



**BITS Pilani**  
Hyderabad Campus

# Theory of Computation (CS F351)

Prof.R.Gururaj  
CS&IS Dept.



**BITS Pilani**  
Hyderabad Campus

# Non-deterministic Finite Automata (Sec. 2.2 of T1)

Prof.R.Gururaj  
CS&IS Dept.

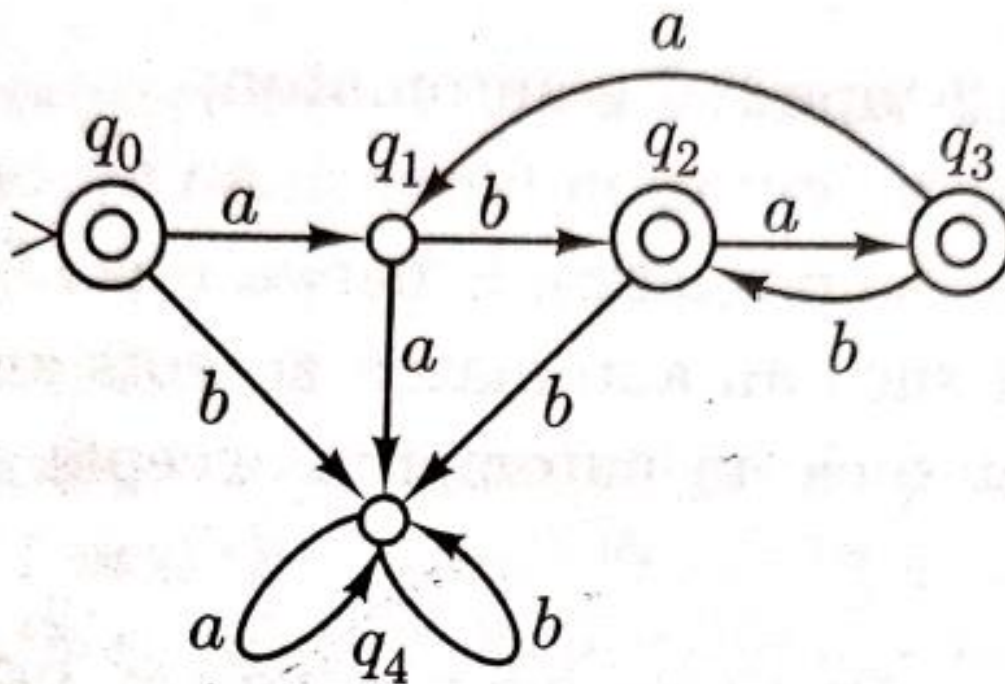
# What is non-determinism in FA?

---

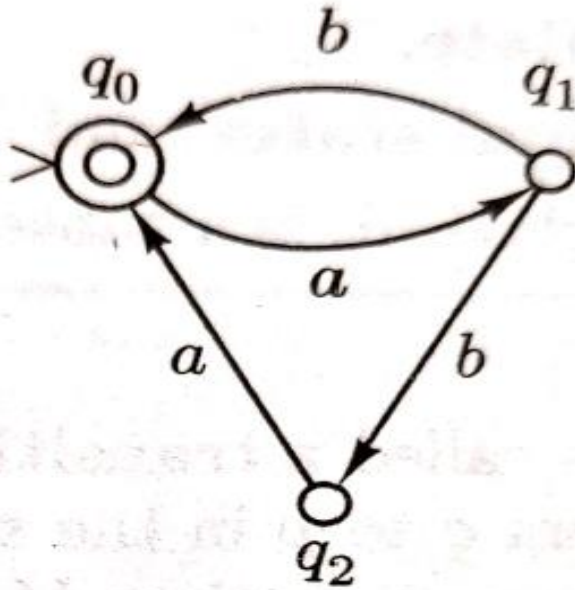
It is a powerful feature.

What is this nondeterminism?

