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★ Assigment - 3

Exercise: find the Eigen Values & Eigen Vectors of following matrices.

1. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ $\left[\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution:- $[A - \lambda I] X = 0$

$[A - \lambda I] = 0$

$\begin{bmatrix} 1 & -6 \\ -2 & 0 \end{bmatrix} = -12$, $\begin{bmatrix} -2 & -3 \\ -1 & 0 \end{bmatrix} = -3$

$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$ $\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} = -6$

$\lambda^3 - [\text{Sum of diagonal element}]\lambda^2 + [\text{Sum of diagonal minors}]\lambda - |A| = 0$

$\lambda^3 - [-1]\lambda^2 + [-21]\lambda - 45 = 0$

$\boxed{\lambda = 3, -3, -1}$ - Eigen values

Eigen Vectors

$\lambda = 3$ $\begin{bmatrix} -5 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-5x_1 + 2x_2 - 3x_3 = 0$ ①

$2x_1 + 2x_2 - 6x_3 = 0$ ②

$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -2 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -5 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -5 & 2 \\ 2 & -2 \end{vmatrix}}$

$\frac{x_1}{-18} = \frac{-x_2}{36} = \frac{x_3}{6}$

$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -18 \\ -36 \\ 6 \end{bmatrix} = \begin{bmatrix} -9 \\ -18 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 1 \end{bmatrix}$ ✓

$\boxed{\lambda = -3}$ $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\cancel{x_1 + 2x_3 - 3x_3 = 0}$

$\cancel{-x_1 - 2x_2 + 3x_3 = 0}$

~~Crammers rule~~ :-

$\cancel{\dfrac{x_1}{\begin{vmatrix} 2 & -3 \\ -2 & 3 \end{vmatrix}}} = \cancel{\dfrac{-x_2}{\begin{vmatrix} 1 & -3 \\ -1 & 3 \end{vmatrix}}} = \cancel{\dfrac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix}}}$

$\cancel{x_1}$

$x_1 + 2x_2 - 3x_3$

$2x_1 + 4x_2 - 6x_3$

Crammers rule :-

$\dfrac{x_1}{\begin{vmatrix} 2 & -3 \\ 4 & -6 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} 1 & -3 \\ 2 & -6 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}}$

$\underline{x_1}$

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2. $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $\left(\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$[A - \lambda I] X = 0$ $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ $\begin{vmatrix} 4 & 1 \\ -2 & 1 \end{vmatrix} = 6$

$[A - \lambda I] = 0$

$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = 0$ $\begin{vmatrix} 4 & 0 \\ -2 & 1 \end{vmatrix} = 4$

$\lambda^3 - [6]\lambda^2 + [11]\lambda - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\boxed{\lambda = 1, 2, 3}$ - Eigen values

$\boxed{\lambda = 1}$ Eigen vectors

$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

Crammer's rule

$3x_1 + 0x_2 + 0x_3 = 0$

$-2x_1 + 0x_2 + 0x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{2} = \frac{x_3}{0}$$

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3 $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ $\left[\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$(A - \lambda I) X = 0$ $\quad \begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0 \quad \begin{vmatrix} 5 & 0 \\ -1 & 3 \end{vmatrix} = 15$

$A - \lambda I = 0$

$\begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} = 0 \quad \begin{vmatrix} 5 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\lambda^3 - [8]\lambda^2 + [15]\lambda - 0 = 0$

$\boxed{\lambda = 0, 5, 3}$ - Eigen values

$\boxed{\lambda = 0}$ Eigen vectors

$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Crammers rule

$5x_1 + 0x_2 + 0x_3 = 0$

$-x_1 + 0x_2 + 3x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 0 \\ -1 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 0 \\ -1 & 0 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{15} = \frac{x_3}{0}$$

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4. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$ $\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$[A - \lambda I] X = 0$ $\begin{vmatrix} 3 & 4 \\ 0 & -2 \end{vmatrix} = -6$ $\begin{vmatrix} 0 & 0 \\ 0 & -2 \end{vmatrix} = 0$

$|A - \lambda I| = 0$

$\begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix} = 0$ $\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0$

$\lambda^3 - [1] \cdot \lambda^2 + [-6] \lambda - 0 = 0$

$\boxed{\lambda = 0, 3, -2}$ - Eigen values

$\boxed{\lambda = 0}$ Eigen vectors

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Crammers rule

$0x_1 + 3x_2 + 4x_3 = 0$

$0x_1 + 0x_3 - 3x_3 = 0$

$\dfrac{x_1}{\begin{vmatrix} 3 & 4 \\ 0 & -3 \end{vmatrix}} = \dfrac{-x_2}{\begin{vmatrix} 0 & 4 \\ 0 & -3 \end{vmatrix}} = \dfrac{x_3}{\begin{vmatrix} 0 & 3 \\ 0 & 0 \end{vmatrix}}$

$\dfrac{x_1}{-9} = \dfrac{-x_2}{0} = \dfrac{x_3}{0}$

5. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

This ~~is~~ matrix has an eigenvalue of 1 without any calculation

Reason :- A square matrix has an eigen value of 1 if there exists a column (or row) in the matrix that consists entirely of ones, and all other corresponding entries in each row (or column) add up to zero