Theory of Computation (CS F351)

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Pushdown Automaton

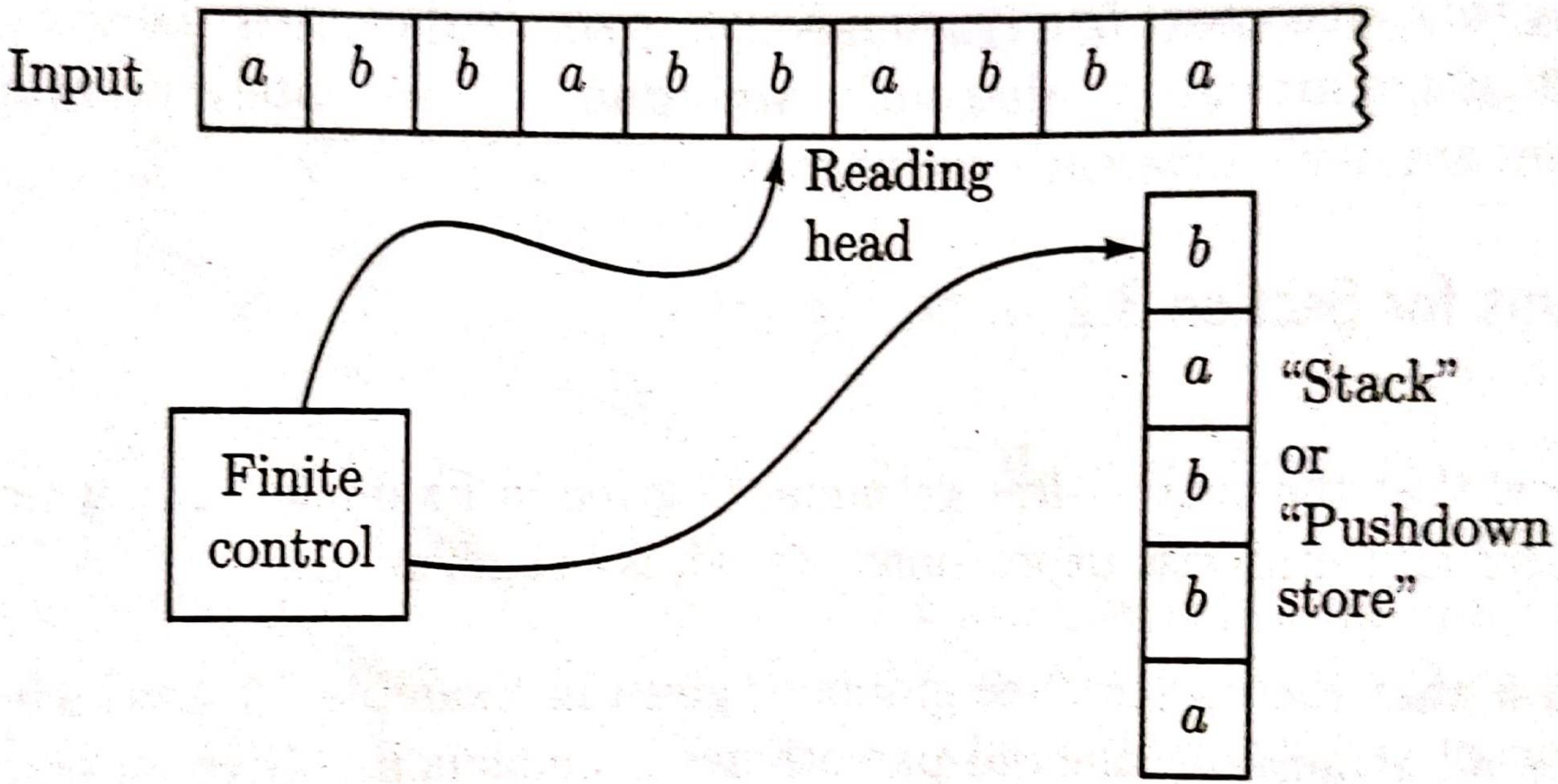
(Chapter-3; Sec-3.3)

Intro to Pushdown Automata (PDA)

Not all CFLs are recognized by FA.

Ex: $L=\{ww^R \mid w \in \Sigma^* \}$

Set of balanced parentheses.


