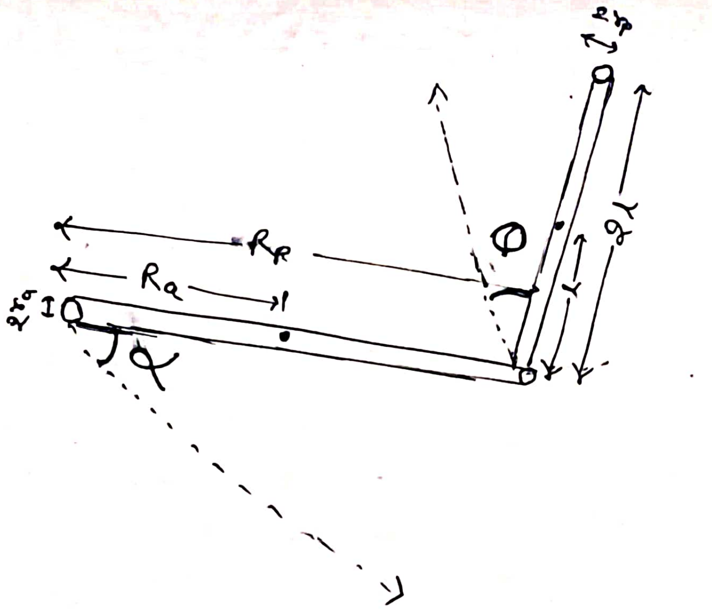

I took all the perameters same as task 1

Because I desined my bike same as pendulum.

The equation describing the x-y cordinates of the pendulum position regaring its base are:-

$$x_p = -2l\sin\theta \quad \text{---} \quad (i)$$

$$y_p = 2l\cos\theta \quad \text{---} \quad (ii)$$

To find velocity components, taking the derivative

$$\dot{x}_p = -2l\dot{\theta}\cos\theta \quad \text{---} \quad (3)$$

$$\dot{y}_p = -2l\dot{\theta}\sin\theta \quad \text{---} \quad (4)$$

The speed of the arm along x-cordinate

$$V_{arm} = -R_p\dot{\alpha} \quad \text{---} \quad (5)$$

using (3) to (5), the velocity components of pendulum along the x and y direction

$$V_x = -R_p\dot{\alpha} - 2l\dot{\theta}\cos\theta \quad \text{---} \quad (6)$$

$$V_y = -2l\dot{\theta}\sin\theta \quad \text{---} \quad (7)$$

Now we know

$$L = E_{K.E.} - E_{P.E.}$$

$$E_{P.E.} = 2g\,M_p\,l\cos\theta \quad ----(8)$$

The total kinetic energy of the system. is equal to sum of kinetic energies of its moving parts as:-

$$E_{KE.} = K.E._{arm} + K.E._{pen} + K.E_{v_x} + K.E._{v_y}$$

$$= \frac{1}{2} \times I\_arm \times \dot{\alpha}^2 + \frac{1}{2} \times I\_pen \times \dot{\theta}^2 + \frac{1}{2} M_p (2l)^2 \dot{\theta}^2 \sin^2\theta$$

$$+ \frac{1}{2} \times M_p \times (R_p)^2 \times \dot{\alpha}^2$$

$$\because KE_{v_x} = \frac{1}{2} M_p V_x^2 \qquad \because KE_{v_y} = \frac{1}{2} M_p V_y^2$$

Now $L = KE - PE$

$$L = \frac{1}{2} I\_arm\,\dot{\alpha}^2 + \frac{1}{2} I\_pen\,\dot{\theta}^2 + \frac{1}{2} M_p (2l)^2 \dot{\theta}^2 \sin^2\theta +$$

$$\frac{1}{2} M_p (R_p)^2 \dot{\alpha}^2 + \frac{1}{2} M_p (2l)^2 \dot{\theta}^2 \cos^2\theta$$

$$+ M_p \dot{\alpha} (2l) (R_p) \dot{\theta} \cos\theta - g M_p (2l) \cos\theta$$

$$L = \frac{1}{2} I\_arm\,\dot{\alpha}^2 + \frac{1}{2} I\_pen\,\dot{\theta}^2 + 2 m_p l^2 \dot{\theta}^2 \sin^2\theta +$$

$$\frac{1}{2} m_p R_p^2 \dot{\alpha}^2 + 2 M_p l^2 \dot{\theta}^2 \cos^2\theta + 2 M_p \dot{\alpha}^2 l R_p \dot{\theta} \cos\theta$$

$$- 2 g m_p l \cos\theta \quad ---.(9)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\alpha}}\right) - \frac{\delta L}{\delta \alpha} = T \quad \text{----} \quad (10)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right) - \frac{\delta L}{\delta \theta} = 0 \quad \text{----} \quad (11)$$

$$L = \frac{1}{2} I\_arm\, \dot{\alpha}^2 + \frac{1}{2} I\_pen\, \dot{\theta}^2 + 2M_p l^2 \dot{\theta}^2 \sin^2\theta +$$
$$\frac{1}{2} M_p R_p^2 \dot{\alpha}^2 + 2M_p l^2 \dot{\theta}^2 \cos^2\theta + 2M_p \dot{\alpha}\, l R_p \dot{\theta} \cos\theta$$
$$- 2g M_p l \cos\theta \quad \text{----} \quad (12)$$

