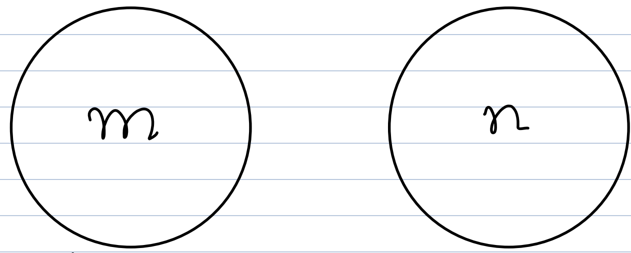
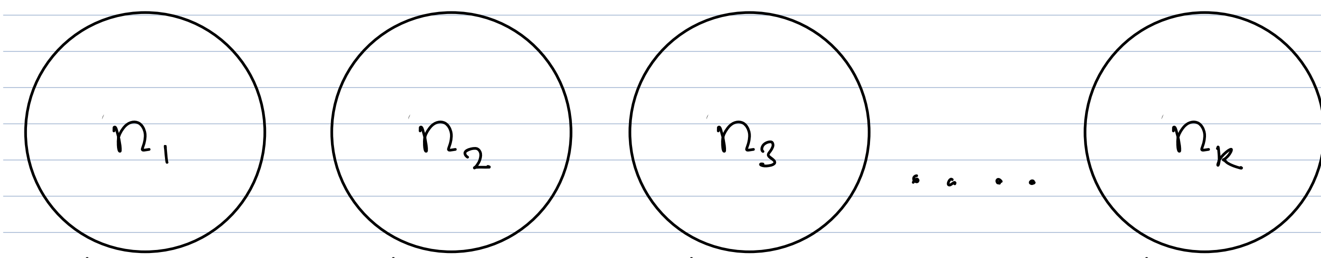
Fundamental Principle of counting : the basic version.



Activity-1 Activity-2 [number of choice are independent of choice made to do activity #1]

total number of ways to do both the activities $= m \times n$



Activity-1 Activity-2 Activity-3 Activity-k

The number of choices for the current activity are independent of choices made in the previous activities!

total number of ways to do all activities together

$= n_1 \times n_2 \times n_3 \times \dots \times n_K.$

Meaning a/b. where a and b are positive numbers.

Division = Repeatitive Subtraction

$a - b$

$a-b-b$

$a-b-b-b$

$a - b - b - b - b$

Quotient : How many times 'b' could be Subtracted from a without making the result of the subtraction negative.

Remainder : Number which remains residue after $a -$ Quotient $\times b$.

Let q be the quotient of division
Let r be the remainder of division

$a = b \cdot q + r$ where $0 \leq r < b$.

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floor (integer or fractional number)
= greatest integer $\leq$ given number

floor(given number)
= greatest integer $\leq$ given number

Ceiling (given number) = least integer $\geq$ given number

$a + ? = b$ $(b - a)$ = Ans. of(?)

$a * ? = b$ (b/a) = Ans. of(?)

$a^{?} = b$ $\log_a b$ = Ans. of(?)

$2^3 = 8, \log_2(8) = 3$

$2^{12} = 4096, \log_2(4096) = 12.$

$\log_2 n = x, \quad 2^x = n.$

Representation: $\log_2(n) = \lg(n)$

$2^0 < x < 2^1 \quad | \quad 1 < x < 2$

$\log_2 2^0 < \log_2 x < \log_2 2^1 \quad | \quad 0 < \log_2(x) < 1$

$2^0 < x < 2^1, \quad 0 < \log_2(x) < 1$

$2^1 < x < 2^2 \quad 1 < \log_2(x) < 2.$

$\vdots$

$2^m < x < 2^{m+1} \quad m < \log_2(x) < m+1$

$\text{floor}(\log_2(x)) = \lfloor \log_2(x) \rfloor = m$

$\text{Ceil}(\log_2(x)) = \lceil \log_2(x) \rceil = m+1$

$x = 2^m \quad | \quad \log_2(x) = m = \lfloor \log_2(x) \rfloor$

$= \lceil \log_2(x) \rceil$

0	100	1000
1	101	
10	110	
11	111	

$563 = 500 + 60 + 3$

$= 5\times10^2 + 6\times10^1 + 3\times10^0$

$1982 = 1000 + 900 + 80 + 2$

$= 1\times10^3 + 9\times10^2 + 8\times10^1 + 2\times10^0$

Decimal Number :

