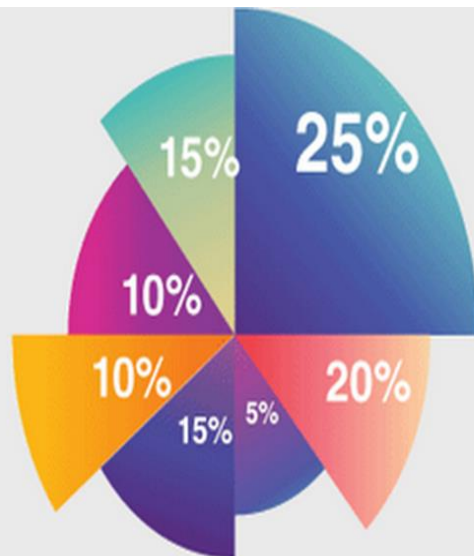


# RATIOS AND PROPORTION



**Ratio and Proportion** are explained majorly based on fractions. When a fraction is represented in the form of  $a:b$ , then it is a ratio whereas a proportion states that two ratios are equal. Here,  $a$  and  $b$  are any two integers. The ratio and proportion are the two important concepts, and it is the foundation to understand the various concepts in mathematics as well as in science.

In our daily life, we use the concept of ratio and proportion such as in business while dealing with money or while cooking any dish, etc. Sometimes, students get confused with the concept of ratio and proportion. In this article, the students get a clear vision of these two concepts with more solved examples and problems.

For example,  $\frac{4}{5}$  is a ratio and the proportion statement is  $\frac{20}{25} = \frac{4}{5}$ . If we solve this proportional statement, we get:

$$\frac{20}{25} = \frac{4}{5}$$

$$20 \times 5 = 25 \times 4$$

$$100 = 100$$

Therefore, the ratio defines the relationship between two quantities such as  $a:b$ , where  $b$  is not equal to 0. Example: The ratio of 2 to 4 is represented as  $2:4 = 1:2$ . And the statement is said to be in proportion here.

## Ratio and Proportion in Maths

The definition of ratio and proportion is described here in this section. Both concepts are an important part of Mathematics. In real life also, you may find a lot of examples such as the rate of speed (distance/time) or price (rupees/meter) of a material, etc, where the concept of the ratio is highlighted.

Proportion is an equation that defines that the two given ratios are equivalent to each other. For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as  $100\text{km/hr} = 500\text{km}/5\text{hrs}$ .

## Ratio Meaning

In certain situations, the comparison of two quantities by the method of division is very efficient. We can say that the comparison or simplified form of two quantities of the same kind is referred to as a ratio. This relation gives us how many times one quantity is equal to the other quantity. In simple words, the ratio is the number that can be used to express one quantity as a fraction of the other ones.

The two numbers in a ratio can only be compared when they have the same unit. We make use of ratios to compare two things. **The sign used to denote a ratio is ‘:’.**

A ratio can be written as a fraction, say  $\frac{2}{5}$ . We happen to see various comparisons or say ratios in our daily life.

Hence, the ratio can be represented in three different forms, such as:

- a to b
- a : b
- $\frac{a}{b}$

**Key Points to Remember:**

- The ratio should exist between the quantities of the same kind
- While comparing two things, the units should be similar
- There should be significant order of terms
- The comparison of two ratios can be performed, if the ratios are equivalent like the fractions

## Definition of Proportion

Proportion is an equation that defines that the two given ratios are equivalent to each other. In other words, the proportion states the equality of the two fractions or the ratios. In proportion, if two sets of given numbers are increasing or decreasing in the same ratio, then the ratios are said to be directly proportional to each other.

For example, the time taken by train to cover 100km per hour is equal to the time taken by it to cover the distance of 500km for 5 hours. Such as  $100\text{km/hr} = 500\text{km}/5\text{hrs}$ .

Ratio and proportions are said to be faces of the same coin. When two ratios are equal in value, then they are said to be in **proportion**. In simple words, it compares two ratios. **Proportions are denoted by the symbol '::' or '='.**

The proportion can be classified into the following categories, such as:

- Direct Proportion
- Inverse Proportion
- Continued Proportion

Now, let us discuss all these methods in brief:

### Direct Proportion

The direct proportion describes the relationship between two quantities, in which the increases in one quantity, there is an increase in the other quantity also. Similarly, if one quantity decreases, the other quantity also decreases. Hence, if “a” and “b” are two quantities, then the direct proportion is written as  $a \propto b$ .

### Inverse Proportion

The inverse proportion describes the relationship between two quantities in which an increase in one quantity leads to a decrease in the other quantity. Similarly, if there is a decrease in one quantity, there is an increase in the other quantity. Therefore, the inverse proportion of two quantities, say “a” and “b” is represented by  $a \propto (1/b)$ .

## Continued Proportion

Consider two ratios to be **a: b** and **c: d**.

Then in order to find the continued proportion for the two given ratio terms, we convert the means to a single term/number. This would, in general, be the LCM of means.

For the given ratio, the LCM of b & c will be bc.

