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Data Science II : HomeWork 1 ①

1] Calculus.

1) Given $y = \log(b.x)$, compute $\frac{dy}{dx}$

→ $y = \log(b.x)$

$\therefore \frac{dy}{dx} = \frac{1}{b.x} \times \frac{dy}{dx}(b.x)$

$\frac{dy}{dx} = \frac{1}{b.x} \times b$

$\frac{dy}{dx} = \frac{1}{x}$

2) Given $y = e^{(x.z)}$, compute $\frac{dy}{dx}$

→ $y = e^{(x.z)}$

$\therefore \frac{dy}{dx} = \frac{dy}{dx}(e^{x.z}) \times \frac{dy}{dx}(x.z)$

$= e^{(x.z)} \times \frac{dy}{dx}(x.z)$

$= e^{(x.z)} \times z.$

$\therefore \frac{dy}{dx} = z.e^{(x.z)}$

3) Given $f = \frac{1}{1+e^{-x}}$, compute $\frac{dy}{dx}$.

$\longrightarrow y = f = \frac{1}{1+e^{-x}} \begin{matrix} \rightarrow u \\ \rightarrow v \end{matrix}$

Applying $\frac{u}{v}$ rule.

$$\therefore \frac{dy}{dx} = \frac{(1+e^{-x}) \cdot \frac{dy}{dx}(1) - (1) \times \frac{dy}{dx}(1+e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{(1+e^{-x}) \cdot 0 - (1) \cdot (0+e^{-x}) \times \frac{dy}{dx}(e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{0 - (1) \cdot (e^{-x}) \cdot (-1)}{(1+e^{-x})^2}$$

$$= \frac{0 + e^{-x}}{(1+e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\therefore \frac{dy}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

4) Given $f = \dfrac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}$, where $i \in \{1, 2, 3, \ldots K\}$.

compute $\dfrac{df}{dz_i}$

$\longrightarrow$ $f = \dfrac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}$, where $i \in \{1, 2, 3, \ldots, K\}$.

Applying u/v rule,

$$\therefore \frac{df}{dz_i} = \frac{\sum_{k=1}^{K} e^{z_k} \times \frac{d}{dz_i}(e^{z_i}) - e^{z_i} \times \frac{d}{dz_i}\left(\sum_{k=1}^{K} e^{z_k}\right)}{\left(\sum_{k=1}^{K} e^{z_k}\right)^2}$$

$$\frac{df}{dz_i} = \frac{\sum_{k=1}^{K} e^{z_k} . e^{z_i} - e^{z_i} \times e^{z_i}}{\left(\sum_{k=1}^{K} e^{z_k}\right)^2}$$

$$\frac{df}{dz_i} = \frac{\cancel{\sum_{k=1}^{K} e^{z_k}} . e^{z_i}\left(\sum_{k=1}^{K} e^{z_k} - e^{z_i}\right)}{\left(\sum_{k=1}^{K} e^{z_k}\right)^2}$$

It can also written as.

$$\frac{df}{dz_i} = \frac{\sum_{k=1}^{K} e^{z_k} . e^{z_i}}{\left(\sum_{k=1}^{K} e^{z_k}\right)^2} - \left(\frac{e^{z_i}}{\sum_{k=1}^{K} e^{z_k}}\right)^2.$$

$$\frac{df}{dz_i} = f - f^2 = f(1-f)$$

5) Given $L = (w.x - y)^2$, compute $\frac{dL}{dw}$

$\longrightarrow$ $L = (w.x - y)^2$.

$\therefore \frac{dL}{dw} = \frac{d}{dw}(wx - y)^2$

$= 2.(wx - y) \times \frac{d}{dw}(wx - y)$

$= 2(w.x - y) \times \left[x.\frac{d}{dw}(w) - \frac{d}{dw}(y)\right]$

$= 2(w.x - y) \times [x - 0]$

$\frac{dL}{dw} = 2.x.(w.x - y)$

6) Given $L = -y.\log(w.x)$, compute $\frac{dL}{dw}$.

