ASEN 2803 Lab 2: Locomotive Crank

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Abstract

The objective is to develop a model to define the ideal relationship between the wheel's rotational motion and the collar's translation. The apparatus uses a collar that moves vertically along a rod, linked to a rotating disk, this relationship was analyzed to study the kinematic relationships. We derived a model that expressed the collar's velocity in terms of the disk's position and velocity. We used MATLAB and pre-defined functions to calculate the vertical velocity and model velocity experienced by the collar for each degree of rotation. The obtained and observed modeled collar velocities are compared and checked for accuracy using angle input 0 to 6 revolutions. The obtained results show that the model was relatively accurate at lower speeds and that at higher speeds, discrepancies due to sensor errors and unmodeled dynamics became more apparent. Improving sensor calibration and refining the model could enhance accuracy.

Color Coding:
Objective
Problem statement
Approach
Results
Conclusion

I. Procedure and Experimental Setups

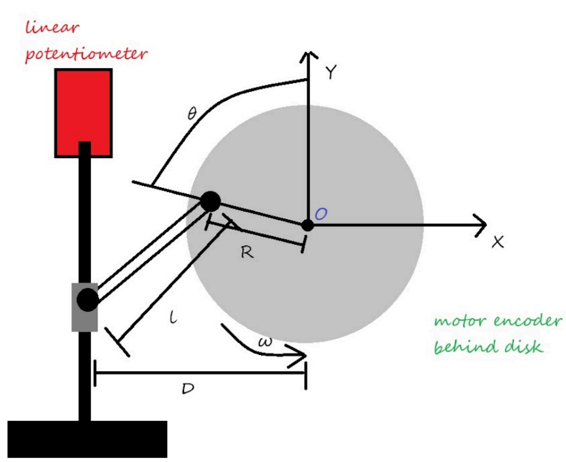

Figure (1)

Nomenclature

- O: The center point of the disk and the origin of the coordinate system used in the derivations.
- X,Y: Reference Axes used in the derivations.
- R: Radial distance between the center of the disk and where the crank bar connects to the disk.
- L: Length of the crank bar between the disk and collar.

- D: Horizontal distance between the center of the disk and the vertical bar where the collar slides.
- θ: Angle between the Y reference and the line that intersects the disk/bar connection and origin.
- ω: Angular velocity of the disk.
- Linear Potentiometer: Measures the height of the collar on the vertical shaft by varying electrical resistance and then recording the resulting difference in voltage.
- Motor Encoder: Measures the angle *(θ)* by recording optical changes in an encoder disk, which is then interpreted by a controller to determine motion.

Measurements

R = 8.1 cm ∓ 5% D = 15.5 cm ∓ 5% L = 25.5 cm ∓ 5%

Motion of Different Voltages

We tested the locomotive crankshaft at 6 different voltages. For all voltages, the speed of the collar was greatest when it is at the bottom of the shaft and slowest at the top, where it underwent a cyclical process in between these velocities and positions. Notably, the average angular velocity of the disk increased with an increase in the voltage supplied. This, in turn, increased the linear velocity of the collar at higher voltages. Overall, for every configuration, we put them through a similar cyclical motion process where varying voltage settings only vary the revolutions rate at which the system went through a complete cycle.

II. Derivations: Free-Body Diagrams and Governing Equations

Refer to the Figure (2) below for Derivations

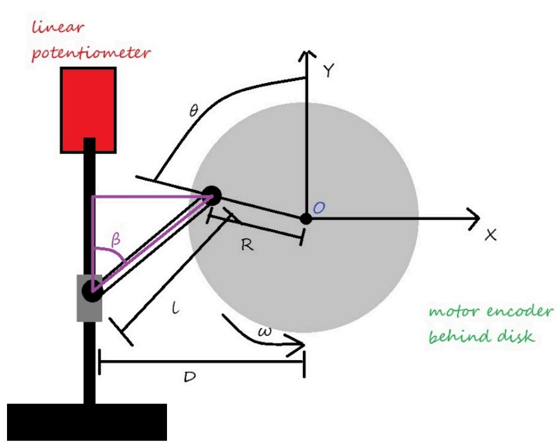

Figure (2)

The angle β can be written in terms of the angle θ as follows:

First, it is possible to construct a right triangle (in purple) with hypotenuse ℓ as shown in Figure 2. The vertical leg of the triangle will have length $d - rsin(\theta)$ and be opposite the angle β.

By the law of sines, it is possible to solve for β in terms of θ and constant geometric aspects of the locomotive crank:

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$\frac{d-rsin\theta}{sin(\beta)} = \frac{\ell}{sin(90)} \Rightarrow sin(\beta) = \frac{d-rsin(\theta)}{\ell}$

$\beta = sin^{-1}\frac{d-rsin(\theta)}{\ell}$

From here, it is possible to find an equation represented v_B as follows:
The vector between the center of the disk and the bar's attachment point

Step 1: $\hat{r} =- rsin(\theta)\hat{\imath} + rcos(\theta)$

Step 2: $v_A = \omega_A \times \hat{r} =- \omega_A rcos(\theta)\hat{\imath} - \omega_A rsin(\theta)\hat{\jmath}$

Step 3: $r_{B/A} = \ell sin(\beta)\hat{\imath} - \ell cos(\beta)\hat{\jmath}$.

