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DS
Assignment 1

Date
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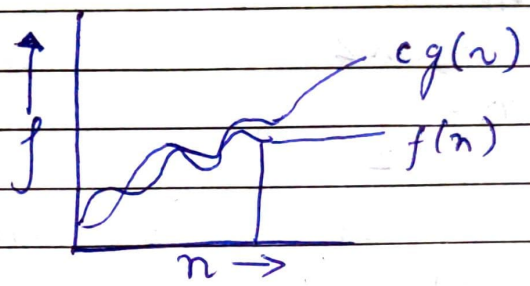
Design and Analysis of algorithm

Q.

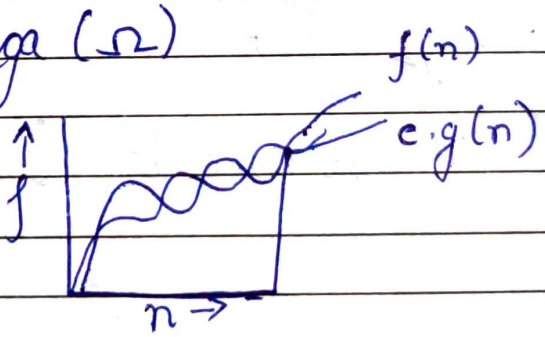
Ans 1 Asymptotic notations to analyse an algorithm running time identifying its behaviour as the input size for the algorithm increases. These notation are used to tell the complexity of an algorithm when input is very large type of asymptotic notation.

1. Big - O n



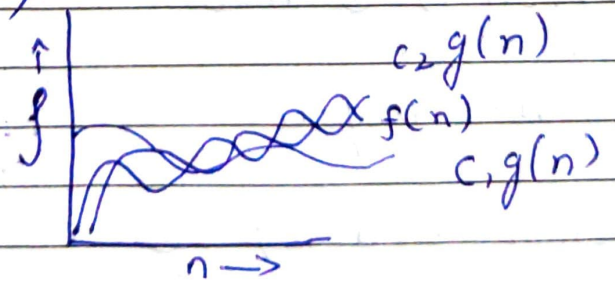
$f(n) = O(g(n))$
if and only if
$f(n) \leq cg(n)$
$\forall n \geq n_0$

2. Big omega (Ω)



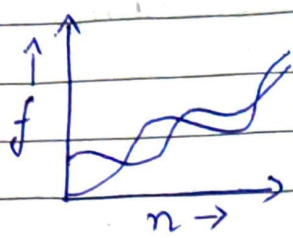
$f(n) = \Omega g(n)$
if and only if
$f(n) \geq cg(n)$
$\forall n > n_0$

3. Theta (θ)



$f(n) = \theta g(n)$
if and only if
$c \cdot g(n) \leq f(n) \leq c_2 g(n)$
$\forall n \geq \max(n_1, n_2)$

4. Small - $O_n(o)$



$f(n) = O(g(n))$
$f(n) < c \cdot g(n) \; \forall n >$

Q

Ans2 $i = 1, 2, 4, 8, \ldots, n$

$2^0, 2^1, 2^2, 2^3, \ldots 2^k$ - GP

$a = 1, r = 2$

$t_n = a r^{k-1}$

$= 1 \times 2^{k-1}$

$n = \frac{2^k}{2}$

$2^k = 2n \quad k = \log_2(2n);$

$K = \log_2(n) + \log_2(2)$

$= \log_2(n) + 1$

$T.C = O(\log_2(n) + 1) = [O(\log n)]$

Q3

Ans3 $T(n) = 3T(n-1) - ①$ $n > 0$

$T(1) = 1$

put $n = n-1$ in eq (1)

$T(n-1) = 3T(n-2) - ②$

put $T(n-1)$ in eq ①

$T(n) = 3(3T(n-2))$

$T(n) = 9T(n-2) - ③$

put $n = n-2$ in eq ①

$T(n-2) = 3T(n-3)$ — ④

put $T(n-2)$ in eq (3)

$T(n) = 9(3T(n-3))$

$T(n) = 27T(n-3)$

$T(n) = 3^k T(n-k)$ — ⑤

$T(1) = 1$

$n - k = 1$

$k = n - 1$ — ⑥

from 5 & 6

$T(n) = 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3} \times 1 = TC = O(3^n)$

Q4.

