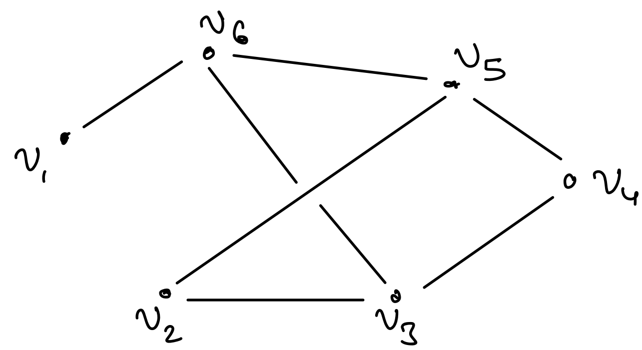


✓ Path in graph: Let $G=(V,E)$ be a graph. Let v_i and v_j be two vertices in V_G. We say that there is a path between v_i and v_j, if there exists an ordered tuple of edges $(e_1, e_2, e_3, --, e_n) \in E_G$ such that first end-point of $e_1 = v_i$, the last end-point of $e_n = v_j$ and for all edges, $e_1, e_2, --, e_k, --, e_{n-1}$, second end-point of e_k = first-end point of e_{k+1}.

✓ Cormen: If there exists a path between v_i to v_j then we denote it as follows

$$v_i \overset{p}{\leadsto} v_j$$

$v_i \overset{p_1}{\leadsto} v_j \mid v_i \overset{p_2}{\leadsto} v_j \mid v_i \overset{p_3}{\leadsto} v_j \mid$

$p1 = (e_1, \text{---})$ tuple of edges.

$p2 = ($

$p3$

$v_i \overset{p}{\leadsto} v_j \mid p = \underline{(e_1, --, e_n)}$ where $e_1 - e_n \in E_G$.
$v_i, v_j \in V_G$.

Cycle in graph: Let $G = (V, E)$ be a graph. Let $v_i \in V_G$. If there exists a path p between v_i to v_i which is NOT a self edge, then such path is known as a cycle.

$$\left(\exists\, v_i \in V_G \ni \exists p\ v_i \overset{p}{\rightsquigarrow} v_i\right) \Rightarrow G \text{ contains a cycle!}$$

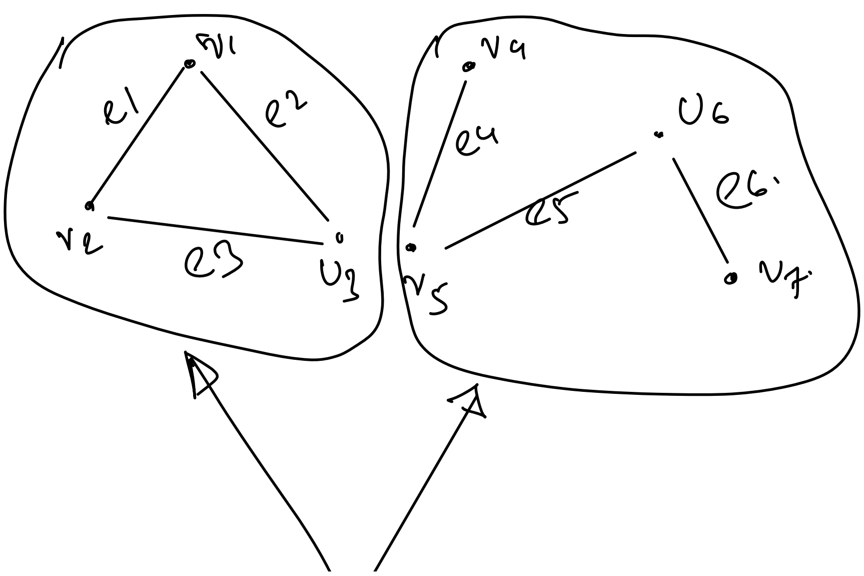
there exists [Existential Quantifier]   belongs   such that   path   implies

---

Connected Graph:

Let $G \equiv (V, E)$ be a graph. If there exists a path between any two distinct vertices v_i and $v_j \in V_G$, then G is a connected graph.

$$G = (V, E) \text{ is connected} \equiv \left[\forall\, v_i, v_j \in V_G,\ v_i \neq v_j,\ \text{if } \exists p,\ v_i \overset{p}{\rightsquigarrow} v_j\right]$$



$V_G = (v_1, — v_1)$

$E_G = (e_1, — e_6)$

$v_1 \to v_3$

e_1

Connected Components of graph
Or forest of graph.

