

Interpretable Strategy Synthesis for Competitive Games

Oral Prelim Presentation

Abhijeet Krishnan

Department of Computer Science
North Carolina State University

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Outline

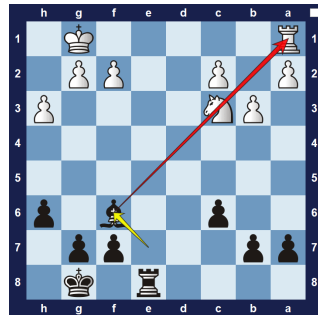
- 1 Overview
- 2 Thesis Statement
 - Motivation
 - RQs
- 3 Research Questions
 - RQ1 – ISS Framework
 - RQ2 – Chess
 - RQ3 – MicroRTS
- 4 Proposed Work & Timeline
- 5 Conclusion

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Overview

- Players tend to identify *strategies* in competitive games
- E.g., the *pin* in chess
- *Good, interpretable* strategies help players improve



Chessfox.com

Figure 1: An example of the *pin* tactic in chess

Overview

Motivating Question

How could we *automatically* learn good, interpretable strategies that help players improve?

- I formalize this question as the problem of *interpretable strategy synthesis*
- I show how we can approach it for *chess*
- I propose an approach for it for the game of *MicroRTS*

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Motivation

Observation

Players tend to identify *strategies* that are a) good and b) communicable when trying to find ways to win in a competitive game.

- Can be communicated to another player

Motivation

Observation

Players tend to identify *strategies* that are a) good and b) communicable when trying to find ways to win in a competitive game.

- Can be communicated to another player
- Help a player decide which actions to take in-game

Examples

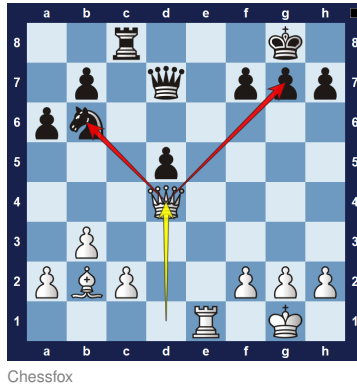


Figure 2: An example of the *fork* tactic in chess

Examples



Go Full Build

Figure 3: A *cannon rush* in progress against a Terran opponent

Examples



Unscriptd, The Times

Figure 4: Boris Becker's tennis service tic

Outcomes

- Esports is a massive industry

Tournament	Game	Prize Pool (USD)
World Blitz Chess Championship	Chess	350,000
IEM Katowice	StarCraft II	500,000

- Better strategies → higher player skill → more earning potential
- Could be used to coach players at all levels of skill
- Could be used to explain artificial agent decisions

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Thesis Statement

Thesis Statement

A *computational model* of a game strategy, along with a *learning method*, could meet the goals of discovering good, communicable strategies and impact the fields of competitive esports and explainable AI.

RQ1

RQ1

How do we formally define the problem of *Interpretable Strategy Synthesis* (ISS)?

- Necessary to clarify terms like *strategy*

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- Need to agree on what “interpretable” means

RQ1

RQ1

How do we formally define the problem of *Interpretable Strategy Synthesis* (ISS)?

- Necessary to clarify terms like *strategy*
- Need to agree on what “interpretable” means
- Useful to compare and contrast approaches across game domains

RQ2

RQ2

How do we approach the problem of ISS for the game of *chess*?

- Well-studied domain with lots of player-derived strategies

RQ3

RQ3

How do we approach the problem of ISS for the game of *MicroRTS*?

- Less-studied domain with fewer player-derived strategies
- Qualitatively different from chess

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RQ1 – ISS Framework

RQ1

How do we formally define the problem of *Interpretable Strategy Synthesis* (ISS)?

The Need for a Framework

Gaps in existing work on strategy synthesis –

- No comparison between multiple algorithms

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Gaps in existing work on strategy synthesis –

- No comparison between multiple algorithms
- No comparison between multiple strategy representations

The Need for a Framework

Gaps in existing work on strategy synthesis –

- No comparison between multiple algorithms
- No comparison between multiple strategy representations
- No comparison across multiple games

The Need for a Framework

Gaps in existing work on strategy synthesis –

- No comparison between multiple algorithms
- No comparison between multiple strategy representations
- No comparison across multiple games
- No definition or evaluation of interpretability

