Birla Institute of Technology and Science-Pilani, Hyderabad Campus
Second Semester 2021-2022
Tutorial-5
Course No CS F351
Course Title: Theory of Computation

General Instructions: *Argue logically. Write it in a manner that explains your logic very clearly. Do not miss steps in between.*

1. Design a Turing machine for accepting $\{a^i b^j c^k \mid i, j, k \geq 1, i = j + k\}$.

   Solution:

   M(w):{

   (a) Read a and change it to X
   (b) Move right until b
   (c) Read the first b and change it to Y
   (d) Move left until X
   (e) Repeat steps (a) to (d) until all b's are exhausted
   (f) After that, it matches $a's$ with $c's$ in a similar manner
   (g) Accepts when the number of $a's$ is equal to the sum of the number of $b's$ and the number of $c's$. **}**

2. Prove that
$$E_{DFA} = \{\langle A \rangle \mid \text{A is DFA and } L(A) = \phi\}$$
   and
$$EQ_{DFA} = \{\langle A, B \rangle \mid \text{ A and B are DFA and } L(A) = L(B)\}$$
   are decidable.

   Solution: $T_1(\langle A \rangle) : \{$

   (a) Mark the start state of A
   (b) Repeat untill no new states get marked
   (c) Mark a state that has a transition coming in to it from any state that is alread marked
   (d) If no accept state is marked, Accept; otherwise Reject

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   $T_2(\langle A, B \rangle) : \{$

   (a) Construct a DFA C such that it accepts $(L(A) \cap \overline{L(B)}) \cup (\overline{L(A)} \cap L(B))$
   (b) Run $T_1(\langle C \rangle)$
   (c) If T_1 accepts, Accept; otherwise Reject

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3. Assume that $A_{TM} = \{\langle M, w \rangle \mid M \text{ is a } TM \text{ which accepts } w\}$ is undecidable. Prove that $E_{TM} = \{\langle M \rangle \mid M \text{ is a } TM \text{ and } L(M) = \phi\}$ is undecidable.

   Solution:

   $M_1(x)$:

(a) If $x \neq w$, reject

(b) IF $x == w$, run M on w and accepts if M does

Let R be a TM which decides E_{TM}. We use R to construct TM S that decides A_{TM}.
$S(\langle M, w \rangle)$:

(a) Use the description of M and w to construct the TM M1 just described.

(b) Run R on $\langle M1 \rangle$

(c) If R accepts (means $L(M1) = \phi$), Reject; if R rejects Accept

The TM S decides A_{TM} which is a contradiction.