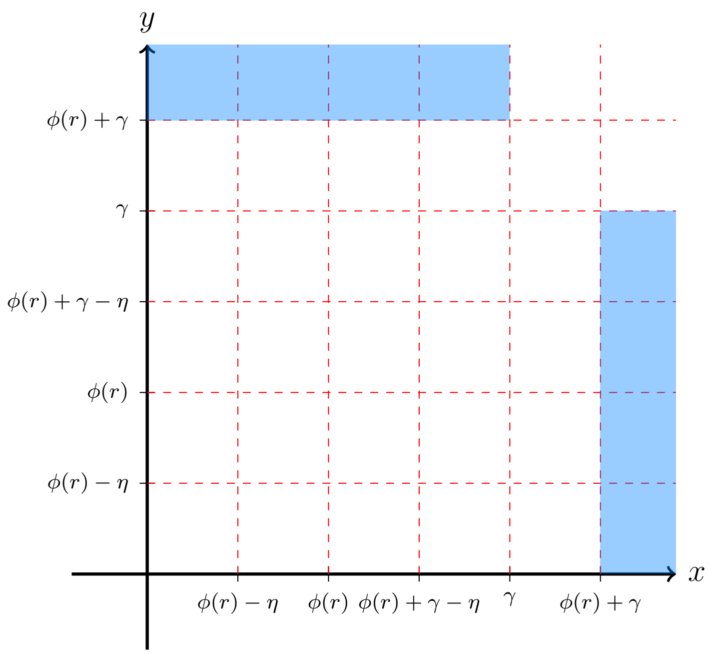
2. Figure 2: $\phi(r)+\gamma > \phi(r)+\gamma-\eta > \phi(r) > \gamma > \phi(r)-\eta$



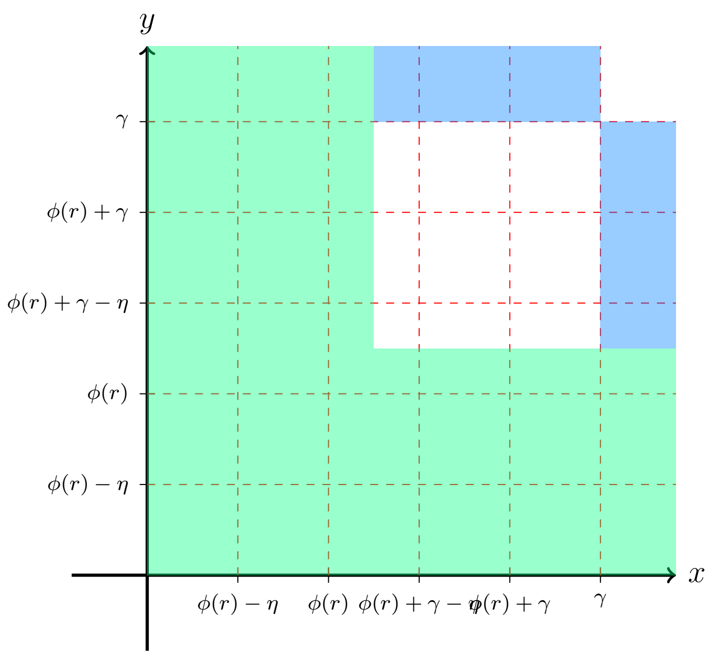
3. Figure 3: $\phi(r)+\gamma > \phi(r)+\gamma-\eta > \gamma > \phi(r) > \phi(r)-\eta$



4. Figure 4: $\phi(r)+\gamma > \gamma > \phi(r)+\gamma-\eta > \phi(r) > \phi(r)-\eta$



5. Figure 5: $\gamma > \phi(r) + \gamma > \phi(r) + \gamma - \eta > \phi(r) > \phi(r) - \eta$



Notice that in the graph immediately above, the green areas are where X_A or X_B are negative. This is impossible given the definition of X_i. In all graphs above, observe that, given the incentive compatibility conditions enlisted above, the blue areas are where there exists no equilibrium.

Also note that given the negative dependence of ϕ on r, the region of non-existence in increasing in r.

4.6 Endogenizing the MFI

In this section, the decisions of the MFI are endogenized. To that end, I suggest that the mechanism begins by the MFI first choosing whether or not to make its program available, denoting this decision by $A \in \{0, 1\}$. If the MFI chooses to *enter* (make its program available), it then chooses an interest rate r. The group members then play the game described above at interest rate r.