The Need for a Framework

Paper	#Domains	#Models	#Algorithms	Interpretability
Spronck, Sprinkhuizen-Kuyper, and Postma [1]	2	1	1	<i>X</i>
Mesentier Silva, Isaksen, Togelius, <i>et al.</i> [2]	1	1	4	✓
Butler, Torlak, and Popović [3]	1	1	1	<i>X</i>
Canaan, Shen, Torrado, <i>et al.</i> [4]	1	1	1	<i>X</i>
Freitas, Souza, and Bernardino [5]	1	1	1	<i>X</i>
Mariño, Moraes, Oliveira, <i>et al.</i> [6]	1	1	1	<i>X</i>
Krishnan and Martens [7]	1	1	1	<i>X</i>
Mariño and Toledo [8]	1	1	1	<i>X</i>
Medeiros, Aleixo, and Lelis [9]	2	1	2	<i>X</i>

Table 1: List of works in ISS

Strategy

Definition (strategy)

Given a game environment \mathcal{G} modeled as a finite, episodic MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$, a *strategy* σ is —

$$\sigma(a|s) \doteq \mathbb{P}[A_t = a | S_t = s], \forall s \in A_\sigma, a \in \mathcal{A}(s)$$

A_σ : set of *applicable* states

- A strategy is similar to an RL policy
- Not necessarily applicable to all states
- Describes a pattern or feature that comes up often in gameplay

Interpretable Strategy Synthesis (ISS)

Definition (ISS)

Given a —

- Game environment \mathcal{G}
- Strategy model \mathcal{M}
- Performance measure $\mathcal{R}_{\mathcal{G}}: \mathcal{M} \rightarrow \mathbb{R}$
- Interpretability measure $\mathcal{I}_{\mathcal{G}}: \mathcal{M} \rightarrow \mathbb{R}$

The problem of ISS is to find a strategy σ^* s.t. —

$$\sigma^* \doteq \arg \max_{\sigma} \mathcal{R}_{\mathcal{G}}(\sigma) \mathcal{I}_{\mathcal{G}}(\sigma), \sigma \in \mathcal{M}$$

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RQ2 – Chess

RQ2

How do we approach the problem of Interpretable Strategy Synthesis for the game of *chess*?

Why Chess?

- Popular “esport” with a *long* competitive history
- Large number of player-discovered strategies to compare against
- Extensive use as a testbed for AI
- Potential for discovery of new strategies using artificial agents [10]

Validating ISS using Chess

- Using our framework, could we measure how “good” existing chess strategies are?

RQ2(a)

Could we represent known chess tactics as a strategy model for chess and develop metrics to show that they suggest better moves than a random baseline?

Validating ISS using Chess

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RQ2(a)

Could we represent known chess tactics as a **strategy model** for chess and develop metrics to show that they suggest better moves than a random baseline?

Strategy Model for Chess

```
tactic(Position,  
        Move) ←  
    feature_1(⋯),  
    feature_2(⋯),  
    ⋮  
    feature_n(⋯)
```

Figure 5: Our chess strategy model expressed in Prolog pseudocode

Strategy Model for Chess

- `Position`: list of predicate features encoding a board state

```
tactic(Position,  
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Figure 5: Our chess strategy model expressed in Prolog pseudocode

Strategy Model for Chess

- `Position`: list of predicate features encoding a board state
- `Move`: predicate encoding of a move

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Strategy Model for Chess

- `Position`: list of predicate features encoding a board state
- `Move`: predicate encoding of a move
- `feature_i`: board state feature encoded as a predicate

```
tactic(Position,  
        Move) ←  
    feature_1(...),  
    feature_2(...),  
    ⋮  
    feature_n(...)
```

Figure 5: Our chess strategy model expressed in Prolog pseudocode

Example

