CS 381                          Final Exam                    Wednesday Dec 14
Fall 2005                       Closed book                   Philips 101 7-9:30pm

All problems should be straight forward. Partial credit will depend on clarity and
conciseness of your answer. Please do not put down correct but irrelevant information.

1. Construct a finite automaton for the set of all strings of 0's and 1's with an even
number of zeros and an odd number of ones. You can draw the state diagram of the finite
automaton or you can specify the delta function by a table. In either case you must
indicate the start state and the set of final states.

2. Give a one or two sentence definition of the symbol $R_{ij}^{k}$ which was used in converting
a finite automaton to a regular expression.

3. Let L=$\left\{a^{n}b^{2^{n}} \mid n \geq 1\right\}$. Prove or disprove that L is a context-free language.

4. Let L=$a^{+}b^{+}c^{+} - \left\{a^{n}b^{n}c^{n} \mid n \geq 1\right\}$. Either give a context-free grammar for L or use the
pumping lemma to prove that L is not context-free.

5. Let $L_{1} = \left\{a^{n}b^{n}c^{n} \mid n \geq 1\right\}$ and $L_{2} = \left\{0^{n}10^{n} \mid n \geq 1\right\}$. Prove or disprove each of the
following statements.
a) $L_{1}$ can be expressed in terms of h, $h^{-1}$, and intersection with regular sets applied to $L_{2}$.
b) $L_{2}$ can be expressed in terms of h, $h^{-1}$, and intersection with regular sets applied to $L_{1}$.

6. Is the class of r.e. sets closed under complement? Give proof of your answer. You
may assume anything proved in class except the result itself.

7. Prove that the halting problem for Turing machines is undecidable.

8. Suppose you could reduce an NP complete problem to a polynomial time problem in
polynomial time. What would be the consequence?
What if the reduction required exponential time?