Team Assignment #3

Course: Quantum Field Theory on a Quantum Computer **Assigned:** Oct 24, 2025 **Due:** Oct 31, 2025 (23:59 IST)

Teams: Work in teams. Submit one PDF report (max 20 pages, including appendix) and a zip with code/data. List all team members on the first page.
Tools: Preferably use QISKIT and Python. Acknowledge AI whenever used. However, we are going to look at what inputs were given as a team and how you went beyond AI.
Reproducibility: Submit commented nbs.

Question 1

Consider the one-dimensional Hamiltonians

$$H_{\text{free}} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\hat{x}^2, \qquad H_{\text{anh}} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\hat{x}^2 + \frac{\lambda}{4}\hat{x}^4,$$

with $\lambda > 0$. In the JLP encoding, the field operator $\hat{x}$ is represented on a uniform grid over the interval $[-x_{\max}, x_{\max}]$ using n_q qubits, giving $N = 2^{n_q}$ points in the grid.

Task 1: Energy Convergence

For a range of values of $(x_{\max}, n_q)$, compute the lowest few eigenvalues

$$E_0(x_{\max}, n_q),\ E_1(x_{\max}, n_q),\ \ldots,\ E_4(x_{\max}, n_q)$$

for both H_{free} and H_{anh}. Study how these eigenvalues change:

1. as $x_{\max}$ is varied at fixed n_q,
2. as n_q is varied at fixed $x_{\max}$.

Task 2: Wavefunction Convergence

For the same parameter choices, compute the corresponding eigenfunctions

$$\psi_k(x_{\max}, n_q), \qquad k = 0, \ldots, 4.$$

Compare the wavefunction shapes:

- for different $x_{\max}$ at fixed n_q,
- for different n_q at fixed $x_{\max}$.

Task 3: Comparison

Summarize how the convergence of the *energy spectrum* and the *wavefunctions* depend on $x_{\max}$ and on n_q. Comment on whether good energy convergence necessarily implies good wavefunction convergence.

Task 4: Nyquist–Shannon Sampling

Read about the Nyquist–Shannon sampling theorem and write a short explanation of how your numerical results reflect the role of sampling in both the x-domain and p-domain. Identify parameter choices for which the representation appears to be "well-balanced."

Question 2

In the lectures, we learnt about the adiabatic theorem. There is a way to speed up the adiabatic state preparation using *Variational Counterdiabatic Driving*. Carry out some research on this topic and examine how well it works for the anharmonic oscillator (don't forget the large λ regime).

Question 3

We saw that for lattice spacing a, the Hamiltonian for 1+1d ϕ^4 theory can be written as

$$H = \sum_n \left[\frac{1}{2}\pi_n^2 + \frac{1}{2}(2+\mu_0^2)\phi_n^2 + g_0\phi_n^4\right] - \kappa\sum_n \phi_n\phi_{n+1}.$$

Here $\kappa = 1$. If $\kappa = 0$ then you can think of this as n decoupled AHO, one at each site n. κ couples these AHOs. Thus, one can think of setting up the adiabatic state preparation by choosing $\kappa \to \kappa(s)$ such that $\kappa(0) = 0$ and $\kappa(1) = 1$ and using $\psi(0)$ appropriately. Explore this possibility and compare with the way we discussed adiabatic state preparation in class, i.e., using g_0 as the ramp with $g_0 \to g_0(s)$ such that $g_0(0) = 0, g_0(1) = g_0$.