Unit 1
Formal Logic
(Lecture 2)

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Today’s Agenda

- What is Predicate logic
- What is Predicate
- What are Quantifiers
- Bounded and free variables
- Quantifiers with negation
- Distributive property of Quantifiers

What is Predicate logic?

Predicate logic is an extension of Propositional logic. It adds the concept of predicates and quantifiers to better capture the meaning of statements that cannot be expressed by propositional logic.

What is Predicate

Consider the statement, “ X is greater than 3″. It has two parts. The first part, the variable “X”, is the subject of the statement. The second part, “is greater than 3”, is the **predicate**.

The statement “ X is greater than 3″ can be denoted by P(X) where P denotes the predicate “is greater than 3” and X is the variable. The predicate P can be considered as a function. It tells the truth value of the statement at X . Once a value has been assigned to the variable X , the statement P(X) becomes a proposition and has a truth or false(tf) value.

- **Example 1:** Let P(X) denote the statement “ X > 10″. What are the truth values of P(11) and P(5)?

  Solution: P(11) is equivalent to the statement 11 > 10, which is True.

  P(5) is equivalent to the statement 5 > 10, which is False.

- **Example 2:** Let R(X,Y) denote the statement “X=Y+1“. What is the truth value of the propositions R(1,3) and R(2,1) ?

  Solution: R(1,3) is the statement 1 = 3 + 1, which is False.

  R(2,1) is the statement 2 = 1 + 1, which is True.

What are Quantifiers

In predicate logic, predicates are used with quantifiers to express the extent to which a predicate is true over a range of elements. There are two types of quantification-

1. **<u>Universal Quantification</u>-** Mathematical statements sometimes assert that **“a property is true for all the values of a variable in a particular domain” Such a statement is expressed using universal quantification.** Universal quantification can be used to form a proposition P(X) which is true for all the values of X in the domain.
It is denoted by $\forall$.

    Example 1: Let P(X) be the statement “ X+2>X “. What is the truth value of the statement for any real number?

    Solution: As X+2 is greater than X for any real number, so P(X) = T for all X or $\forall x P(x)=T$.

2. **<u>Existential Quantification</u>-** Existential quantification can be used to form a proposition that is true if and only if P(X) is true for at least one value of X in the domain.

    Example : Let P(X) be the statement “ X> 5″. What is the truth value of the statement for all real numbers?

    Solution: P(X) is true for all real numbers greater than 5 and false for all real numbers less than 5. So,

    $\exists x P(X)=T$

Bounded and free variables

If a variable is bounded by a Quantifier than it is called as bounded variable. Variables which is not bounded by any quantifier is called as free variable.

For example: $\forall x P(x) + y + z = 10$

Here, x is bounded variable while y and z are free variables.

Quantifiers with negation

1. $\sim \forall x P(x) = \exists x \sim P(x)$
2. $\sim \exists x P(X) = \forall x \sim P(x)$

Distributive property of Quantifiers

1. $\forall x(P(x) \wedge Q(x)) = \forall xP(x) \wedge \forall xQ(x)$
2. $\exists x(P(x) \vee Q(x)) = \exists xP(x) \vee \exists xQ(x)$

"Universal quantifier is distributive over conjunction but not on disjunction operator while existential quantifier is distributive over disjunction but not on conjunction operator".