1) How many digits it has ?
2) For every digit, which choice between 0-9 is made. [Most significant digit cannot be zero]

One digit number

$\boxed{0-9}$

$\times 10^0$

two digit numbers

$1-9$ d_2	$0-9$ d_1

$d_1 \times 10^0$ +

$d_2 \times 10^1$

$\boxed{1}$ $= 1 \times 10^0$

$\times 10^0$ $= 1 \times 1 = 1$

$\boxed{8}$ $= 8 \times 10^0$

$\times 10^0$ $= 8 \times 1 = 8$

$\boxed{9}$ $= 9 \times 10^0$

$= 9 \times 1 = 9$

$\times 10^0$

$= d_1 \times 10^0 + d_2 \times 10^1$

$= d_1 \times 1 + d_2 \times 10$

$= d_1 + 10 \cdot d2$

5	4

$= 4 + 10 \times 5 = 54$

9	1

$= 1 + 10 \times 9 = 91$

Three digit numbers :

1–9	0–9	0–9
d3	d2	d1

d1 $\times 10^0$ +

d2 $\times 10^1$ +

d3 $\times 10^2$

$(d_3 d_2 d_1) = d_1 \times 10^0 + d_2 \times 10^1 + d_3 \times 10^2$

$= d_1 \times 1 + d_2 \times 10 + d_3 \times 100$

$= d_1 + 10 . d_2 + 100 \cdot d3$

8	5	2

$= 2 + 10 \cdot 5 + 100 \cdot 8$

$= 2 + 50 + 800$

$= 52 + 800$

$= 852$

1-9		0-9	0-9	0-9	0-9	0-9
d_n		d_5	d_4	d_3	d_2	d_1

$$(d_n d_{n-1} -- d_1) = d_1 \times 10^0 + d_2 \times 10^1 + d_3 \times 10^2 + \dots + d_n \times 10^{n-1}$$

Binary Number System:

One digit binary number.

0-1
d_1

$\times 2^0 = d_1 \times 1 = d_1$

1

$\rightarrow \times 2^1 = 1 \times 2^0 = 1 \times 1 = 1$

two digit binary number

1	0-1
d_2	d_1

$d_1 \times 2^0 = d_1 \times 1 = d_1$

$d_2 \times 2^1 = d_2 \times 2 = 2 \cdot d_2$

$d_1 + 2 \cdot d_2$

1	1

$\rightarrow \times 2^0 = 1 \times 2^0 = 1 \times 1 = 1$

$\rightarrow \times 2^1 = 1 \times 2^1 = 1 \times 2 = 2$

$(11)_2 = (3)_{10}$

three digit binary number.

1	0-1	0-1
d_3	d_2	d_1

$\times 2^0 = d_1 \times 2^0 = d_1$

$\times 2^1 = d_2 \times 2^1 = 2d_2$

$\times 2^2 = d_3 \times 2^2 = 4d_3$

$$d_1 + 2d_2 + 4d_3 = (d_3 d_2 d_1)_2$$

1	0	1

$\times 2^0 = 1 \times 2^0 = 1$

$+$

$\times 2^1 = 0 \times 2^1 = 0$

$+$

$\times 2^2 = 1 \times 2^2 = 4$

$\overline{5}$

$$(101)_2 = (5)_{10}$$

Exercise:

