2803: Dynamics Lab Report

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This lab aimed to design a re-configurable roller coaster as a pilot simulator using Virtual Reality (VR) goggles. To perform this lab, we each focused on different track elements, including a loop, a zero-G parabola, a banked turn, and a braking section. The end objective is to produce a program in MATLAB that would calculate the G-forces (normal, lateral, and tangential) experienced by the cart for each track element. The results showed that G-forces remained within the required limits for a semi-realistic yet safe training environment by selecting different track heights and curvatures.

I. Derivations: Free-Body Diagrams and Governing Equations

A. Loop

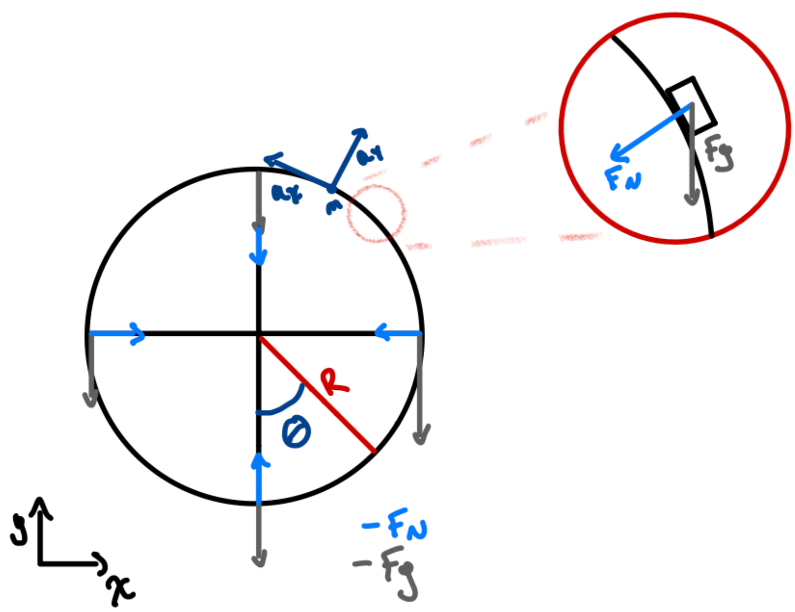

Figure 1. Loop Free Body and Acceleration Diagrams

By revolving the cart in a radial direction, we can express the gravitational force component as the cosine component of gravity, where θ starts at the loop's entrance, contributing to the normal force at different points around the loop. We then manipulated the centripetal equation to be in terms of the normal force - derived in Appendix 1.1.4. We placed the acceleration diagram at 160° and made separate free-body diagrams (FBD) for the bottom, top, and midpoints of the loop. We created a loop function in our MATLAB code, which implemented Appendix 1.1.5, to find all the G forces acting around the loop. We then plotted these forces against the track length to achieve the graph shown in [Figure 9]. (refer to appendix (1.1) for derivations of the loop equations).