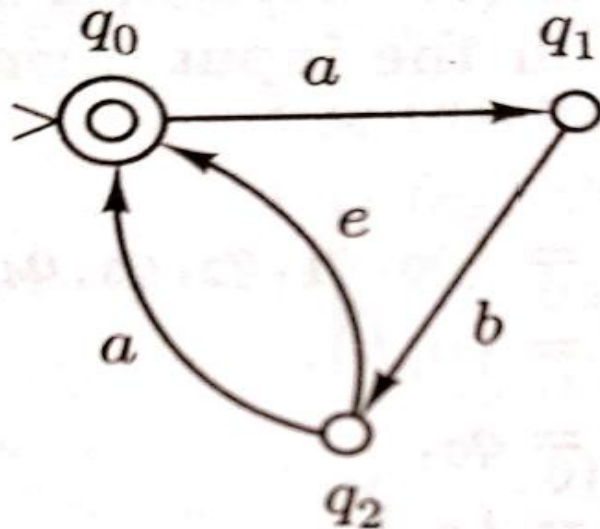
## A Deterministic Finite Automaton (DFA)



A Non-deterministic Finite Automaton (NFA) for the same language



NDFA for the same language with null transition



# Description of an NDFA



**Definition 2.2.1:** A nondeterministic finite automaton is a quintuple  $M = (K, \Sigma, \Delta, s, F)$ , where

$K$  is a finite set of **states**,

$\Sigma$  is an alphabet,

$s \in K$  is the **initial state**,

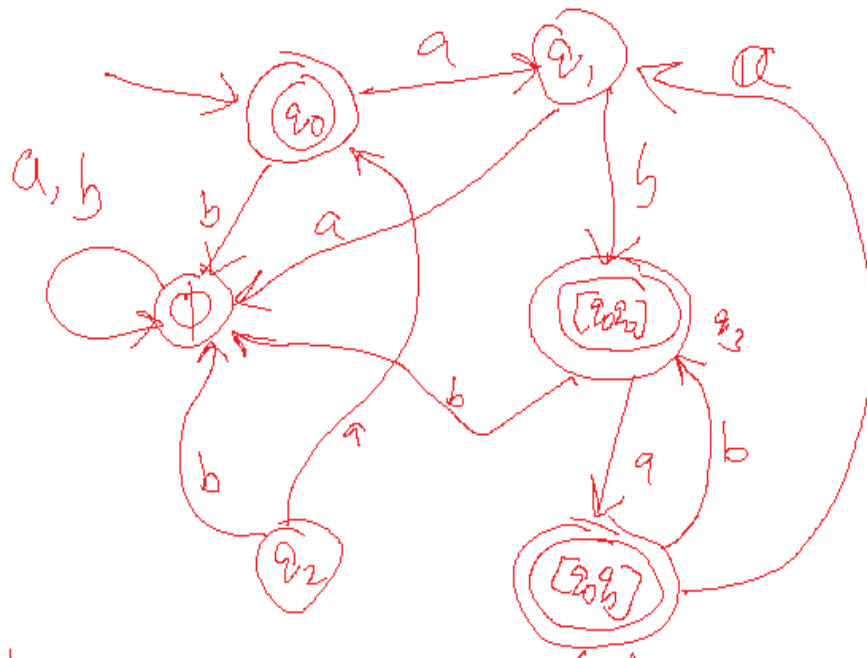
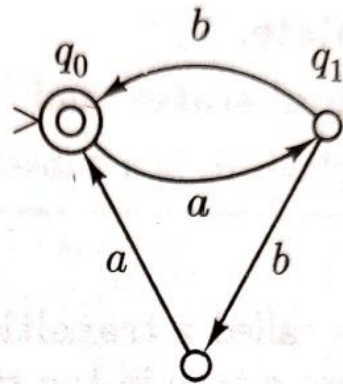
$F \subseteq K$  is the set of **final states**, and

$\Delta$ , the **transition relation**, is a subset of  $K \times (\Sigma \cup \{e\}) \times K$ .

- ❖ NDFA are not meant as realistic models for computers.
- ❖ They simply notational generalization of FA, as they simplify the description of these FA.
- ❖ Meaning that we can draw a FA for a language with ease (less number of states and edges)
- ❖ Further, non-determinism is a non-essential feature of FA.
- ❖ Ever NDFA can be converted to equivalent DFA.
- ❖ We have algorithms that can convert NDFA to its equivalent DFA.