$\longrightarrow$ $L = -y.\log(w.x)$

Applying Chain Rule

$\frac{dL}{dw} = -y.\frac{d}{dw}(\log(w.x)) + \log(w.x).\frac{d}{dw}(-y).$

$= -y.\frac{1}{w.x} \times \frac{d}{dw}(w.x) + \log(w.x).0$

$= -y.\frac{1}{w.x} \times \cancel{w}x + 0.$

$\frac{dL}{dw} = \frac{-y}{w}$

7) Given $y = w^T \cdot x$, where $y \in \mathbb{R}$, $w \in \mathbb{R}^D$ & $x \in \mathbb{R}^D$, compute $\nabla_w y$.

$\longrightarrow$ $y = w^T \cdot x$, $y \in \mathbb{R}$, $w \in \mathbb{R}^D$ & $x \in \mathbb{R}^D$

$$\therefore \nabla_w y = \begin{bmatrix} \frac{\partial y}{\partial w_1} \\ \frac{\partial y}{\partial w_2} \\ \vdots \\ \frac{\partial y}{\partial w_3} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_D \end{bmatrix}$$

8) Given $y = w^T \cdot w$, where $y \in \mathbb{R}$, $w \in \mathbb{R}^D$, compute $\nabla_w y$

$\longrightarrow$ $y = w^T \cdot w$, $y \in \mathbb{R}$, $w \in \mathbb{R}^D$

$$\therefore \nabla_w Y = \begin{bmatrix} 2w_1 \\ 2w_2 \\ 2w_3 \\ \vdots \\ 2w_D \end{bmatrix}$$

$$= 2 \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_D \end{bmatrix}$$

$$\nabla_w Y = 2w$$

9) Given $L = (W^T \cdot x - y)^2$, where $w \in \mathbb{R}^D$ and $x \in \mathbb{R}^D$ compute $\nabla_w L$.

→ $L = (W^T \cdot x - y)^2$, where $W \in \mathbb{R}^D$ and $x \in \mathbb{R}^D$.

$$\therefore \nabla_w L = 2 \cdot \frac{d}{dw} (W^T \cdot x - y)^2$$

$$= 2 \cdot (W^T \cdot x - y) \times \frac{d}{dw} (W^T \cdot x - y)$$

$$= 2 (W^T \cdot x - y) \times (x_i - 0)$$

$$\therefore \nabla_w L = 2 (W^T \cdot x - y) \times (x_i)$$

$$\nabla_w L = 2 (W^T \cdot x - y) \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix}$$

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Data Science - II : HomeWork - 1

2] Linear Algebra

1) Create a matrix $W \in \mathbb{R}^{3\times 2}$. Write down W and W^T

→ Let us consider W as,

$$W = \begin{bmatrix} 8 & 9 \\ 16 & 18 \\ 24 & 27 \end{bmatrix}_{3\times 2}$$

$$\therefore W^T = \begin{bmatrix} 8 & 16 & 24 \\ 9 & 18 & 27 \end{bmatrix}_{2\times 3}$$

2) Given $W = [2, 1, 1]$, $X = [1, 2, 0]$, compute $W \cdot X^T$

→ $W = [2, 1, 1]_{1\times 3}$, $X = [1, 2, 0]_{1\times 3}$.

$$\therefore X^T = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}_{3\times 1}$$

$$W \cdot X^T = [2, 1, 1]_{1\times 3} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}_{3\times 1}$$

$$W \cdot X^T = [4]_{1\times 1}$$

3) Given $w = [0, 2, 1]$, $x = [2, 1, 1]$. compute $w^T \cdot x$

$\longrightarrow$ $w = [0, 2, 1]_{1\times 3}$, $x = [2, 1, 1]_{1\times 3}$

$$\therefore w^T = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}_{3\times 1}$$

$$w^T \cdot x = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}_{3\times 1} [2, 1, 1]_{1\times 3}$$

$$w^T \cdot x = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}_{3\times 3}$$

4) Given $X = [2, 1, 1; 1, 0, 1]$, $w = [1, 0, 1]$. compute $X \cdot w^T$

$\longrightarrow$ $X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$ $w = [1, 0, 1]_{1\times 3}$

$$w^T = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_{3\times 1}$$

$$X \cdot w^T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}_{2\times 1}$$

5) Given $X = [2,1,1;1,0,1]$, $W = [1,0;1,1]$
Compute $W \cdot X$

$\rightarrow$ $X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$ $W = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2}$

$\cancel{X \cdot W}$ $W \cdot X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$W \cdot X = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}_{2\times 3}$.