Time Complexity and Asymptotic Notations:

How to measure time complexity of an algorithm?

- Algorithm design $\longrightarrow$ Correctness $\longrightarrow$ Efficiency.
- Most imp. parameter to measure the efficiency is Time!
- Parameter to measure complexity = Time!
- Measure:

1) Absolute running time
2) Prof. Donald Knuth:

- Number of fundamental CPU operations (Data movement + ALU)
- fundamental number of CPU ops $(\underline{N})$
- $\underline{T(N)} = N^2$.

ALGORITHM: INSERTION-SORT(A, N)		Cost per st	#Nr	Total cost
S1:	j = 1	C1	1	$C1$
S2:	while j < N	C2	N	$C2 \cdot N$
S3:	key := A[j]	C3	$N-1$	$C3 \cdot N - C3$
S4:	i := j-1	C4	$N-1$	$C4 \cdot N - C4$
S5:	while i > -1 and A[i] > key	C5	$\frac{N^2}{2} + \frac{N}{2} - 1$	$\frac{C5 \cdot N^2}{2} + \frac{C5 \cdot N}{2} - C5$
S6:	A[i+1] = A[i]	C6	$\frac{N^2}{2} - \frac{N}{2}$	$C6 \cdot \frac{N^2}{2} - C6 \frac{N}{2}$
S7:	i = i-1	C7	$\frac{N^2}{2} - \frac{N}{2}$	$C7 \cdot \frac{N^2}{2} - C7 \frac{N}{2}$
S8	A[i+1] := key	C8	$N-1$	$C8 \cdot N - C8$

s9:	$j := j+1$	$c9$	$N-1$	$c9 \cdot N - c9$

Total cost = $T(N)$

$$= C1 + C2 \cdot N + C3 \cdot N - C3 + C4 \cdot N - C4 + \frac{C5}{2} \cdot N^2 - \frac{C5}{2} \cdot N - C5 + \frac{C6}{2} \cdot N^2 - \frac{C6}{2} N - C6 + \frac{C7}{2} \cdot N^2 - \frac{C7}{2} \cdot N + C8 \cdot N - C8 + C9 \cdot N - C9$$

$$= \left(\frac{C5 + C6 + C7}{2}\right) N^2 + \left(C2 + C3 + C4 - \frac{C5}{2} - \frac{C6}{2} - \frac{C7}{2} + C8 + C9\right) N + \left(C1 - C3 - C4 - C5 - C6 - C8 - C9\right)$$

$$= A \cdot N^2 + B \cdot N + C$$

where $A = \left(\frac{C5 + C6 + C7}{2}\right)$

$B = \left(C2 + C3 + C4 - \frac{C5}{2} - \frac{C6}{2} - \frac{C7}{2} + C8 + C9\right)$

$C = \left(C1 - C2 - C4 - C5 - C6 - C8 - C9\right)$

$$\boxed{T(N) = AN^2 + B \cdot N + C}$$

$$AN^2 >> \boxed{BN + C}$$

$A=5 \quad B=7 \quad C=9$

$N=100.$

$700 \quad 9$

$5\cdot(100)^2$

$5\times(10000)$

$\underline{50000}$

$\underline{709}$

$N=10^6.$

$5\underline{(10^6)}^2$

$7\cdot10^6+9$

$70,00,000+9$

$5\,|\,10^{12}.$

$\boxed{70,00,009}$ 13 NTC.

$10^9=1B=100$ Crore.

$5\cdot10^{12}=5\cdot10^3\cdot10^9.$

$=(5\times1000)\times10^9$

$=(5000)\cdot(100)$ Crore

$=(5\ L)$ Crore

1L. Crone

$$T(N) = AN^2 + (BN + C)$$

$$= AN^2 + \epsilon$$

$$T(N) \approx \boxed{A} N^2$$

1. $a \propto b$ $\qquad$ $a \propto b$.

