Group Assignment - 3

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Question 1

Given : Consider $R(A_1, A_2, A_3, \ldots, A_n)$ to be a relation with functional dependencies defined as

$$A_{(i-1)i/2+1}A_{(i-1)i/2+2}\ldots A_{(i-1)i/2+i} \rightarrow A_{(i-1)i/2+i+1}\ldots A_nA_1\ldots A_i(i-1)/2$$

For $i > 3$ and till $\frac{(i-1)i}{2} + i = n$

1.1

The given condition $\frac{(i-1)i}{2} + i = n$ must be satisfied for some $i \in N$for the functional dependencies to be possible. Thus

$$n = \frac{i(i+1)}{2}$$

1.2

For some i,

$$A_{\frac{i(i-1)}{2}+1}A_{\frac{i(i-1)}{2}+2}\ldots A_{\frac{i(i-1)}{2}+i} \rightarrow A_{\frac{i(i-1)}{2}+i+1}A_{\frac{i(i-1)}{2}+i+2}\ldots A_nA_1A_2\ldots A_{\frac{i(i-1)}{2}}$$

This can also be written as

$$\begin{aligned} A_{\frac{i(i-1)}{2}+1}A_{\frac{i(i-1)}{2}+2}\ldots A_{\frac{i(i-1)}{2}+i} \rightarrow A_{\frac{i(i-1)}{2}+i+1}A_{\frac{i(i-1)}{2}+i+2}\ldots A_nA_1A_2\ldots A_{\frac{i(i-1)}{2}}A_{\frac{i(i-1)}{2}+1}A_{\frac{i(i-1)}{2}+2}\ldots A_{\frac{i(i-1)}{2}+i} \\ \implies A_{\frac{i(i-1)}{2}+1}A_{\frac{i(i-1)}{2}+2}\ldots A_{\frac{i(i-1)}{2}+i} \rightarrow A_i \forall i \\ \implies A_{\frac{i(i-1)}{2}+1}A_{\frac{i(i-1)}{2}+2}\ldots A_{\frac{i(i-1)}{2}+i} \text{ is a key } \forall i \end{aligned}$$

Thus the number of keys equals i for which $\frac{(i-1)i}{2} = n$, that is

$$i = \frac{-1+\sqrt{1+8n}}{2}$$

1.3

1. No information is provided on the atomicity of the attributes, we assume all the attributes are atomic and thus the relation is in the First Normal Form
2. From the functional dependencies given we can conclude
    1. Every attribute is a prime attribute, that is each attribute A_x belongs to a key. This can clearly be seen as $\frac{i(i-1)}{2} + 1 \leq x \leq \frac{i(i-1)}{2} + i$ for some i.
    2. Also each attribute A_x belongs to exactly one key as each of the sets $[\frac{i(i-1)}{2} + 1, \frac{i(i-1)}{2} + i]$ are disjoint

    From the above to conclusion we can further conclude that any of the functional dependencies above no longer hold if one of the attributes is removed. Thus the relation R is in the Second Normal Form.
3. Since every attribute in R is a prime attribute, the relation is in Third Normal Form

4. For every $X \to Y$, X is a key. Thus the relation R is in Boyce-Codd Normal Form

1.4

The given set of functional dependencies can be broken down into

$$\begin{aligned} F \ &= \{A_1 \to A_2, A_1 \to A_3, \dots, A_1 \to A_n, \\ &A_2A_3 \to A_1, A_2A_3 \to A_4, \dots, A_2A_3 \to A_n, \\ &\dots, \\ &A_{\frac{i(i-1)}{2}+1} A_{\frac{i(i-1)}{2}+2} \dots A_{\frac{i(i-1)}{+}i} \to A_1, \dots, A_{\frac{i(i-1)}{2}+1} A_{\frac{i(i-1)}{2}+2} \dots A_{\frac{i(i-1)}{+}i} \to A_n\} \end{aligned}$$

This set consists of (n-1)i dependencies many of which are clearly redundant such as $A_1 \to A_2A_3 \ \& \ A_2A_3 \to A_4 \implies A_1 \to A_4$ which is already a part of the set.

Consider the set of functional dependencies G such that

$$\begin{aligned} G = \{&A_1 \to A_2, A_1 \to A_3, \dots, A_1 \to A_n \\ &A_2A_3 \to A_1, \\ &\dots, \\ &A_{\frac{i(i-1)}{2}+1} A_{\frac{i(i-1)}{2}+2} \dots A_{\frac{i(i-1)}{+}i} \to A_1\} \end{aligned}$$

This set of functional dependencies has $n + i - 2$ dependencies. Also $G = G^+ = F^+$ thus the number of dependencies in G cannot be reduced. Also for every $X \to Y$, X has the minimum number of attributes, thus G is the minimal cover of F.

Since $G \subseteq F$, All our conclusion from F still hold, thus G is in Boyce-Codd Normal Form.