Unit 1
Formal logic
(Lecture 3)

By Ananya Pandey

1. $\sim\forall x P(x) = \exists x \sim P(x)$
2. $\sim\exists x P(X) = \forall x \sim P(x)$

Example on negating the quantifiers

Negate the following:

(a) There is a honest politician.

Solution: Assume H(x)= x is honest

x= All politicians

We conclude that above sentence can be represented as $\exists xH(x)$, Now we want a negation of this statement so as we know $\sim\exists xP(X) = \forall x\sim P(x)$

$\forall x\sim H(x)$ it means that- For all x and if x is a politician than x(politician) will be dishonest. So negation will be: All the politicians are dishonest.

(b) All americans eat cheeseburgers.

Solution: Assume, C(x)= x eats cheeseburgers

Where, x= All americans

We conclude that above sentence can be represented as $\forall xC(x)$, Now we want a negation of this statement so as we know $\sim\forall xP(x) = \exists x\sim P(x)$

$\exists x\sim C(x)$ it means that- there exists an x and if x is an american than x does not eats cheeseburgers.

Example on negating the quantifiers

(c) $\forall x(x^2>x)$

Solution: $\sim\forall x(x^2>x)$

$\exists x\sim(x^2>x)$

$\exists x(x^2<=x)$

(d) $\exists x\ (x^2=2)$

solution: $\sim\exists x(x^2=2)$

$\forall x\sim(x^2=2)$

$\forall x(x^2!=2)$

Some important examples/properties:

(1) $\sim\forall x(P(x)\wedge Q(x)) = \exists x\sim(P(x)\wedge Q(x)) = \exists x(P(x)vQ(x))$

(2) $\sim\forall x(P(x)\text{->}Q(x)) = \exists x\sim(P(x)\text{->}Q(x)) = \exists x\sim(\sim P(x)vQ(x)) = \exists x(P(x)\wedge\sim Q(x))$ V.IMP NOTE: P->Q=~PvQ

(3) $\sim\exists x(P(x)\wedge Q(x)) = \forall x\sim(P(x)\wedge Q(x)) = \forall x(\sim P(x)v\sim Q(x)) = \forall x(P(x)\text{->}\sim Q(x))$

Logical translation of the english statement using quantifiers

(a) Every student in this class has taken the course OS.

Solution: Let O(x): x has taken the course OS.
S(x): x is a student in the class.
Where, x: All students in the class
We can conclude that: $\forall x\ (S(x) \rightarrow O(x))$

(b) Some of the student in this class has taken the course OS.

Solution: Let O(x): x has taken the course OS.
S(x): x is a student in the class.
Where, x: All students in the class
We can conclude that: $\exists x\ (S(x) \wedge O(x))$

VERY VERY IMPORTANT

- If question has “every” “all” words, than use ‘$\forall$’ quantifier and also use ‘P->Q’.
- If question has “some” word, than use ‘$\exists$’ quantifier and also use ‘P^Q’.

Logical translation of the english statement using quantifiers

(C) Every student in this class has taken either OS or DBMS.

Solution: Let O(x): x has taken the course OS.
D(x): x has taken the course DBMS.
Where, x: All students in the class
S(x): x is a student in the class.
We can conclude that: $\forall x\ (S(x) \rightarrow (O(x) \vee D(x))$

(d) Some student in this class has taken either OS or DBMS.

Solution: Let O(x): x has taken the course OS.
D(x): x has taken the course DBMS.
Where, x: All students in the class
S(x): x is a student in the class.
We can conclude that: $\exists x\ (S(x) \wedge (O(x) \vee D(x))$

Logical translation of the english statement using quantifiers

(e) Some real numbers are rational.

Solution: Real(x): x is a real number
Rational(x): x is a rational
We can conclude: $\exists x\ (Real(x) \wedge Rational(x))$

(f) Not all that glitter is gold.

Solution: For this question it would be easier to first make a logical translation for "all glitter is gold" and than do negation of it, because it can also be seen as Not(All that glitter is gold). So, the logical translation of "all glitter is gold". $\forall x\ (glitter(x) \to gold(x))$. Now negation will be $\exists x \sim(glitter(x) \to gold(x))$
$\exists x \sim(\sim glitter(x) \vee gold(x)) = \exists x (glitter(x) \wedge \sim gold(x))$

Logical translation of the english statement using quantifiers

(g) Gold and silver ornaments are precious.

Solution: $\forall x \ ((gold(x) \lor silver(x)) \rightarrow precious(x))$