Substituting L in equation (10)

$$\left(\frac{\delta L}{\delta \alpha}\right) = 0 \quad \text{----} \quad (13)$$

$$\left(\frac{\delta L}{\delta \dot{\alpha}}\right) = I\_arm\, \dot{\alpha} + M_p R_p^2 \dot{\alpha} + 2M_p l R_p \dot{\theta} \cos\theta \quad \text{----} \quad (14)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\alpha}}\right) = I\_arm\, \ddot{\alpha} + M_p R_p^2 \ddot{\alpha} + 2M_p l R_p \ddot{\theta} \cos\theta$$
$$- 2M_p l R_p \dot{\theta}^2 \sin\theta \quad \text{----} \quad (15)$$

From (10), (13), (15) equations

$$I\_arm\, \ddot{\alpha} + M_p R_p^2 \ddot{\alpha} + 2M_p l R_p \ddot{\theta} \cos\theta - 2M_p l R_p \ddot{\theta} \sin\theta = T$$

$$I_{arm} \ddot{\alpha} + M_p R_p^2 \ddot{\alpha} + 2M_p l R_p \dot{\theta} \cos\theta - 2M_p l R_p \ddot{\theta} \sin\theta = T$$

$$\left(\frac{\delta L}{\delta \theta}\right) = -2M_p \dot{\alpha} l R_p \dot{\theta} \sin\theta + 2gM_p l \sin\theta \quad \text{---} (16)$$

$$\left(\frac{\delta L}{\delta \dot{\theta}}\right) = I\text{.pen}\, \dot{\theta} + 4M_p l^2 \dot{\theta} \sin^2\theta + 4M_p l^2 \dot{\theta} \cos^2\theta + 2M_p l \dot{\alpha} R_p \cos\theta$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = I\_pen\, \ddot{\theta} + 4M_p l^2 \ddot{\theta} + 2M_p l R_p \ddot{\alpha} \cos\theta - \dot{\alpha}\dot{\theta}\sin\theta \quad \text{---} (17)$$

From (11), (16), (17) equations

$$I\_pen\, \ddot{\theta} + 4M_p l^2 \ddot{\theta} + 2M_p l R_p \ddot{\alpha} \cos\theta - \dot{\alpha}\dot{\theta}\sin\theta + 2M_p \dot{\alpha} l R_p \dot{\theta} \sin\theta - 2gM_p l \sin\theta = 0$$

Now we have two equations:-

$$\boxed{I\_arm\, \ddot{\alpha} + M_p R_p^2 \ddot{\alpha} + 2M_p l R_p \ddot{\theta} \cos\theta - 2M_p l R_p \dot{\theta}^2 \sin\theta = T}$$

$$\boxed{I\text{-}pen\, \ddot{\theta} + 4M_p l^2 \ddot{\theta} + 2M_p l R_p \ddot{\alpha} \cos\theta - \dot{\alpha}\dot{\theta}\sin\theta + 2M_p \dot{\alpha} l R_p \dot{\theta} \sin\theta - 2gM_p l \sin\theta = 0}$$

Using these two equation I got $\ddot{\alpha}$ and $\ddot{\theta}$ which is used in octave I will provide in pdf also.

$$\ddot{a} = \frac{-T \cdot K_1 \cos\theta - K_1^2 \dot{\theta}^2 \sin\theta \cos\theta + 2K_1K_6 \dot{\alpha} \dot{\theta} \sin\theta - K_5K_6 \sin\theta - \dot{\alpha}\dot{\theta}K_6 \sin\theta}{-K_1^2 \cos^2\theta - K_6K_7}$$

$$\ddot{\theta} = \frac{\cos\theta \sin\theta [(K_1\dot{\alpha}\dot{\theta} - 2K_1^2\dot{\alpha}\dot{\theta} + K_1K_5)] - K_7T - K_1K_7\dot{\theta}^2 \sin\theta}{K_1^2 \cos^2\theta - K_7K_6}$$

Where

$$K_1 = 2 M_p l R_p$$

$$K_2 = 2 M_p l$$

$$K_3 = M_p R_p l^2$$

$$K_4 = M_p l^2$$

$$K_5 = 2 g M_p l$$

$$K_6 = I\_arm + K3$$

$$K_7 = I\_pen + 4K4$$