This paper models the MFI as a non-profit that is genuinely invested in the development of the communities it operates with. This is essentially in line with a lot of MFIs especially in the early days. The stated aims and observed actions of the Grameen MFI as well as the early days of SKS (before its commercialization) do match up to this description. Suppose the MFI faces a cost of acquiring capital denoted by c per dollar loaned. Let $\sigma(r)$ correspond to a strategy profile played by the members of the group at interest rate r and $\pi\big(\sigma(r)\big)$ as the implied repayment rate, then, the MFI's utility function is defined as follows:

$$U_{MFI}(r, \sigma(r)) = 1\Bigg\{r \cdot \pi\big(\sigma(r)\big) \geq c\Bigg\} \cdot A + 0.1(1 - A)$$

This basically states that if the MFI is able to recover the cost of providing loans (in expectation) upon entering, it gets a utility of 1. To the contrary, if it unable to do so upon entering, the utility of the MFI has been normalized to 0. If the MFI chooses not to enter it receives 0.1 utils, this implies that the MFI would rather choose not to enter if it cannot recover costs. I consider the equilibrium in the new *game with endogenous MFI* and start by defining the strategies in this game. Notice that given the set up of the game played among group members, each state of the world is realized with known probability. In each of these states, in any sub-game perfect equilibrium of the game played by group members, either full repayment or no repayment occurs owing to the complementarity of repayments in the game. There are finite number of states in which either repayment occurs in full or not. Thus, there are a finite number of repayment rates that are possible denote in the set:

$$\{\rho : \rho = \mathbb{P}(A),\ \forall A \in \Lambda\}$$

Since the MFI is allowed to not offer its program, the option for it to set an infinite interest rate is excluded. Further, restrict set of interest rates to

$$r \in \left\{\frac{c}{\mu^2}, \frac{c}{\mu^2 + \mu(1-\mu)}, \frac{c}{\mu^2 + 2\mu(1-\mu)}, c\right\}$$

This assumption is made to ensure existence of equilibrium for a reasonably large set of parameter values. Recall that the non-existence region is increasing in the interest rate. Since I focus on equilibria that correspond to either the maximum or the minimum repayment rate, I demonstrate that this restriction is not severe.

Definition 15 (Strategy profile of game with endogenous interest rates)**.** *A (pure) strategy profile* $\left(A, r, \left(\sigma(r)\right)_{\forall r}\right)$ *lists the decision to enter, the interest rate chosen by the MFI and a strategy of pressure and repayments by group members at each interest rate that may be set by the MFI* $\sigma(r) = \left\{\delta_A^\lambda(r), \delta_B^\lambda(r), d^\lambda(.;r) : \forall \lambda \in \{0,1\}^2\right\}$.

Definition 16 (Extension of admissible equilibrium to game with endogenous interest rate)**.** *A strategy profile* $\left(A, r, \left(\sigma(r)\right)_{\forall r}\right)$ *is an equilibrium if the following conditions hold:*

1. $\forall r,\ \sigma(r)$ *is a admissible equilibrium in the game played by group members*
2. r *is a best response for the MFI given* $\left(\sigma(r)\right)_{\forall r}$,

$$i.e.\ \ U_{MFI}(1, r, \left(\sigma(r)\right)_{\forall r}) \geq U_{MFI}(1, r', \left(\sigma(r)\right)_{\forall r}),\quad \forall r'$$

3. $A = 1$ *iff* $r \cdot \pi\big(\sigma(r)\big) \geq c$

Lemma 17. *The game with endogenous interest rates contains an admissible equilibrium iff there exists an admissible equilibrium in every group game at all interest rates* $r \in \left\{\frac{c}{\mu^2}, \frac{c}{\mu^2+\mu(1-\mu)}, \frac{c}{\mu^2+2\mu(1-\mu)}, c\right\}$.

Proof: If the game with endogenous interest rates contains an admissible equilibrium, then, by definition, there must exist an admissible equilibrium in every group game at all interest rates r. So this direction is trivial.

Suppose now that $\forall r$, an admissible equilibrium exists. Denote this by $\{\sigma(r)\}_{\forall r}$. Now suppose there exists some $\tilde{r}$ such that $\tilde{r} \cdot \sigma(\tilde{r}) \geq c$. Then, $\{A = 1, \tilde{r}, \{\sigma(r)\}_{\forall r}\}$ constitutes an equilibrium. If such an interest rate $\tilde{r}$ does not exist, then $\forall r$, $\{A = 0, r, \{\sigma(r)\}_{\forall r}\}$ constitutes an equilibrium. ■

The multiplicity of equilibrium in the game played among group members at any given interest rate further complicates the equilibrium of this game. This is because which equilibrium would get *selected* in the game played by group members at any given interest rate is unknown.