Step 4: $v_B = v_A + \omega_R \times \hat{r}_{B/A} = (- \omega_A rcos(\theta))\hat{\imath} - \omega_A rsin(\theta))\hat{\jmath} + \omega_R \ell cos(\beta)\hat{\imath} - \omega_R \ell sin(\beta)\hat{\jmath}$

Step 5: $v_B = (- \omega_A rcos(\theta) + \omega_R \ell cos(\beta))\hat{\imath} + (- \omega_A rsin(\theta) - \omega_R \ell sin(\beta))\hat{\jmath}$

Point B is restricted by the bar to vertical movement, so it's $\hat{\imath}$ component must be trivial. This equality can be used to solve for the value of ω_R:

Step 6: $- \omega_A rcos(\theta) + \omega_R \ell cos(\beta) = 0 \Rightarrow \omega_A rcos(\theta) = \omega_R \ell cos(\beta) \Rightarrow \omega_R = \frac{\omega_A rcos(\theta)}{\ell cos(\beta)}$

Step 7: $v_B = (0)\hat{\imath} + (- \omega_A rsin(\theta) - \omega_R \ell sin(\beta))\hat{\jmath} = (\frac{\omega_A rcos(\theta)}{\ell cos(\beta)} \ell sin(\beta) - \omega_A rsin(\theta))\hat{\jmath} + 0\hat{\imath}$

Step 8: $v_B =- \omega_A(rcos(\theta)tan(\beta) + rsin(\theta))\hat{\jmath} + 0\hat{\imath}$

The MATLAB function LCSMODEL returns the value of v_B for a given $\theta$and geometric constraints using equations derived above. As follows

```
function v_mod = LCSMODEL(R, D, L, theta, w)
   % Compute beta (angle of connecting rod)
   beta = asin((D - R * sind(theta) ) / L);

   % Compute vertical velocity of the collar
   v_mod = -w .* (R .* cosd(theta) .* tand(beta) + R .* sind(theta));
end
```

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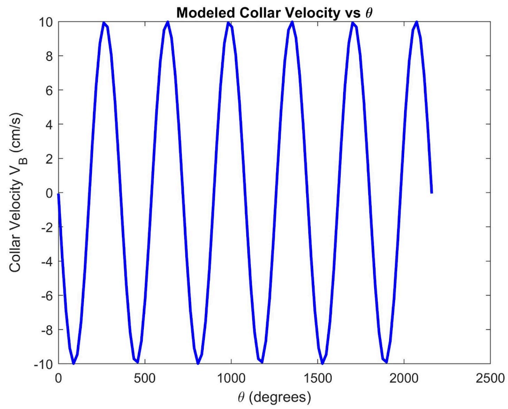

Figure (3)

We tested the accuracy of this function by computing the collar velocity (v_B) as a function of θ for a set of reasonable input parameters. Given the mode of motion, we expected to see a sinusoidal curve, which was exactly what has been calculated in **Figure 2**. This indicated the validity of our derivations and the LCS MODEL function.

III. Results and Analysis

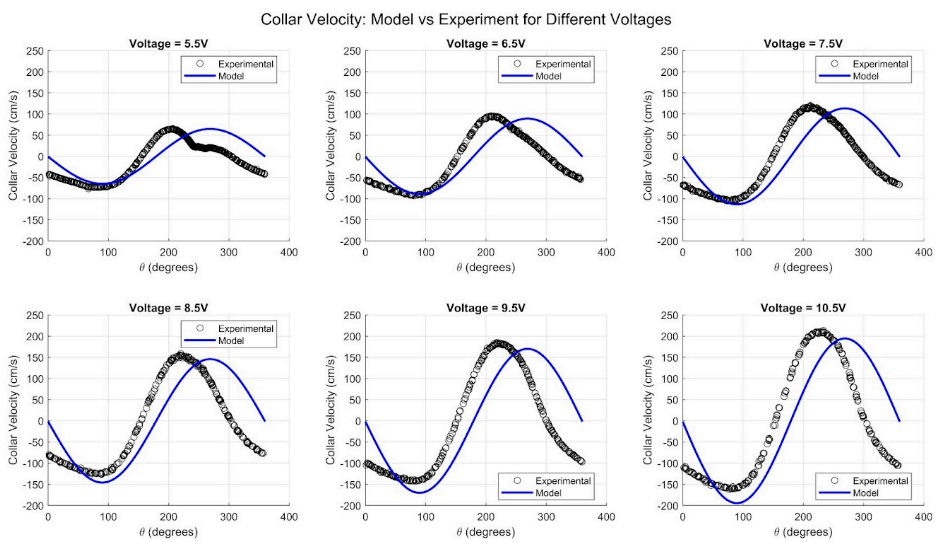

Figure (4)

Figure 4 shows a comparison of our MATLAB model, and experimental data of the apparatus. The model is shown by the blue line, and the data is shown by the black points.