1		0-1	0-1	0-1	0-1
d_n		d_4	d_3	d_2	d_1

derive formula for the same

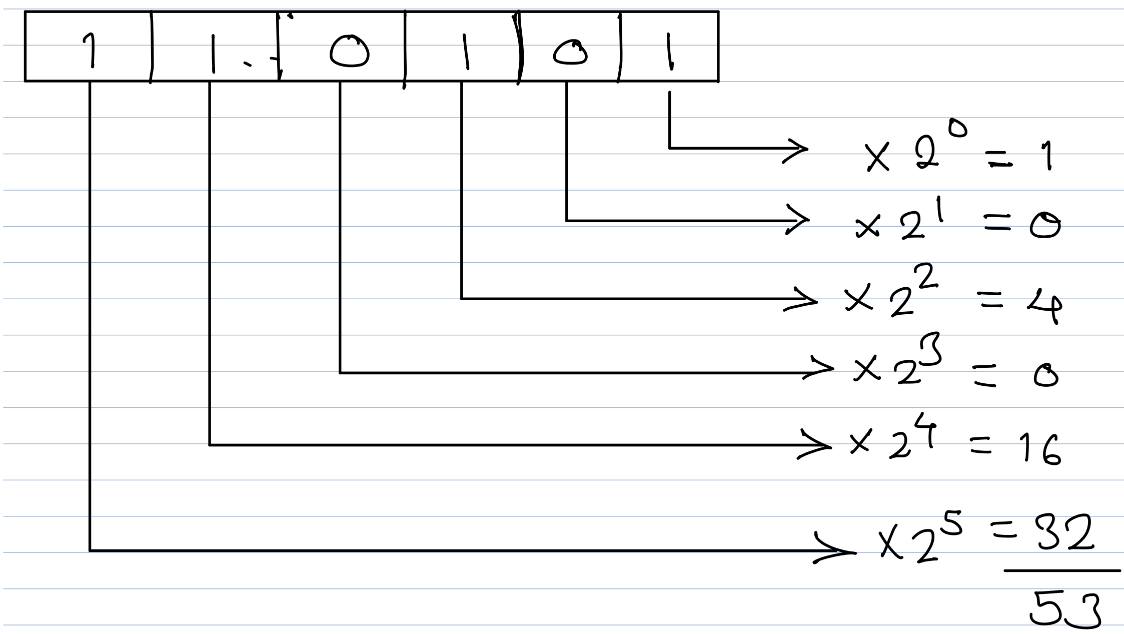
1	1	0	1	0	1

$\longrightarrow$ convert into decimal,

1		0-1	0-1	0-1	0-1
d_n		d_4	d_3	d_2	d_1

$$(d_n d_{n-1} d_{n-2} \cdots d_1)_2$$

$$= d_1 \times 2^0 + d_2 \times 2^1 + \cdots + d_n \times 2^{n-1}$$



$$(110101)_2 = (53)_{10}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(5+3)^2 = 5^2 + 2\times5\times3 + 3^2$$

$$= 25 + 30 + 9$$

$$= 55 + 9$$

$$= 64$$

$$a^n - 1 = \underline{(a-1)} \cdot (a^{n-1} + a^{n-2} + a^{n-3} + \cdots + 1)$$

$$a^2 - 1 = (a-1)(a+1)$$

$$a^3 - 1 = (a-1)\cdot(a^2 + a + 1)$$

$$a^4 - 1 = (a-1)\cdot(a^3 + a^2 + a + 1)$$

$$(a-1)\cdot(a^{n-1} + a^{n-2} + a^{n-3} + \dots + 1)$$

$$= a\cdot a^{n-1} + a\cdot a^{n-2} + a\cdot a^{n-3} + \dots + a$$
$$- a^{n-1} - a^{n-2} - a^{n-3} + \dots - 1$$

$$= a^n + \cancel{a^{n-1}} + \cancel{a^{n-2}} + \dots + \cancel{a}$$
$$- \cancel{a^{n-1}} - \cancel{a^{n-2}} + \dots \cancel{-} 1$$

$$= a^n - 1$$

Let $(d_n d_{n-1} \dots d_1)_{10}$ be any decimal number

$(d_n d_{n-1} \dots d_1)_{10}$

$$= d_1 \times 10^0 + d_2 \times 10^1 + d_3 \times 10^2 + \dots + d_{n-1}\cdot 10^{n-2} + d_n \cdot 10^{n-1}$$

