UNIT 1
FORMAL LOGIC

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Today’s Agenda

- Propositions
- Propositions variables and connectives
- Argument/Inference
- Rule of inference

Why we need to learn discrete structures?

The mathematics of modern computer science is built almost entirely on discrete math, in particular combinatorics and graph theory. This means that in order to learn the fundamental algorithms used by computer programmers, students **will need a solid background** in these subjects.

What actually are PROPOSITION?

•Any declarative statement with a truth value assigned to it is called as proposition.

•For example: My name is ananya pandey. It is a declarative statement and it can have truth value too therefore it is a proposition. If we can’t assign truth value than you can’t say it is a proposition.

•Some examples of Propositions are given below −

•"Man is Mortal", it returns truth value “TRUE”

•"12 + 9 = 3 – 2", it returns truth value “FALSE”

•The following is not a Proposition −

•"A is less than 2". It is because unless we give a specific value of A, we cannot say whether the statement is true or false

•What is propositional logic?

•**Propositional Logic** is concerned with statements to which the truth values, “true” and “false”, can be assigned. The purpose is to analyze these statements either individually or in a composite manner.

Proposition variables and connectives

- A propositional logic consists of propositional variables and connectives. We denote the propositional variables by alphabetical letters (A, B, c,d etc). The connectives connect the propositional variables.
- In propositional logic we make use of connectives to combine two or more compound propositions/statements. Generally we use five connectives which are −
- OR/Disjunction ($\vee$)
- AND/conjunction ($\wedge$)
- Negation/ NOT ($\neg$)
- Implication / if-then ($\rightarrow$)
- If and only if ($\Leftrightarrow$).

Connectives

OR (⋁)/Disjunction– The OR operation of two propositions A and B (written as A⋁B) is true if at least any of the propositional variable A or B is true.The truth table is as follows −

A	B	AVB
T	T	T
T	F	T
F	T	T
F	F	F

AND (^)/Conjunction − The AND operation of two propositions A and B (written as A∧B) is true if both the propositional variable A and B is true.

A	B	A^B
T	T	T
T	F	F
F	T	F
F	F	F

Negation (¬)− The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

A	~A
T	F
F	T

Implication / if-then (→) − An implication A→B (~AVB) is the proposition “if A, then B”. It is false if A is true and B is false. The rest cases are true.

For ex: P: NM is PM of India.

Q: You will get into iits.

P->Q: If NM is PM of india than you will get into iits.

A	B	A->B
T	T	T
T	F	F
F	T	T
F	F	T

If and only if (⇔) − A⇔B is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

A	B	A<->B
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies

A Tautology is a formula which is always true for every value of its propositional variables.

For example $[(A \rightarrow B) \wedge A] \rightarrow B$ is a tautology.

A	B	A->B	(A->B)^A	[(A->B)^A]->B
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Contradictions

A Contradiction is a formula which is always false for every value of its propositional variables.

Example − Prove

$(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$ is a contradiction.

A	B	AVB	~A	~B	(~A)^(~B)	(AVB)^[(~A)^(~B)]
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

Contingency

A Contingency is a formula which has both some true and some false values for every value of its propositional variables.

Propositional Equivalences

Two statements X and Y are logically equivalent if any of the following two conditions hold −

- The truth tables of each statement have the same truth values.
- The bi-conditional statement $X \Leftrightarrow Y$ is a tautology.
- For example : $\neg(A \lor B)$ and $[(\neg A) \land (\neg B)]$ are equivalent

Inverse, Converse, and Contra-positive

Implication / if-then is also called a conditional statement. It has two parts −

- Hypothesis, p
- Conclusion, q

As mentioned earlier, it is denoted as $p \rightarrow q$.

Inverse − An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the statement is “If p, then q”, the inverse will be “If not p, then not q”. Thus the inverse of **$p \rightarrow q$ is $\neg p \rightarrow \neg q$**

Example − The inverse of “If you do your homework, you will not be punished” is “If you do not do your homework, you will be punished.”

Converse − The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is “If p, then q”, the converse will be “If q, then p”. The converse of $p \rightarrow q$ is $q \rightarrow p$.

Example − The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do your homework”.

Contra-positive − The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is “If p, then q”, the contra-positive will be “If not q, then not p”. The contra-positive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$

Example − The Contra-positive of " If you do your homework, you will not be punished” is "If you are punished, you did not do your homework”.

Equivelence

$$p \to q = \sim p \vee q$$
$$p \to q = \sim q \to \sim p$$
$$p \vee q = \sim p \to q$$
$$p \wedge q = \sim (q \to \sim p)$$
$$\sim (p \to q) = p \wedge \sim q$$
$$(p \to q) \wedge (p \to r) = p \to (q \wedge r)$$
$$(p \to r) \wedge (q \to r) = (p \vee q) \to r$$
$$(p \to q) \vee (p \to r) = p \to (q \vee r)$$
$$(p \to r) \vee (q \to r) = (p \wedge q) \to r$$
$$p \leftrightarrow q = (p \to q) \wedge (q \to p)$$
$$p \leftrightarrow q = \sim p \leftrightarrow \sim q$$
$$p \leftrightarrow q = (p \wedge q) \vee (\sim p \wedge \sim q)$$
$$\sim (p \leftrightarrow q) = p \leftrightarrow \sim q$$

De Morgan's Law:

$\sim(p \wedge q) = \sim p \vee \sim q$

$\sim(p \vee q) = \sim p \wedge \sim q$

P: Ravi has a computer

Q: A phone

P^Q: Ravi has a computer and a phone

~(P^Q): ~P V~Q= Ravi doesnot have computer or ravi doesnot have a phone.

Argument/Inference: If a set of premises {p1,p2,p3…..pn} yields another proposition ‘Q’ (this ‘Q’ is a conclusion) than whole process is called as argument.

An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction

Check whether the below premises are valid or not?

(1) $p \to q$

$q \to r$

$\sim r$

$\sim p$

(2) $\sim p$

$p \vee q$

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q

(3) $p \to q$

$q \to r$

$\sim p$

$\sim r$

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Now, using transitivity Rule

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline \therefore P \to R \end{array}$$

Now,

$$\left.\begin{array}{l} P \to R \\ \sim R \text{ is valid} \\ \hline \therefore \sim P \end{array}\right\}$$

from Modus Tollen

$\sim R \to T$

$\sim P \vee R$

$T \vee F \to T$

b) The conclusion $p \to q$ follows from the premises $\{\sim p, p \vee q\}$

$$\begin{array}{l} \sim p \\ p \vee q \\ \hline \therefore q \\ \hline \end{array}$$

by using Disjunctive Syllogism

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$P \to Q$
$Q \to R$
$\sim P$

By transitivity $P \to Q$
$Q \to R$
$\therefore P \to R$

Now, $P \to R$
$\sim P$ valid
$\therefore R$

$\sim P \vee R$
$T \vee T$
$\sim R . F$

But $\sim R$ can't be concluded $\therefore$ This argument is false. bcz It is a fallacy of assuming Inverse