

Example predicates.

$$P(n) : 1 + 2 + 3 + \dots + n = \frac{n \cdot (n+1)}{2}$$

Domain of predicate = Set of natural numbers.

$$\{1, 2, 3, 4, \dots\}$$

$$n = 4$$

$$P(n=4) : 1 + 2 + 3 + 4 = \frac{4 \cdot (4+1)}{2}$$

$$\text{LHS: } P(n=4) \equiv 10$$

$$\text{RHS: } P(n=4) \equiv \frac{4 \times 5}{2} = 2 \times 5 = 10$$

$$\text{LHS: } P(n=4) = \text{RHS: } P(n=4)$$

$$\therefore P(n=4) \equiv \text{True.}$$

$$P(n=6) : 1 + 2 + 3 + 4 + 5 + 6 = \frac{6 \cdot (6+1)}{2}$$

$$\text{LHS: } P(n=6) = 21$$

$$\text{RHS: } P(n=6) = \frac{6 \cdot (6+1)}{2} = 3 \cdot 7 = 21$$

$$\therefore \text{LHS: } P(n=6) = \text{RHS: } P(n=6)$$

$$\therefore P(n=6) \equiv \text{True.}$$

$\forall n, P(n) \equiv$  the predicate  $P(n)$  holds for all values in its domain.

For any natural number,  $N$ , the sum of first  $N$  natural numbers is  $\frac{N \cdot (N+1)}{2}$

Generalisation of universal quantifier.

If, somehow, we are able to show that for some arbitrary natural number  $n_0$ ,  $P(n=n_0)$  is true. i.e.  $1+2+3+\dots+n_0 = \frac{n_0(n_0+1)}{2}$

then we can invoke Generalisation of universal quantifier.

& say that  $\forall n. P(n) \equiv \text{True}$

& then

$1+2+3+\dots+n = \frac{n(n+1)}{2}$  can be used as formula.

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Sharpening exercises:

Compound proposition:

1] AND ing: [Conjunction]

Stmnt 1: New Delhi is capital of India.

Stmnt 2: India won its first cricket world cup in 1983.

New Delhi is capital of India and

India won its first cricket world cup in 1983.

} Stmnt.

{ Proposition 1 AND Proposition 2 }  $\equiv$  Proposition

Proposition-1

Proposition-2

Prop-1 AND Prop-2.

F

F

F

F

T

F

T

F

F

T

T

T

$p, q, r, s, t$  : Proposition

Let  $p$  be any proposition.

Let  $q$  be any other proposition.

$p$  and  $q$  |  $p \wedge q$

$p$

$q$

$p \wedge q$

F

F

F

F

T

F

T

F

F

T

T

T.

2] OR ing : [Disjunction]

$\wedge$  = ANDING

Let  $p$  &  $q$  be any two propositions.

$\vee$  = ORing,

$p$  or  $q \longrightarrow$  Proposition

$p$

$q$

$p \vee q$

F

F

F

F

T

T

T

F

T

T

T

T

## 3] NOT (Negation)

Let  $p$  be any proposition.

It is not the case that  $p$ .

Mumbai is a capital of MH.

It is not the case that Mumbai is a capital of MH.

Mumbai is not a capital of MH.

$p$	not $p$ ( $\neg p$ )
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F	T
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T	F
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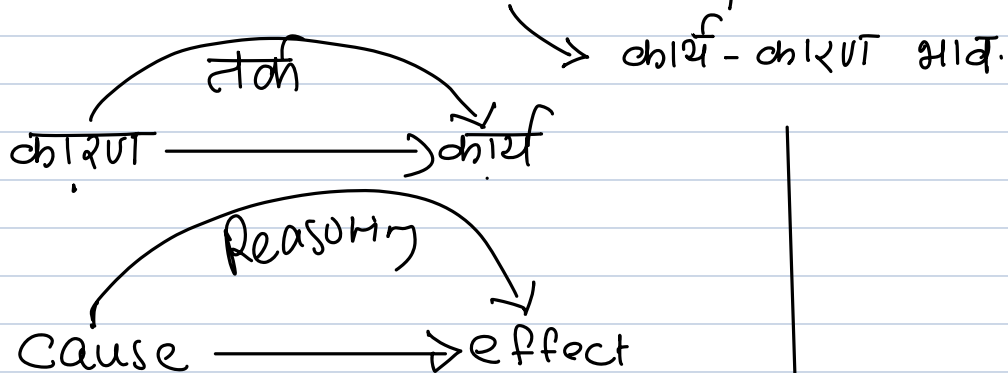
## 4] Implication:

Deductive Reasoning,

Logic = तर्कशास्त्र.

शास्त्र, Science,

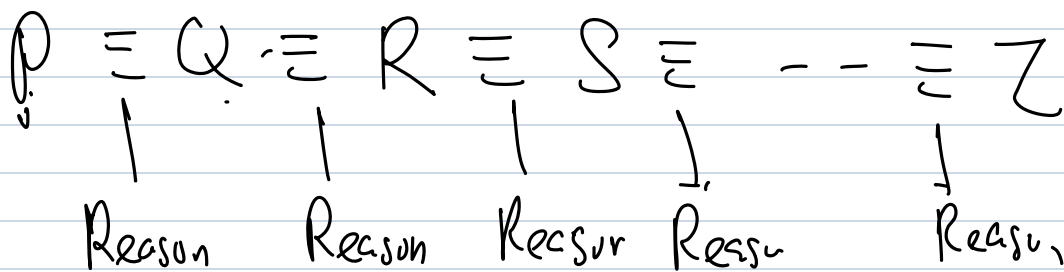
→ Principle of Causality



$p \in \text{True / False} \rightarrow$

$\{ \underbrace{p_1, p_2, p_3 \dots p_n}_{\text{Truth values}} \} \xrightarrow{\text{Reasoning}} Q$

Reasoning  $\leq$  Valid



Let  $p$  &  $q$  be any two proposition.

If  $p$  then  $q$  = Proposition

$p$	$q$	If $p$ then $q$
F	F	T
F	T	T
T	F	F
T	T	T

Politician: [If I get elected then I will construct a bridge.]

$p$ : Mr. X gets elected

$q$ : Bridge is constructed

F	F	T
F	T	T
T	F	F
T	T	T

1) If  $p$  then  $q$

2) If  $p, q$

3)  $p$  implies  $q$

4) When  $p$  then  $q$ .

5)  $p$  is a sufficient condition for  $q$

6) Necessary condition for  $p$  is  $q$ .

7)  $p$  only if  $q$

8)  $q$  if  $p$ .

9)  $q$  is necessary for  $p$ .

10) Sufficient condition for  $q$  is  $p$ .

11)  $q$  whenever  $p$ .

12)  $q$  unless  $\neg p$ .

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$p$ :  $ABCD$  is a square

$q$ :  $ABCD$  is a quadrilateral.

1) If  $ABCD$  is a square then  $ABCD$  is a quadrilateral.

2) If  $ABCD$  is a square,  $ABCD$  is a quadrilateral.

3)  $ABCD$  is a square implies  $ABCD$  is a quadrilateral.

4) When  $ABCD$  is a square then  $ABCD$  is a quadrilateral.

5)  $ABCD$  is a square is a sufficient condition for  $ABCD$  to be a quadrilateral.

6) Necessary condition for  $ABCD$  to be square is  $ABCD$  be a quadrilateral.

7)  $ABCD$  is square only if  $ABCD$  is a quad.

8)  $ABCD$  is quad. if  $ABCD$  is square

9)  $ABCD$  is quad. is a necessary cond. for  $ABCD$  to be a square

10) Suff. cond for  $ABCD$  to be a quad. is  $ABCD$  is a square

11)  $ABCD$  is a quad. whenever  $ABCD$  is square

12)  $ABCD$  is a quad. unless  $ABCD$  is not a square.

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