

Assignment 2

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$$-u''(x) + \sigma u(x) = f$$

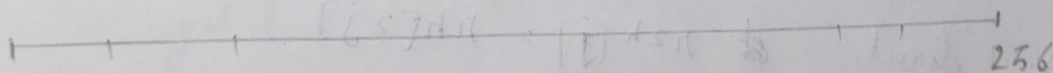
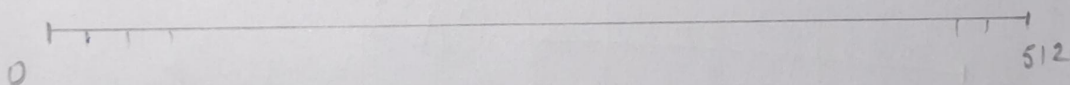
using central differencing

$$\frac{-u_{i+1} + 2u_i - u_{i-1}}{h^2} + \sigma u_i = f_i$$

$$u_i (2 + \sigma h^2) = f_i h^2 + u_{i+1} + u_{i-1}$$

$$u_i = \frac{f_i h^2 + u_{i+1} + u_{i-1}}{(2 + \sigma h^2)}$$

Grid detail $n = 512$



function created

① Gauss_iteration.c (input v & f) $Av = f$ or $Ax = b$

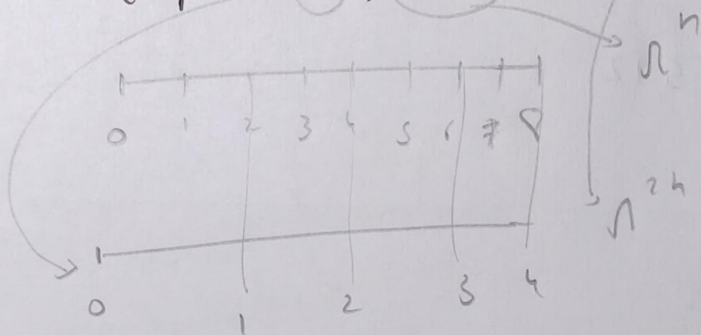
$$x[i] = (x[i+1] + x[i-1] + b[i] \times h^2) / (2 + h^2)$$

upto new iteration

② residual.c (input n, x, v, f) $r = f - Av$
or $r = b - Ax$

$$r[i] = b[i] - \frac{(-x_{i+1} - x_{i-1} + 2x_i) + x_i}{h^2}$$

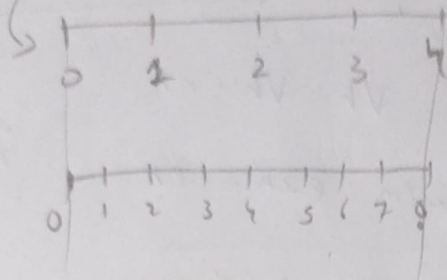
③ Restriction (input m, r_{2h}, r_{2h})



either direct $r_{2h}[j] = r_h[2j]$

or full weight $r_{2h}(j) = \frac{d_h[2j-1] + 2 \times d_h[2j] + d_h[2j+1]}{4}$

Prolongation (int m , n^h , n^{2h})