$$\neg \boxed{\forall x.} \, P(x) \equiv \exists x. \neg P(x)$$

$$\neg \left(\forall x. P(x) \right) \equiv \exists x. \neg P(x)$$

$G = (V, E)$ is connected $\equiv [\forall v_i, v_j \in V_G, v_i \neq v_j,$ ADV
if $\exists p, v_i \overset{p}{\rightsquigarrow} v_j]$

$\neg \left(G = (V, E) \text{ is connected} \right) \equiv [G = (V, E)]$ is not connected

$\neg \left(\forall (v_i, v_j) \in V_G, \exists p \, (v_i \overset{p}{\rightsquigarrow} v_j) \right) \equiv$

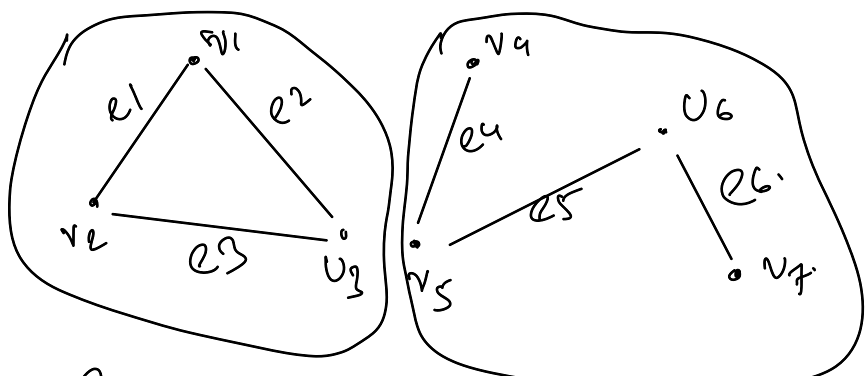
$\exists (v_i, v_j) \in V_G \ni \left(\exists p \, (v_i \overset{p}{\rightsquigarrow} v_j) \equiv F \right)$.

$\exists (v_i, v_j) \in V_G \ni v_i \overset{p}{\not\rightsquigarrow} v_j$

Let $G \equiv (V, E)$ be a graph. Let V' be a non-empty subset of V. If $\forall v_i, v_j \in V'$, $\exists p \; v_i \overset{p}{\rightsquigarrow} v_j$ then V' is a connected component.

Let $G \equiv (V, E)$ be a graph. Let $V_S \subseteq V$ and $V_S \neq \phi$ and let $E_S \subseteq E$ and $E_S \neq \phi$ such that for any two vertices in V_S, there exists a path and all edges belonging to any path of any two vertices in V_S belong to E_S then (V_S, E_S) is a connected component or a forest of G.

Let $G = (V, E)$ be a graph. Let $E_S \subseteq E$ and $E_S \neq \emptyset$ and V_S is set of all vertices such that each vertex is V_S is an end point of at least one edge in E_S. Then $(V_S, E_S) \subseteq (V, E)$



$G = (\{v1, v2, v3, v4, v5, v6, v7\}, \{e1, e2, e3, e4, e5, e6\})$.

$G_1 = (\{v1, v2, v3\}, \{e1, e2, e3\})$ | G_1 is a forest.

$G2 = (\{v4, v5, v6, v7\}, \{e4, e5, e6\})$. | G_2 is a connected component

$G \mid G_1 \mid G_2$. $G_1 \subseteq G \mid G_2 \subseteq G$.

$\underline{\{e1, e3, e6\}} \subseteq \{e1, e2, e3, e4, e5, e6\}$.

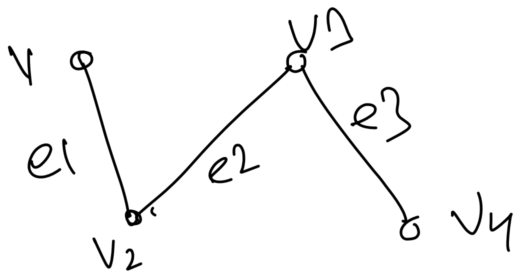
$\underline{\{v1, v2, v3, v6, v7\}} \subseteq V$

derive

$(\{v1, v2, v3, v6, v7\}, \{e1, e3, e6\}) \subseteq G.$

Construct a subgraph of G.

1) Select subset $E_S \neq \emptyset$, of E_G. $(E_S \subseteq E_G)$.
2) Collect all end points of edges in ES without repetition in set V_S
3) $(V_S, E_S) \subseteq G.$



$\{e1, e3\}$

$\{v1, v2, v3, v4\}$