Ans

$T(n) = 2T(n-1) - 1$ — ①

$T(1) = 1$

put $n = n-1$ in eq ①.

$T(n-1) = 2T(n-2) - 1$

put $T(n-1)$ in eq ①

$T(n) = 2(2T(n-2) - 1) - 1$

$T(n) = 4T(n-2) - 2 - 1$ → ②

put $n = n-2$ in eq ①

$T(n-2) = 2T(n-3) - 1$

put $T(n-2)$ in eq ②

$T(n) = 4(2T(n-3) - 1) - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$ — ③

$T(n) = 2^k [T(n-k)] - 2^{k-1} - 2^{k-2} - 2^1 - 2^0$ — ④

$T(1) = 1$

$n - k = 1$

$k = n - 1$ — ⑥

from ④ & ⑤

$T(n) = 2^{n-1} [T(n-(n-1))] - 2^{n-2} - 2^{n-3} \cdots 2^0$

$= 2^{n-2} - 2^{n-1} - 2^{n-3} - 1$

$= \frac{1}{2} [2^n - [2^n - 1]]$

$= \frac{1}{2} \times 1 = \frac{1}{2}$ $[T.C = O(1)]$

Q.

Ans 5 $n = 1, 3, 6 \ldots - K \rightarrow A.P$

$T.C = \frac{K(K+1)}{2}$

$O(\frac{K^2 + K}{2})$

$O(K^2)$ $[T.C = O(n^2)]$

Q6.

Ans

```
void function(int n) {
    int i, count = 0;          -①
    for (i=1; i*i<=n; i++)
        count++;               -n
}
```

$1 + 1 + (n+1)^2 + n + n$

$2 + n^2 + 2n + 1 + 2n$

$n^2 + 4n + 3$

$O(n^2 + 4n + 3)$

$O(n^2)$ $\quad [T.C = O(n^2)]$

Q.7.

Ans

```
void function (int n) {
  int i, k, count = 0;
  for(i = n/2 ; i<=n; i++)          - O(n)
   for (j=1 ; j<=n ; j=j*2 )        - log(n)
   for(k=1; k<=n; k=k*2)
        count ++; }                 - log(n)
```

$T.C = \log(n) * \log(n)$

$= \log^2(n)$

$= O[\log^2(n)]$

Q8

Ans8

```
function (int n) {
  if (n==1) return ;  -> 1
  for(i=1 to n)        ]  n*n
    for(j=1 to n)      ]
      print ("*");  -> 1

  }
}
function (n-3) -> n*n^2
}
```

$1 + n^2 + 1 + n^3$

$n^3 + n^2 + 2 \quad O(n^3)$

Q9.

Ans9
```
for(i=1 to n)
    for(j=1; j<=n; j=j+1)
        print("*");
    y
y
```

$T.C = \log n\left(\frac{n+1}{2}\right)$

$= O\left(\frac{n+1}{2}\log n\right)$

$= O(n \log n)$

i	j	times
1	$1 \to n$	$\frac{n+1}{2}$
2	$1 \to n$	$n+1/2$
⋮		
n	$1 \to n$	$n+1/2$

Q10

Ans10

$n^k \leq ca^n$

$a^n + n^k \leq ca^n \to a^n$

$a^n + n^k \leq a^n(c-1)$

$\frac{a^n + n^k}{a^n} \leq (c-1)$

$c \geq 1 + \frac{n^k}{a^{n_0}} + 1$

$c \geq 2 + \frac{n_0^k}{a^n}$

$c \geq 2 + \frac{n_0}{1.5^n}$

$n_0 = 1$

$c \geq 2 + \frac{1}{1.5}$

$c \geq 3.0 + 1$

$c \geq 4$

Q11.