$\boxed{a = k \cdot b}$ $\qquad$ $a = k \cdot b$

$\boxed{T(N) \propto N^2}$ $\qquad$ A, B, C

$$T(N) = \boxed{A} N^2$$

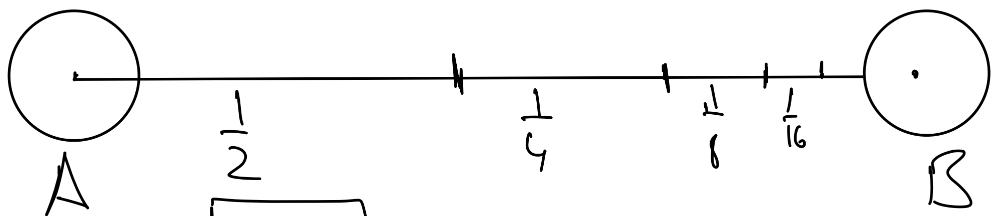
$$\boxed{T(N) \propto N^2}$$

$$T(N) = AN^2 + \boxed{BN + \epsilon}$$

$$T(N) \propto N^2$$

Asymptotic Notations!

Frog = ବେଙ୍ଗ! immortal = ଅମର



$R(N) = \boxed{\frac{1}{2^N}}$

N - - -

$| \rightarrow 0$

R(N) = lower bound

l u $\overset{R}{>}$ R(N)

$\frac{1}{2^N} > 0 \longrightarrow 0$

N

$0 + \epsilon$

$0 +$

$$0 < \frac{1}{2^N} < 0+\epsilon$$

$0+0.1$
$0+0.01$
$0+0.001$
$0+0.00001$
$0+0.0000000000000000001$

N

$$\frac{1}{2^N} < 0.0000000000000000001$$

$(\underline{\forall}\, \epsilon>0)(\underline{\exists}\, n \in N)$ such that
$0 < \frac{1}{2^n} < 0+\epsilon$ is true

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limiting Process +
let $\{a_n\}$ be a seq.
$\lim \{a_n\} = L$ iff

SELF STUDY.

$(\forall \epsilon > 0)(\exists n \in \mathbb{N})$ such that:

$L - \epsilon < a_n < L + \epsilon$; $\{a_n\} \neq L$.

$0 < |a_n - L| < \epsilon$.

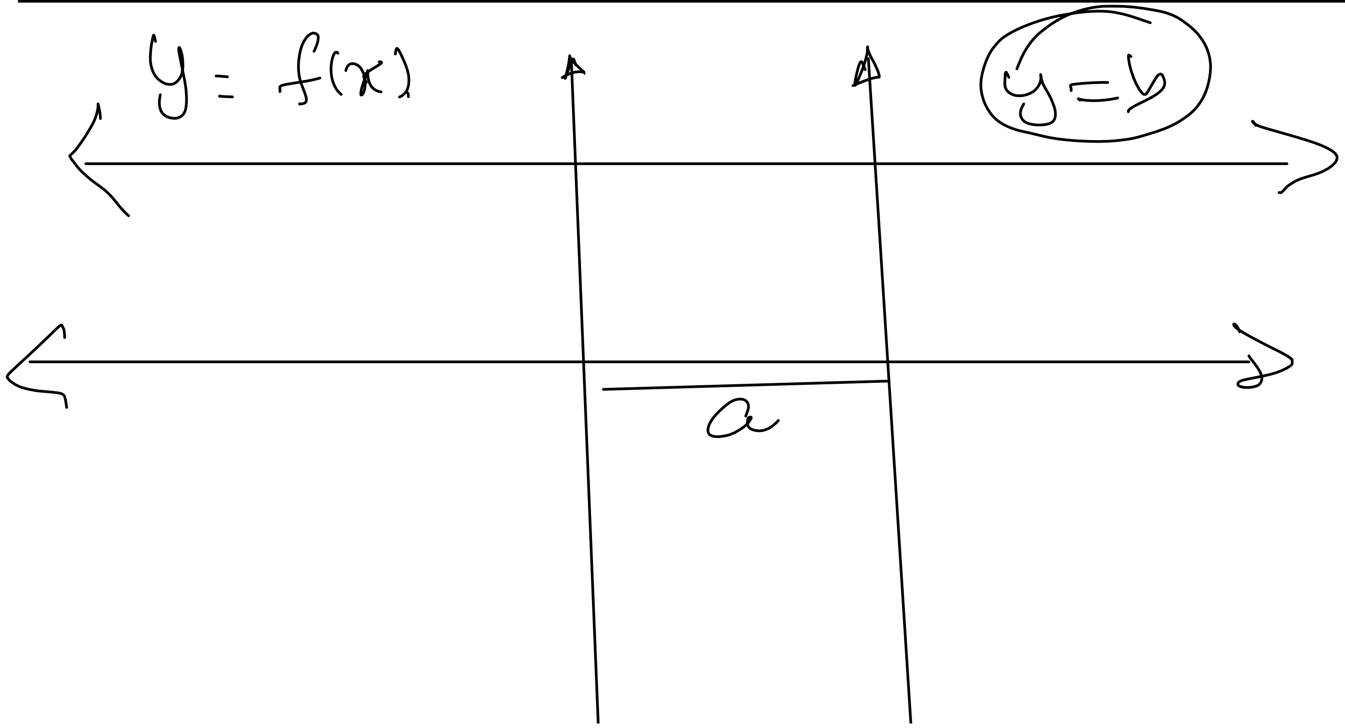
$f: \mathbb{R} \to \mathbb{R}$.