B. Parabola

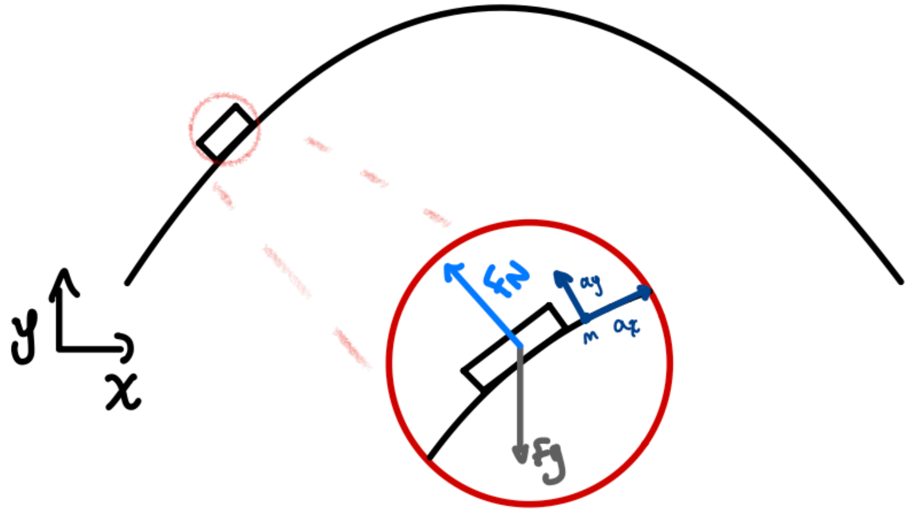

Figure 2. Parabola Free Body and Acceleration Diagrams

We applied gravity and a force normal to the track to create the free-body diagram for the parabola. We used a reference frame fixed to the ground and expressed the car's direction in terms of an angle from the horizontal in this reference frame. We obtained the angle by taking the arctangent of the derivative of the parabola (appendix 1.2.1- 1.2.4 shows this derivation). To create a zero-g parabola, we assumed the normal force applied by the track is zero. We can then parameterize the parabola starting from acceleration in the horizontal and vertical directions, with no horizontal acceleration and a vertical acceleration equal to gravity. Integrating the acceleration yields parameterized velocity and position equations. Rewriting in terms of horizontal and vertical position allows us to solve for instantaneous radius of turn (which was given) - appendix 1.2.1 & 1.2.9. Taking the centripetal acceleration from gravity in this reference frame to be equal to the cosine component of gravity, we checked against the velocity (appendix 1.2.7) and radius of the curve at each point using MATLAB and arrived at a normal force (appendix 1.2.8) for each point. This model gave us a track-induced acceleration in terms of g on the order of 10^{-16} [Figure 10], which we take to be equal to zero. (Refer to appendix 1.2.10 for details on g force components.)

C. Braking

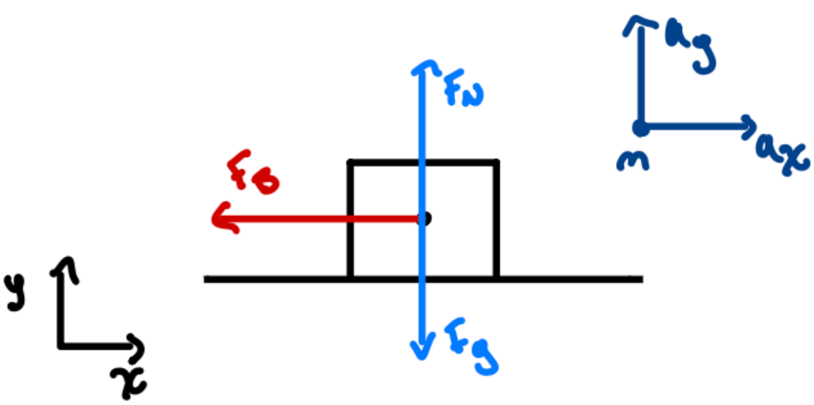

Figure 3. Braking Free Body and Acceleration Diagrams

The analysis of the Free Body Diagram for the braking section began with applying Newton's Second Law to account for the decelerating force acting on the cart - in appendix 1.3.1. The cart is supposed to constantly stop in the braking section, so we modeled the tangential force (braking force) as a constant force that decelerates the cart to a stop. We drew the acceleration diagram to view the cart's deceleration in the negative horizontal direction. By setting a constant deceleration to -30 ms^{-2}, we created a constant g-force the cart experienced throughout the braking process. The constant deceleration did not exceed G-force limits. Utilizing this constant deceleration, the length of the track was computed using a distance kinematic equation- refer appendix 1.3.2 for details. We implemented a braking function using MATLAB to compute and plot the deceleration forces along the braking path, as visualized in [Figure 12]. This analysis verified that the braking system provided a smooth deceleration without exceeding acceptable g-force limits for safety and comfort. (refer to 2.4 for more details on the g force components)

D. Transition

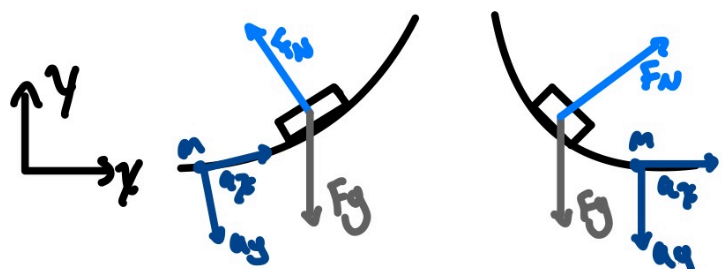

Figure 4. Transition Free Body and Acceleration Diagrams

The transition sections ensure smooth, realistic roller coaster motion between the significant track elements. These sections help gradually adjust the g-forces to avoid abrupt changes in acceleration. To analyze the forces in the transition sections, we consider the gradual shift in normal force as the cart moves from one element to another. Since the transitions mirrored a portion of the loop, the normal force acting on the cart also varies just like a loop (Equation in appendix 1.2.4). A circular transition from a sloped track to a flat track ensures that forces remain continuous without sharp changes that could lead to high jerks. The transition sections are strategically placed before and after key elements such as loops, parabolic sections, and banked turns to allow for a gradual shift in velocity and g-forces. In MATLAB, we implemented transition sections by using the height of where the track is and either an increasing or decreasing slope depending on other track element requirements. The full rollercoaster plots show the g-forces enacted by the transition sections. (refer to 2.4 for more details on the g-forces)

