Name :- Abhijeet Bisen
Enrollment-Id :- 2301010003

_/_/_

* Final Assignment

Q.1 Find the rank of a matrix A by reducing in Row echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} \quad R_2 \longleftrightarrow R_3 \quad \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_2 \to -\frac{R_2}{4} \quad A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_1 \to R_1 - 2R_2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_4 \to R_4 + 4R_2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$R_3 \to -\frac{R_3}{3}$$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & -2 \end{bmatrix}$$

$R_1 \rightarrow R_1 + R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of Matrix is '3'

__/__/__

Q.2 Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$. find the rank & nullity of T

Solution:- Since the maximum degree of Polynomial

$T = 2$ so $\dim(P_2) = 3$

Kernel

So a subset of Kernel T is $T(A) = 0$

$$(a-b) + (b-c)x + (c-a)x^2 = 0$$

$$\boxed{a = b = c = t \text{ (let)}}$$

New matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of Kernel is 1, because there is only one independent parameter as 't'

Acc. to rank nullity theorem:-

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

$$\text{rank}(T) + 1 = 4$$

So rank of T is 3, & nullity is 1

_/_/__

Q. 3 Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigen values & eigenvectors of A^{-1} & $A + 4I$.

Solution :- $A - \lambda I = 0$

$$\det \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda) = \pm 1$$

$$\lambda = 1, 3$$

For $\lambda = 1$ $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$x - y = 0$$

$x = y$ let $x = k$

$y = k$

Eigen vector $v_1 = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans

__/__/__

Ex. 4. Solve by Gauss - Seidel Method (Take three iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y + 10z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0, y(0) = 0, z(0) = 0$.

Solution eqn $\Rightarrow$ $x^{K+1} \Rightarrow \dfrac{7.85 + 0.1x^{K+1} - 0.3z^K}{3}$

$y^{K+1} \Rightarrow \dfrac{-19.3 - 0.1x^{K+1} - 0.3z^K}{7}$

$z^{K+1} \Rightarrow \dfrac{71.4 - 0.3x^{K+1} + 0.2y^{K+1}}{10}$

we know $x(0) = 0, y(0) = 0, z(0) = 0$

<u>Iteration - 1</u>

$x(1) = \left(\dfrac{7.85 + 0.1y(0) + 0.2z(0)}{3}\right) = \left(\dfrac{7.85}{3}\right) = 2.62$

$y(1) = \left(\dfrac{-19.3 - 0.1x(1) - 0.3z(0)}{7}\right) =$

$= \left(\dfrac{-19.3 - 0.1(2.62) - 0.3(0)}{7}\right) = -2.74$

$z(1) = \left(\dfrac{71.4 - 0.3x(1) - 0.2y(1)}{10}\right)$

$= \left(\dfrac{71.4 - 0.3(2.62) - 0.2(-2.74)}{10}\right) = 6.84$

__/__/__

Iteration 2 :

$$x(2) = \left(\frac{7.85 + 0.1y(1) + 0.2z(1)}{3}\right) = \left(\frac{7.85 + 0.1(-2.74) + 0.2(6.64)}{3}\right)$$

$$= 2.76$$

$$y(2) = \left(\frac{-19.3 - 0.1x(2) - 0.3z(1)}{7}\right) = \left(\frac{-19.3 - 0.1(2.76) - 0.3(6.64)}{7}\right)$$

$$= -2.73$$

$$z(2) = \left(\frac{71.4 - 0.3x(2) + 0.2y(2)}{10}\right) = \left(\frac{71.4 - 0.3(2.76) - 0.2(-2.73)}{10}\right)$$

$$= 6.86$$

Iteration 3:

$$x(3) = \left(\frac{7.85 + 0.1y(2) + 0.2z(2)}{3}\right) = \left(\frac{7.85 + 0.1(-2.73) + 0.2(6.86)}{3}\right)$$

$$= 2.763$$

$$y(3) = \left(\frac{-19.3 - 0.1x(3) - 0.3z(2)}{7}\right) = \left(\frac{-19.3 - 0.1(2.763) - 0.3(6.86)}{7}\right)$$

$$= -2.730$$

$$z(3) = \left(\frac{71.4 - 0.3x(3) - 0.2y(3)}{10}\right) = \left(\frac{71.4 - 0.3(2.763) - 0.2(-2.730)}{10}\right)$$

$$= 6.862$$

Iteration	x	y	z
0	0	0	0
1	2.62	-2.74	6.64
2	2.76	-2.73	6.86
3	2.763	-2.730	6.862

_/_/__

Q.5 Define consistent and inconsistent system of equations. Hence solve the following system of equation if consistent $x + 3y + 2z = 0$; $2x - y + 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$

Consistent
(atleast one solution)

Dependent (infinite sol^n)

Independent (unique sol^n)

Inconsistent
(No solution)

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$A = \left[\begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 + 2R_2 \end{array}$$

$$A = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} \rho(A) \Rightarrow 2 \\ \rho(A:B) \Rightarrow 2 \\ n \Rightarrow 3 \end{array}$$

$\rho(A) = \rho(A:B) \neq n$

Consistent but infinite sol^n.