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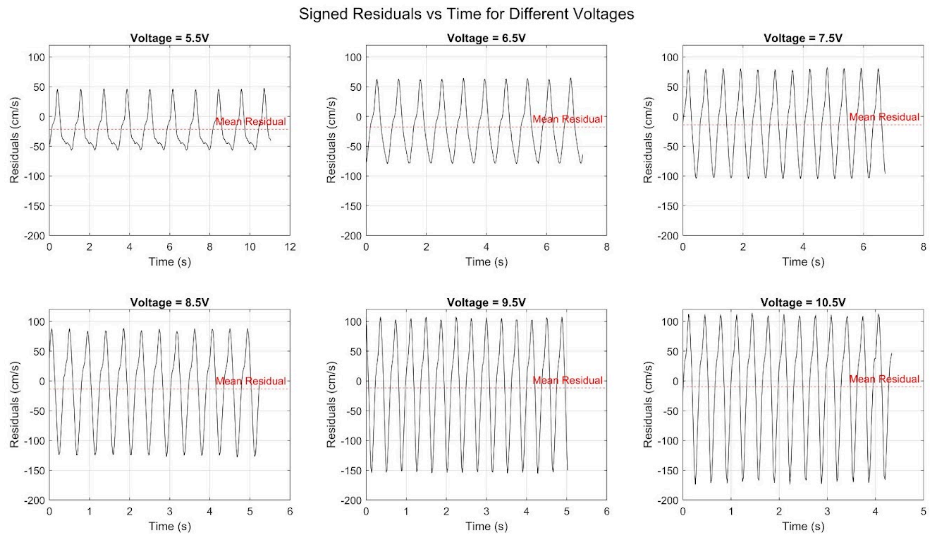

Figure (5)

Figure 5 shows the positive and negative residual error vs time between our model and the experimental data for the different voltage trials. The residual error is the difference between the velocities of the two datasets.

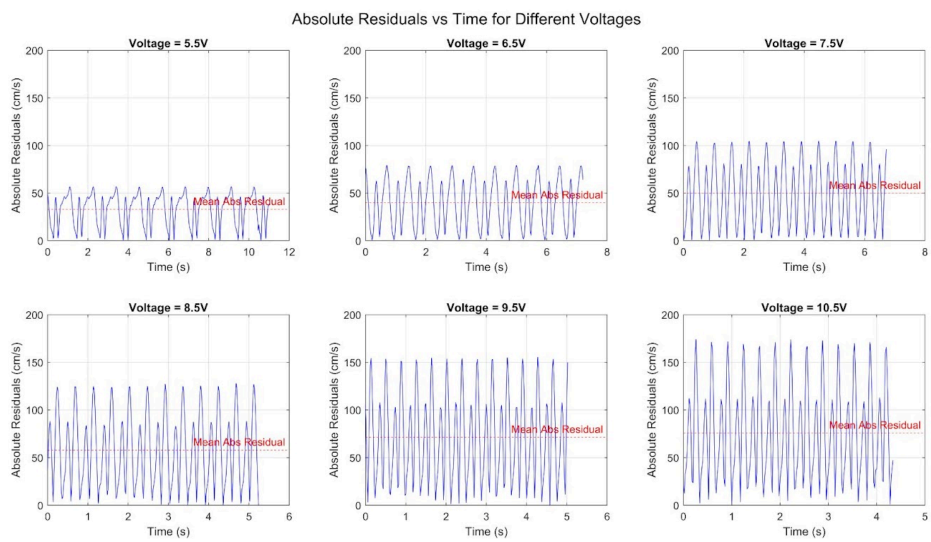

Figure (6)

Figure 6 reflects the absolute value in the difference between the experimental and modeled collar velocities (cm/s) for different voltage settings. With the increase in voltages, the absolute residuals increased and time taken for each revolution decreased.

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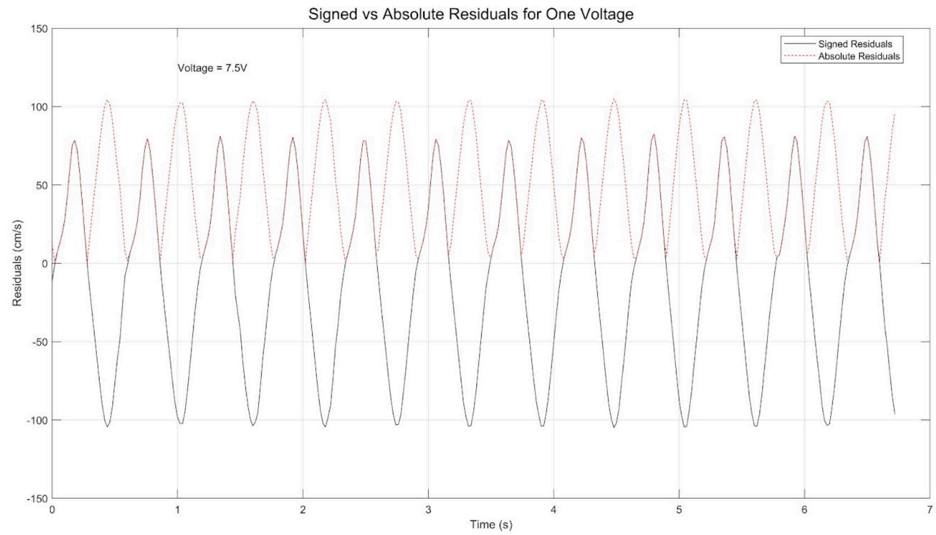

