Name :- Abhijeet Bisen
Enrollment Id :- 2301010003

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Assignment - 1

(a) Test for Consistency & solve

(i) $2x-3y+7z=5, \quad 3x+y-3z=13, \quad 2x+19y-47z=32$

Solution :-

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - \frac{3}{2} R_1 \\ R_3 = R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 13.5 & -14.5 & 14.5 \\ 0 & 22.5 & -54.5 & 27 \end{bmatrix} \quad R_2 = \frac{2}{27} R_2$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1 & -1.074 & 1 \\ 0 & 22.5 & -54.5 & 27 \end{bmatrix} \quad R_3 = R_3 - 22.5 R_2$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1 & -1.074 & 1 \\ 0 & 0 & -1.93 & 4 \end{bmatrix} \quad R_3 = \frac{-1}{1.93} R_3$$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1 & -1.074 & 1 \\ 0 & 0 & 1 & -2.07 \end{bmatrix}$$

$\boxed{z = -2.07}$

$1y - 1.074(-2.07) = 1$

$y + 2.20158 = 1$

$\boxed{y = -1.20158}$

$2x - 3(-1.20158) + 7(-2.07) = 5$

$2x + 3.60474 - 14.49 = 5$

$2x - 10.88526 = 5$

$2x = 15.88526$

$\boxed{x = 7.94263}$

(ii) $2x - y + 3z = 8, \; -x + 2y + z = 4, \; 3x + y - 4z = 0$

Solution : $\begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$ $R_2 = R_2 + R_1$, $R_3 = R_3 - \frac{3}{2} R_1$

$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 1 & 4 & 12 \\ 0 & 3.5 & -7.5 & -12 \end{bmatrix}$ $R_3 = R_3 - \frac{7}{2} R_2$

$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & -21 & -60 \end{bmatrix}$ $R_3 = \frac{-1}{21} R_3$

$\begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & 1 & 20/7 \end{bmatrix}$

$\boxed{z = \frac{20}{7}}$

$y + 4\left(\frac{20}{7}\right) = 12$

$y + \frac{80}{7} = 12$

$\boxed{y = \frac{64}{7}}$

$2x - \left(\frac{64}{7}\right) + 3\left(\frac{20}{7}\right) = 8$

$2x - \frac{64}{7} + \frac{60}{7} = 8$

$2x - \frac{4}{7} = 8$

$2x = \frac{60}{7} + \frac{4}{7}$

$2x = \frac{64}{7}$

$\boxed{x = \frac{32}{7}}$

(iii) $4x - y = 12, \quad -x + 5y - 2z = 0, \quad -2x + 4z = -8$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 0 & -8 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - \left(-\frac{1}{4}\right) R_1 \\ R_3 = R_3 - (-1/2) R_1 \end{array}$$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 21/4 & -2 & 3 \\ 0 & 1 & 0 & -4 \end{bmatrix} \quad R_2 = \frac{4}{21} R_2$$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 1 & -8/21 & 3/7 \\ 0 & 1 & 0 & -4 \end{bmatrix} , R_3 = R_3 - R_2$$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 1 & -8/21 & 3/7 \\ 0 & 0 & 8/21 & -25/7 \end{bmatrix} \quad R_3 = \frac{21}{8} R_3$$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 1 & -8/21 & 3/7 \\ 0 & 0 & 1 & -25/8 \end{bmatrix} \quad \boxed{z = \frac{-25}{8}}$$

$y - \frac{8}{21}\left(\frac{-25}{8}\right) = \frac{3}{7}$

$y + \frac{25}{21} = \frac{3}{7}$

$y = \frac{3}{7} - \frac{25}{21}$

$\boxed{y = \frac{-16}{21}}$

$4x - \left(\frac{-16}{21}\right) = 12$

$4x + \frac{16}{21} = 12$

$4x = 12 - \frac{16}{21}$

$4x = \frac{236}{21}$

$\boxed{x = \frac{59}{21}}$

(b) For what values of λ and μ the given system of equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has (i) no solution, (ii) a unique solution and (iii) infinite number of solutions.

Solution (i) $\Rightarrow \Delta A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix}$

$= 1(2\lambda-6) - 1(3\lambda-3) + 1(2-6)$

$= 2\lambda - 6 - 3\lambda + 3 + 2 - 6$

$= 2\lambda - 7$

$\Delta A \neq 0$ for unique solution

$2\lambda - 7 \neq 0$

$\boxed{\lambda \neq \frac{7}{2}}$

(ii) If $\det(A) = 0$

M is minor

$M_1 : \begin{vmatrix} 2 & 3 \\ 2 & \lambda \end{vmatrix} = 2\lambda - 6$

$M_2 : \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3 - 1 = 2$

$M_3 : \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$

system to have no solution then all minor determinants must be zero & $\det(A) = 0$

so,

$2\lambda - 6 = 0$ $\boxed{\lambda = 3}$

(iii) for infinitely many solution atleast one of the correspond minor determinants is nonzero

we have $\lambda = 3$ it satisfies $\det(A) = 0$

~~Solution~~

~~$AX = B$~~

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(c) find for what values of λ ~~of~~ the given equation $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution and solve them completely in each case.