4.7 Monotone Maximal Repayment Equilibrium

Define the highest probability of repayment in equilibrium at interest rate r:

$$i.e. \quad \bar{\pi}(r) = \max\left\{\pi(\sigma(r)) : \sigma(r) \text{ is an admissible equilibrium}\right\}$$

Definition 18 (Monotone maximum repayment equilibrium)**.** *Any admissible equilibrium strategy profile that helps achieve the lowest interest rate while maintaining the highest repayment rate. Thus,* $\left(A, r, \left(\sigma(r)\right)_{\forall r}\right)$ *is a monotone maximal equilibrium iff*

1. $\forall r$, $\pi(\sigma(r)) = \bar{\pi}(r)$
2. $\forall r' < r$, $\pi(\sigma(r')) \times r' < c$

Lemma 19. *In any monotone maximal repayment equilibrium, the following properties hold:*

1. $\forall r$, *if full repayment occurs in state* λ, *then full repayment also occurs in all* $\lambda' \geq \lambda$ *where every group member's investment outcome is at least as successful;*
2. $\forall \lambda$, *if full repayment occurs at interest rate* r, *then full repayment also occurs at any lower interest rate* $r' < r$.

Proof: Both parts are proved by contradiction.

1. Suppose in a monotone maximal repayment equilibrium, there exists some r for which for some $\lambda' \geq \lambda$ such that full repayment occurs in λ but not in λ'. Given the assumption on the repayment norm $\tilde{x}_i$, $\tilde{x}_i(\lambda') \leq \tilde{x}_i(\lambda)$, $\forall i$. Then, by concavity, $\forall i$

$$u(w + \lambda_i R) - u(w + \lambda_i R - r\tilde{x}_i(\lambda)) \geq u(w + \lambda_i' R) - u(w + \lambda_i' R - r\tilde{x}_i(\lambda'))$$

   Thus, if full repayment is incentive compatible in λ, it is also incentive compatible in λ'. Thus, there exists an equilibrium with full repayment in both λ and λ'. Since $\mathbb{P}(\lambda') > 0$, such a equilibrium sustains a higher repayment rate. This violates the first condition of monotone maximal repayment equilibrium, $\forall r, \pi(\sigma(r)) = \bar{\pi}(r)$.

2. Suppose in a monotone maximal repayment equilibrium, there exists some λ in which repayment occurs at interest rate r but not at interest rate $r' < r$. Then $\pi(\sigma(r')) < \pi(\sigma(r))$. Given $u(.)$ is increasing,

$$u(w + \lambda_i R) - u(w + \lambda_i R - r\tilde{x}_i(\lambda)) \geq u(w + \lambda_i R) - u(w + \lambda_i R - r'\tilde{x}_i(\lambda))$$

   Thus, there exists some equilibrium for which repayment would occur in λ violating $\pi(\sigma(r')) = \bar{\pi}(r')$. ■

4.8 Planner's problem

To formally study any inefficiencies in this setup, planner's problem is now described. Basically, the notion of the planner's problem is that of a utilitarian planner who seeks to maximize the consumer surplus subject to meeting the non-negative profit condition of the MFI. Define $g : \{0,1\}^2 \to \{0,1\}$, the planner's repayment choice function which maps each state to a binary decision of either full compliance with the norm or non-compliance (pay nothing). Thus, the probability of repayment for a given choice of $g(.)$ is $\mathbb{E}_\lambda\big[g(\lambda) \cdot 1(\lambda \neq (0,0))\big]$. That is, the probability of arriving at states at which the planner chooses to repay and norm suggested repayment amounts leads to full repayment.

Then, the planner's problem can be written as:

$$\max_{A,r,g} A\big[\mathbb{E}U_A + \mathbb{E}U_B\big] \cdot 1\Big\{r \cdot \mathbb{E}_\lambda\big[g(\lambda) \cdot 1(\lambda \neq (0,0))\big] \geq c\Big\} + \big(1 - A\big)2u(w)$$

The planner is allowed to choose repayment behavior $g : \{0,1\}^2 \to \{0,1\}$ as any function from the state space to the binary repayment decision (there are finitely many of these), the gross interest rate $r \in \left\{\frac{c}{\rho} : \rho = \mathbb{P}(A) > 0,\ \forall A \in \Lambda\right\}$ as well as whether or not to offer the program $A \in \{0,1\}$. Define:

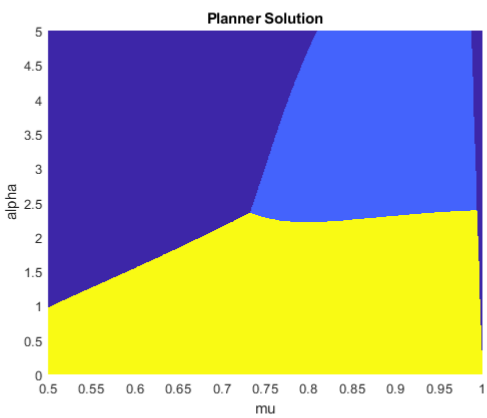
$$\mathbb{E}U_i(r, g(.)) = \sum_{\lambda} \mathbb{P}(\lambda)\big[g(\lambda)\cdot U_i(\delta^\lambda = (0,0), d^\lambda = (1,1); \lambda) + (1-g(\lambda))\cdot U_i(\delta^\lambda = (0,0), d^\lambda = (0,0); \lambda)\big]$$

Since there are a finite set of options to choose from, existence in trivial.