Figure (7)

Figure 7 shows the difference in the absolute value and signed values of the residual errors for the 7.5 volt configuration.

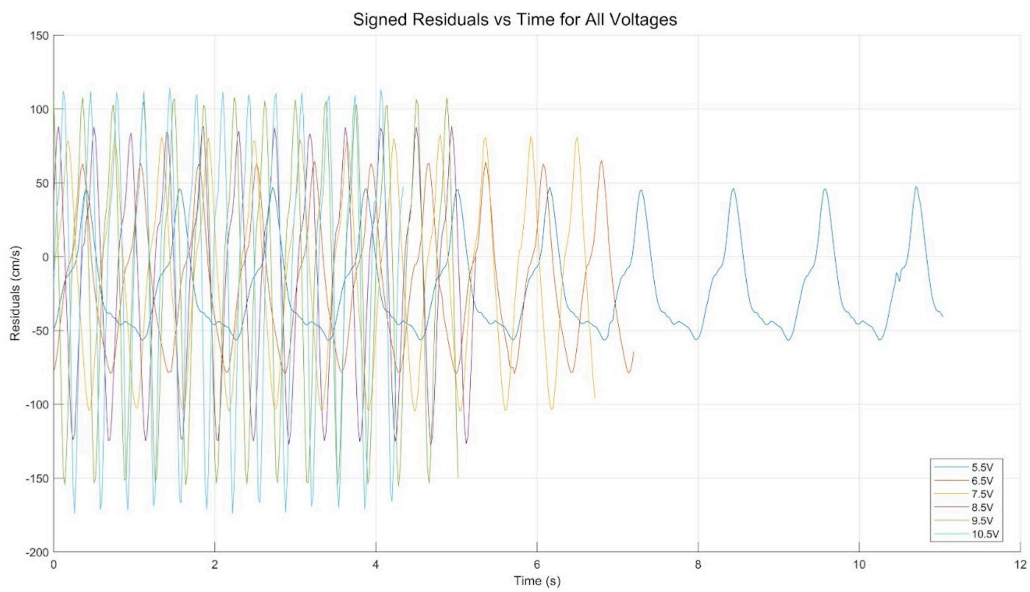

Figure (8)

Figure 8 shows how the residuals change over time and voltage by plotting every configuration on one plot.

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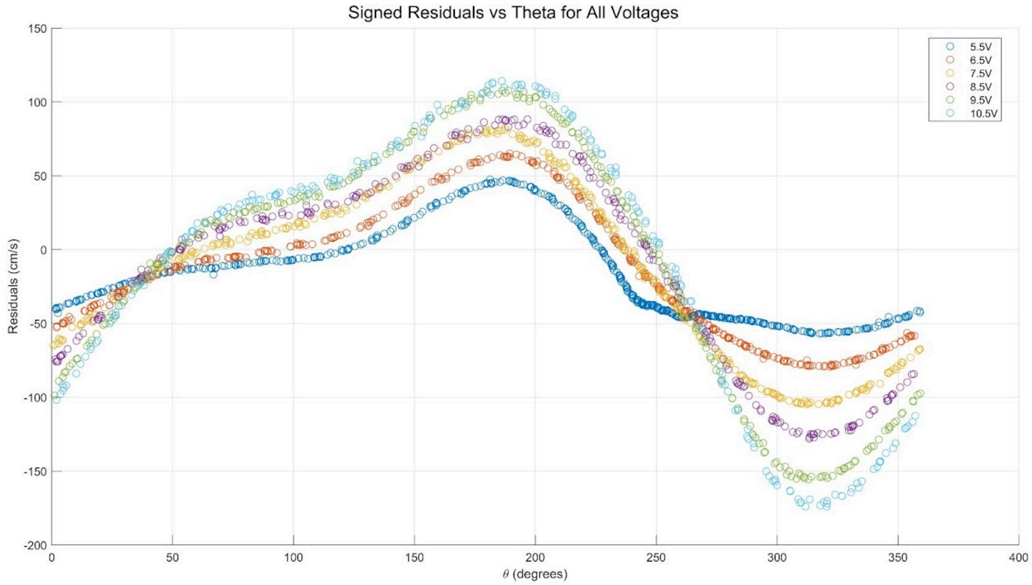

Figure (9)

Figure 9 shows how the residuals change over differing angles and voltage by plotting every configuration on one plot.