Ans Time Complexity = $O(n)$

The execution of different code lines here are:-

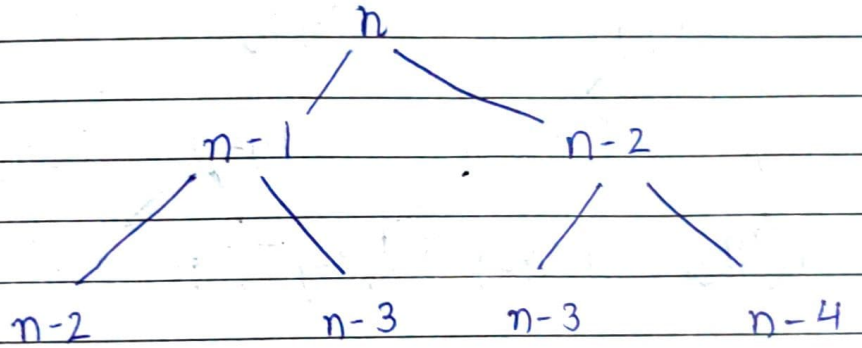
1. while $\Rightarrow (n-1)$
2. $i = i + j \Rightarrow (n)$

3) j++ => (n)

T.C = n + n + n - 1
= 3n - 1
T.C = O(3n - 1)
T.C = O(n)

Q12.
Ans The main working of fabonacci series is

$$f(n) = f(n-1) + f(n-2)$$



$T(n) = 1 + 2 + 4 + \cdots 2^n$

$a = 1, r = 2$

$$\frac{a(r-1)}{r-1} = \frac{1(2^{n+1} - 1)}{2-1} = 2^{n+1}$$

$$T(n) = O(2^{n+1}) = O(2^{r} * 2^{1}) = O(2^n)$$

Q13.
Ans O(n(log n))

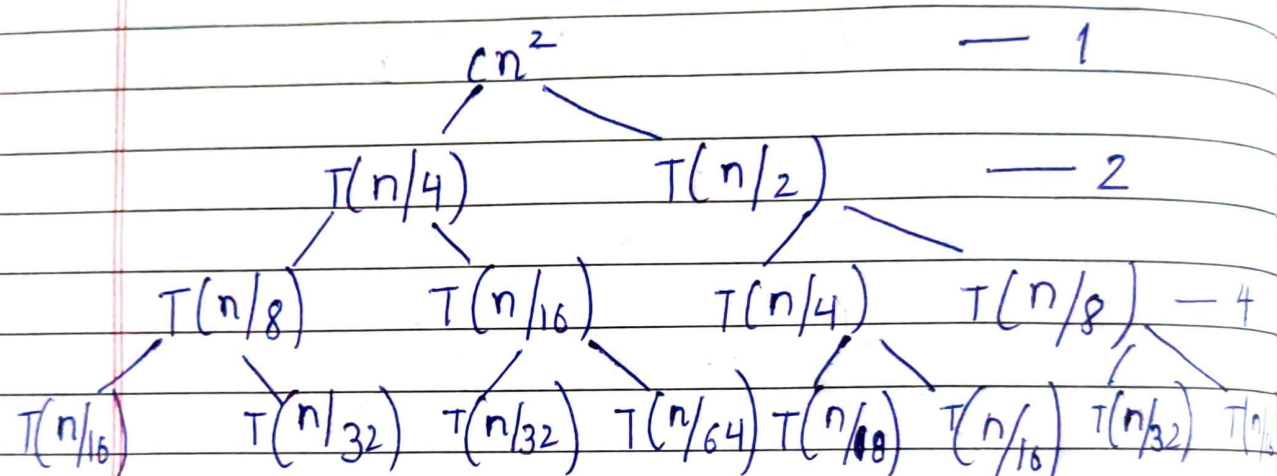
int n;

```
for( int i=0; i<n; i++ ){
    for( j=1; j<=n; j++ ){
        for( k=1; k<=n; k++ ){
            printf( "*" );
        }
    }
}
```