E. Banked turn

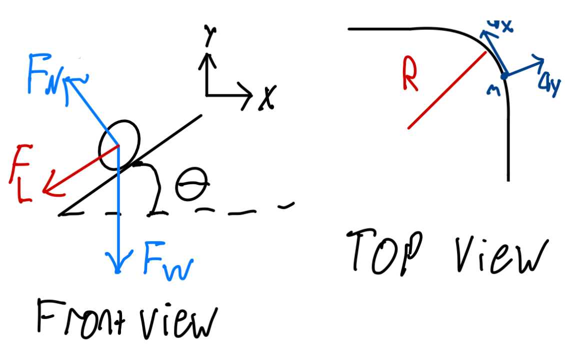

Figure 5. Banked Turn Free Body and Acceleration Diagrams

The banked turn portion is the only rollercoaster section where lateral forces could occur. However, as seen in the derived equations (appendix 1.5.3), we aimed to make the lateral force 0N during the banked turn. We arbitrarily put the banked turn at 100 meters and set the turn's slope to 60 degrees. Using these parameters, we set the lateral force equal to zero and found a radius, $50\cot(\pi/3)$ or roughly 28.9 meters, that would result in a curve with zero lateral force. With no lateral force, the only force contributing to G-forces acting upon the cart is the normal force. The normal force (appendix 1.5.6) is greater than that of a flat surface because it contributes to the cart's weight and the centripetal acceleration. There is also no tangential force because there is no friction or breaking. However, a lateral force is acting on the cart during the transition into the turn. The transition banks the tracks 60 degrees; during this process, the weight of the cart is supported by both the normal force and the lateral force (appendix 1.5.7). The transition decreases the normal force slightly (relative to a flat track) and is the only time lateral force occurs throughout the rollercoaster. This transition occurs right before and after the turn, producing lateral forces. The normal force decreases during the transition but increases during the turn, and there is no tangential force throughout.

II. Performance Analysis

A. Whole Track

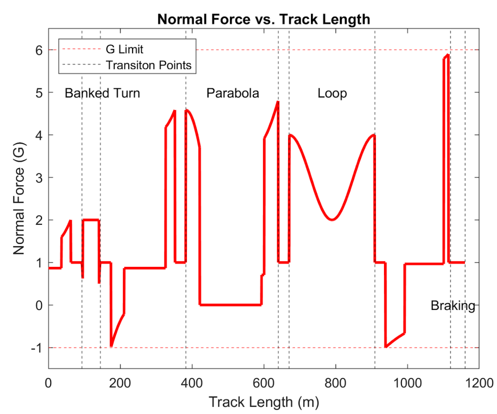

Figure 6. Normal G-Forces throughout entire track

This is the normal force felt throughout the whole length of the track, which varies significantly. Each transition shows where the track elements occur, along with the transitions at the beginning and end of the track elements. The other parts of the track result from flat portions, sloped portions, and circular transitions between them. The flat portions have a constant normal force of 1, the sloped portions have a constant normal force of $cos\theta$, and the transitions are the small portion of a circular loop (equal to theta of the slope), so the normal Gs are similar to that experienced in the loop. The normal G-forces are not constant over the transition sections, and they can be seen in the curved sections of the plot.

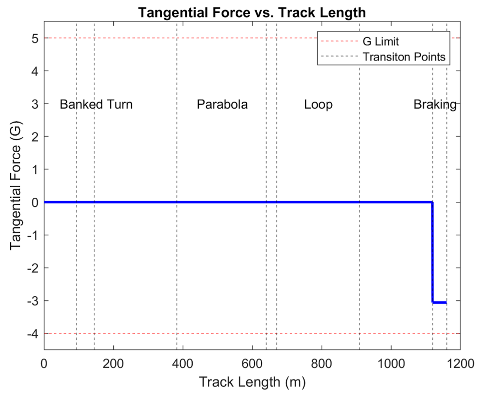

Figure 7. Tangential G Forces vs Track length

We assume there is zero tangential force for all sections except the braking section because there is no friction, brakes, or boosters anywhere on the track except the braking section. The braking section of the track is the only place where tangential acceleration occurs, and it pushes the rider back.