Experiment	Voltage	Mean_Error	Std_Deviation
{'Test1_5pt5V' }	5.5	-21.562	29.611
{'Test1_6pt5V' }	6.5	-17.56	43.552
{'Test1_7pt5V' }	7.5	-14.011	57.706
{'Test1_8pt5V' }	8.5	-13.302	67.821
{'Test1_9pt5V' }	9.5	-11.978	83.703
{'Test1_10pt5V'}	10.5	-10.057	89.918

Figure (10)

Figure 10 is a table of the mean and standard deviation of the error for each experiment.

Our model demonstrated adequate accuracy, particularly at lower voltages. However, as the voltage would increase, there were more discrepancies in our model in comparison to the experimental as seen in **Figure 2**. As the mean error is constantly negative, its likely due to misalignment between the model alignment and the system. Additionally, as the voltage increased, the standard deviation also increased drastically; this was most likely influenced by noise. This noise was most likely due to sensor error and friction between the collar and the rod.

IV. Conclusions and Recommendations

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Over the course of this project, our team was able to successfully model the Locomotive Crank. We used derivations by hand and created them in Matlab. Throughout this project, we began with mathematical derivations modeling the system at individual points in time and used these derivations to create Matlab code that could model the Locomotive Crank's motion over time. One of the most challenging aspects of this project was to write the code in a way such that errors due to incorrect derivations were easily correctable. Despite the added difficulty, this allowed us to work on our code before having double-checked the derivations, making the overall process more efficient.

Our model is of adequate accuracy, and is capable of modeling the types of motion seen in the Locomotive Crank, especially at low voltages. Our model could be improved by taking into account more external forces. Such as friction forces throughout the assembly, Individual part weights and inertias, and by taking more precise measurements of equipment. Overall, while the model provided reasonable accuracy at low voltages, addressing the misalignment of our graphs and adding a frictional component into the velocity of our model can drastically improve the accuracy and reliability of our data across more operating conditions. Our group learned about the overall importance of doing derivations by hand, before beginning code, and how important the overall code structure is, and it's ease of corrections when mistakes are made.

Color Coding:
Summary of work
Comments on experiment
Recommendations to improve the model
What we learned from the work

V. Member Contributions

Abhi Siwakoti: Code, Calculations, Figures, Diagrams, Results and Analysis.

Tori Field: Abstract, Derivations, Conclusions and Recommendations.

Tyler Hall: Procedure and Experimental Setups, Diagrams & Figures, Results & Analysis.

VI. Acknowledgments

We would like to thank our professor and TAs who helped us complete the project.

VII. References

1. "ASEN 2803 Spring 2025 Lab 2: Locomotive Crank," report, 2025.

2. "ASEN 2803: Locomotive Crankshaft Lab Spring 2025," lab manual, season-01 2025.

[1] Abhi Siwakoti, ASEN 2803, CU Boulder
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3. Locomotive Crank Lab Slides
4. Locomotive Crank Video Resources

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Appendix

1. Derivations:

$$sin(\beta) \;=\; \frac{d - r\,sin(\theta)}{l}$$

$$\beta \;=\; sin^{-1}\left(\frac{d - r\,sin(\theta)}{l}\right)$$

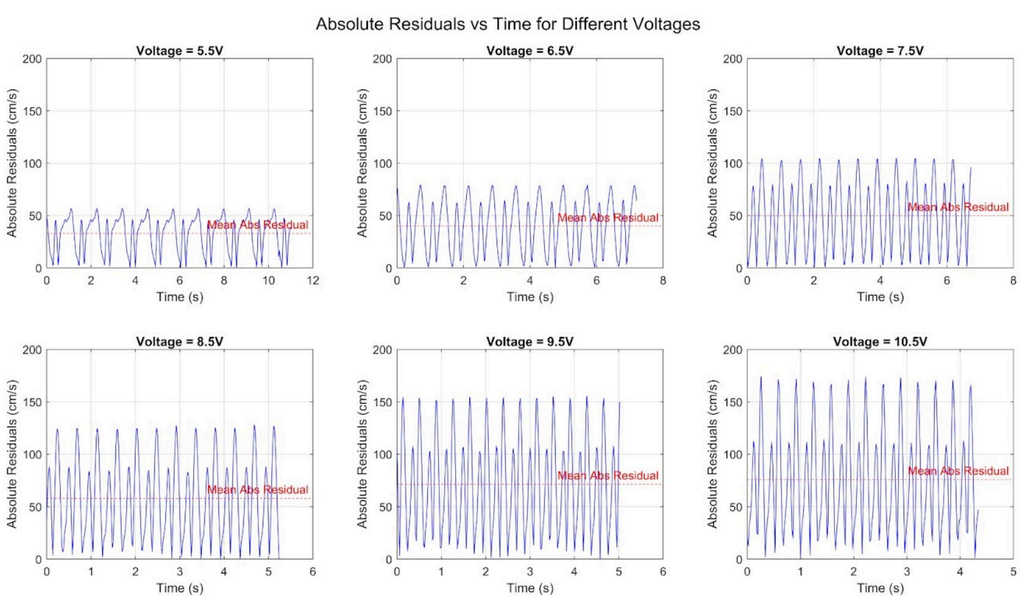
$$v_{Collar_{MODEL}} = -\,\omega(rcos(\theta)tan(\beta) \,+\, rsin(\theta))$$