Solution:- $\det A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}$

$= 1(2\times10-4\times4) - 1(1\times10-4\times1) + 1(1\times4-2\times1)$

$= 1(20-16) - 1(10-4) + 1(4-2)$

$= 4-6+2=0$

$\det(A) = 0$

$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ \lambda & 2 & 4 \\ \lambda^2 & 4 & 10 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & 4 \\ 1 & \lambda^2 & 10 \end{bmatrix}$

$A_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda \\ 1 & 4 & \lambda^2 \end{bmatrix}$

$\det(A_1) = 1(20-4\lambda^2) - 1(10\lambda-4\lambda^2) + 1(4\lambda-2\lambda^2)$

$= 20 - \cancel{4\lambda^2} - 10\lambda + \cancel{4\lambda^2} + 4\lambda + 2\lambda^2$

$= 4\lambda - 10$

$= \boxed{\lambda = \frac{5}{2}}$

$\det(A_2) = 1(10\lambda - 4\lambda^2) - 1(10-4) + 1(\lambda-\lambda^2)$

$= 10\lambda - 4\lambda^2 - 10 + 4 + \lambda - \lambda^2$

$= -3\lambda^2 + 11\lambda - 6$

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$$= -3\lambda^2 + 11\lambda - 6 = 0$$

$$-3\lambda^2 + 11\lambda - 6 = 0$$

$$(\lambda - 1)(3\lambda - 6) = 0$$

$$\boxed{\lambda = 1} \text{ or } \boxed{\lambda = 2}$$

$$\det(A_3) = 1(2\lambda^2 - 16) - 1(\lambda^2 - 4) + 1(4 - 2\lambda)$$

$$= 2\lambda^2 - 16 - \lambda^2 + 4 + 4 - 2\lambda$$

$$= \lambda^2 - 2\lambda - 16$$

$$= \lambda^2 - 2\lambda - 16 = 0$$

$$(\lambda - 4)(\lambda + 4) = 0$$

$$\boxed{\lambda = 4} \text{ or } \boxed{\lambda = -4}$$

The values of lamda for which the given equations have a solution are

$\lambda = \frac{5}{2}, 1, 2, 4, -4$

(d)

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(d)

Solution $A \quad X = B$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{matrix} \\ R_2 = R_2 - 2R_1 \\ R_3 = R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} \begin{matrix} R_2 = -\frac{1}{7} R_2 \\ \\ R_1 = R_1 - 3R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2/7 \\ 0 & 1 & -8/7 \\ 0 & -14 & 16 \end{bmatrix} \quad R_3 + 14R_2$$

$$\begin{bmatrix} 1 & 0 & -2/7 \\ 0 & 1 & -8/7 \\ 0 & 0 & 2 \end{bmatrix}$$

$x - \frac{2}{7} z = 0 \qquad x - 0 = 0 \qquad x = 0$

$y - \frac{8}{7} z = 0 \qquad y - 0 = 0 \qquad y = 0$

$2z = 0 \qquad \boxed{z = 0}$

Solution coefficient matrix is $\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$

$$\Delta = \begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix}$$

$$\Delta = 3\begin{vmatrix} -2 & -3 \\ 4 & \lambda \end{vmatrix} - 1\begin{vmatrix} 4 & -3 \\ 2\lambda & \lambda \end{vmatrix} - \lambda\begin{vmatrix} 4 & -2 \\ 2\lambda & 4 \end{vmatrix}$$

$$= 3(-2\lambda - (-12)) - (4\lambda + 6\lambda) - \lambda(16 - 4\lambda)$$

$$= -6\lambda + 36 - 4\lambda - 6\lambda + 16\lambda^2$$

$$= 16\lambda^2 - 16\lambda + 36$$

$$\Delta = 0$$

$$16\lambda^2 - 16\lambda + 36 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 16,\ b = -16,\ c = 36$$

$$\lambda = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \cdot 16 \cdot 36}}{2 \cdot 16}$$

$$\lambda = \frac{16 \pm \sqrt{-2048}}{32}$$

$$\lambda = \frac{16 \pm 16i\sqrt{2}}{32}$$

$$\lambda = \frac{1 \pm i\sqrt{2}}{2}$$