Thus, multiplying the first ratio by c and the second ratio by b, we have

First ratio- ca : bc

Second ratio- bc : bd

Thus, the continued proportion can be written in the form of **ca: bc: bd**

## Ratio and Proportion Formula

Now, let us learn the Maths ratio and proportion formulas here.

### Ratio Formula

Assume that, we have two quantities (or two numbers or two entities) and we have to find the ratio of these two, then the formula for ratio is defined as;

$$\mathbf{a: b \Rightarrow a/b}$$

where a and b could be any two quantities.

Here, “a” is called the first term or **antecedent**, and “b” is called the second term or **consequent**.

Example: In ratio 4:9, is represented by 4/9, where 4 is antecedent and 9 is consequent.

If we multiply and divide each term of ratio by the same number (non-zero), it doesn't affect the ratio.

Example:  $4:9 = 8:18 = 12:27$

## Proportion Formula

Now, let us assume that, in proportion, the two ratios are **a:b** & **c:d**. The two terms '**b**' and '**c**' are called '**means or mean term**,' whereas the terms '**a**' and '**d**' are known as '**extremes or extreme terms**.'

$$a/b = c/d \text{ or } a : b :: c : d$$

**Example:** Let us consider one more example of a number of students in a classroom. Our first ratio of the number of girls to boys is 3:5 and that of the other is 4:8, then the proportion can be written as:

$$3 : 5 :: 4 : 8 \text{ or } 3/5 = 4/8$$

Here, 3 & 8 are the extremes, while 5 & 4 are the means.

**Note:** The ratio value does not affect when the same non-zero number is multiplied or divided on each term.

## Important Properties of Proportion

The following are the important properties of proportion:

- Addendo – If  $a : b = c : d$ , then  $a + c : b + d$
- Subtrahendo – If  $a : b = c : d$ , then  $a - c : b - d$
- Dividendo – If  $a : b = c : d$ , then  $a - b : b = c - d : d$
- Componendo – If  $a : b = c : d$ , then  $a + b : b = c + d : d$

- Alternendo – If  $a : b = c : d$ , then  $a : c = b : d$
- Invertendo – If  $a : b = c : d$ , then  $b : a = d : c$
- Componendo and dividendo – If  $a : b = c : d$ ,

then  $a + b : a - b = c + d : c - d$

## Difference Between Ratio and Proportion

To understand the concept of ratio and proportion, go through the difference between ratio and proportion given here.

Sl. No	Ratio	Proportion
1	The ratio is used to compare the size of two things with the same unit	The proportion is used to express the relation of two ratios
2	It is expressed using a colon (:), slash (/)	It is expressed using the double colon (::) or equal to the symbol (=)
3	It is an expression	It is an equation
4	Keyword to identify ratio in a problem is “to every”	Keyword to identify proportion in a problem is “out of”

## Fourth, Third and Mean Proportional

If  $a : b = c : d$ , then:

- $d$  is called the fourth proportional to  $a, b, c$ .
- $c$  is called the third proportion to  $a$  and  $b$ .
- Mean proportional between  $a$  and  $b$  is  $\sqrt{ab}$ .

## Comparison of Ratios

If  $(a : b) > (c : d) = (a / b > c / d)$

The compounded ratio of the ratios:  $(a : b), (c : d), (e : f)$  is  $(ace : bdf)$ .

## Duplicate Ratios

If  $a:b$  is a ratio, then:

- $a^2 : b^2$  is a duplicate ratio
- $\sqrt{a} : \sqrt{b}$  is the sub-duplicate ratio
- $a^3 : b^3$  is a triplicate ratio

## Ratio and Proportion Tricks

Let us learn here some rules and tricks to solve problems based on ratio and proportion topics.

- If  $u/v = x/y$ , then  $uy = vx$
- If  $u/v = x/y$ , then  $u/x = v/y$
- If  $u/v = x/y$ , then  $v/u = y/x$
- If  $u/v = x/y$ , then  $(u + v)/v = (x + y)/y$
- If  $u/v = x/y$ , then  $(u-v)/v = (x-y)/y$



- If  $u/v = x/y$ , then  $(u+v)/(u-v) = (x+y)/(x-y)$ , which is known as componendo - Dividendo Rule
- If  $a/(b+c) = b/(c+a) = c/(a+b)$  and  $a+b+c \neq 0$ , then  $a=b=c$

## Ratio and Proportion Summary

- Ratio defines the relationship between the quantities of two or more objects. It is used to compare the quantities of the same kind.
- If two or more ratios are equal, then it is said to be in proportion.
- The proportion can be represented in two different ways. Either it can be represented using an equal sign or by using a colon symbol.(i.e)  $a:b = c:d$  or  $a:b :: c:d$
- If we multiply or divide each term of the ratio by the same number, it does not affect the ratio.
- For any three quantities, the quantities are said to be in continued proportion, if the ratio between the first and second quantity is equal to the ratio between the second and third quantity.
- For any four quantities, they are said to be in continued proportion, if the ratio between the first and second quantities is equal to the ratio between the third and fourth quantities