$$v_{Residuals} = v_{Collar_{MODEL}} - v_{Collar_{EXP}}$$

$$v_{Residuals_{ABS}} = \left|v_{Collar_{MODEL}} - v_{Collar_{EXP}}\right|$$

$r \;=\; Distance\ from\ the\ rotation\ axis\ to\ point\ A\ (cm)$
$d \;=\; Horizontal\ distance\ from\ the\ vertical\ shaft\ to\ the\ disk\ center\ (cm)$
$l \;=\; Length\ of\ the\ connecting\ bar\ from\ A\ to\ B\ (cm)$
$\theta \;=\; Angular\ position\ of\ the\ disk\ (°)$
$\omega \;=\; Angular\ velocity\ of\ the\ disk\ (rad/s)$
$\beta \;=\; Angle\ of\ the\ connecting\ rod\ with\ respect\ to\ the\ horizontal\ (°)$

Figures:



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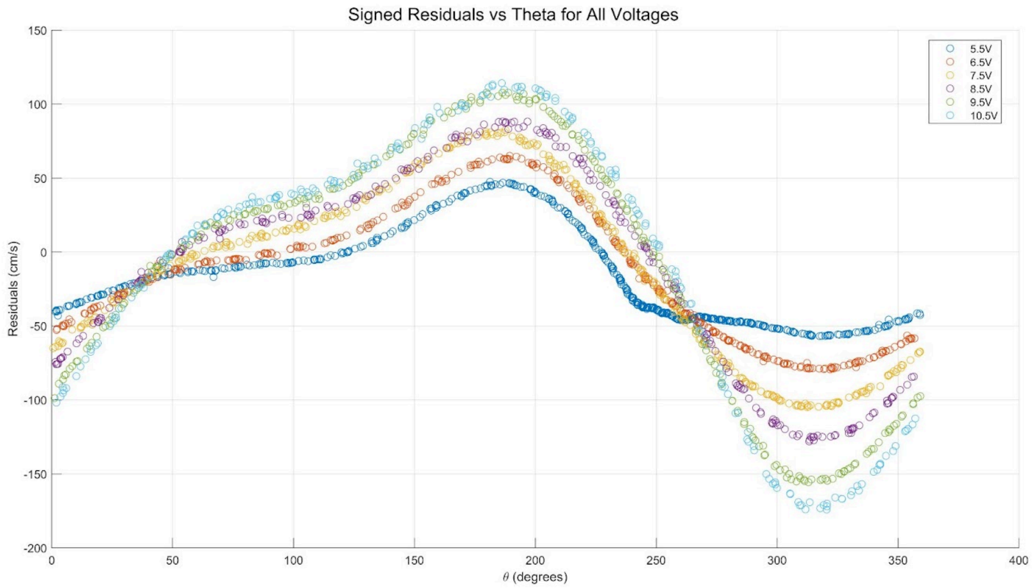