```
fork(Position,Move)  $\leftarrow$   
    legal_move(Position,Move),  
    move(Move,_,To,_),  
    make_move(Position,Move,NewPosition),  
    can_capture(NewPosition,To,ForkSquare1),  
    can_capture(NewPosition,To,ForkSquare2),  
    different(ForkSquare1,ForkSquare2).
```

Figure 6: An interpretation of the *fork* tactic from the chess literature using our predicate vocabulary.

Why This Model?

- Chess tactics are used by human players to think about chess [11] and useful in chess education [12]
- FO-logic used extensively to model chess patterns [13]–[19]
- Logic rules are acknowledged to be interpretable and have a long history of research [20]

Validating ISS using Chess

- Using our framework, could we measure how “good” existing chess strategies are?

RQ2(a)

Could we represent known chess tactics as a strategy model for chess and develop **metrics** to show that they suggest better moves than a random baseline?

Metrics

Definition

Divergence of a tactic from a set of examples P is the average difference in *evaluation* between the moves suggested by the tactic and the ground truth move.

$$\text{Divergence}_E(\sigma, P) \doteq \frac{1}{|P_A|} \sum_{(s, a_1) \in P_A} \sum_{a_2 \in \mathcal{A}(s)} \sigma(a_2|s) d_E(s, a_1, a_2) \quad (1)$$

Validating ISS using Chess

- Using our framework, could we measure how “good” existing chess strategies are?

RQ2(a)

Could we represent known chess tactics as a strategy model for chess and develop metrics to show that they suggest better moves than a random baseline?

Evaluating Chess Tactics [21]

- Obtain tactics learned using PAL [19]
- Translate to chess strategy model
- Obtain example set of positions and moves from online games played by human beginners
- Measure divergence using strong and weak chess engines
- Compare to random baseline

Results

- Tactics have lower divergence when measured using a weak chess engine, higher divergence when measured with a strong engine
- Tend to suggest moves similar to a human beginner

Learning Chess Strategy Models

- How do we automatically learn “good” chess strategy models?

RQ2(b)

Do the chess strategies learned using inductive logic programming outperform a random baseline in how closely their divergence scores approximate a beginner player?

Learning Chess Strategies using ILP [7]

- Inductive Logic Programming (ILP) is a symbolic machine learning technique
- Translate the ISS for chess problem into an ILP problem
- Modify an existing ILP system to learn chess strategies
- Evaluate the learned chess strategies using divergence metrics
- Compare to random, strong/weak engine baselines

Results

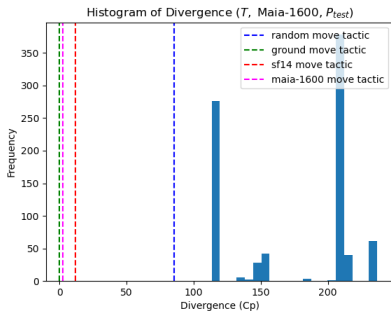


Figure 7: Divergence histogram for T evaluated using Maia-1600

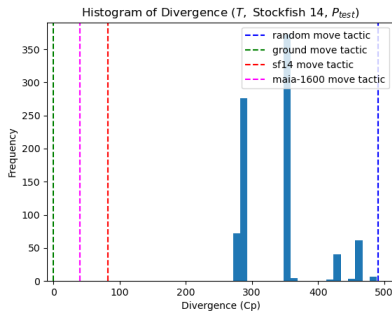


Figure 8: Divergence histogram for T evaluated using Stockfish 14

Conclusion

- The chess strategies learned are better than a random baseline when evaluated using a strong engine

Improving the ILP Learning Method

- The learned chess strategies merely outperform a random baseline, could we improve upon them?

RQ2(c)

Do the chess strategies learned by an ILP system incorporating the changes of the new predicate vocabulary and precision/recall-based constraints produce moves better than those learned by an ILP system without these modifications?

Improvements using Precision/Recall-based Constraints [22]

- Limit the search space of chess strategies using constraints based on precision and recall of the synthesized strategies
- Introduce a new predicate vocabulary
- Conduct ablative study to measure impact of contributions
- Use similar evaluation setup to previous work

Results

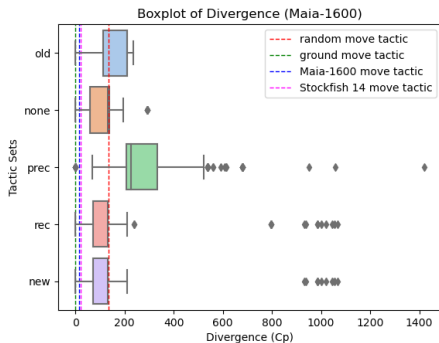


Figure 9: Boxplot of tactic divergence (evaluated using Maia-1600) for each system

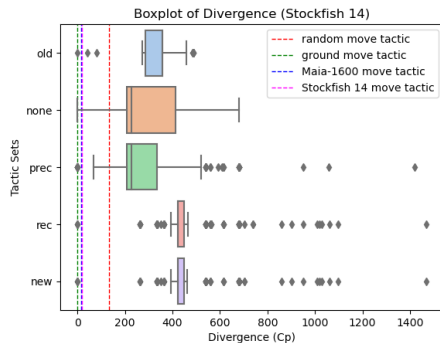


Figure 10: Boxplot of tactic divergence (evaluated using Stockfish 14) for each system

Conclusion

- Statistically significant ($p < 0.01$) improvement in divergence due to new predicate vocabulary
- Mixed results for precision and recall constraints
 - Recall constraint improves divergence when measured using weak engine
