

**Birla Institute of Technology and Science-Pilani, Hyderabad Campus**  
**Second Semester 2021-2022**  
**Tutorial-2**  
**Course No CS F351**  
**Course Title: Theory of Computation**

**General Instructions:** *Argue logically. Write it in a manner that explains your logic very clearly. Do not miss steps in between.*

1. Consider the following languages over the alphabet  $\Sigma = \{a, b\}$

$$L_1 = \{a^n \mid n \geq 1\}$$

$$L_2 = \{b^n \mid n \geq 1\}$$

Describe the following languages as per the set notation as well as the precise definition in english.

- $L_3 = \overline{L_1}$
- $L_4 = (L_1 L_2)^+$

2. Give DFA's on  $\Sigma = \{0, 1\}$  accepting the following strings.

- (a)  $\{0^n \mid n \geq 0, n \neq 3\}$
- (b) The set of all strings divisible by 5
- (c)  $\{01^4 x 1^3 \mid x \in \{0, 1\}^*\}$

3. Prove that if  $L$  is regular then the language

$$Pre(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$$

is regular.

4. Let  $L_1$  and  $L_2$  be the languages. Then  $L_1 L_2 = \{xy \mid x \in L_1, y \in L_2\}$  and  $L_1^R = \{x^R \mid x \in L_1 \text{ where } x^R \text{ is the string obtained by reversing } x\}$ . Prove that  $\forall L_1, L_2, (L_1 L_2)^R = L_2^R L_1^R$ . The equality  $(xy)^R = y^R x^R$  where  $x$  and  $y$  are string on any alphabet can be used without proving. By using induction try to prove this equality by yourself.