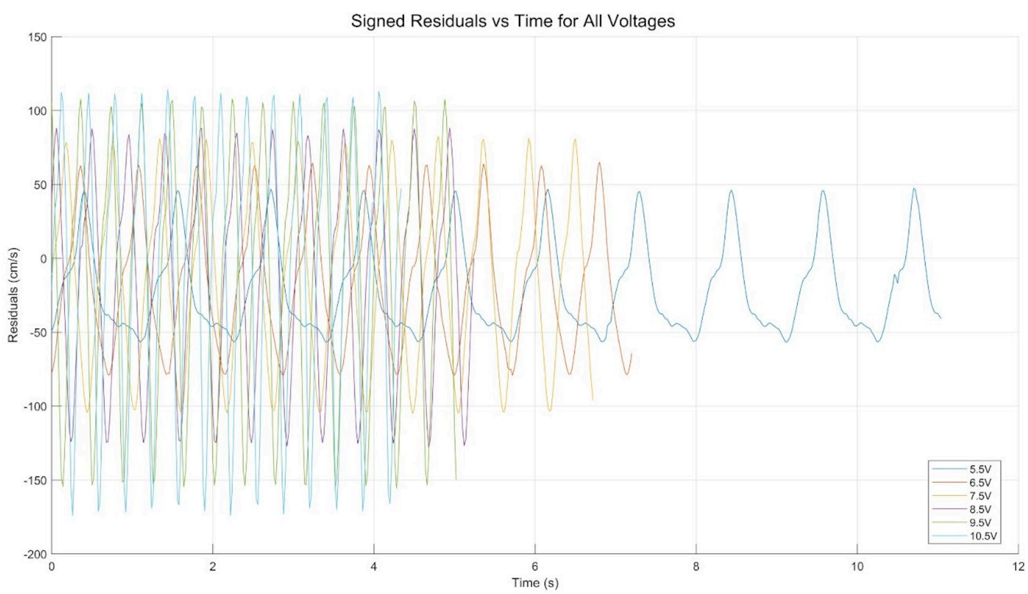

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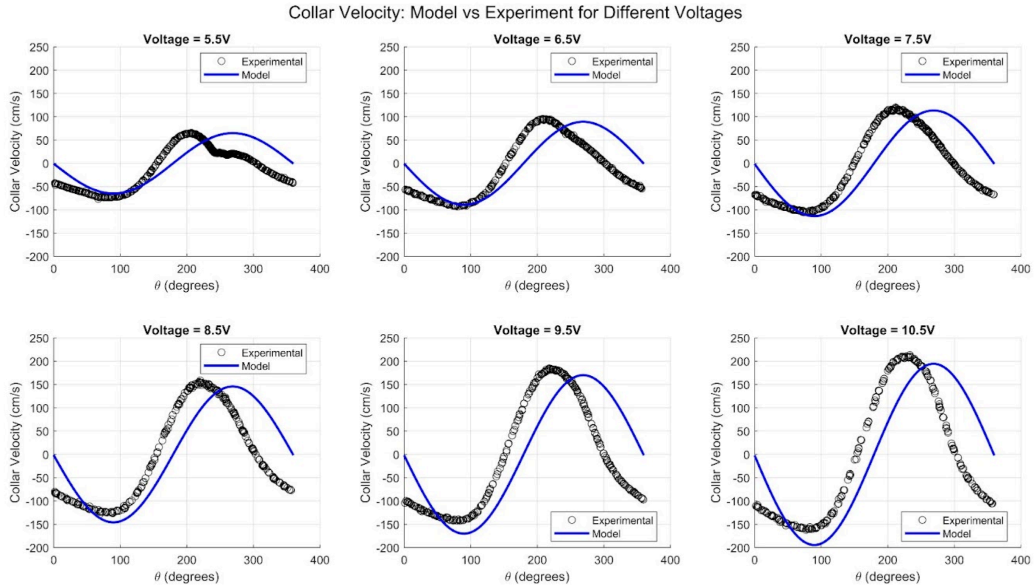