$\lim_{x \to a} f(x) = L$ iff:

$(\forall \epsilon > 0)(\exists \delta(\epsilon) > 0)$ such that:

$0 < |x - a| < \delta \Rightarrow$

$|f(x) - L| < \epsilon$.



$x = a$

$x = 0$

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let $f(x)$ be a function $(\mathbb{R} \to \mathbb{R})$
$f(x)$ is said to have a vertical asymptote at $x = a$ if curve of $f(x)$ goes as near as possible to $x = a$ but never touches $x = a$

let $f(x)$ be a function $(\mathbb{R} \to \mathbb{R})$
$f(x)$ is said to have a horizontal asymptote at $y = b$ if curve of $f(x)$ goes as near as possible to $y = b$ but never touches $y = b$.

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upper bound, lower bound.
Sequence of numbers:

$a_n \to$	1	2	9	16	25	36	49
$n \to$	1	2	3	4	5	6	7.

$a_n = n^2$

a_n	1	4	2	7	11	3	6	25
n	1	2	3	4	5	6	7	8

$n \in \mathbb{N}$ = Domain.

$\{a_n\}$

$a_n = n^2$

$n^2/2$ $a_3 = \frac{9^2}{2} =$ 5?

$\{a_n\} \to$ Sequence

$a_n = \langle \text{formula} \rangle$

$\{a_n\} : \mathbb{N} \to \mathbb{R}$

T_n

$$\boxed{a_n = n^2}$$

$T(n)$ $T_n =$

$f(x)$ $f(n)$ f_n

$\{T_n\}$

input

Analyse the behavior of T_n as n becomes larger & larger!

n

$$\boxed{\{T_n\}}$$

$n \to T_n$

$\boxed{T(n) = An^2 + Bn + C.}$

$= 5n^2 + 7n + 9.$

$n \to \infty$

$Tn \to \infty$

$\boxed{T'(n) = n^2}$

we can find constant K such that

$An^2 + Bn + C \leq K \cdot n^2$ after n crosses certain value.

$5n^2 + 7n + 9.$

$6n^2.$

n

$1 + 2 + 3 + 4 + 5 + - - + N-1$

j=1 j=2 j=N-1

N+1

1 2

$\leq \frac{(N-1) \cdot N}{2} + (N-1).$