Unit 1
Formal logic
Lecture 4

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Rules of Inference for Quantified Statements

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	***Name***
$\dfrac{\forall x P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x P(x)}$	Existential generalization

Show that the premises “Everyone in this discrete mathematics class has taken a course in computer science” and “Marla is a student in this class” imply the conclusion “Marla has taken a course in computer science.”

Solution: Let $D(x)$ denote “x is in this discrete mathematics class,” and let $C(x)$ denote “x has taken a course in computer science.” Then the premises are $\forall x(D(x) \rightarrow C(x))$ and $D(\text{Marla})$.
The conclusion is $C(\text{Marla})$.
The following steps can be used to establish the conclusion from the premises.

Step	Reason
1. $\forall x(D(x) \rightarrow C(x))$	Premise
2. $D(\text{Marla}) \rightarrow C(\text{Marla})$	Universal instantiation from (1)
3. $D(\text{Marla})$	Premise
4. $C(\text{Marla})$	Modus ponens from (2) and (3)

Show that the premises “A student in this class has not read the book,” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not read the book.”

Solution: Let $C(x)$ be “x is in this class,” $B(x)$ be “x has read the book,” and $P(x)$ be “x passed the first exam.” The premises are $\exists x(C(x) \land \neg B(x))$ and $\forall x(C(x) \rightarrow P(x))$. The conclusion is $\exists x(P(x) \land \neg B(x))$.

These steps can be used to establish the conclusion from the premises.

Step	Reason
1. $\exists x(C(x) \land \neg B(x))$	Premise
2. $C(a) \land \neg B(a)$	Existential instantiation from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \land \neg B(a)$	Conjunction from (6) and (7)
9. $\exists x(P(x) \land \neg B(x))$	Existential generalization from (8)

Logic Programming

Prolog (from Programming in Logic), developed in the 1970s by computer scientists working in the area of artificial intelligence, is an example of such a language. Prolog programs include a set of declarations, consisting of two types of statements, **Prolog facts and Prolog rules**. **Prolog facts** define predicates by specifying the elements that satisfy these predicates. **Prolog rules** are used to define new predicates using those already defined by Prolog facts.

Consider a Prolog program given facts telling it the instructor of each class and in which classes students are enrolled. The program uses these facts to answer queries concerning the professors who teach particular students. Such a program could use the predicates instructor(p, c) and enrolled(s, c) to represent that professor p is the instructor of course c and that student s is enrolled in course c, respectively. For example, the Prolog facts in such a program might include:

```
instructor(chan,math273)
instructor(patel,ee222)
instructor(grossman,cs301)
enrolled(kevin,math273)
enrolled(juana,ee222)
enrolled(juana,cs301)
enrolled(kiko,math273)
enrolled(kiko,cs301)
```

A new predicate teaches(p, s), representing that professor p teaches student s, can be defined using the Prolog rule

teaches(P,S) :- instructor(P,C), enrolled(S,C)

which means that teaches(p, s) is true if there exists a class c such that professor p is the instructor of class c and student s is enrolled in class c. (Note that a comma is used to represent a conjunction of predicates in Prolog. Similarly, a semicolon is used to represent a disjunction of predicates.)

Prolog answers queries using the facts and rules it is given.

For example, using the facts and rules listed, the query

(1)?enrolled(kevin,math273) produces the response

yes

(2)?enrolled(X,math273) produces the response

kevin

kiko

(3) ?teaches(X,juana)

patel

grossman

(4) a) ?instructor(chan,math 273) yes

b) ?instructor(patel,cs301) no

c) ?enrolled(X,cs301) juana kiko

d) ?enrolled(kiko,Y) math273 cs301

e) ?teaches(grossman,Y) cs301

f)?enrolled(kevin,ee222) no

g) ?enrolled(kiko,math273) yes

h) ?instructor(X,cs301) grossman

i) ?teaches(X,kevin) chan

Suppose that Prolog facts are used to define the predicates mother(M, Y) and father(F , X), which represent that M is the mother of Y and F is the father of X, respectively. Give a Prolog rule to define the predicate sibling(X, Y), which represents that X and Y are siblings (that is, have the same mother and the same father).

```
sibling(X,Y) : -  mother(M,X),mother(M,Y),father(F,X),father(F,Y)
```

