13 August 2023 11:00

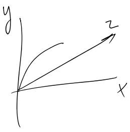
Linear Algebra:

✓ Highest power 1
Sequential
$y = mx + c$
✓ constant

slope constt.
✓ $y \to x, z$

$ax + by + cz + d = 0$

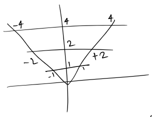
$y = mx + c$



$y = 2$, $x = 6$

$y = \sin x$ Taylor series

$$\boxed{y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} - - - - -}$$



$y = |x| = (\sqrt{x})^2$

$\boxed{y = |x|}$ $y \in (0, \infty)$
$x \in (-\infty, +\infty)$

$|-2| = 2$
$|2| = 2$

$\boxed{y = (\sqrt{x})^2}$

for a single $y \to 2x$

$$y = \begin{cases} +x & [0, \infty) \\ -x & (-\infty, 0) \end{cases}$$

$(0,1)$ $0 < x < 1$ $[0,1)$
$[0,1]$ $0 \le x \le 1$
discrete $\{0,1\}$ $x = 0, x = 1$

a, b, c
$\underbrace{}$
scalors

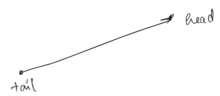
Vectors:

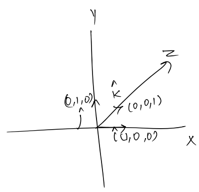
✓ → 1. Physics: any quantity with magnitude & dirn. $(\vec{v})$ ⟶

✓ → 2. Computer Science: any ordered pair $a = [2, 3]$ $[1, 2, 3]$

→ 3. Mathematics: Phy.
CS.

10 North
$\vec{V}$

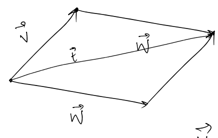




{ basis of co-ordinate system:
$\hat{i}, \hat{j}, \hat{k}$ $5\hat{i}$ $-5\hat{i}$

$10\hat{i} + 4\hat{j}$

$\vec{V}$ $\vec{w}$ $\vec{V} + \vec{w} = \vec{t}$

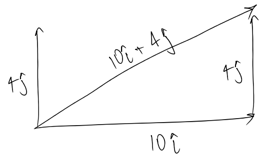


$\vec{v} + \vec{w} = \vec{t}$
$\vec{w} + \vec{v} = \vec{t}$

2 rules
→ 1. Satisfy rule of vector addition.
→ 2. Scaling of a vector

$\vec{v} = 5\hat{i}$

$\vec{V}$ ⟶ = ⟶ $2\vec{v}$ $2\vec{v} = 10\hat{i}$

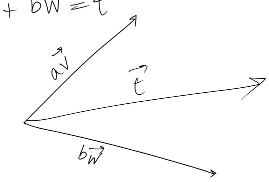


$|\vec{V}| = 13$ $2\vec{v} = (5\hat{i} + 12\hat{j}) \times 2$
$= 10\hat{i} + 24\hat{j}$

$\vec{V} = x\hat{i} + y\hat{j}$
$|\vec{V}| = \sqrt{x^2 + y^2}$

Linear Combination of vectors:

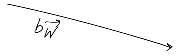
$a\vec{V} + b\vec{W} = \vec{t}$



Case I: $a \to$ constant $b \to$ var. $(a = 1)$

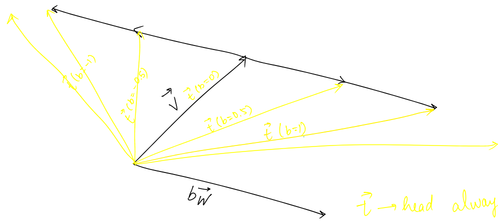
$\vec{a}$ $\quad$ $\underbrace{A, B, C}_{\text{Matrix}}$

vect.