Input
a
b
b
a
b
b
a
b
b
a
Reading
head
b
a
b
b
a
"Stack"
or
"Pushdown
store"
Finite
control

Definition 3.3.1: Let us define a **pushdown automaton** to be a sextuple $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

K is a finite set of **states**,

Σ is an alphabet (the **input symbols**),

Γ is an alphabet (the **stack symbols**),

$s \in K$ is the **initial state**,

$F \subseteq K$ is the set of **final states**, and

Δ, the **transition relation**, is a finite subset of $(K \times (\Sigma \cup \{e\}) \times \Gamma^*) \times (K \times \Gamma^*)$.

innovate achieve lead

Intuitively, if $((p, a, \beta), (q, \gamma)) \in \Delta$, then M, whenever it is in state p with β at the top of the stack, may read a from the input tape (if $a = e$, then the input is not consulted), replace β by γ on the top of the stack, and enter state q. Such a pair $((p, a, \beta), (q, \gamma))$ is called a **transition** of M; since several transitions of M may be simultaneously applicable at any point, the machines we are describing are nondeterministic in operation. (We shall later alter this definition to define a more restricted class, the deterministic pushdown automata.)

To **push** a symbol is to add it to the top of the stack; to **pop** a symbol is to remove it from the top of the stack. For example, the transition $((p, u, e), (q, a))$ pushes a, while $((p, u, a), (q, e))$ pops a.

Example 3.3.1: Let us design a pushdown automaton M to accept the language $L = \{wcw^R : w \in \{a,b\}^*\}$. For example, $ababcbaba \in L$, but $abcab \notin L$, and $cbc \notin L$. We let $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where $K = \{s, f\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{a, b\}$, $F = \{f\}$, and Δ contains the following five transitions.

(1) $((s, a, e), (s, a))$
(2) $((s, b, e), (s, b))$
(3) $((s, c, e), (f, e))$
(4) $((f, a, a), (f, e))$
(5) $((f, b, b), (f, e))$

State	Unread Input	Stack	Transition Used
s	$abbcbba$	e	-
s	$bbcbba$	a	1
s	$bcbba$	ba	2
s	$cbba$	bba	2
f	bba	bba	3
f	ba	ba	5
f	a	a	5
f	e	e	4

$L=\{ww^R \mid w \in \Sigma^*\}$

$$M = (K, \Sigma, \Gamma, \Delta, s, F),$$

where $K = (s, f)$, $\Sigma = \{a, b\}$, $F = \{f\}$, and Δ is the set of the following five transitions.

(1) $((s, a, e), (s, a))$
(2) $((s, b, e), (s, b))$
(3) $((s, e, e), (f, e))$
(4) $((f, a, a), (f, e))$
(5) $((f, b, b), (f, e))$

L={w | w ϵ∑* and has equal number of ***a***s and ***b***s }

Let $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where $K = \{s, q, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, c\}$, $F = \{f\}$, and Δ is listed below.

(1) $((s, e, e), (q, c))$
(2) $((q, a, c), (q, ac))$
(3) $((q, a, a), (q, aa))$
(4) $((q, a, b), (q, e))$
(5) $((q, b, c), (q, bc))$
(6) $((q, b, b), (q, bb))$
(7) $((q, b, a), (q, e))$
(8) $((q, e, c), (f, e))$

State	Unread Input	Stack	Transition	Comments
s	$abbbabaa$	e	-	Initial configuration.
q	$abbbabaa$	c	1	Bottom marker.
q	$bbbabaa$	ac	2	Start a stack of a's.
q	$bbabaa$	c	7	Remove one a.
q	$babaa$	bc	5	Start a stack of b's.
q	$abaa$	bbc	6	
q	baa	bc	4	
q	aa	bbc	6	
q	a	bc	4	
q	e	c	4	
f	e	e	8	Accepts.

FA to Pushdown Automata (PDA)

Every FA can be trivially viewed as a PDA without stack operations.

To be precise, let $M = (K, \Sigma, \Delta, s, F)$ be a nondeterministic finite automaton, and let M' be the pushdown automaton $(K, \Sigma, \emptyset, \Delta', s, F)$, where $\Delta' = \{((p, u, e), (q, e)) : (p, u, q) \in \Delta\}$.

PDA and CFG

(Chapter-3; Sec-3.4)

PDA and CFG

The class of languages accepted by PDA is exactly the class of Context –Free Languages.

Each CFL is accepted by some PDA.

For every grammar we must be able to construct a PDA.

Rules for constructing PDA from CFG

If $G = (V, \sum, R, S)$

We must be able to construct M such that L(M)=L(G)

$M = (\{p,q\}, \sum, V, \Delta, p, \{q\})$

Where-

$\Delta =$ (1) $((p,e,e), (q,S))$

(2) $((q,e,A), (q, x))$

for each rule $A \rightarrow X$ in R

(3) $((q,a,a), (q,e))$

for each $a \epsilon \sum$

Summary

What is PDA
Description of PDA
Operations of a PDA
Tabular representation of the operations of the PDA.
Correspondence between PDA and FA.
PDA Corresponding to the CGF