# Converting a NDFA without null moves, to equivalent DFA



*States*

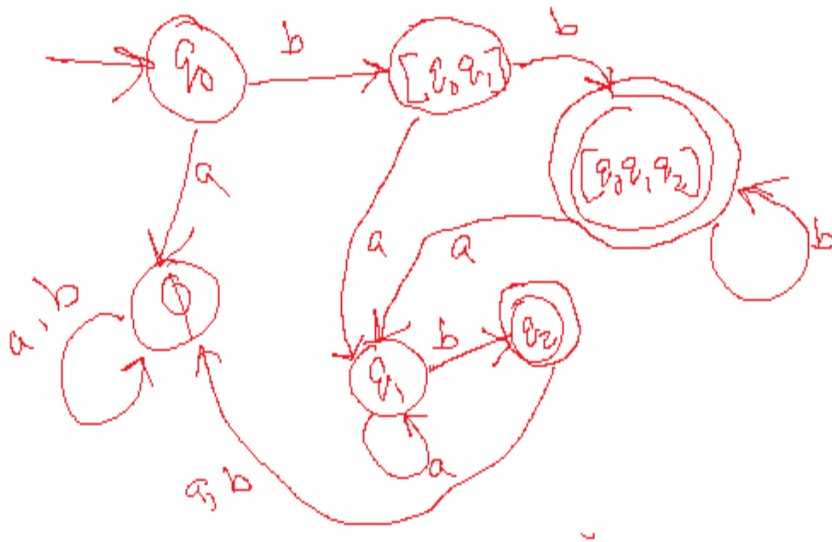
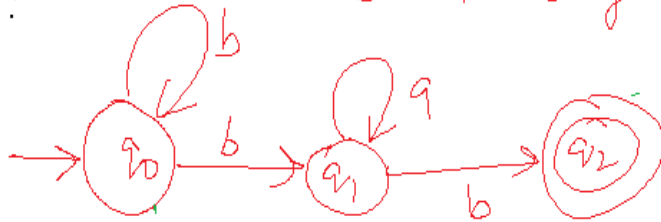
state	a	b
$q_0$	$q_1$	$\phi$
$q_1$	$\phi$	$\phi$
$q_2$	$\phi$	$[q_0 q_1]$
$q_3$	$q_2$	$q_0$
$q_4$	$[q_0 q_1]$	$(q_0, a) \cup (q_2, a)$ $\{q_1\} \cup \{\phi\}$ $= [q_1]$
$q_5$	$[q_0 q_1]$	$(q_0, b) \cup (q_2, b)$ $\phi \cup \phi = \phi$

# Converting a NDFA without null moves, to equivalent DFA



Ex: 2:

Convert the following to DFA.