 - Precision constraint improves divergence when measured using strong engine

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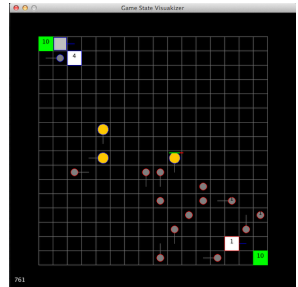
RQ3 – MicroRTS

RQ3

How do we approach the problem of Interpretable Strategy Synthesis for the game of *MicroRTS*?

Why MicroRTS?

- Simplified real-time strategy game for AI research [23]
- Popular genre for esports titles
- Active research community
- Qualitatively different from chess – *real-time, partially observable*



Google Code Archive

Figure 11: A MicroRTS game in progress

Learning MicroRTS Strategies using ASP

RQ3(a)

How does an ASP-based approach towards developing a synthesizer for the *SynProS competition* compare to other synthesizers in this competition?

SynProS Competition

- SynProS: **S**ynthesis of **P**rogrammatic **S**trategies
- Research competition [24] to test ISS approaches for MicroRTS with a fixed strategy model
- MicroRTS strategy model represented as a production in a **context-free grammar (CFG)**

$$\begin{aligned}
 S_1 &\rightarrow C S_1 \mid S_2 S_1 \mid S_3 S_1 \mid \epsilon \\
 S_2 &\rightarrow \text{if } (S_5) \text{ then } \{C\} \mid \text{if } (S_5) \text{ then } \{C\} \text{ else } \{C\} \\
 S_3 &\rightarrow \text{for (each unit } u) \{S_4\} \\
 S_4 &\rightarrow C S_4 \mid S_2 S_4 \mid \epsilon \\
 S_5 &\rightarrow \text{not } B \mid B \\
 B &\rightarrow b_1 \mid b_2 \mid \dots \mid b_m \\
 C &\rightarrow c_1 C \mid c_2 C \mid \dots \mid c_n C \mid c_1 \mid c_2 \mid \dots \mid c_n \mid \epsilon
 \end{aligned}$$

ASP-based Approach

- **A**nswer **S**et **P**rogramming (ASP) is a declarative programming paradigm (like Prolog)
- Has been used to model and generate game levels [25], [26]
- Has been used to model visualization design preferences and optimize data layouts based on them [27]

ASP-based Approach

- Proposal to replicate approach in [27]
- Model MicroRTS strategy using ASP
- Train a regression model to predict strategy win rate given feature encoding
- Convert regression function to ASP to optimize newly generated strategies

Interpretability Factors for MicroRTS Strategies

- How do we define the interpretability of a MicroRTS strategy?
- How do we make MicroRTS strategies *more* interpretable?

RQ3(b)

Which features of a MicroRTS strategy model have a statistically significant correlation with the interpretability of said strategy?

Task Design

- Conduct a *human-grounded* [28] evaluation using a simplified task to quantify interpretability
- Use a forward simulation/prediction task
- Subjects presented with —
 - Strategy
 - Game state
 - Options for future states (1 correct, 3 incorrect)
- Task is to predict the expected future state from current state if strategy is followed
- Generate tasks using ASP model of MicroRTS strategy

Obtaining Significant Factors

- Train decision tree model to predict whether strategy will be correctly simulated
- Obtain significant factors by measuring Gini index [29]

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Proposed Work & Timeline

- Spring 2022

RQ1 Framework (EAAI-22) - *Completed*

RQ2(a) Validity (EAAI-22) - *Completed*

- Fall 2022

RQ2(b) Study (AIIDE-22 SG Workshop) - *Completed*

- Spring 2023

RQ2(c) Study (under review) - *Completed*

RQ3(a) Dataset Assembly - *In progress*

RQ3(a,b) ASP Modeling

Proposed Work & Timeline

- Summer 2023

RQ3(a) Study (ACG-23)

RQ3(b) Obtaining Strategies

- Fall 2023

RQ3(b) Task Design

RQ3(b) Study Approval

- Spring 2024

RQ3(b) Analysis (EXTRAAMAS '24)

- Complete dissertation (early Spring '24)
- Defend thesis (Mid-late Spring '24)
- Graduate (May '24)

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Conclusion

- Goal: investigate approaches to the problem of interpretable strategy synthesis for games
- Defined a framework for ISS
- Approached ISS using ILP for chess
- Proposal to approach ISS for MicroRTS using ASP
- Outcomes –
 - Benefit esports industry in providing better analytics for player performance
 - Benefit explainable AI research in investigating new approaches to generate policy explanations

Thank You!

Questions?

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