Case I: $a \to$ constant $b \to$ var. $(a=1)$

{ addition rule

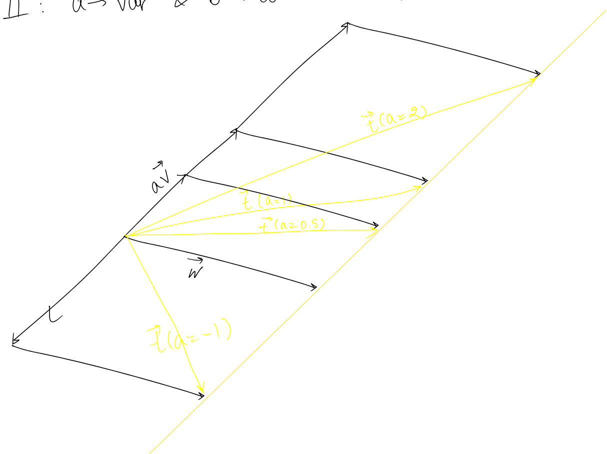


$\vec{v} + b\vec{w} = \vec{t}$

$\vec{t} \to$ head always & always lie on $\vec{w}$'s direction

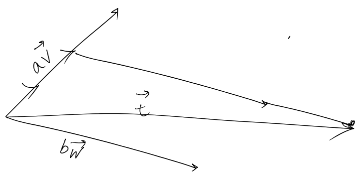
Case II: $a \to$ var & $b \to$ constant $(b=1)$

$a\vec{v} + \vec{w} = \vec{t}$



Case III: $a \to$ var $b \to$ var

$\vec{t} = a\vec{v} + b\vec{w}$



$a = 0.5$

$b = 1.5$

$\vec{t} \to$ reach every point in 2D plane.

Span of vector: All the points that my vect. can (Range) reach is called span of $\vec{v}$.

$b\vec{w} \to$ line

Matrix:

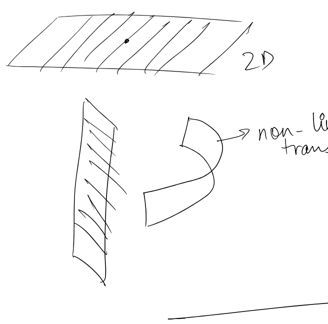
what is a matrix?

Its a function that does linear transformations

$(100 \times 100) \times (100 \times 8)$

$(100 \times 8)^T \to (8 \times 100) \times (100 \times 10)$

$= (8 \times 10)^T \longrightarrow (10 \times 8)$



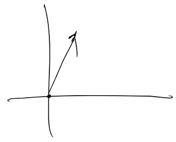
2 rules for linear transformation:

→ 1. The st. line should remain parallel after

near
formation

1. The st. line should remain parallel after transformation.
2. Origin shouldn't change.

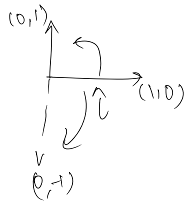
$\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$
$\hat{i} \quad \hat{j}$



$\vec{V} = \hat{i} + \hat{j}$

$$M = \begin{array}{cc} \left[\begin{pmatrix}1\\3\end{pmatrix} \right. & \left. \begin{pmatrix}2\\4\end{pmatrix}\right] \\ \hat{i}_N & \hat{j}_N \end{array}$$

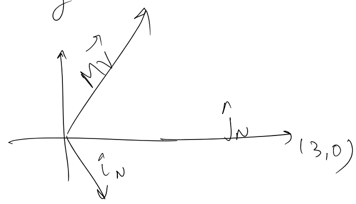
$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$



$M\vec{V} = ? \quad = 3\hat{i} + 7\hat{j}$

Matrix: It changes your $(\hat{i}, \hat{j}, \hat{k})$ basis of co-ordinate system.

$$M = \begin{array}{c} \begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix} \\ \hat{i}_N \quad \hat{j}_N \end{array}$$



$\begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix}$

$\vec{V} = -1\hat{i} + 2\hat{j}$

$M\vec{V}$

$\vec{V} = -1\hat{i} + 2\hat{j} = -1(\text{dir}^n \text{ of } \hat{i}) + 2(\text{dir}^n \text{ of } \hat{j})$

$M\vec{V} = -1(\text{transformed } \hat{i}_N) + 2(\text{transformed } \hat{j}_N) = -1\hat{i}_N + 2\hat{j}_N$

$= -1\begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}3\\0\end{bmatrix}$

$= \begin{bmatrix}-1\\2\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix} = \begin{bmatrix}5\\2\end{bmatrix}$

$= 5\hat{i} + 2\hat{j}$

(nx

$$\begin{array}{c} \begin{bmatrix}a & b\\c & d\end{bmatrix} \\ \hat{i}_N \quad \hat{j}_N \end{array} \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}ax + by\\cx + dy\end{bmatrix}$$

$M\vec{V} \Rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}ax\\cx\end{bmatrix} + [$

$= \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$) \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$\underline{n}) \times (\underline{m} \times p) \Rightarrow n \times p$

