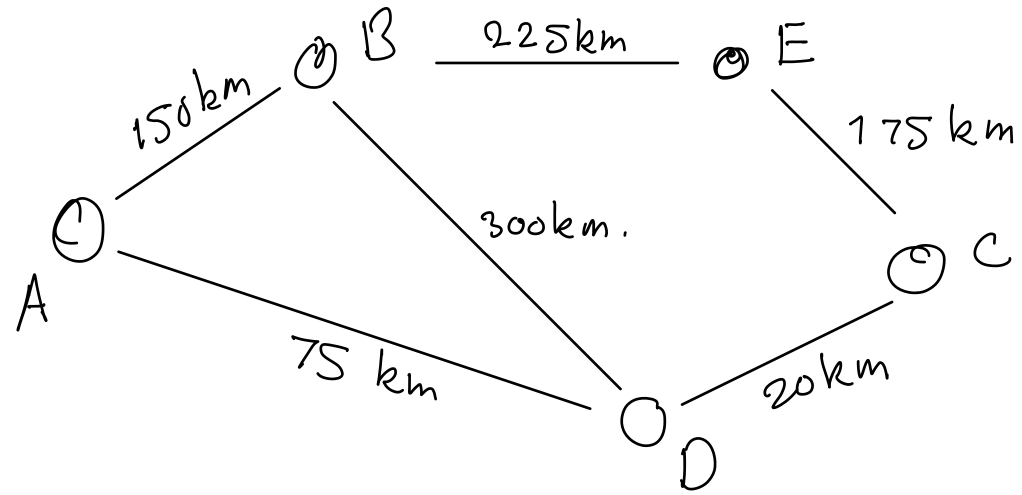
Problem:- There are five cities. Some cities are directly connected via road. For some other cities, we must take the connecting roads.

- In general, there are multiple ways to travel from one city to the another
- If we are given the cost of the travel in terms of the distance between all all cities that are directly connected then can we find the shortest route from any one city to the another!



- In order to model problems, many times we must form a set of entities involving in the problem.
- And we have to depict some kind of relationship between these entities

belonging to same set!

- Entities in our problem are cities viz. A, B, C, D, E!

Cities = {A, B, C, D, E}.

form a relation between some pair of cities in the same set!

This scenario can be mathematically depicted!

Set A, Set B.

$$A \times B = \{ (x, y) \mid x \in A, y \in B \}$$

$R : A \to B$ is any non-empty subset of $A \times B$!

Set A, Set A.

$R : A \to A$, Non-empty subset of $A \times A$, $R(A)$.

$A = \{ \text{Entities} \}$

Non-empty subset of $A \times A$.

Proposition → Mathematical Structure

Predicate → Mathematical Structure

Set → Mathematical Structure

Relation on sets → Mathematical Structure

→ Let set A be a non-empty set. Consider Relation on set A.

→ Mathematical Structure

Function → Mathematical Structure

Permutation, Combination, Group. Ring. Fields.

D.F.A. NFA, PDA D.P.D.A. ND.P.A.

Turing

$\text{Cities} = \{ A, B, C, D, E \}$

$R(\text{Cities}) \subseteq \text{Cities} \times \text{Cities}$

$A \to A, B, C, D, E.$

$B \to A, B, C, D, E$

$C \to A, B, C, D, E$

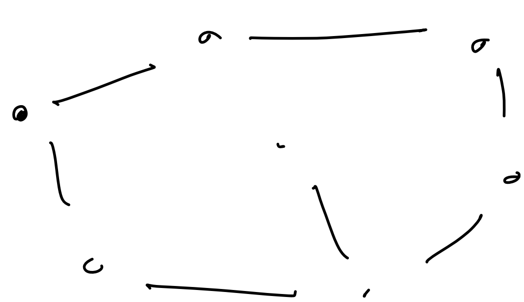
$D \to A, B, C, D, E$

$E \to A, B, C, D, E$

$R : \text{Cities} \to \text{Cities}$ as $R(\text{Cities})$

$= \{(A,B), (A,D), (B,A), (B,D), (B,E), (C,D), (C,E), (D,A), (D,B), (D,C), (E,B), (E,C)\}.$

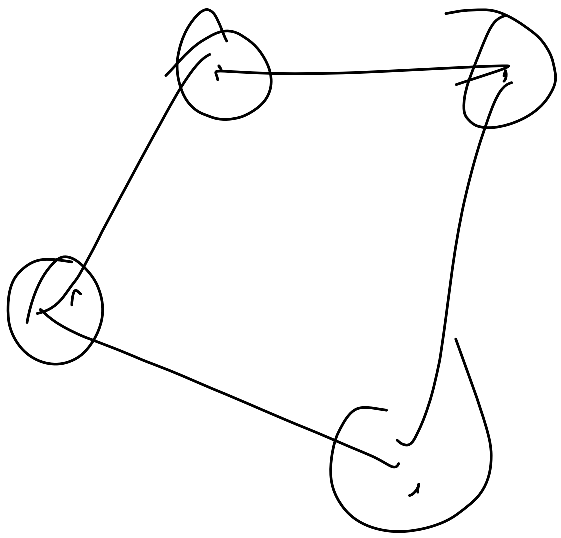
$\text{Cost} : R(\text{Cities}) \to \text{Real}$



Let V be a non-empty set of vertices

Relationship between the vertices of set V, is called a set of edges.

Graph = (V, E)



Graph G is defined as an ordered pair of sets (V, E) where V is a set of vertices and E is a set of edges. In fact, E is a relation on V. More elaborately E is a non-empty subset of V×V.

