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★ Assignment - 2

Are the following sets of vectors linearly independent or dependent ?

1) [1 0 0], [1 1 0] [1 1 1]

Solution :-

$a1[1 0 0] + a2[1 1 0] + a3[1 1 1] = [0 0 0]$

$1a1 + 1a2 + 1a_3 = 0$

$0a1 + 1a2 + 1a_3 = 0$

$0a1 + 0a2 + 1a_3 = 0$

$a_1 + a_2 + a_3 = 0$ $\quad$ $a_1 + 0 + 0 = 0$ $\quad$ $a_1 = 0$

$a_2 + a_3 = 0$ $\quad$ $a_2 + 0 = 0$ $\quad$ $a_2 = 0$

$a_3 = 0$

Only solution is the trivial solution that the vectors are linearly independent

2) [7 -3 11 -6], [-56 24 -88 48]

Solution :-

$a1[7 -3 11 -6] + a_2[-56 24 -88 48] = [0 0 0 0]$

$7a_1 - 5a_2 = 0$ — (1)

$-3a_1 + 24a_2 = 0$ — (2) $\longrightarrow$ $C1 = 8C2$ (5)

$11a1 - 88a_2 = 0$ — (3)

$-6a_1 + 48a_2 = 0$ — (4)

Substitute 5 eqn in (1)

$7(8C_2) - 56C_2 = 0$

$0 = 0$

If we take $C_2 = 1$, then $C1 = 8$ therefore vectors are linearly dependent

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3) $[-1\ 5\ 0], [16\ 8\ 3], [-64\ 56\ 9]$

Solution :- $a_1[-1\ 5\ 0] + a_2[16\ 8\ 3] + a_3[-64\ 56\ 9] = [0\ 0\ 0]$

$-1a_1 + 16a_2 - 64a_3 = 0$ (1)

$5a_1 + 8a_2 + 56a_3 = 0$ (2)

$0a_1 + 3a_2 + 9a_3 = 0$ (3) $\quad a_2 = -3a_3$ (4)

substitute (4) in (2)

$5a_1 + 8(-3a_3) + 56a_3 = 0$

$5a_1 - 16a_3 = 0$

$a_1 = \frac{16}{5}a_3$ (5)

substituting eqn (4) & eqn (5) in (1) eqn

$-1\left(\frac{16}{5}a_3\right) + 16(-3a_3) - 64a_3 = 0$

$-\frac{16}{5}a_3 - 48a_3 - 64a_3 = 0$

$-\frac{176}{5}a_3 = 0 \quad \boxed{a_3 = 0}$

substitution $a_3 = 0$ in (4) eqn

$a_2 = 0$

substitute $a_3 = 0$ in eqn (5) $\boxed{a_1 = 0}$

Only solution is the trivial solution that the vectors are linearly independent

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4) $[1\ -1\ 1],\ [1\ 1\ -1],\ [-1\ 1\ 1]\ [0\ 1\ 0]$

Solution :- $a_1[1\ -1\ 1] + a_2[1\ 1\ -1] + a_3[-1\ 1\ 1]$
$+ a_4[0\ 1\ 0] = [0\ 0\ 0]$

$a_1 + a_2 - a_3 + 0 = 0$ ①
$-a_1 + a_2 + a_3 + a_4 = 0$ ②
$a_1 + a_2 + a_3 + 0 = 0$ ③

① + ③ $\quad 2a_1 = 0 \quad a_1 = 0$

$a_1 = 0$ in eqn ②
$a_2 + a_3 + a_4 = 0$

$a_2 = 0$ in eqn ③ $\quad a_3 = 0$

$a_2 = 0 \quad a_1 = 0 \quad a_3 = 0$ in eqn ②

$a_4 = 0$

solution is trivial show that the vectors are linearly independent.

5) $[2\ -4], [1\ 9], [3\ 5]$

Solution:- $a_1[2\ -4] + a_2[1\ 9] + a_3[3\ 5] = [0\ 0]$

$2a_1 + a_2 + 3a_3 = 0$ — (1) $\qquad a_1 = -(a_2 + 3/2\, a_3)$ — (3)

$-4a_1 + 9a_2 + 5a_3 = 0$ — (2)

substitute (3) in (2)

$-4\left(-a_2 - \frac{3}{2}a_3\right) + 9a_2 + 5a_3 = 0$

$4a_2 + 6a_3 + 9a_2 + 5a_3 = 0$

$13a_2 + 11a_3 = 0$

This is linearly dependent

6) $[3\ -2\ 0\ 4], [5\ 0\ 0\ 1], [-6\ 1\ 0\ 1], [2\ 0\ 0\ 3]$

Solution:- $a_1[3\ -2\ 0\ 4] + a_2[5\ 0\ 0\ 1] + a_3[-6\ 1\ 0\ 1]$
$+ a_4[2\ 0\ 0\ 3] = [0\ 0\ 0\ 0]$

$3a_1 + 5a_2 - 6a_3 + 2a_4 = 0$

$-2a_1 + a_3 = 0$

$\boxed{a_4 = 0}$

$4a_1 + a_2 + a_3 + 3a_4 = 0$

substitute $a_4 = 0$ in eqn (1)

$3a_1 + 5a_2 - 6a_3 = 0$

from second equation $c_3 = 2c_1$

substitute $a_3 = 2a_1$ in eqn (1)

$3a_1 + 5a_2 - 6(2a_1) = 0$

$3a_1 + 5a_2 - 12a_1 = 0$

$a_2 = \frac{9}{5}a_1$

$\boxed{c_1 = c_2 = c_3 = c_4 = 0}$

This is linearly independent

from third eqn $4a_1 + a_2 + a_3 = 0$ substitute a_2 & a_3

$4a_1 + \frac{9}{5}a_1 + 2a_1 = 0$

$c_1\left(4 + \frac{19}{5}\right) = 0 \qquad c_1\left(\frac{49}{5}\right) = 0 \qquad c_1 = 0$

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7) $[3\ 4\ 7], [2\ 0\ 3], [8\ 2\ 3], [5\ 5\ 6]$

Solution:-

$$A = \begin{pmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{pmatrix} \quad R_2 \to R_2 - 3R_1, \quad R_3 \to R_3 - 7R_1$$

$$A = \begin{pmatrix} 3 & 3 & 8 & 5 \\ 0 & -6 & -22 & -10 \\ 0 & -11 & -53 & -29 \end{pmatrix} \quad R_2 \rightleftharpoons R_3, \quad R_2 \to -\tfrac{1}{6}R_2, \quad R_3 \to R_3 - \left(-\tfrac{11}{6}\right)R_2$$

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 0 & 1 & 11/3 & 5/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

System is inconsistent

vectors is linearly independent

8)

$$A = \begin{bmatrix} 6 & 0 & 12 \\ 0 & -1 & 3 \\ 3 & 2 & 0 \\ 1 & 7 & -19 \\ 4 & 0 & 8 \\ 23 & 5 & -11 \end{bmatrix} \quad R_1 \rightleftharpoons R_4, \quad R_1 \to R_1/6, \quad R_3 \to R_3 - 3R_1, \quad R_6 \to R_6 - \tfrac{1}{6}R_1, \quad R_5 \to R_5 - 4R_1$$

$$A = \begin{bmatrix} 1 & 7/6 & -19/6 \\ 0 & -1 & 3 \\ 0 & -7/2 & 9 \\ 0 & -7 & 4 \\ 0 & -7/6 & 152/6 \\ 0 & -97/6 & 137/6 \end{bmatrix}$$

System is inconsistent

Vectors is linearly independent