$\begin{matrix} by \\ dy \end{bmatrix} \Rightarrow \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} \Rightarrow (ax + by)\hat{i} + (cx + dy)\hat{j}$

$M\vec{v} \Rightarrow x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} \\ d \end{bmatrix} = [\,[x\,]$

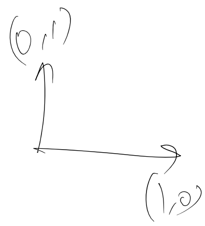
$M =$

$M = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

ro

c



It tells th

$\dots y] \quad [cx + dy]$

$= \begin{bmatrix} 10 & 30 \\ 7 & 2 \end{bmatrix}$ $\quad M_2 M_1 \vec{v} \quad M_1 M_2 \vec{v}$

$M_1 M_2 \neq M_2 M_1$

$f(g(x))$

$A(BC) = (AB)C$

$\dots 2 \; \dots 4] \quad M_2 = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Matrix formulation:

$\begin{bmatrix} \dots & -1 \\ \dots & 0 \end{bmatrix} M_1 \qquad M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} M_2$

...tation 90°
...nti clockwise

Shear matrix

$M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

rotation
clock wise

Determinants:

$M = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$

... change in area of the basis of co-ordinate system.

Rank of a matrix: no. of unique dimensions in o/p linear transformation.

matrix:

inv matrix = a

S

$\vec{v}.\vec{w} =$

$\vec{v}$

Area

$\vec{v} \times \vec{w} = |\vec{v}||\vec{w}$

Eigen vectors an

Eigen values:

$$M = \begin{bmatrix} 10 & 1 \\ 0 & 1 \end{bmatrix}$$

$\vec{v} \rightarrow$ eigen vect.

Eigen $\vec{v}$ are

span even afte

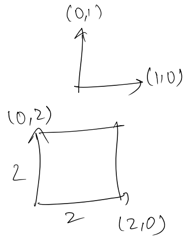
$$M\vec{v} = \lambda \vec{v}$$

system.

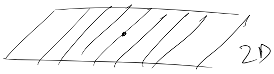
linear transformation.

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$= 4$



$\frac{\text{adj}(M)}{|M|}$ $|M| = 0$. inv.(M)



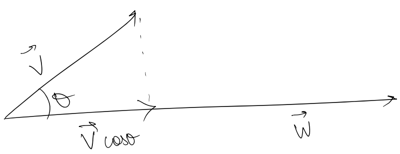
Dot product:

b/w $\vec{V}$'s.

similarity b/w 2 vectors $\underbrace{\text{(Scalor)}}_{\text{output}}$

Projection of a $\vec{v}$ wrt another $\vec{v}$.

$|\vec{v}||\vec{w}|\cos\theta$



$= 10\hat{i} + 2\hat{j}$

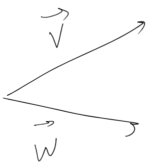
$\vec{w} = 2\hat{i} + 5\hat{j}$ $= \vec{v}.\vec{w} = 20 + 10 = 30$

Cross product:

B/w 2 vectors.

b/w 2 $\vec{v}$.

$|\sin\theta\ \hat{n}$



d

$\vec{v}$ that remain on their

r linear transformation.

x : scalor / eigen value]

$M\vec{v} - \lambda I\vec{v} = 0$

$(M - \lambda I)\vec{v} = 0$

$\det(M - \lambda I) = 0$

$\det\left(\begin{bmatrix} 10-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix}\right) = 0$

$(10-\lambda)(1-\lambda) = 0$

$\lambda_1 = 1, \ \lambda_2 = 10$

$\begin{bmatrix} 10 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 1$

$\begin{bmatrix} 10x+y \\ y \end{bmatrix} =$

$10x + y =$

$y = -$

Any $\vec{v}$ in dir^n of $x($

& eigen value i

$(\vec{v} = 0)$

$\vec{v} = a\hat{i} + b\hat{j}$

Let my $\vec{v} = x\hat{i} + y\hat{j}$

x & y are scalar values

$\begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 10 & 1 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$\begin{bmatrix} 10a + b \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ -9a \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -9x \end{bmatrix}$

$10a + b = a$

$b = -9a$

$= a \begin{bmatrix} 1 \\ -9 \end{bmatrix}$

$= x$

$9x$

$= x \begin{bmatrix} 1 \\ -9 \end{bmatrix}$

$= a(\hat{i} - 9\hat{j})$

$\hat{i} - 9\hat{j})$ is your eigen vector.

$\lambda = 1$.