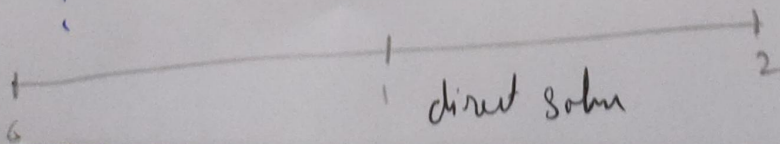
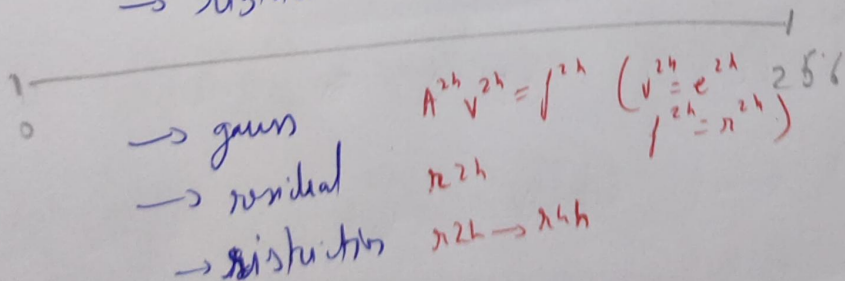
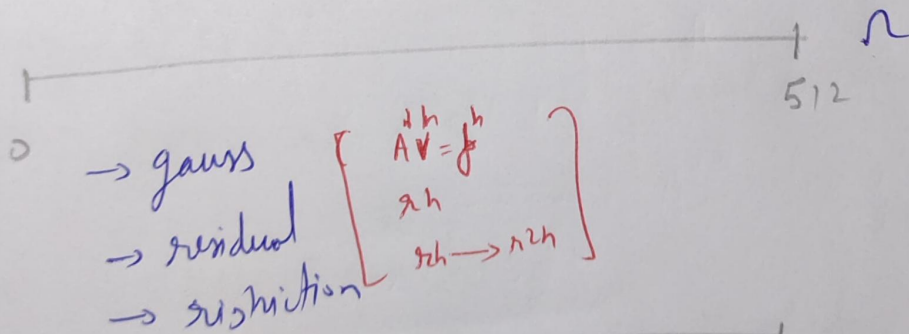


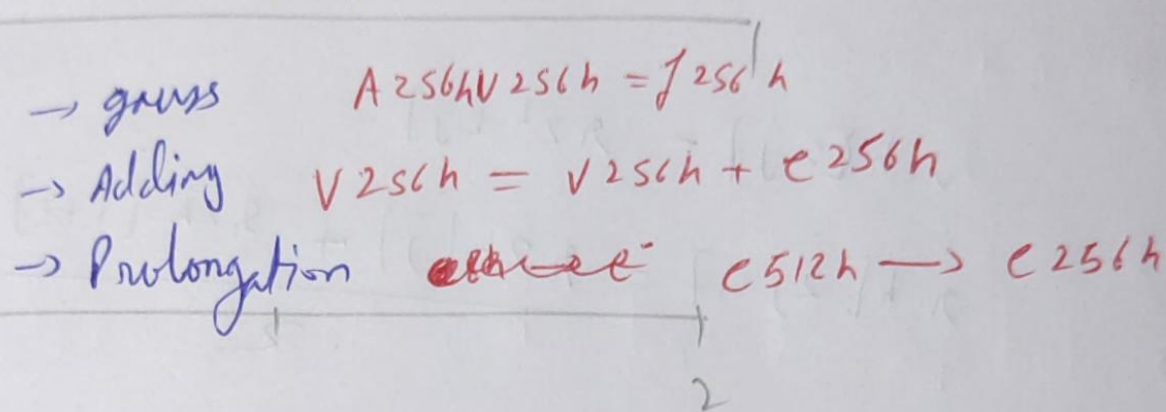
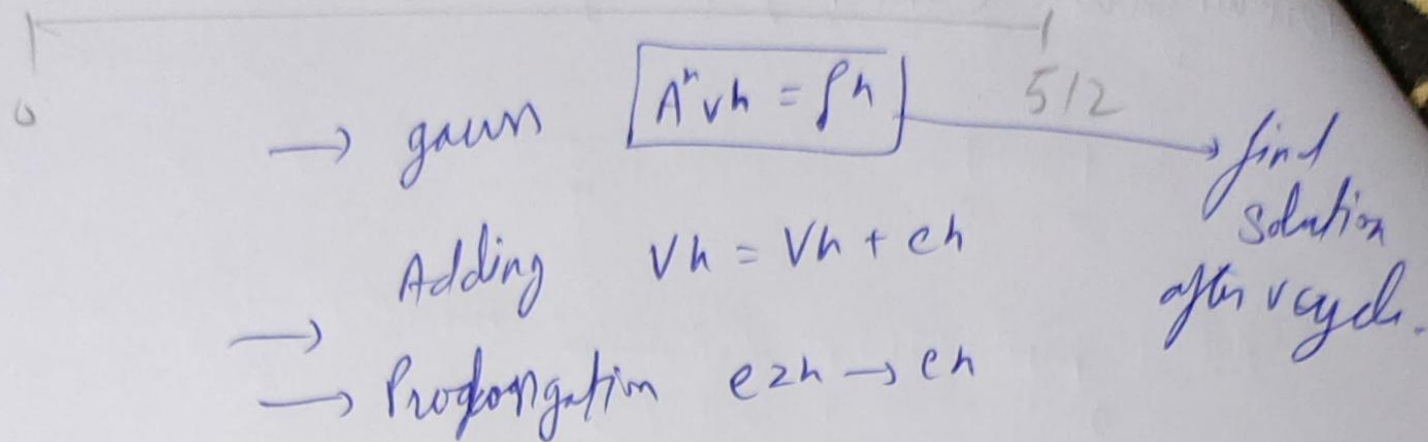
$$n^h[2j] = n^{2h}[j]$$