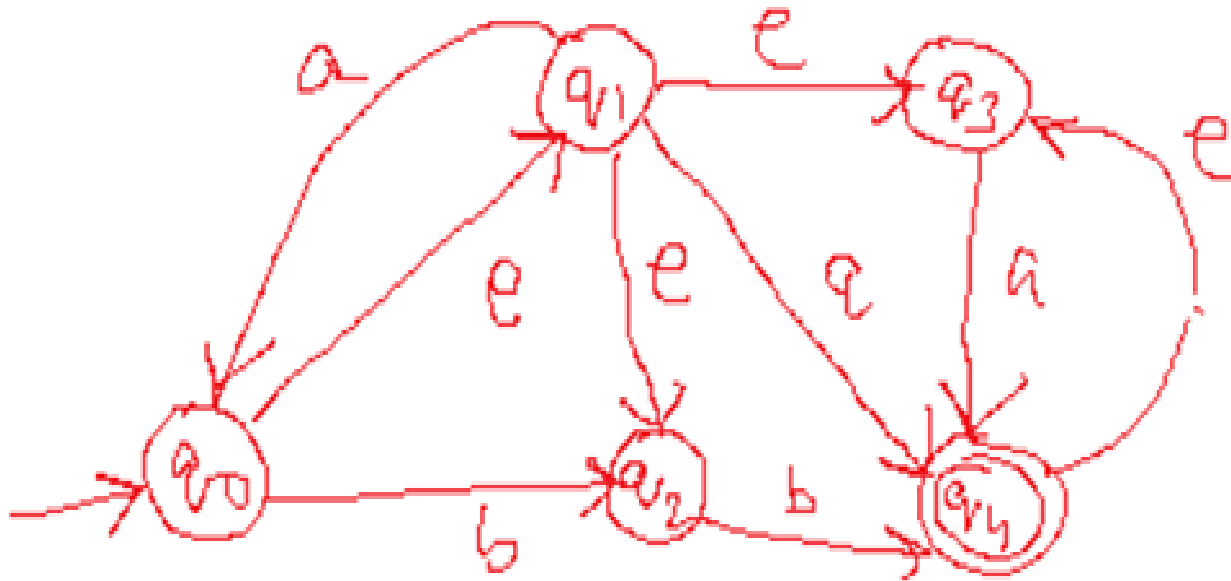


State	a	b
$\phi$	$\phi$	$\phi$
$\Rightarrow q_0$ (A)	$\phi$	$[q_0, q_1] \checkmark$
$q_1$ (B)	$q_1$	$q_2$
$(q_2)$ (C)	$\phi$	$\phi$
$[q_0, q_1]$ (D)	$q_1$	$[q_0, q_1, q_2] \checkmark$
<u><math>[q_0, q_1, q_2]</math> (E)</u>	$\phi \cup q_1 \cup \phi = \{q_1\}$	$\{q_0, q_1, q_2\} \cup \{q_2\} = [q_0, q_1, q_2] \checkmark$