Q.6. Determine whether the function $T: P_2 \to P_2$ is linear transformation or not. Where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

Solution :-

1) Additive

$$T(u+v) = T(u) + T(v)$$

$$u = a_1 + b_1 x + c_1 x^2$$

$$v = a_2 + b_2 x + c_2 x^2$$

$$T(u+v) \Rightarrow T\big((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2\big)$$

$$\Rightarrow (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$$

$$\Rightarrow (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$$

$$\Rightarrow T(u) + T(v)$$

Hence Proved

2) Homogeneity

$$T(ku) \Rightarrow kT(u)$$

$$T(k(a+bx+cx^2))$$

$$T(ka + kbx + kcx^2)$$

$$\Rightarrow (ka + kb + kc + 1) + (ka + kb + kc + 1)x + (ka + kb + kc + 1)x^2$$

$$\Rightarrow k(a+1) + k(b+1)x + k(c+1)x^2$$

$$\Rightarrow kT(u)$$

Hence Proved

Hence it is a linear Transformation

Q.7 Determine the whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is where ~~f(...)~~ = ~~(...)~~ not a basis determine the dimension and the basis of the sub space spanned by S

Solution:- $a(1,2,3) + b(3,1,0) + c(-2,1,3) = (0,0,0)$

$$a + 3b - 2c = 0$$

$$2a + b + c = 0$$

$$3a + 3c = 0$$

$$c = -a \quad , \quad b = -a$$

Only one solution is possible i.e. $a = b = c = 0$ so linearly independent

Since dimension of $V_3(R)$ is 3 and S also contains 3 vector and $S \rightarrow L$ then it spans $V_3(R)$ making it a basis for $V_3(R)$.

_/_/_

Q.8 Using Jacobi's method (perform 3 iterations), solve $3x + 6y + 2z = 23$, $-4x + y - z = +15$, $x - 3y + 7z = 16$, with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

⇒ first eqn $x = \frac{1}{3}(23 + 6y - 2z)$

Second eqn $y = (-15 + 4x + z)$

third eqn $z = \frac{1}{7}(16 - x + 3y)$

$x(0) = 1$, $y(0) = 1$, $z(0) = 1$

Iteration - 1

$x(1) \Rightarrow \frac{(23 + 6 - 2)}{3} = 9$

$y(1) = (-15 + 4 + 1) = -10$

$z(1) = \frac{(16 - 1 + 3)}{7} = \frac{18}{7}$

Iteration - 2

$x(2) = \frac{1}{3}\left(23 + 6(-10) - \frac{36}{7}\right) \Rightarrow 10.62$

$y(2) = \left(-15 + 4(10.62) + \frac{18}{7}\right) \Rightarrow 30.04$

$z(2) = \frac{1}{7}\left(16 - 10.62 + 3(30.4)\right) = 13.8$

_/_/_

Iteration - 3 -

$$x(3) = \left(\frac{23 + 6(30.4) + 2(13.8)}{3}\right) = 59.27$$

$$y(3) = \left(-15 + 4(59.27) + 13.8\right) = 235.88$$

$$z(3) = \left(\frac{16 - 59.27 + 3(235.88)}{7}\right) = 67.4$$

So after three iteration $x(3) = 59.27$, $y(3) = 235.88$, $z(3) = 67.4$

Q.9. Explain one application of matrix operations in image processing with example.

Solution:- Image rotation is a geometric transformation that maps the position of every object in the image to a new location. This transformation can be achieved using matrix operations.

Let's consider a simple example where we want to rotate an image by 90° clockwise. The rotation matrix for this operation is:

$$R = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

If we have pixel at position (x, y) in the image, we can represent this as a column vector:

$$P = \begin{bmatrix} x \\ y \end{bmatrix}$$

The new position p' of the pixel, after rotation Can be calculatated by multiplying the rotation matrix R with the pixel position P :

$$P' = R \cdot P$$

Q.10 Give a brief description of Linear transformation for computer Vision for rotating 2D image

Solution :- Linear Transformation are mathematical operations that map one set of vectors to another set of vectors. In Computer vision, they're widely used for various image processing tasks, including image rotation.

When rotating an image around a fixed point, we can represent the rotation using a rotation matrix. This matrix encodes the transformation that needs to be applied to each pixel's coordinates (x,y) to achieve the desired rotation.