$$n^h[2j+1] = \frac{n^{2h}[j] + n^{2h}[j+1]}{2}$$

⑤ V-cycle (input n , k) $\rightarrow (512, 2)$

$$\text{level} = \frac{\log n}{\log 2} - 1, \quad \text{level} = 8$$





algorithm

How thing works

output

Level ①

Solve $A^h v^h = f^h$

given $(n, n_{eh}, A^h, v^h, f^h), (v[0].m, v[0].r, v[0].v, v[0].f)$
 \hookrightarrow give v^h

② find residual of main eqⁿ (in v^h)

residual $(n, r^h, A^h, v^h, f^h), (v[0].m, v[0].r, v[0].v, v[0].f)$
 \hookrightarrow gives r^h

③ restrict the residual $h \rightarrow 2h \quad n^h \rightarrow n^{2h}$
 $n=8$ to $m=\frac{n}{2}=4$.

restriction $(\frac{n}{2}, r^{2h}, r^{2h}), (v[0].m, v[0].r, v[0].v, v[0].f)$
 \hookrightarrow given r^{2h}

Level ②

④ solve error eqⁿ

$$A^{2h} e^{2h} = r^{2h}$$

given $(\frac{n}{2}, n_{eh}, A^{2h}, e^{2h}, r^{2h}), (v[0].m, v[0].r, v[0].v, v[0].f)$
 \hookrightarrow we get e^{2h} error.

⑤ find residual in error equation (in e^{2h})

residual $(\frac{n}{2}, r_{e2h}, A^{2h}, e^{2h}, r^{2h})$

$r_{e2h} \rightarrow$ residual in error equation

$(v[0].m, v[0].r, v[0].v, v[0].f)$

①

$$r_{e2h} = r^{2h} - A^{2h} e^{2h}$$

5) restrict the error $\frac{r_{2h} - r_h}{n^{2h} - n^{4h}}$
 $n=8, \frac{n}{4} = 2 \rightarrow (n^{2h})$

restrict $(\frac{n}{4}, r_{2h}, r_{4h})$
 $(v(1).m, v(1).n, v(2).f) \rightarrow r_{4h}$ residual of error at n^{4h} grid.

7) find solution
 Solve error of error e_1^n ($A^{4h} e_{4h} = r_{4h}$)
 $e_{4h} \Rightarrow$ is the error in error e_1^n at n^{4h} grid

direct-method $(\frac{n}{4}, A_{4h}, r_{4h}, e_{4h})$
Gauss-iter $(v(2).m, v(2).v, v(2).f) \rightarrow$ give e_{4h}
 error in error e_1^n

8) return level 2
 Prolonging the error in error e_1^n
 Prolongation $(\frac{n}{4}, e_{4h}, e_{2h})$
 $(v(2).m, v(2).v, v(1).e)$

\rightarrow get e_{2h} (8)
 error in error at n^{2h} grid

9) $e_{2h} = e_{2h} + e_{2h}$ (adding error)
 $v(1).v = v(1).v + v(1).e$
 $A_{2h} e_{2h} = r_{2h}$

10) Solving error equation
 Gauss iteration $(\frac{n}{2}, r_{2h}, A_{2h}, e_{2h}, r_{2h})$
 $(v(1).m, v(1).v, v(1).f) \rightarrow$ gives e_{2h}