4.9 Simulation Exercise

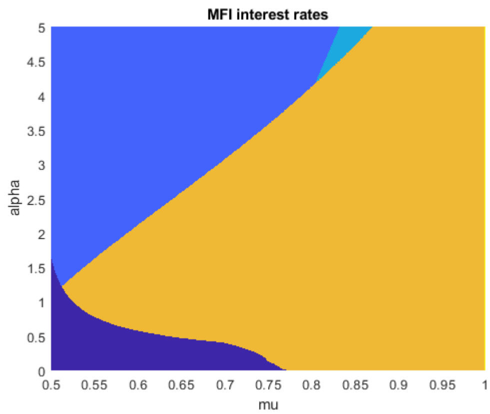
Within this illustration, I set $\theta = 0.5,\ c = 1.06,\ R = 3,\ w = 5,\ \beta = 0.85$ while allowing the probability of the lottery resulting in high return, $\mu \in (0.5, 1)$ and risk aversion of the group members, $\alpha \in (0, 5)$.

The solution to the planner's problem:



Here, the yellow region corresponds to combinations of (μ, α) for the MFI sets $r = \frac{c}{\mu^2 + 2\mu(1-\mu)}$ and the repayment occurs so long as at least one group member succeeds. The blue region corresponds to combinations of (μ, α) for which the MFI sets $r = \frac{c}{\mu^2}$ and the repayment occurs so long as both group member succeeds. Finally, the purple region is where the MFI does not offer an microfinance program.

The monotone maximal repayment equilibrium:



Here, the orange region corresponds to combinations of (μ, α) for the MFI sets $r = \frac{c}{\mu^2+2\mu(1-\mu)}$ and the repayment occurs so long as at least one group member succeeds. The indigo region corresponds to combinations of (μ, α) for which the MFI sets $r = \frac{c}{\mu^2}$ and the repayment occurs so long as both group member succeeds. The blue region is where the MFI does not offer an microfinance program. Finally, the purple region is where no equilibrium exists.

The main take away intended from this exercise is that when the equilibrium exists, repayment is higher (no smaller) under the monotone maximal repayment equilibrium that what would be efficient. This has implications on the risk sharing between the MFI and the group members.

5 Selection Mechanism

In what follows, institutional details are used to motivate a selection mechanism motivated by the observed practices among these microfinance programs. I first present the observed features that I wish to include in informing the selection mechanism.

1. MFIs build relationships with their clients. Mohammed Yunus himself remarks in his memoirs, *Banker to the poor*, an important feature of the founding days of microfinance includes bankers riding through villages on bicycles acquainting themselves with the villagers and forging relationships with them. Further, Stiglitz and Haldar point out that in its early days, SKS (a large MFI in India) had set itself up on the principles of Grameen MFI (the model NGO MFI founded by Mohammed Yunus). This included

"community-building practices traditionally associated with microfinance, such as regular group and center meetings, and relationship-building visits from MFI workers."

2. MFI use this relationship to encourage repayment. As discussed in Stiglitz, Haldar (2013), the extra information gained by the MFIs in building the relationships with its clients and in building a strong group identity allows the MFI to distinguish between strategic default and a genuine inability to repay.

3. MFI's prefer keeping interest rates as low as possible. Consider the following quote from the then acting Managing Director of the Grameen MFI, Ratan Kumar Nag "We are now charging the highest 20.0 per cent interest against a loan though the ceiling of interest is 27.0 per cent fixed by the Microcredit Regulatory Authority (MRA)." He also suggested that the loan recovery rate was 99.05% in 2016. MFI that operate with an altruistic purpose may take pride in keeping interest rates low in order to be more inclusive and accessible to people who most require access to credit markets.

These features of the market that have not been explicitly incorporated in the model serve to select the equilibrium with the highest repayment (and highest pressure). Note here that given the non-negativity of profits, the only way to reduce the interest rates is by ensuring repayment rates are high. This will be formalized in the following assumption.

Assumption 20 (Selection Mechanism)**.** *In the event of multiplicity of equilibria, the one with the highest pressure and repayment (in each relevant sub-game) is chosen.*

As anecdotal evidence, consider the case of SKS microfinance in Andhra Pradesh, India. Initially, set up to closely mimic the model of microfinance, by late 2000s, they decide to commercialize and shift to for-profit lending in a highly competitive market for microfinance. As Ballem et al from Microsave (a competitor to SKS) recount: "Most MFIs are mono-service credit companies providing standard basic joint liability group (JLG) loans to customers. There has been only a limited focus on clients; be it in terms of assessing their capacity to repay or in developing appropriate products to suit their needs. Microsave has often observed that despite the MFI management's protestation to the contrary, most clients see MFIs as just another source of credit, rather than institutions interested in client welfare. The rapid influx of capital resulted in rapid expansion in scale without adequate investment in building customer relationships. This, combined with intense competition amongst MFIs and the resultant multiple lending, led to a situation where clients refer to MFIs as "Monday MFI," "Tuesday MFI," etc., depending on their collection schedules. This clearly demonstrates the lack of relationship between MFIs and their clients."

