* Logistic Regression

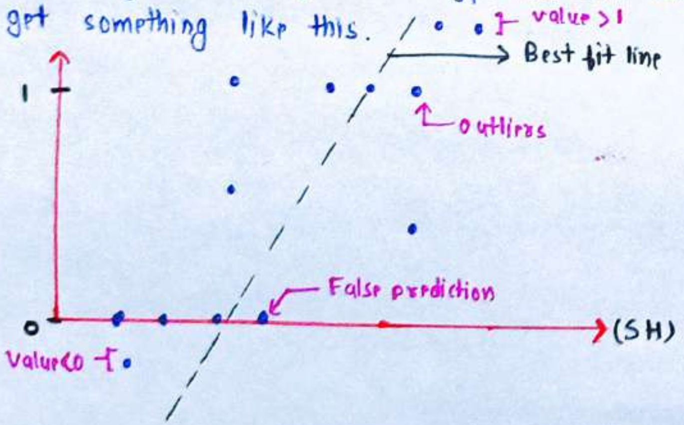
- Supervised algorithm
- It is a regression model which tries to predict whether given data point belong to category '1' or '0' using binomial function.

A. Why not use Linear Regression to classify?

1. Linear regression deals with continous value while logistic regression deals with discrete values.
2. If we try to classify using threshold on contnous value, it fails to do it properly when new value or outlier added as threshold shifts.
3. As it is binary classification value should either be 1 or 0 but as data is continous it doesnt happen.

Ex-. Say if we want to classify if student pass or fail with criteria: if study hr (SH) > 4 it is pass else fail. [1 = pass, 0 = fail]

- When we plot this data and try to get best fit line using linear regression hypothesis line we get something like this.



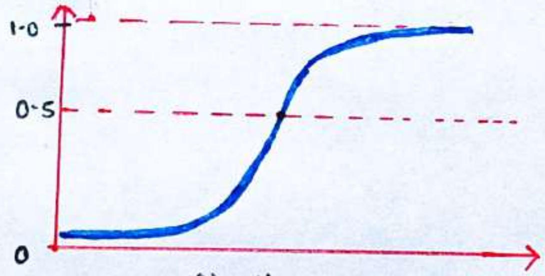
- We can observe that introducing outliers will lead to even worse regression line.
- Also few values even dont belong to the binar classes 1 or 0. (value>1, value<0)
- High chances of false prediction.
- As being type of regression logistic also uses the same hypothesis function as linear regression i.e,

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

B. Using Sigmoid function with hypothesis function.

- Applying Sigmoid function (g) over hypothesis function ($h_\theta(x)$) helps to resolve the issue of values going over 1 or going less than 0 not belonging to any classes by limiting the regression line between 1 and 0.

$$\boxed{g = \frac{1}{1+e^{-z}}}$$



Sigmoid function(g)

$\therefore$ $z = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$

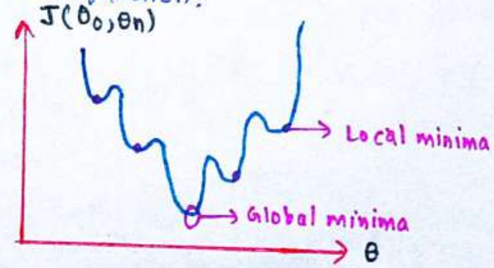
$h_\theta(x) = g(z)$

$$\Rightarrow \boxed{h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 x_1 + \dots \theta_n x_n)}}}$$

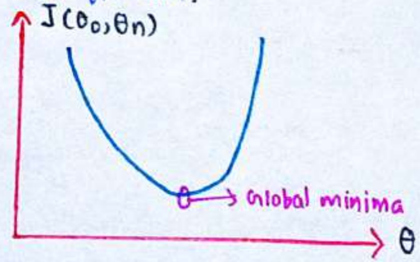
- But Sigmoid function is not a convex function. So when we try to minimize our cost function to reach global minima it fails to do so as of presence of local minima.
- If we use L2 Loss function, and update it with sigmoid applied hypothesis function we get,

$$\boxed{J(\theta_0,\theta_n) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{1+e^{-(\theta_0+\theta_1 x_1 + \dots \theta_n x_n)}} - y^i\right]^2}$$

and this cost function yields an non convex function.



- But we need a convex function like this one to reach the global minima and have best fit line.



- To achieve this we update our cost function first to problistic function then to log probability function.