$(d_1 d_2 \dots d_n)$

$$= d_n \cdot 10^0 + d_{n-1}\cdot 10^1 + d_{n-2}\cdot 10^2 \dots + d_3 \cdot 10^{n-3}$$
$$+ d_2 \cdot 10^{n-2} + d_1 \times 10^{n-1}$$

$$(1\ 4\ 2\ 5) = 5\times 10^0 + 2\times 10^1 + 4\times 10^2 + 1\times 10^3$$

$$(5\,2\,4\,1) = 1\times 10^0 + 4\times 10^1 + 2\times 10^2 + 5\times 10^3$$

$$d_1\times 10^0 + d_2\times 10^1 + d_3\times 10^2 + - - + d_{n-1}\cdot 10^{n-2} + d_n\cdot 10^{n-1}$$

$$-\ d_1\times 10^{n-1} + d_2\times 10^{n-2} + d_3\times 10^{n-3} \quad + d_{n-1}\cdot 10^1 + d_n\, 10^0$$

$$d_1\cdot(10^{n-1} - 1) + d_2(10^{n-2} - 10^1)$$

$$+ d_3(10^{n-3} - 10^2) + - - + d_k\cdot(10^{n-k} - 10^{k-1})$$

$$d_1\cdot(10^k - 1) + d_2\cdot(10^l - 1) + d_3(10^l - 1 \quad)$$

$$d_1\cdot(10-1)(\text{---}) + d_2\cdot(10-1)\cdot(\text{---})$$

$$+ \ - \ - \ + d_n(10-1)(\text{---} \quad)$$

$$9\left[d_1\times\text{---} + d_2\times\text{---} + - + d_n\times\text{---}\right]$$

$$(5+3)^2 = 5^2 + 2\cdot 5\cdot 3 + 3^2$$

$$5^6 - 1 = (5-1)(5^5 + 5^4 + 5^3 + 5^2 + 5^1 + 1)$$

$(a+b)^2$ logic -

$a^n - 1$

How to convert binary into decimal

Most significant digit least significant digit

b_n	b_{n-1}	b_{n-2}	. . .	b_3	b_2	b_1

$b_1 \times 2^0$ +

$b_2 \times 2^1$ +

$b_3 \times 2^2$ +

$b_{n-2} \times 2^{n-3}$ +

$b_{n-1} \times 2^{n-2}$ +

$b_n \times 2^{n-1}$

$$= b_1 \times 2^0 + b_2 \times 2^1 + b_3 \times 2^2 + \dots + b_{n-2} \times 2^{n-3} + b_{n-1} \times 2^{n-2} + b_n \times 2^{n-1}$$

$13 = 1 \times 10 + 3$

$13 = \underline{6} \times 2 + 1$

$= [3 \times 2 + 0] \times 2 + 1$

$= 3 \times 2 \times 2 + 0 \times 2 + 1$

$3 = 1 \times 2 + 1$

$= [1 \times 2 + 1] \times 2 \times 2 + 0 \times 2 + 1$

$= 1 \times 2 \times 2 \times 2 + 1 \times 2 \times 2 + 0 \times 2 + 1$

$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 1$

$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= (1101)_2.$ L long method

$2 \overline{)13} \,(1$

$2 \overline{)6} \,(0$

$2 \overline{)3} \,(1$

$2\overline{)\,1\,}\,(1$

0

$(1101)_2$

$a = bq + r$

$a = 9 \cdot 2 + 0$

$= 9 \cdot 2 + 1$

$17 = 8 \times 2 + 1$

$(8/2 \quad 8 = 4 \times 2 + 0)$

$17 = (4 \times 2 + 0) \times 2 + 1$

$= 4 \times 2 \times 2 + 0 \times 2 + 1$

$(4/2 \quad 4 = 2 \times 2 + 0)$

$= (2 \times 2 + 0) \times 2 \times 2 + 0 \times 2 + 1$

$= 2 \times 2 \times 2 \times 2 + 0 \times 2 \times 2 + 0 \times 2 + 1$

$(2/2 \quad 2 = 1 \cdot 2 + 0)$

$= (1 \cdot 2 + 0) \times 2 \times 2 \times 2 + 0 \times 2 \times 2 + 0 \times 2 + 1$