It is particularly interesting to note that this commercialization was followed by a collapse of the industry to the point of government intervention. Repayment rates plummeted to 10 - 15% while interest rates soared (as suggested by Ballem and colleagues). This is suggestive of the role of the relationship between MFIs and its clients in ensuring repayment is high and interest rates are low.

6 Endogenous project selection

Armed with the above selection mechanism, I now proceed to endogenize project selection in this environment. As before, the MFI first decided whether or not to offer its program, its decision is encoded by $A \in \{0, 1\}$. Conditional of offering the program, the MFI sets interest rates r. Having observed this, participants of the program simultaneously choose between a low risk-low reward investment technology (henceforth denoted 'safe') and a high risk-high reward investment technology (henceforth denoted 'risky'). The outcomes of the investments are then realized based on the distribution implied by the chosen investment technologies. Consequently, participants simultaneously choose whether or not to apply pressure and subsequently whether or not to repay the norm suggested amounts. I start by defining the different investment technologies as well as a generalized version of the norm that can account for the different types of lotteries.

The two different investment technologies will be denoted by L_1 and L_2. Investment technology 1 (L_1) has stochastic returns governed by probability law:

$$\text{return in } L_1 = \begin{cases} R_i, & \text{with probability } \mu \\ \frac{R_i}{2}, & \text{with probability } 1 - \mu \end{cases}$$

while investment technology 2 (L_2) has stochastic returns governed by probability law:

$$\text{return in } L_2 = \begin{cases} 2R_i, \text{ with probability } \mu \\ 0, \text{ with probability } 1-\mu \end{cases}$$

A crucial feature of these technologies is that L_2 (risky) has a higher excepted return and higher variance than L_1 (safe). Since participants now choose between investment technologies, the state space (of investment outcomes) has also expanded. States continue to be denoted by $\lambda \in \{0, 0.5, 1, 2\}^2$ but the state space has expanded. Define a new social repayment norm that accounts for the different levels of outcomes that may be realized.

$$\tilde{x}_i(\lambda) = \begin{cases} \theta, \text{ if if } \lambda_i = \lambda_j = 0 \\ \frac{\theta}{\lambda_j}, \text{ if } \lambda_i = 0, \ \lambda_j \neq 0 \\ 1 + (1 - \frac{\theta}{\lambda_i}), \text{ if } \lambda_i \neq 0, \ \lambda_j = 0 \\ 1, \text{ if } \lambda_i \neq 0, \ \lambda_j \neq 0 \end{cases}$$

where $j = \{A, B\} \backslash i$. In the suggested specification, the norm takes into account the different successful (non-zero) investment outcomes and proposes larger co-payment amounts in states with higher outcomes.

When the MFI chooses to offer the program, at each interest rate r, the participants (A and B) choose lotteries L_A and L_B. The choice of projects now effects the distribution of the state λ. $\mathbb{P}(.|L_A, L_B)$ denotes the distribution over states (investment realizations) implied by the choice of projects (tables 2, 3, 4 and 5 describe these). For each interest rate r and corresponding choice of L_A and L_B, the group repayment game is played (as described in section 4).

As established in section 5, the MFI is modeled as a non-profit that seeks to seeks to set the lowest interest subject to meeting its zero-profit condition.

$$r \in \left\{ \frac{c}{\mu^2}, \frac{c}{\mu^2 + \mu(1-\mu)}, \frac{c}{\mu^2 + 2\mu(1-\mu)}, c \right\}$$

As before, the set of allowed interest rates is restricted.

Definition 21 (Strategy profile of game with endogenous project selection)**.** *A (pure) strategy profile* $\Big(A, r, \big(L_A(r), L_B(r), \sigma(r, L_A, L_B)_{\forall L_A, L_B}\big)_{\forall r \in \mathscr{R}}\Big)$ *lists the decision to enter, the in-*

Table 2: $L_A = L_B = L_1$

λ	$\mathbb{P}(\lambda)$
(0.5,0.5)	$(1-\mu)^2$
(0.5,1)	$\mu(1-\mu)$
(1,0.5)	$\mu(1-\mu)$
(1,1)	μ^2

Table 3: $L_A = L_1,\ L_B = L_2$

λ	$\mathbb{P}(\lambda)$
(0.5,0)	$(1-\mu)^2$
(0.5,2)	$\mu(1-\mu)$
(1,0)	$\mu(1-\mu)$
(1,2)	μ^2