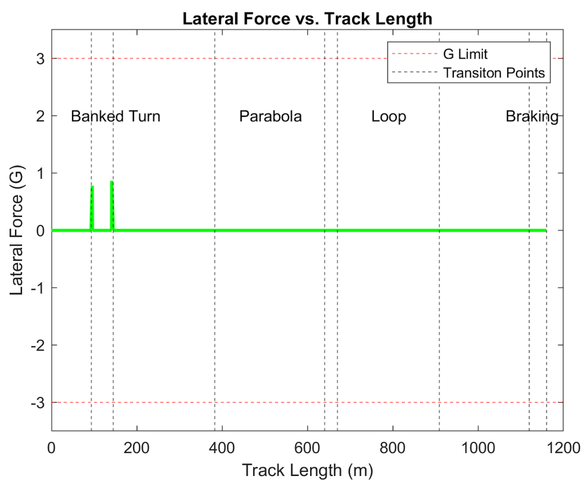

Figure 8. Lateral force vs track length

Since there are no sideway movements on the rollercoaster except for the banked turn, there are no other lateral forces. The transition sections for the banked turn that banks the track produces the only lateral forces throughout the track (explained in I-D) .

B. Loop Section

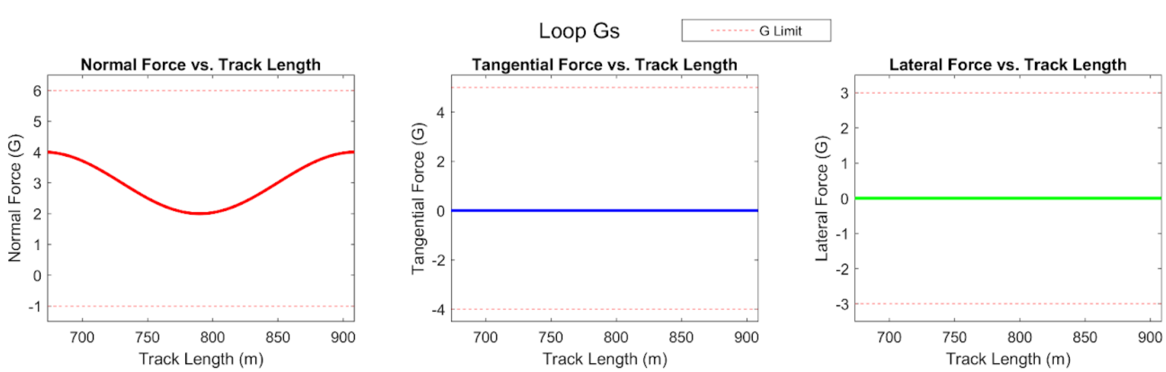

Figure 9. Normal force, tangential force, and lateral force vs track length for the loop section.

By equating velocity in terms of change in height and taking the gravitational force component acting radially and making it a function of height, while taking track length as a function of height (refer to Appendix 1.1.6), mapping it to each other using the common h variable. No sideways movement/forces exist; lateral force is zero throughout the loop, and the tangential graph is zero throughout (explained in I-D).

C. Parabola Section

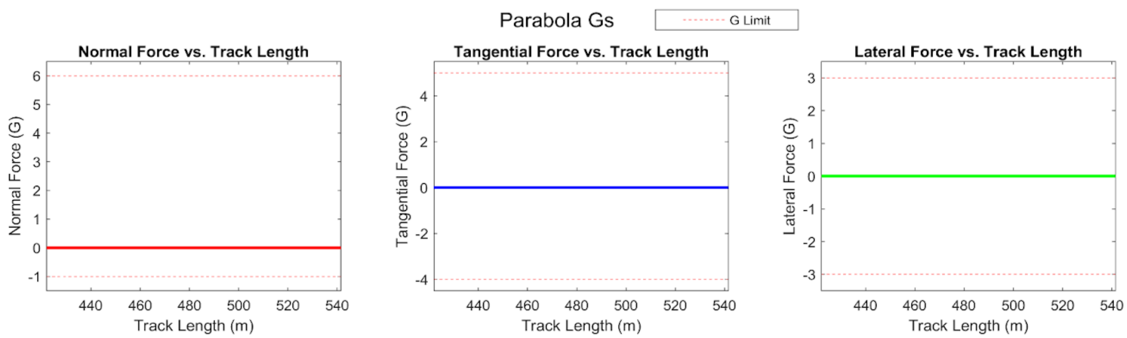

Figure 10. Normal force, tangential force, and lateral force vs track length for the parabola section.