$= 1 \times 2 \times 2 \times 2 \times 2 + 0 \times 2 \times 2 \times 2 + 0 \times 2 \times 2$

$\vdots$

$+0\times 2+1$

$(1/2 \quad 1 = 0\cdot 2+1)$

$= (0\cdot 2+1)\times 2\times 2\times 2\times 2 + 0\times 2\times 2\times 2 + 0\times 2\times 2 + 0\times 2 + 1$

$= 1\times 2\times 2\times 2\times 2 + 0\times 2\times 2\times 2 + 0\times 2\times 2 + 0\times 2 + 1$

$= 1\times 2^4 + 0\times 2^3 + 0\times 2^2 + 0\times 2^1 + 1\times 1$

$= \underline{1}\times 2^4 + \underline{0}\times 2^3 + \underline{0}\times 2^2 + \underline{0}\times 2^1 + \underline{1}\times 2^0$

$= (10001)_2$

$1 \quad 0 \quad 0 \quad 0 \quad 1$

$\times 2^0 = 1$

$\times 2^1 = 0$

$\times 2^2 = 0$

$\times 2^3 = 0$

$\times 2^4 = 16$

$(147)_{10}$

$2 \overline{)147} (1$ $\quad (10010011)_2$

Q = 73 $\quad = (147)_{10}$.

$2 \overline{)73} (1$

Q = 36

$2 \overline{)36} (0$

Q = 18

$2 \overline{)18} (0$

Q = 9

$2 \overline{)9} (1$

Q = 4

$2 \overline{)4} (0$

Q = 2

$2 \overline{)2} (0$

1

$2 \overline{)1} (1$ short method

0

$17 = \underline{\underline{8}} \times \underline{\underline{2}} + 1$

Q D R

$[8 = 4\times 2 + 0]$

$= [4\times 2 + 0]\times 2 + 1$

Q D R

$= \underline{\underline{4}}\times \underline{2\times 2} + \underline{0\times 2} + 1$

$[4 = 2\times 2 + 0]$

Q D R

$= [\underline{\underline{2\times 2}} + 0]\times \underline{\underline{2\times 2}} + 0\times 2 + 1$

$= \underline{\underline{2}}\times \underline{2\times 2\times 2} + \underline{0\times 2\times 2 + 0\times 2} + 1$

$= [2 = 1\times 2 + 0]$

Q D R

$= [\underline{1\times 2} + 0]\times \underline{2\times 2\times 2} +$

$0\times 2\times 2 + 0\times 2 + 1$

$= 1\times \underline{2\times 2\times 2\times 2} + \underline{0\times 2\times 2\times 2} +$

$0\times 2\times 2 + 0\times 2 + 1$

$= [1 = 0\times 2 + 1]$

Q D R

$= [$ [illegible]

$= (0 \times 2 + 1) \times 2 \times 2 \times 2 \times 2$
$+ 0 \times 2 \times 2 \times 2 +$
$0 \times 2 \times 2 + 0 \times 2 + 1$

$= 1 \times 2 \times 2 \times 2 \times 2 +$
$0 \times 2 \times 2 \times 2 +$
$0 \times 2 \times 2 +$
$0 \times 2 +$
1×2^0

$= 1 \times 2^4 +$
$0 \times 2^3 +$
$0 \times 2^2 +$
$0 \times 2^1 +$
1×2^0

$= (10001)_2.$