Table 4: $L_A = L_2,\ L_B = L_1$

λ	$\mathbb{P}(\lambda)$
(0,0.5)	$(1-\mu)^2$
(0,1)	$\mu(1-\mu)$
(2,0.5)	$\mu(1-\mu)$
(2,1)	μ^2

Table 5: $L_A = L_B = L_2$

λ	$\mathbb{P}(\lambda)$
(0,0)	$(1-\mu)^2$
(0,2)	$\mu(1-\mu)$
(2,0)	$\mu(1-\mu)$
(2,2)	μ^2

terest rate chosen by the MFI, the projects to be chosen and the strategy of pressure and repayments by group members at each interest rate that may be set by the MFI, i.e. for all $L_A, L_B \in \{L_1, L_2\},\ \sigma(r, L_A, L_B) = \Big\{\delta_A^\lambda(r, L_A, L_B), \delta_B^\lambda(r, L_A, L_B), d^\lambda(r, L_A, L_B) : \forall\lambda \in \{0, 0.5, 1, 2\}^2\Big\}$.

Payoff to household i associated with a strategy profile is:

$$U_i\Big(A, r, \big(\sigma(r, L_A, L_B)\big)_{\forall r}\Big) = A \times \sum_\lambda \mathbb{P}(\lambda; L_A, L_B) \times U_i\big(\delta_A^\lambda, \delta_B^\lambda, d^\lambda(\delta_A^\lambda, \delta_B^\lambda); \lambda\big) + (1 - A) \times u(w)$$

Definition 22.

$$V_i\Big(r, L_i, L_j, \sigma(r, L_i, L_j)\Big) = \sum_\lambda \mathbb{P}(\lambda | L_i, L_j) \times U_i\big(\sigma(r, L_i, L_j) | \lambda\big)$$

Definition 23 (Admissible equilibrium in a game with endogenous project selection)**.** *A strategy profile* $\tilde{\sigma} = \Big(A, r, \big(L_A(r), L_B(r), \sigma(r, L_A, L_B)_{\forall L_A, L_B}\big)_{\forall r \in \mathscr{R}}\Big)$ *is an admissible equilibrium if the following conditions hold:*

1. $\forall r \in \mathscr{R},\ \forall L_A, L_B,\ \sigma(r, L_A, L_B)$ *is admissible equilibrium in the group repayment game.*

2. $\forall r \in \mathscr{R},\ \ L_i$ *and* L_j *are simultaneous best responses given the strategy in the group repayment game, i.e.*

$$V_i\Big(r, L_i, L_j, \sigma(r, L_i, L_j)\Big) \geq V_i\Big(r, L', L_j, \sigma(r, L', L_j)\Big), \quad \forall i \text{ and } \forall L'$$

3. r *and* A *satisfy the zero profit condition, i.e.*

$$A = \begin{cases} 1, & \text{if } r \cdot \pi\big(\sigma(r, L_A, L_B)\big) \geq c \\ 0, & \text{otherwise} \end{cases}$$

The selection mechanism in assumption 20 still does not yield in a unique equilibrium. This is due to the multiplicity in the lottery selection stage. One candidate selection rule at this stage could be: selection towards the risky lotteries. This is the equilibrium depicted in the simulation exercises to follow (since it is least likely to yield the conclusion obtained).

6.1 Planners's Problem

Start by defining $h : \{0, 1/2, 1, 2\}^2 \to \{0, 1\}$ and consequently the planner's problem as:

$$\max_{A,r,h(.),L_A,L_B} A\big[\mathbb{E}U_A(r,h(.),L_A,L_B) + \mathbb{E}U_B(r,h(.),L_A,L_B)\big] \cdot 1\Big\{r \cdot \mathbb{E}_\lambda\big[h(\lambda) \cdot 1(\lambda \neq (0,0)); L_A, L_B\big] \geq c\Big\}$$
$$+ \big(1 - A)2u(w)$$

where

$$\mathbb{E}U_i(r,h(.)) = \sum_\lambda \mathbb{P}(\lambda; L_A, L_B)\big[h(\lambda) \cdot U_i(\delta^\lambda = (0,0), d^\lambda = (1,1); \lambda) + (1-h(\lambda)) \cdot U_i(\delta^\lambda = (0,0), d^\lambda = (0,0); \lambda)\big]$$

where the actions of the planner are such that interest rate $r \in \mathfrak{R}$, the choice of lotteries $L_A, L_B \in \{L_1, L_2\}$ and $h : \{0, 1/2, 1, 2\}^2 \to \{0, 1\}$ such that when $\mathbb{P}(\lambda; L_A, L_B = 0)$ then $h(\lambda) = 0$. Again owing to the finiteness of the problem, a solution exists.

6.2 Setting $\phi(L_A, L_B, r)$

The continuation values for different lottery choices are now estaiblished.

