Topics: Normal distribution, Functions of Random Variables

1. **The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
   - **A. 0.3875**
   - **B. 0.2676**
   - **C. 0.5**
   - **D. 0.6987**

Ans: We have to find the probability that the service manager cannot meet his commitment i.e probability of work done on car greater than 50(as work begins after 10 min, so 60-10=50).

$P(X>50)= 1-P(X<50)$

$= 1-0.73$

$= 0.26$

2. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.**
   - **A. More employees at the processing center are older than 44 than between 38 and 44.**
   - **B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Ans: Both the statements are true.

a) There are about 63 employees older than 44 and 137 employees between 38 and 44. So the statement is true.

b) The training program under the age of 30 has approximately 36 employees. So this statement is also true.

3. **If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.**

Ans: Distributions:

a) $2\,X_1$

Mean= 2μ

Varaince = $4\sigma^2$

b) $X_1 + X_2$

Mean= 2μ

Variance = $2\sigma^2$

Both a) and b) have same mean but variance differ. The Varaince of a) is twice than the b).

4. **Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Ans: D. Mean=100

Variance=20

a= Mean-Z(Variance)

b=Mean+Z(Variance)

As the confidenceinterval is 99% so value of z is 2.576

hence a= 100-2.576(20) = 48.5

b= 100+2.576(20) = 151.5

5. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits**

are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains a 95% probability for the annual profit of the company.

Ans. As both the divisions are independent, the sum of independent normal random variables results in a normal random variable with mean equal to the sum of the individual means and variance equal to the sum of the individual variances.

So, Total for profits

Mean = 5+7= 12 *45= 540 million rupees

Variance = 9+15=25

Std Deviation = 5*45= 225 million rupees

For 95% probability, z= 1.960

Range= 99 to 981 million rupees

540 -1.96(225) =99

540 +1.96(225) = 981

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans. for 5th percentile, z= 1.675

Profit = 540-1.675(225) = 169.875 million rupees

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans. As both the divisions have profit mean greater than 0 so neither divisions have the probability of mking loss in a given year.