**Problem Statement 1:**

In one state, 52% of the voters are Republicans, and 48% are Democrats. In a second state, 47% of the voters are Republicans, and 53% are Democrats. Suppose a simple random sample of 100 voters are surveyed from each state.

What is the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state?

Solution

For this analysis, let P1 = the proportion of Republican voters in the first state, P2 = the proportion of Republican voters in the second state, p1 = the proportion of Republican voters in the sample from the first state, and p2 = the proportion of Republican voters in the sample from the second state. The number of voters sampled from the first state (n1) = 100, and the number of voters sampled from the second state (n2) = 100.

The solution involves four steps.

* Make sure the sample size is big enough to model differences with a normal population. Because

n1P1 = 100 \* 0.52 = 52, n1(1 - P1) = 100 \* 0.48 = 48,

n2P2 = 100 \* 0.47 = 47, n2(1 - P2) = 100 \* 0.53 = 53

are each greater than 10, the sample size is large enough.

* Find the mean of the difference in sample proportions: E(p1 - p2) = P1 - P2 = 0.52 - 0.47 = 0.05.
* Find the standard deviation of the difference.

σd = sqrt{ [ P1(1 - P1) / n1 ] + [ P2(1 - P2) / n2 ] }   
σd = sqrt{ [ (0.52)(0.48) / 100 ] + [ (0.47)(0.53) / 100 ] }   
σd = sqrt (0.002496 + 0.002491) = sqrt(0.004987) = 0.0706

* Find the probability. This problem requires us to find the probability that p1 is less than p2. This is equivalent to finding the probability that p1 - p2 is less than zero. To find this probability, we need to transform the random variable (p1 - p2) into a z-score. That transformation appears below.

zp1 - p2 = (x - μp1 - p2 ) / σd = = (0 - 0.05)/0.0706 = -0.7082

Using Normal Distribution Calculator, we find that the probability of a z-score being -0.7082 or less is 0.24.

Therefore, the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state is 0.24.