

* Standard Normal Distribution

Let x be a random variable which belongs to a Gaussian distribution having ' μ ' mean and ' σ ' standard deviation.

Then we can transform this ' x ' to ' y ' which will belong to standard Normal Distribution having $\mu=0$ and $\sigma=1$, this process is called standardization and done by calculating Z-score.

$x \in \text{Gaussian}$
(μ, σ)

Transformation
(standardization)
using Z-score.
 $y \in \text{Normal standard}$

→ x to y called transformation.

→ y to x called inverse transformation.

$$Z \text{ score} = \frac{x_i - \mu}{\sigma / \sqrt{n}}$$

here,

$$\sigma / \sqrt{n} = \text{Standard error}$$

where n is sample size.

as in case of Gaussian to Normal standard transformation, we transform each value or features so ' n ' tends to be 1, as area under probability distribution as 1.

So, Z-score formula for this specific transformation case is ($n=1$),

$$Z \text{-score} = \frac{x_i - \mu}{\sigma}$$

Ex → $x = \{1, 2, 3, 4, 5\}$, $\mu = 3$, $\sigma = 1.414$

Z-scores,

for 1 its -1.414 as $(1-3/1.414)$

similarly for 2 its -0.707 ,

for 3 its 0, for 4 its 0.707 , for 5 its 1.414

∴ $y = \{-1.414, -0.707, 0, 0.707, 1.414\}$

Comparing x and y we can conclude that after standardization score has decreased leading to faster calculation and easy visualization of data.

→ This standardization helps us build models efficiently as it decreases the scope for those datasets where each features range vary widely.

ex -

Age (yr)	Height (cm)	Weight (kg)
20	180	72
22	190	94
24	160	82

Here all these features have huge scope, so we should standardize it first.

→ There is another way to scale random variables which is called Normalization.

→ In case of standardization range of scaled values determined by the formula outcome.

→ In case of Normalization range of scaled variable can be determined in advance.

One method to do Normalization is min-max normalization which gives the scaled variables within range of 0 to 1.

And is used mostly in deep learning.

Another example is that pixels of image are kept within range of 0-255.

→ Normalization and standardization are part of procedure called Feature scaling.

→ Basically in normalization we are shifting our mean close to Z-score, same with standardization.

→ Area under the curve can be calculated by Z-score and Z-table.