INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
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ME 543 Computational Fluid Dynamics
Computer Assignment – 1

Solve the following partial differential equation using below said iterative schemes with specified boundary conditions as shown in the figures.

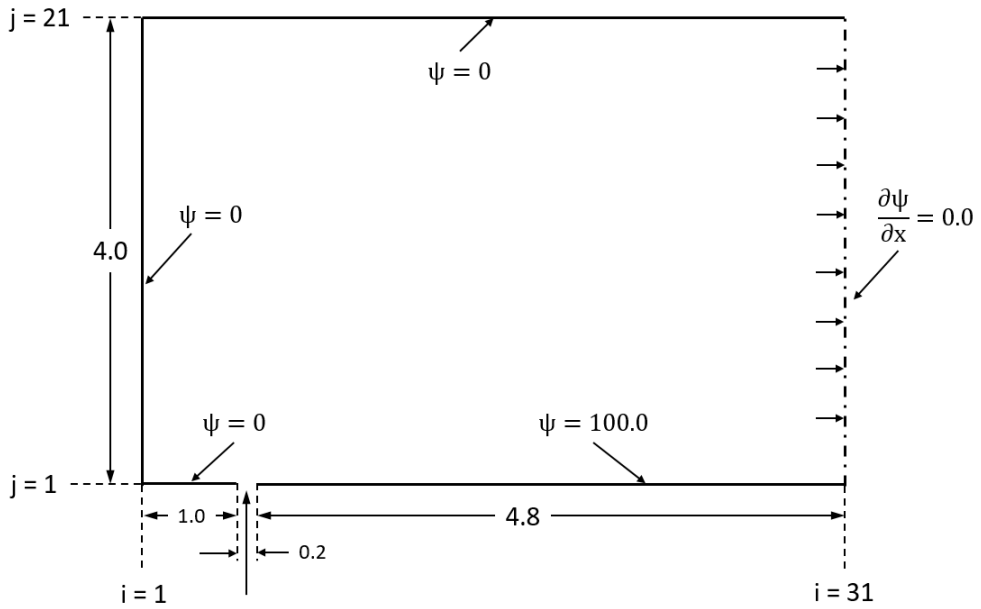
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \epsilon = \sqrt{\frac{\sum_{\substack{i=2,M-1 \\ j=2,N-1}} \left(\phi^{k+1} - \phi^{k}\right)^2}{(M-2)\times(N-2)}}$$

1. Jacobi iterative method
2. Point Gauss-Seidel iterative method
3. Point Successive Over Relaxation (PSOR) method
4. Line Gauss-Seidel iterative method (TriDiagonal Matrix Algorithm)
5. Alternating Direction Implicit method (ADI)

Discretize the PDE using finite difference method with uniform grid $M \times N$. Write the code such a way so that you can input the values of $N, M, \Delta x, \Delta y$. Here $\Delta x, \Delta y$ may be different, so use $= \frac{\Delta x}{\Delta y}$.

Submit the soft copy of the results in terms of contours, and report on discretized algebraic equation of each iterative method, comparison study of number of iterations and time taken to converge up to $\epsilon < 10^{-6}$. Plot ω vs number of iterations for PSOR method and find ω_{opt} from the plot. Email the soft copy of the code along with report.

First Problem: $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$



Second Problem: $$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

The exact solution of the problem is given below to validate your results.

$$T(x,y) = T_c + (T_h - T_c)\left[1 - 2\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \frac{\sinh\left(\frac{n\pi y}{L}\right)}{\sinh\left(\frac{n\pi H}{L}\right)} \sin\left(\frac{n\pi x}{L}\right)\right]$$