Level 1
 11) Prolonging error e_{2h} to e_h
 Prolongation $(\frac{n}{2}, e_{2h}, e_h), (v(1).m, v(1).v, v(1).e) \rightarrow$ give e_h

(12)

$$V_h = V_h + e_h$$

(adding error)

→ V_h

$$V(i).V = V(i).V + V(i).e$$

(13)

solving $AV_h = f_h$

gauss($n, \text{new}, A_h, v_h, f_h$)

$(V(i).m, \gamma, V(i).V, V(i).f)$

→ gauss(V_h) find solution

Line 10 = 8

(2)

$n = 4$

S	(13)	(1) S
A	(1)	(2) Ri
P	(1)	(3) Ri
S	(13)	(4) S
A	(1)	(5) Ri
P	(8)	(6) Ri

(3)

$n = (2)$

(7) first

$P \rightarrow \text{Pivot}$
 $A \rightarrow \text{add}$
 $S \rightarrow \text{Sub}$

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 (2) Residual
 (3) Right

Full algorithm step with

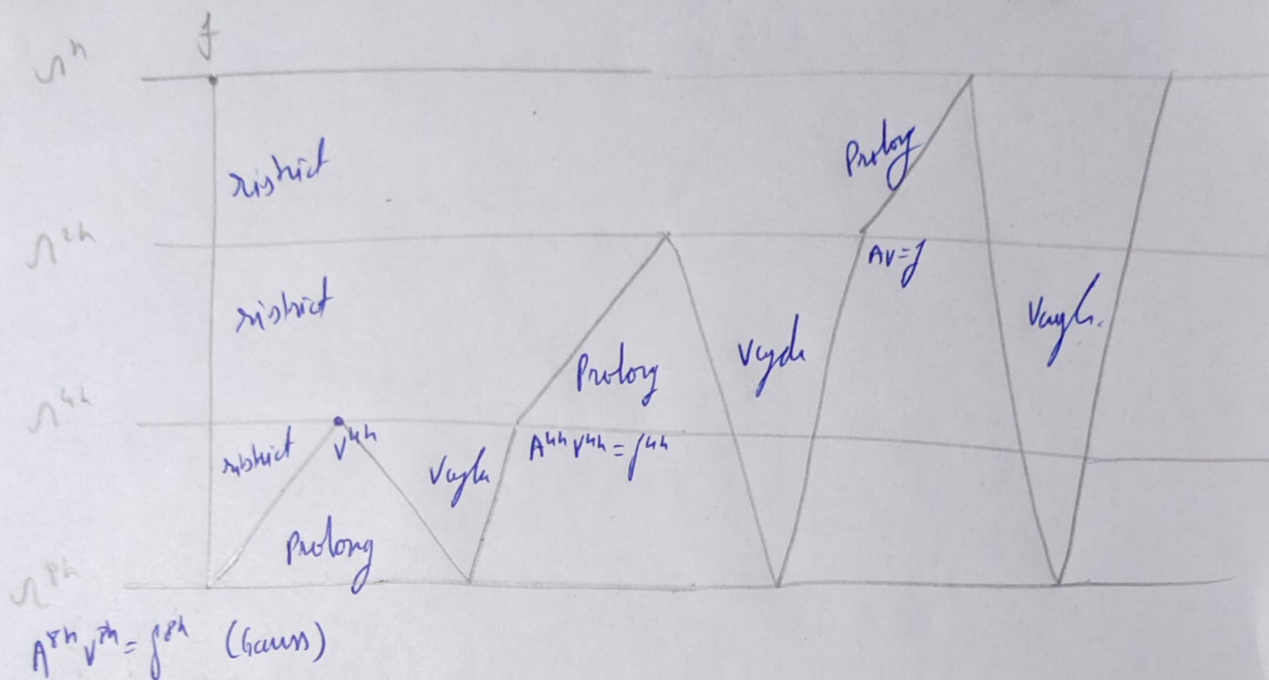
for 2 level

$n = 8$

(3)

FMG \therefore let $n = 16$, level = 3

Q2



$(n_{\text{cell}})_0 = 2 \rightarrow$ running Vayls twice each time (at each level).

finally after FMG completion running Vayls till residual becomes less than 10^{-6} .