A) Given NLPP

$$Z = 4x_1 - x_1^2 + 2x_1x_2 - 2x_2^2$$

Constraints $\quad 2x_1 + x_2 - S_1 = 6$

$\quad x_1 - 4x_2 - S_2 = 0$

making S_1 and S_2 are basic variables in initial solution, with x_1, x_2 are non-basic variables.

$$X_B = (S_1, S_2) \quad X_{NB} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(i) $S_1 = -6 + 2x_1 + x_2$

$S_2 = x_1 - 4x_2$

$$Z = 4x_1 - x_1^2 + 2x_1x_2 - 2x_2^2$$

$$\frac{\partial z}{\partial x_1}\bigg|_{at\,(0,0)} = 4 - 2x_1 + 2x_2 = 4$$

$$\frac{\partial z}{\partial x_2}\bigg|_{at\,(0,0)} = 2x_1 + 4x_2 = 0.$$

$$X_B = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad X_{NB} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1 = 3 - \frac{1}{2}x_2 + \frac{1}{2}s_1.$

$s_2 = 3 - \frac{1}{2}x_2 + \frac{1}{2}s_1 - 4x_2$

$= 3 - \frac{9}{2}x_2 + \frac{1}{2}s_1$

$z = 4\left(3 - \frac{1}{2}x_2 + \frac{1}{2}s_1\right) - \left(3 - \frac{1}{2}x_2 + \frac{1}{2}s_1\right)^2$

$+ 2\left(3 - \frac{1}{2}x_2 + \frac{1}{2}s_1\right)x_2 - 2x_2^2$

$z = 9 + x_2 - s_1 + \frac{3}{2}x_2 s_1 - \frac{13}{4}x_2^2 - \frac{1}{4}s_2^2$

d. z.w x_2 and s_1

$\left.\frac{\partial z}{\partial x_2}\right|_{\substack{x_2=0 \\ s_1=0}} = 1 + \frac{3}{2}s_1 - \frac{13}{2}x_2 = 1.$

$\left.\frac{\partial z}{\partial s_1}\right|_{\substack{x_2=0 \\ s_1=0}} = -1 + \frac{3}{2}x_2 - \frac{1}{2}s_1 = -1$

Compute minimum ratio.

$\min\left\{\frac{3}{|-\frac{1}{2}|}, \frac{3}{(-9/2)}\right\} = \frac{6}{9}$

$u_1 = \frac{1}{2}\frac{\partial z}{\partial x_2} = \frac{1}{2} + \frac{3}{4}s_1 - \frac{13}{4}x_2$

$X_B = (x_1, x_2, s_2) \quad X_B = (s_1, u_1).$

expressing basic variables and z interms of X_{NB}

$$x_2 = \frac{2}{13} + \frac{3}{13}s_1 - \frac{4}{13}u_1$$

$$x_1 = \frac{38}{13} - \frac{3}{26}s_1 + \frac{2}{13}u_1$$

$$s_2 = \frac{30}{13} - \frac{27}{26}s_1 + \frac{18}{13}u_1$$

$$z = 9 + \frac{1}{13}\left[2+s_1-4u_1\right] - s_1 + \frac{3}{26}(2+3s_1-4u_1)s_1$$

$$-\frac{1}{52}(2+3s_1-u_1)^2 - \frac{1}{4}s_1^2$$

Again,

$$\left.\frac{\partial z}{\partial s_1}\right|_{\substack{s_1=0\\ u_1=0}} = \frac{3}{13} - 1 + \frac{3}{26}(2-4u_1) + \frac{18}{26}s$$

$$-\frac{6}{52}(2+3s_1-4u_1) - \frac{1}{2}s_1$$

$$= \frac{9}{13}$$

$$\left.\frac{\partial z}{\partial u_1}\right|_{\substack{s_1=0\\ u_1=0}} = -\frac{4}{13} - \frac{12}{36}s_1 + \frac{8}{52}\left[2+3s_1-4u_1\right]$$

$$\approx 0$$

since both $(\alpha, \beta) \leq 0$ the optimal value

of z is at $s_1 = 0$ $v_1 = 0$ in current

objective function

$$z^* = 9 + \frac{2}{13} - \frac{2}{52} = \frac{474}{52}$$

hence optimal Solution of NLPP.

is $x_1 = \frac{38}{13}$ $x_2 = 2/13$

$z = 9.115$.