The parabola section only experiences the force of gravity and is therefore in freefall. As a result, all the normal, tangential, and lateral forces are zero, as shown.

D. Banking Turn

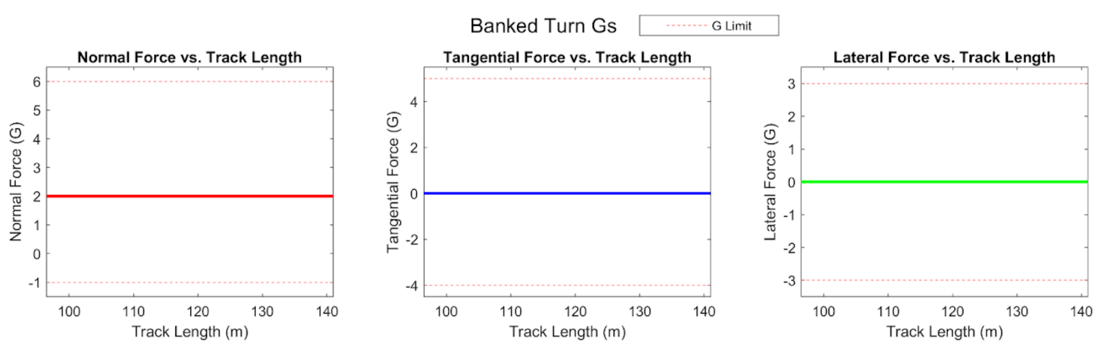

Figure 11. Normal force, tangential force, and lateral force vs track length for the banked turn section.

The banking of the turn at 60 degrees and a turning radius of roughly 28.9 meters causes the lateral force to be zero because the normal force fully supports the weight. The normal force is roughly 2 because it supports gravitational and centripetal forces (refer I-E for more information), and the tangential force remains zero (as explained in I-D).

E. Braking

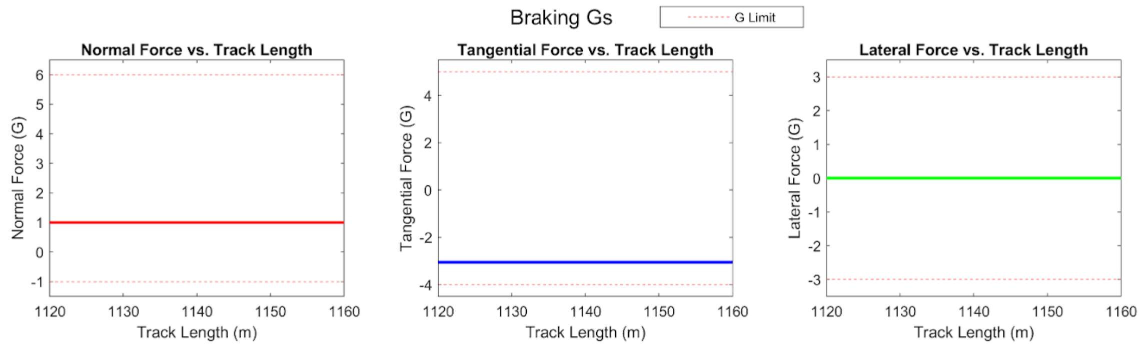

Figure 12. Normal force, tangential force, and lateral force vs track length for the braking section.

The braking section has the normal force that counteracts gravity; therefore, normal force vs track length has one G-force, lateral force is zero (as explained in I-D), and tangential force is around 3 G-forces backwards (relative to the rider) as the braking force acts tangentially in the opposite direction of motion.

III. Conclusion

Our results demonstrated that track element designs allow us to have a level of control over the G-forces that the cart experiences; we kept them within safe and reasonable limits. The loop provided a dynamic variation in G-forces, with the highest forces occurring at the bottom due to centripetal acceleration. The zero-G parabola successfully achieved an extended period of weightlessness, which validated our calculations. The banked turn minimized lateral G-forces by appropriately selecting the banking angle, reducing the impact on the rider. The braking section effectively stopped the cart while staying under the G-force limits. Our MATLAB simulation made designing a pilot simulator roller coaster with controlled forces feasible. However, future refinements could enhance realism and applicability, such as incorporating frictional losses and variable braking forces.