6.2.1 Both play safe

$$\phi(Safe, Safe, r)_1 = \beta\Big\{\mu\big[u(w + R - r)\big] + (1 - \mu)\big[u(w + 0.5 \cdot R - r)\big] + \phi(Safe, Safe, r)_1\Big\} \tag{8}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big] \tag{9}$$

$$\phi(Safe, Safe, r) = \phi(Safe, Safe, r)_1 - \phi_0 \tag{10}$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1 - \beta} \tag{11}$$

$$\phi(Safe, Safe, r)_1 = \frac{\beta EU^{SS}(norm)}{1 - \beta} \tag{12}$$

where

$$EU^{SS}(norm) = \mu\big[u(w + R - r)\big] + (1-\mu)\big[u(w + 0.5 \cdot R - r)\big] \tag{13}$$

and hence

$$\phi(Safe, Safe, r) = \frac{\beta\big[EU^{SS}(norm) - u(w)\big]}{1-\beta} \tag{14}$$

6.2.2 Both go risky

$$\begin{aligned}\phi(Risky, Risky, r)_1 =& \beta\Big\{\mu^2\big[u(w + 2R - r) + \phi(Risky, Risky, r)_1\big] \\ &+ \mu(1-\mu)\big[u(w + 2R - (2 - 0.5\theta)r) + \phi(Risky, Risky, r)_1\big] \\ &+ (1-\mu)\mu\big[u(w - 0.5\theta \cdot r) + \phi(Risky, Risky, r)_1\big] + (1-\mu)^2\big[u(w) + \phi_0\big]\Big\}\end{aligned} \tag{15}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big] \tag{16}$$

$$\phi(Risky, Risky, r) = \phi(Risky, Risky, r)_1 - \phi_0 \tag{17}$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta} \tag{18}$$

$$\phi(Risky, Risky, r)_1 = \frac{\beta EU^{RR}(norm) + \beta\phi_0(1-\mu)^2}{1 - \beta\mu(2-\mu)} \tag{19}$$

where

$$\begin{aligned}EU^{RR}(norm) = \mu^2\big[u(w + 2R - r)\big] + \mu(1-\mu)\big[u(w + 2R - (2 - 0.5\theta)r)\big] \\ + (1-\mu)\mu\big[u(w - 0.5\theta r)\big] + (1-\mu)^2\big[u(w)\big]\end{aligned} \tag{20}$$

and hence

$$\phi(Risky, Risky, r) = \frac{\beta EU^{RR}(norm) + (\beta - 1)\phi_0}{1 - \beta\mu(2-\mu)} \tag{21}$$

6.2.3 A goes risky - B goes safe

$$\begin{aligned}\phi(Risky, Safe, r)_1 =&\beta\Big\{\mu^2\big[u(w+2R-r)+\phi(Risky, Safe, r)_1\big]\\&+\mu(1-\mu)\big[u(w+2R-r)+\phi(Risky, Safe, r)_1\big]\\&+(1-\mu)\mu\big[u(w-\theta r)+\phi(Risky, Safe, r)_1\big]\\&+(1-\mu)^2\big[u(w-2\theta r)+\phi(Risky, Safe, r)_1\big]\Big\}\end{aligned} \tag{22}$$

$$\phi_0 = \beta\Big[u(w)+\phi_0\Big] \tag{23}$$

$$\phi(Risky, Safe, r) = \phi_1^{RS} - \phi_0 \tag{24}$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta} \tag{25}$$

$$\phi(Risky, Safe, r)_1 = \frac{\beta EU^{RS}(norm)}{1-\beta} \tag{26}$$

where

$$EU^{RS}(norm) = \mu\big[u(w+2R-r)\big]+(1-\mu)\mu\big[u(w-\theta r)\big]+(1-\mu)^2\big[u(w-2\theta r)\big] \tag{27}$$

and hence

$$\phi(Risky, Safe, r) = \frac{\beta\big[EU^{RS}(norm)-u(w)\big]}{1-\beta} \tag{28}$$

6.2.4 A goes safe - B goes risky

$$\begin{aligned}\phi(Safe, Risky, r)_1 =&\beta\Big\{\mu^2\big[u(w+R-r)+\phi(Safe, Risky, r)_1\big]\\&+\mu(1-\mu)\big[u(w+R-(2-\theta)r)+\phi(Safe, Risky, r)_1\big]\\&+(1-\mu)\mu\big[u(w+0.5R-r)+\phi(Safe, Risky, r)_1\big]\\&+(1-\mu)^2\big[u(w+0.5R-(2-\theta)r)+\phi(Safe, Risky, r)_1\big]\Big\}\end{aligned} \tag{29}$$

$$\phi_0 = \beta\Big[u(w) + \phi_0\Big] \tag{30}$$

$$\phi(Safe, Risky, r) = \phi_1^{SR} - \phi_0 \tag{31}$$

The solution to this system is attained at:

$$\phi_0 = \frac{\beta u(w)}{1-\beta} \tag{32}$$

$$\phi(Safe, Risky, r)_1 = \frac{\beta EU^{SR}(norm)}{1-\beta} \tag{33}$$

where

$$\begin{aligned} EU^{SR}(norm) = \mu^2\big[u(w+R-r)\big] + \mu(1-\mu)\big[u(w+R-(2-\theta)r)\big] \\ + (1-\mu)\mu\big[u(w+0.5R-r)\big] + (1-\mu)^2\big[u(w+0.5R-(2-2\theta)r)\big] \end{aligned} \tag{34}$$