Suppose that Prolog facts are used to define the predicates mother(M, Y) and father(F, X), which represent that M is the mother of Y and F is the father of X, respectively. Give a Prolog rule to define the predicate grandfather(X, Y), which represents that X is the grandfather of Y .

```
grandfather(X,Y):- mother(M,Y),father(X,M) ; father(F,Y),father(X,F)
```

Precedence of Logical Operators

Operators	Precedence
~	1
^	2
v	3
->	4
<->	5

TABLE 6 **Logical Equivalences.**

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

(a)Show that $\neg(p \lor (\neg p \land q))$ and $\neg p \land \neg q$

$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg(\neg p \land q)$ by the second De Morgan law
$\equiv \neg p \land [\neg(\neg p) \lor \neg q]$ by the first De Morgan law
$\equiv \neg p \land (p \lor \neg q)$ by the double negation law
$\equiv (\neg p \land p) \lor (\neg p \land \neg q)$ by the second distributive law
$\equiv F \lor (\neg p \land \neg q)$ because $\neg p \land p \equiv F$
$\equiv (\neg p \land \neg q) \lor F$ by the commutative law for disjunction
$\equiv \neg p \land \neg q$ by the identity law for F

(b)Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

$(p \land q) \rightarrow (p \lor q) \equiv \neg(p \land q) \lor (p \lor q)$ by Example 3
$\equiv (\neg p \lor \neg q) \lor (p \lor q)$ by the first De Morgan law
$\equiv (\neg p \lor p) \lor (\neg q \lor q)$ by the associative and commutative laws for disjunction
$\equiv T \lor T$ by Example 1 and the commutative law for disjunction
$\equiv T$ by the domination law

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

Solution: Let p be the proposition “It is sunny this afternoon,” q the proposition “It is colder than yesterday,” r the proposition “We will go swimming,” s the proposition “We will take a canoe trip,” and t the proposition “We will be home by sunset.” Then the premises become $\neg p \land q, r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$. The conclusion is simply t. We need to give a valid argument with premises $\neg p \land q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$ and conclusion t.

Step	Reason
1. $\neg p \land q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

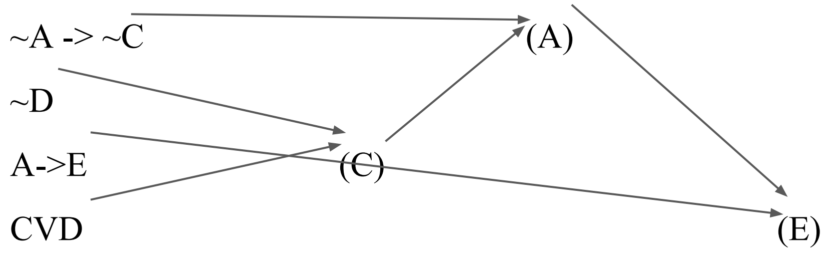
Show that the premises “If you send me an e-mail message, then I will finish writing the program,” “If you do not send me an e-mail message, then I will go to sleep early,” and “If I go to sleep early, then I will wake up feeling refreshed” lead to the conclusion “If I do not finish writing the program, then I will wake up feeling refreshed.”

Solution: Let p be the proposition “You send me an e-mail message,” q the proposition “I will finish writing the program,” r the proposition “I will go to sleep early,” and s the proposition “I will wake up feeling refreshed.” Then the premises are $p \to q$, $\neg p \to r$, and $r \to s$. The desired conclusion is $\neg q \to s$. We need to give a valid argument with premises $p \to q$, $\neg p \to r$, and $r \to s$ and conclusion $\neg q \to s$.

Step	Reason
1. $p \to q$	Premise
2. $\neg q \to \neg p$	Contrapositive of (1)
3. $\neg p \to r$	Premise
4. $\neg q \to r$	Hypothetical syllogism using (2) and (3)
5. $r \to s$	Premise
6. $\neg q \to s$	Hypothetical syllogism using (4) and (5)

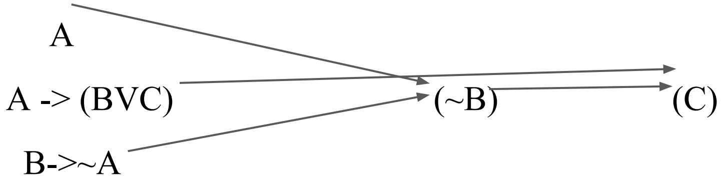
Very Important to NOTE:
P->Q = ~PVQ = ~Q -> ~P (Contrapositive)
Q->P(Converse) = ~P -> ~Q (Inverse)