One of this model's main limitations is assuming that friction and air resistance will be negligible. However, in reality, friction between the track and the cart's wheels and air resistance reduces the cart's velocity throughout the track length. Assuming no friction leads to discrepancies between the predicted and actual G-forces experienced in a real-world application. Including frictional (throughout the whole track) and aerodynamic forces in future models, iterations would improve accuracy by accounting for these energy losses. Additionally, since we modeled the braking system to have a constant deceleration force, it does not accurately reflect how a roller coaster would slow down in real life. In practice, a roller coaster has a varied braking system, which leads to a smoother, more controlled stop. Future improvements should include a varied braking system that better simulates real-world roller coaster braking mechanisms and enhances safety analysis.

Member Contributions

McCathy Devine - Banked turn math, code, and report and code for transition and braking sections.
Abhishek Siwakoti- Braking section report, transitions report, FBDs, derivations, loop report, loop code.
Alex Zhang- Parabola math, code, report, and report formatting.
Saketh Guttapalli- Loop derivations, loop code, performance analysis, review and edits of the report.

Appendix

1. Derivations:

1.1 Loop (derivations go 1-4)

1) $F_c = F_N + F_G$
2) $F_c = F_N + cos(180 - \theta) * F_G$
3) $F_c = F_N + F_G cos(\theta)$
4) $F_N = \frac{mV^2}{R} - mgcos(\theta)$
5) $g_N = \frac{V^2}{g} - cos\theta$ (normal g- force), tangential and lateral forces are 0

   $(track\ length \frac{\theta}{360} * 2 * \pi R,\ where\ \theta = cos^{-1}(\frac{R - \Delta h}{R})$)

1.2 Parabola

1) $y(x) = -\frac{1}{2} g\, (\frac{x}{v_0 cos\theta_0})^2 + tan(\theta_0)x$
2) $y'(x) = -g\, (\frac{x}{v_0 cos\theta_0}) + tan(\theta_0)$
3) $y''(x) = (\frac{-g}{v_0 cos\theta_0})$
4) $\theta = tan^{-1}(y'(x))$, referenced against horizontal
5) $\theta_0 = 45^\circ$, initial launch angle, arbitrarily chosen
6) $v_0 = \sqrt{2g(h_0 - h_1)}$, as per lab slides
7) $v = v_0 - \sqrt{2g(y - h_1)}$, velocity at start of element - velocity change due to height change
8) $N = \frac{mv^2}{R} - mgcos(\theta)$, sum of forces - force of gravity
9) $R = \frac{(1 + (y'(x))^2)^{\frac{3}{2}}}{y''(x)}$, as per lab slides
10) The g's for normal, tangential, and lateral force components are 0

1.3 Braking

1) $F_b = -ma$
2) $\frac{v_i^2}{2a} = s,\ v_i = \sqrt{2g(h_o - h_1)},\ where\ h_0 - h_1 = 125m\ (total\ height\ of\ the\ coaster)$

1.4 Transition Section

1) $F_N = \frac{mV^2}{R} - mgcos(\theta)$

1.5 Banked Turn

1) $\sum F_Z = Ncos\theta - Lsin\theta - mg = 0$
2) $\sum F_r = Nsin\theta + Lcos\theta = \frac{mv^2}{r}$
3) For L = 0 $N = \frac{mg}{cos\theta}$ so $\frac{mg}{cos\theta} \times sin\theta = \frac{mv^2}{r}$

Substituting $v = \sqrt{2g\Delta h}$ we get $r = \frac{2\Delta h}{tan\theta}$

4) $r = 50cot(pi/3)$
5) $\sum F_Z = L = Ncot\theta - \frac{mg}{sin\theta} = 0$
6) $\sum F_r = L = \frac{mv^2}{rcos\theta} - Ntan\theta = 0$
7) $Ncot\theta - \frac{mg}{sin\theta} = \frac{mv^2}{rcos\theta} - Ntan\theta$
8) $N(cot\theta + tan\theta) = mg(\frac{1}{sin\theta} + \frac{2\Delta h}{rcos\theta})$
9) Gs experienced in Normal direction: $N = \frac{(\frac{1}{sin\theta}+\frac{2\Delta h}{rcos\theta})}{(cot\theta+tan\theta)}$
10) Transition: $N = mgcos\theta$, $L = mgsin\theta$ - Theta varies from 0 to $\Pi/3$ over 4 meters