# Converting a NDFA with null moves, to equivalent DFA



Ex.1



Computing  $E(q)$  for each  $q \in K$

$$E(q_0) = \{q_0 q_1 q_2 q_3\}$$

$$E(q_1) = \{q_1 q_2 q_3\}$$

$$E(q_2) = \{q_2\}$$

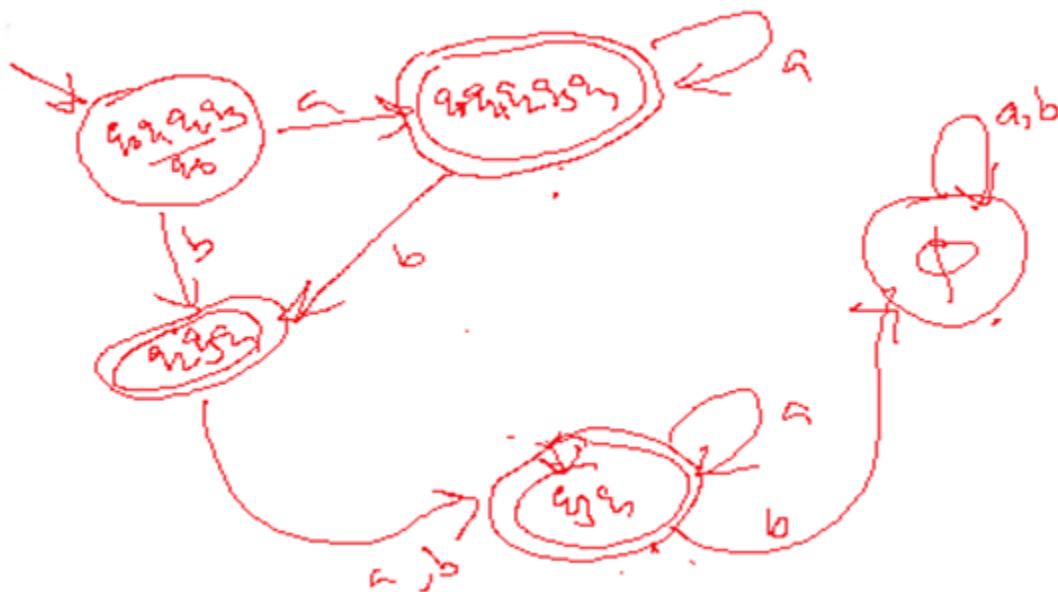
$$E(q_3) = \{q_3\}$$

$$E(q_4) = \{q_3 q_4\}$$

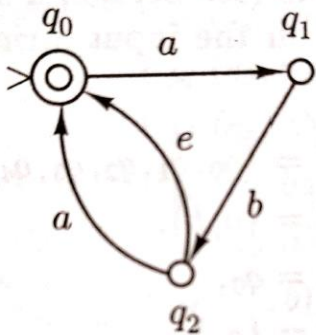
## Defining the transitions of DFA

State	a	b
$\phi$	$\phi$	$\phi$
$q_0$ <u><math>[q_0 q_1 q_2 q_3]</math></u>	$(q_0 a) \cup (q_1 a) \cup (q_2 a) \cup (q_3 a)$ $\phi \cup \{q_0\} \cup \{\phi\} \cup \{q_4\}$ $= \{q_0 q_4\} = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_3) = \{q_4\} \cup \{q_3 q_4\}$ $= [q_0 q_1 q_2 q_3 q_4]$	$(q_0 b) \cup (q_1 b) \cup (q_2 b) \cup (q_3 b)$ $\{q_1\} \cup \phi \cup \{q_4\} \cup \phi = q_2 \cup q_3$ $= [q_2 q_3 q_4]$
$q_1$ <u><math>[q_1 q_2 q_3]</math></u>	$(q_1 a) \cup (q_2 a) \cup (q_3 a)$ $q_0 \cup \phi \cup q_4 = q_0 q_4$ $= [q_0 q_1 q_2 q_3 q_4]$	$(q_1 b) \cup (q_2 b) \cup (q_3 b)$ $\phi \quad q_1 \quad \phi = (q_4) = [q_3 q_4]$
<u><math>q_2</math></u>	$\phi \cup$	$q_4 = E(q_2) = [q_3 q_4]$
<u><math>q_3</math></u>	<u><math>[q_3 q_4]</math></u>	$\phi$
$F$ <u><math>[q_3 q_4]</math></u>	<u><math>[q_3 q_4]</math></u>	$\phi$
$F$ <u><math>[q_0 q_1 q_2 q_3]</math></u>	<u><math>[q_0 q_1 q_2 q_3 q_4]</math></u>	$(q_2 q_3 q_4)$
$vf$ <u><math>[q_2 q_3 q_4]</math></u>	<u><math>[q_3 q_4]</math></u>	<u><math>[q_3 q_4]</math></u>

## Final equivalent DFA



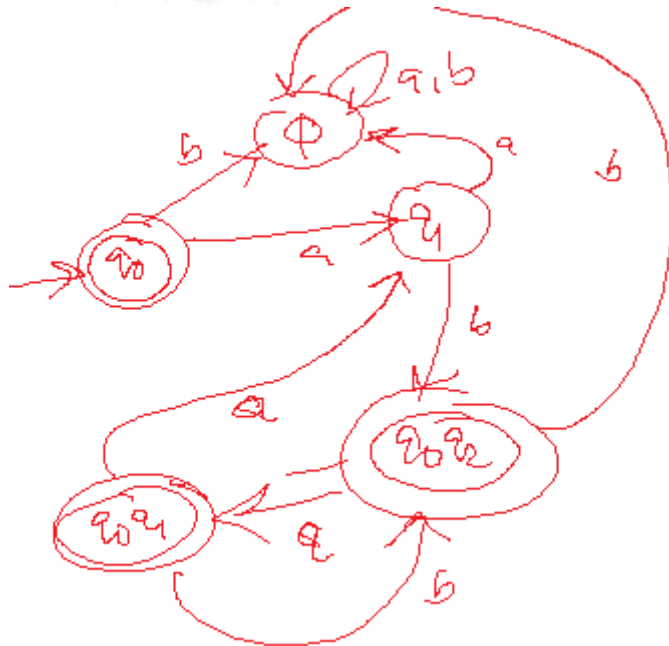
# Converting a NDFA with null moves, to equivalent DFA



$$E(q_0) = \{q_0\}$$

$$E(q_1) = \{q_1\}$$

$$E(q_2) = \{q_0, q_2\} \rightarrow F$$



State	a	b
$\phi$	$\phi$	$\phi$
$q_0 = E(q_0) = q_0$	$q_1$	$\phi$
$q_1 = E(q_1) = q_1$	$\phi$	$q_2 = E(q_2) = [q_0, q_2]$
$q_2 = E(q_2) = [q_0, q_2]$	$(q_0, a) \cup (q_2, a)$ $q_1 \cup q_0$ $[q_0, q_1]$	$(q_0, b) \cup (q_2, b)$ $\phi \cup \phi$ $\phi = \phi$
$[q_0, q_1]$	$(q_0, a) \cup (q_1, a)$ $q_1 \cup \phi$ $= q_1$	$(q_0, b) \cup (q_1, b)$ $\phi \cup q_2 = [q_1]$ $= E(q_1) = [q_0, q_2]$