A graph G=(V,E) is known as a weighted graph if we associate a cost function

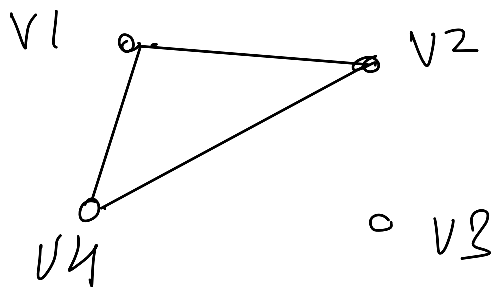
$cost : E \to \mathbb{R}$.

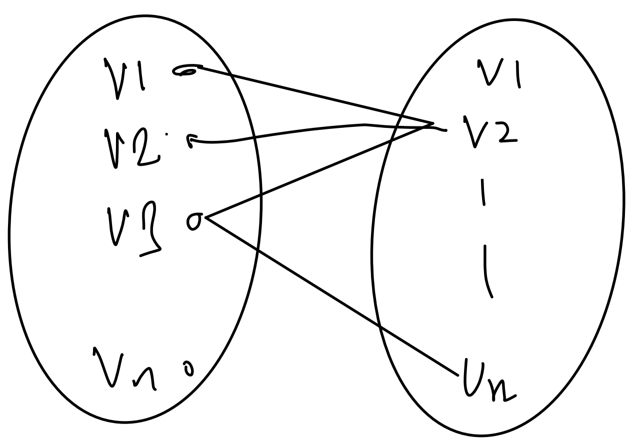
cost (edges = real number

$\|$

is known as the weight of the edge.

Isolated vertex and self edge.

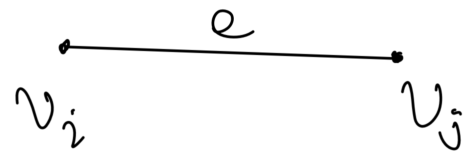




Let $G = (V, E)$ be a graph.

Let v_i and $v_j \in V$.

Let there be edge between V_i to V_j



we say that

1) there exists an edge (e) between v_i and v_j
   or

2) Edge e is incident on v_i.
   Edge e is incident on v_j.

3) One of the end-points of edge e is v_i.
   or v_i is an end point of edge e.
   [Same for v_j]

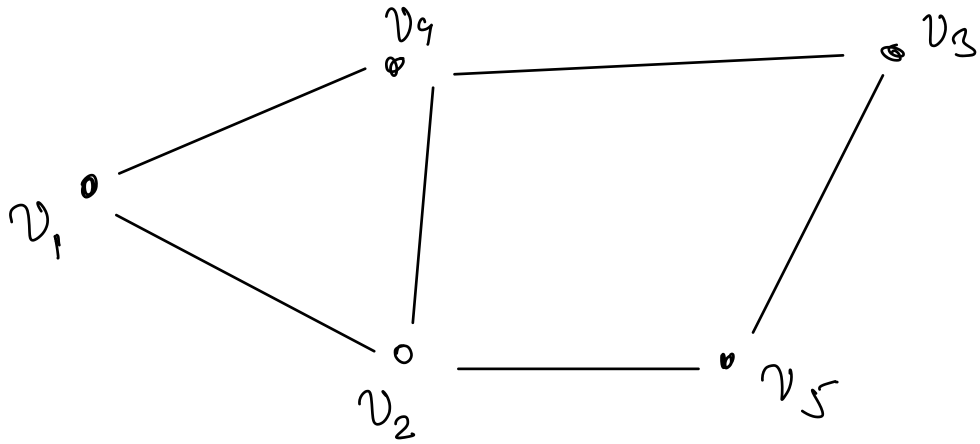
4) Edge e, emerges ~~of~~ from v_i
   Edge e, emerges from v_j.

Degree of Vertex:

Let $G = (V, E)$ be a graph.

Let vertex $v \in V$.

A degree of vertex v = total number of edges emanating from it / or incident it / emerging from it.



$\text{Degree}(v_1) = 2.$

$\text{Degree}(v_2) = 3$

$\text{Degree}(v_3) = 2.$

$\text{Degree}(v_4) = 3$

$\text{Degree}(v_5) = 2.$

In any graph degree(v) is either an odd number or an even number.

Theorem = In any graph, total number of vertices having an odd degree is Even!

try to prove this theorem.

Let $G = (V, E)$ be a graph

Let $v_1, v_2, \dots, v_n$ be the vertices of the graph.

$d(v_1) + d(v_2) + \dots + d(v_n) = 2 \times$ Number of edges.

$\underbrace{\boxed{\text{Summation}(d(v))}}_{\text{Even degree}} + \underbrace{\boxed{\text{Summation}(d(v))}}_{\text{odd degree}}$

$= 2 \times$ Number of edges.

$\underbrace{\text{Summation}(d(v))}_{\text{odd degree}} = 2 \times \text{Number of edges} - \underbrace{\text{Summation}(d(v))}_{\text{Even degree.}}$

$=$ Even number $-$ Even number

$\underbrace{\text{Summation}(d(v))}_{\text{odd degree}} =$ Even number

Problem: Prove that total number of people who have shook the hand odd number of times is even

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first human ——— Comment:

$V = \{$ All people who shook hands at least once $\}$

$E = \{$ there is an edge between any two people in V if they have shook their hands $\}$

$d(\text{person})$ = total number of hand shakes of that person.

total number of vertices with odd degree must be even.

→ path in graph. — Cycle in graph

connected graph – connected components / forest

Tree | 3 | binary tree

| Binary tree → C.S. angle

→ Binary Search Tree

→ Node

Programming (90-95%)