Q14

ans

$$T(n) = T(n/4) + T(n/2) + cn^2$$



— 8

$$T(n) = c\left(n^2 + 5\left(\frac{n^2}{16}\right) + 25\left(\frac{n^2}{256}\right) + \cdots\right)$$

$$\text{ration} = 5/16 \qquad \frac{n^2}{1 - 5/16} = O(n^2)$$

Q15

ans

```
int fun (int n){
    for (int i=1; i<=n; i++){
        for(int j=1; j<=n; j=j*i){
            O(1); }
```

i	j	times
1	1→n	n
2	1→n	n/2
⋮		
n		n/n

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \cdots \frac{n}{n}$$

$$= n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right)$$

$$T(n) = n \log(n)$$

Q16 Ans16 $i = 2, 2^c, 2^{c^2}, 2^{c^3} \cdots 2^{c^{\log_c \log(n)}}$

The last term has to be $<= n$

$2^{c \log_c \{\log(n)\}} = 2^{\log n} = n$

There are in total $\log_c(\log(n))$ iterations each take a constant amount of time to run

$$T.C = O(\log(\log n))$$

Q18

Ans 18

a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n = \log(n!)$
$< n^2 < 2^n < 2^2 < 4^n < n!$

b) $1 < \log\log(n) < \sqrt{\log(n)} < \log n < n < 4n < 2(2^n)$
$< \log(2n) < 2\log(n) < n < n\log n = \log(n!) < n^n$

c) $96 < \log_2(n) = \log_8(n) < n\log_6(n) = n\log_2(n) = \log$
$(n!) < 5n < 8n^2 < 7n^3 < 8^{2n}$

Q
Ans 19

```
Int fun (int arr [N], key){
  for (i = 0 to n-1){
      if (ARRAY [i] = key){
        return i; }
      }
  return -1;
}
```

Q20

Ans a. Iterative Insertion Sort

```
void InsertionSort (int arr[], int n){
    int i, temp, j;
  for (int i = 1; i <= n-1; i++)
     temp = arr[i];
     j = i-1;
    while (j >= 0 && arr[j] > temp){
      arr[j+1] = arr[j];
       j = j-1; }
    arr[j+1] = temp;
```

Recursive Insertion Sort

```
void Insertion sort (int arr[], int n){
    if(n<2)
        return n;
    Insertion Sort (arr, n-1);
    last = arr[n-1], j = n-2;
    while [j>=0 && arr [j]>temp) {
            arr [j+1] = arr[j];
            j= j-1; }
    arr [j+1] = last;
}
```

Q.21

Algorithm	Best case	Avg. case	Worst case
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n^2)$	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quick	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Heap	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Q.22
Ans

Algorithm	Inplace	Stable	Online
Bubble	✓	✓	x
Selection	✓	x	x
Insertion	✓	✓	✓
Merge	x	✓	x
Quick	x	x	x
Heap	✓	x	x

Q23. Iterative Binary Search

```
int BinarySearch (int arr[], int l, int r, int x)
    while (l <= r) {
        int m = (l + r) / 2;
        if (arr[m] = x;
            return m;
        else if (arr[m] < n)
            l = m + 1
        else u = m - 1; }
        return -1;
    }
```

Recursive Binary Search

```
int BinarySearch (int arr[], int l, int r, int
    if (l > r)
    return -1;
    int m = (l + r) / 2;
    if (arr[m] = n;
        return m;
    else if (arr[m] < n)
    return BinarySearch (arr, m+1, r, n)
      else
    return BinarySearch (arr, l, m-1, n);
    }
```

Time Complexity	Space Complexity
Linear (Recursive) → O(n)	O(1)
Binary (Recursive) → O(n)	O(log n)
Linear (Iterative) → O(1)	O(1)
Binary (Iterative) → O(1)	O(1)

Q24

Ans Recurrence relation for binary search

$$T(n) = T(n/2) + 1.$$