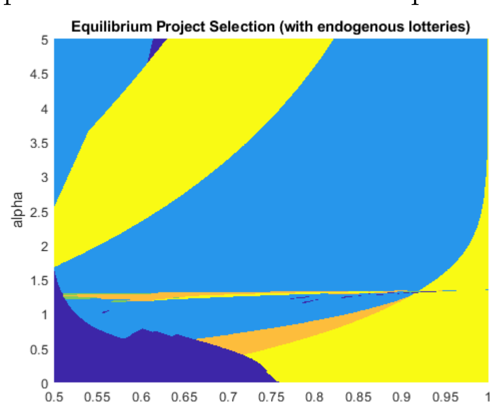
and hence

$$\phi(Safe, Risky, r) = \frac{\beta\big[EU^{SR}(norm) - u(w)\big]}{1-\beta} \tag{35}$$

6.3 Simulation Exercise with endogenous lotteries

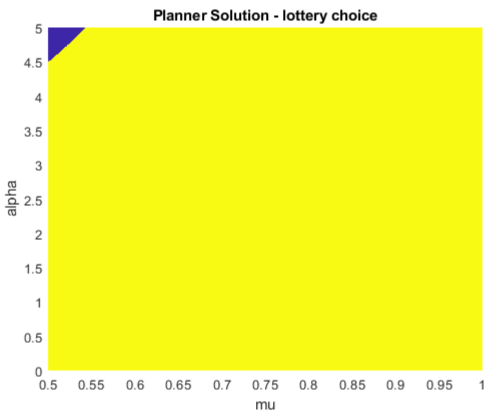
As before, within these illustrations I set $\theta = 0.5,\ c = 1.06,\ R = 3,\ w = 5,\ \beta = 0.85$ while allowing $\mu \in (0.5, 1)$ and $\alpha \in (0, 5)$.

I present below the results of equilibrium project selection:



Here, the purple region corresponds the parameter values for which an equilibrium does not exist. The blue region corresponds to parameter values for which both group members choose the safe lottery. The green region corresponds to parameter values for which group member A chooses the safe lottery while B chooses the risky lottery. The orange region corresponds to parameter values for which group member B chooses the safe lottery while A chooses the risky lottery. Finally, the yellow region is where both group members choose the risky lottery.

Now, compare this the results of the planner's problem:



In the above figure, the yellow region corresponds to the parameter values for which both group members choose the risky lottery while the purple region corresponds to both choosing the safe lottery.

This thus demonstrates that increased risk levied onto the group members by market forces that result in equilibrium with low interest rates leads to sub-optimal lottery choice. Specifically, it shows that group members are inefficiently driven away from high risk high reward lotteries.

7 Conclusion

I now summarize the some results established in this paper:

1. The high repayment rates induced by peer pressure, often considered a measure of success of the JLL microfinance program, inefficiently transfers the risk associated with the investments towards the participants and away from the MFI. This shift in the bearing of risk causes equilibrium repayment rates to be higher than what would be

socially efficient. Thus, this is able to explain the predominantly high repayment rates observed in the data when microfinance institutions invest in building relationships with its clients.

2. The model also sheds light on the dramatic non-repayment observed in Andhra Pradesh where microfinance institutions sought to commercialize the process and veered away from building relationships with their clients. That is to say, the market was unable to *select* equilibria with high repayment.

3. On endogenizing lottery selection, I demonstrate that when choosing between high risk high reward (risky) and low risk low reward (safe) loans, this inefficient allocation of risk induces participants to pick the low risk-low reward investment in equilibrium for a larger set of parameter values than what would be socially efficient. Thus, peer pressure causes over investment in safe loans and under investment in high risk-high reward loans. Consequently, the (local) economy underinvests in welfare increasing investments.

Finally the model highlights the issues of multiplicity of equilibria generated by the strategic complementarity in repayments. Consequently, I find that the social pressure is not sufficient in generating high repayments which as often assumed in the literature (with Besley, Coate (1995) being an exception). It is only when compounded with the MFI's preference for low interest rates that high repayments are generated. To conclude, it this top down pressure which include pressure from both: the MFI and the peers, that sustains high repayments in this model. This inefficiently shifts excessive risk towards the group members which results in an inefficient choice of projects (lotteries).

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