(B) { (~A->~C), ~D, (A->E), (CvD) } -> E



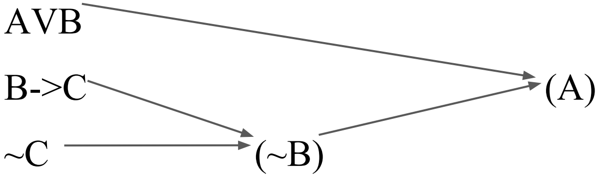
VALID

(C) { A, (A->(BVC)), (B->~A) } -> C



VALID

(D) { (AVB), (B->C), ~C } -> ~A



NOT VALID

Let P (x), Q(x), and R(x) be the statements “x is a teacher,” “x is arrogant,” and “x is rude,” respectively. Express each of these statements using quantifiers; logical connectives; and P (x), Q(x), and R(x), where the domain consists of all people. a) No teachers are arrogant. b) All arrogant people are rude. c) No teachers are rude.

(a) $\forall x(P(x) \rightarrow \sim Q(x))$

(b) $\forall x(Q(x) \rightarrow R(x))$

(c) $\forall x(P(x) \rightarrow \sim R(x))$

Let P (x), Q(x), and R(x) be the statements “x is a clear explanation,” “x is satisfactory,” and “x is an excuse,” respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P (x), Q(x), and R(x). a) All clear explanations are satisfactory. b) Some excuses are unsatisfactory. c) Some excuses are not clear explanations.

(a) $\forall x(P(x) \rightarrow Q(x))$

(b) $\exists x(R(x) \wedge \sim Q(x))$

(c) $\exists x(R(x) \wedge \sim P(x))$

. Let P (x), Q(x), R(x), and S(x) be the statements “x is a baby,” “x is logical,” “x is able to manage a crocodile,” and “x is despised,” respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P (x), Q(x), R(x), and S(x). a) Babies are illogical. b) Nobody is despised who can manage a crocodile. c) Illogical persons are despised. d) Babies cannot manage crocodiles. ∗e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

(a) ∀x(P(x)-> ~Q(x))
(b) ¬∃x(S(x) ∧ R(x)) **** note v.imp
(c) ∀x(~Q(x)-> S(x))
(d) ∀x(P(x)-> ~R(x))

Consider these statements, of which the first three are premises and the fourth is a valid conclusion. “All hummingbirds are richly colored.” “No large birds live on honey.” “Birds that do not live on honey are dull in color.” “Hummingbirds are small.” Let P (x), Q(x), R(x), and S(x) be the statements “x is a hummingbird,” “x is large,” “x lives on honey,” and “x is richly colored,” respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and P (x), Q(x), R(x), and S(x).

(a) ∀x(P(x)-> S(x))
(b) ¬∃x(Q(x) ∧ R(x)) *****note v.imp
(c) ∀x(¬R(x) → ¬S(x))
(d) ∀x(P (x) → ¬Q(x))

“All lions are fierce.” “Some lions do not drink coffee.” “Some fierce creatures do not drink coffee.”Let P (x), Q(x), and R(x) be the statements “x is a lion,” “x is fierce,” and “x drinks coffee,” respectively.Assuming that the domain consists of all creatures, express the statements in the argument using quantifiers and P (x), Q(x), and R(x)

(a) $\forall x(P(x) \rightarrow Q(x))$
(b) $\exists x(P(x) \land \neg R(x))$
(c) $\exists x(Q(x) \land \neg R(x))$.

Express the statements “Some student in this class has visited Mexico” and “Every student in this class has visited either Canada or Mexico” using predicates and quantifiers.We introduce M(x), which is the statement “x has visited Mexico.” If the domain for x consists of the students in this class, We introduce S(x) to represent “x is a student in this class.We let C(x) be “x has visited Canada.”

(a) $\exists x(S(x) \land M(x))$
(b) $\forall x(S(x) \rightarrow (C(x) \lor M(x)))$.

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a)Someone in your class can speak Hindi.

b)Everyone in your class is friendly.

c)There is a person in your class who was not born in California.

d)A student in your class has been in a movie.

e) No student in your class has taken a course in logic programming.

Solution: $\exists x(C(x) \land H(x))$

$\forall x(C(x) \rightarrow F(x))$

$\exists x(C(x) \land \sim N(x))$

$\exists x(C(x) \land M(x))$

$\sim \exists x(C(x) \land L(x))$