Example predicates.

$P(n): \quad 1+2+3+\ldots+n = \frac{n \cdot (n+1)}{2}.$

Domain of predicote = set of natuval numbers.

$\{1, 2, 3, 4 \ldots \}.$

$n = 4$

$P(n=4): \quad 1+2+3+4 = \frac{4 \cdot (4+1)}{2}$

LHS: $P(n=4) \equiv 10$

RHS: $P(n=4) \equiv \frac{4 \times 5}{2} = 2 \times 5 = 10$

LHS. $P(n=4)$ = RHS: $P(n=4)$

$\therefore P(n=4) \equiv$ True.

$P(n=6): \quad 1+2+3+4+5+6 = \frac{6 \cdot (6+1)}{2}.$

LHS: $P(n=6) = 21$

RHS: $P(n=6) = \frac{6 \cdot (6+1)}{2} = 3 \cdot 7 = 21$

$\because$ LHS: $P(n=6)$ = RHS: $P(n=6)$

$\therefore P(n=6) \equiv$ True.

$\forall n. P(n) \equiv$ the predicate $P(n)$ holds for all values in its domain.

for any natural number, N, the sum of first N natural numbers is $\frac{N \cdot (N+1)}{2}$

Generalisation of universal quantifier.

If, somehow, we are able to show that for some arbitrary natural number n_0, $P(n = n_0)$ is true i.e. $1 + 2 + 3 + \dots + n_0 = \frac{n_0(n_0+1)}{2}$

then we can invoke Generalisation of universal quantifier.

& say that $\forall n. P(n) \equiv True$

& then

$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ can be used as formula.

Sharpening exercises:

Compound proposition:

1] AND ing: [Conjunction]

Stmt 1: New Delhi is capital of India.

Stmt 2: India won its first cricket world cup in 1983.

$\left.\boxed{\begin{array}{l}\text{New Delhi is capital of India and} \\ \text{India won its first cricket world cup in 1983.}\end{array}}\right\}$ Stmt.

$\left\{ \underset{2}{\text{Proposition 1}} \text{ AND } \underset{2}{\text{Proposition 2}} \right\} \equiv$ Proposition

Proposition-1	Proposition-2	Prop-1 AND Prop-2.
F	F	F
F	T	F
T	F	F
T	T	T

p, q, r, s, t : Proposition

Let p be any proposition.

Let q be any other proposition.

p and q | $p \wedge q$

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T.

2] ORing : [Disjunction]

$\wedge$ = ANDING

$\vee$ = ORing.

Let p & q be any two propositions.

P or q $\longrightarrow$ Proposition

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

3] NOT (Negation)

Let p be any proposition.

It is not the case that p.

Mumbai is a capital of MH.

It is not the case that Mumbai is a capital of MH

Mumbai is not a capital of MH.

p	not p ($\neg p$)
F	T
T	F

4] Implication :

Deductive Reasoning.

Logic = तर्कशास्त्र.

शास्त्र, Science,

→ Principle of Causality → कार्य - कारण भाव.

$$\text{कारण} \xrightarrow{\text{तर्क}} \text{कार्य}$$

$$\text{Cause} \xrightarrow{\text{Reasoning}} \text{effect}$$

$p \equiv$ True / False

$$\underbrace{\{P_1, P_2, P_3, \ldots, P_n\}}_{\text{Truth values}} \Rightarrow Q$$

Reasoning $\equiv$ Valid

$P \equiv Q \equiv R \equiv S \equiv \cdots \equiv Z$

Reason Reason Reason Reason Reason

Let p & q be any two proposition.

$\boxed{\text{If } p \text{ then } q} = \text{Proposition}$

p	q	If p then q
F	F	T
F	T	T
T	F	F
T	T	T

Politician: [If I get elected then I will construct a bridge.]

p: Mr. X gets elected

q: Bridge is constructed

F	F	T
F	T	T
T	F	F
T	T	T

1) If p then q

2) If p, q

3) p implies q

4) when p then q.

5) p is a sufficient condition for q

6) Necessary condition for p is q.

7) p only if q

8) q if p.

9) q is necessary for p.

10) Sufficient condition for q is p.

11) q whenever p.

12) q unless $\neg p$.

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p: ABCD is a square

q: ABCD is a quadrilateral.

1) If ABCD is a square then ABCD is a quadrilateral.

2) If ABCD is a square, ABCD is a quadrilateral.

3) ABCD is a square implies ABCD is a quadrilateral.

4) when ABCD is a square then ABCD is a quadrilateral.

5) ABCD is a square is a sufficient condition for ABCD to be a quadrilater.

6) Necessary condition for ABCD to be square is ABCD be a quadrilateral.

7) ABCD is square only if ABCD is a quad.

8) ABCD is quad. if ABCD is square

9) ABCD is quad. is a necessary cond. for ABCD to be a square

10) Suff. cond for ABCD to be a quad. is ABCD is a square

11) ABCD is a quad. whenever ABCD is square

12) ABCD is a quad. unless ABCD is not a square.