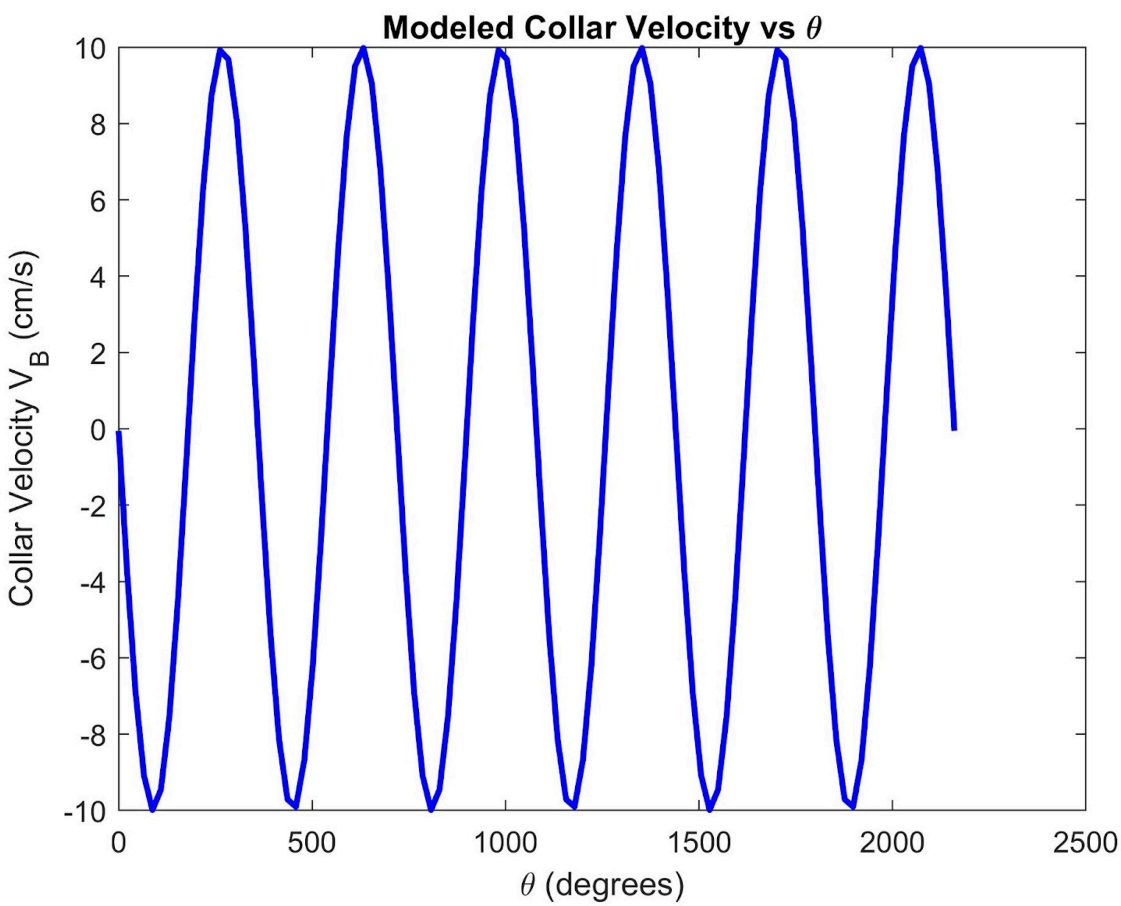

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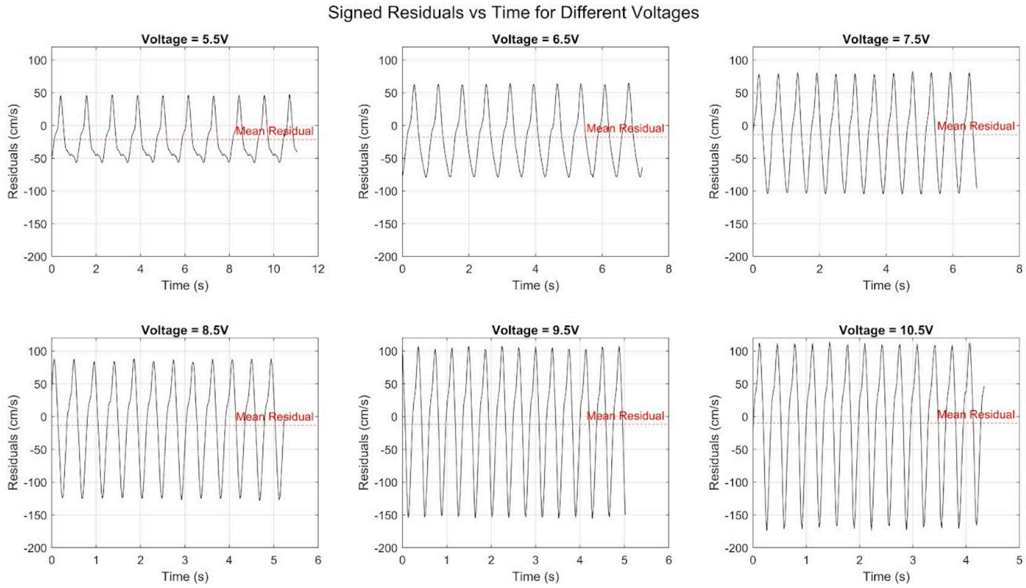

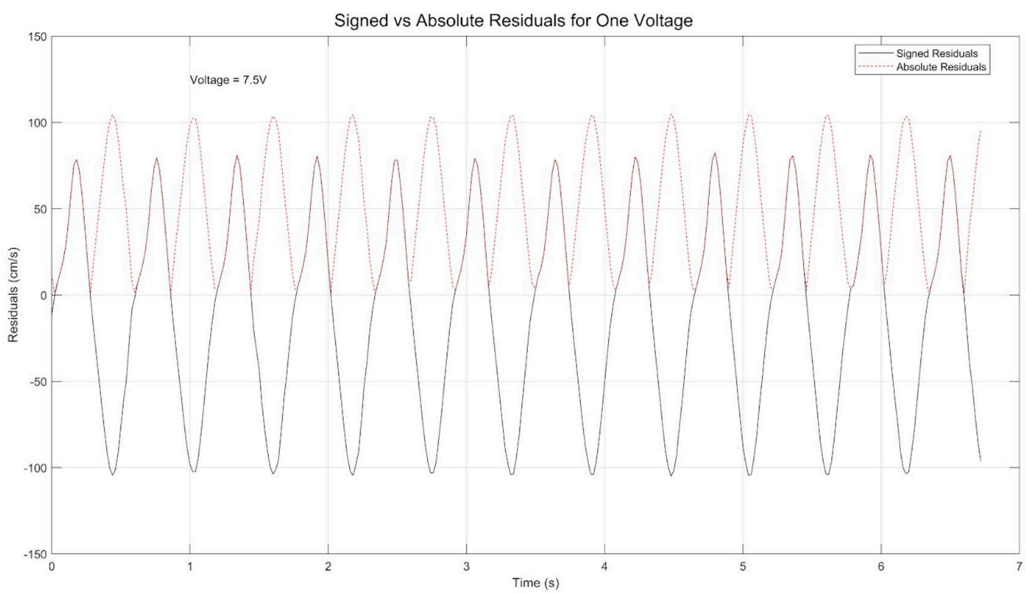

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