



Clocks

- 1) The circumference of the circular dial of a clock is divided into 60 equal divisions or parts. These equal divisions are known as minute spaces or minute divisions.
- 2) A clock has two hands, i.e. the hour hand and the minute hand. The hands move around the circular dial. The hour hand is smaller and slower than the minute hand. The hour hand shows time in hours and the minute hand shows time in minutes.
- 3) In an hour, the hour hand moves forward one clock number and covers 5-minute spaces while the minute hand moves forward twelve clock numbers and covers 60-minute spaces. Thus, in one hour, the minute hand gains 55-minute spaces over the hour hand.
- 4) The hour hand covers the circumference of the clock in twelve hours, so the angle traced by hour hand in twelve hours is 360° . So, the speed of the hour hand is 0.5° per minute.

5) The minute hand covers the circumference of the clock in one hour, so the angle traced by minute hand in one hour (60 minutes) is 360° . So, the speed of minute hand is 6° per minute.

6) In every hour, the hour hand and minute hand coincide once, i.e. if both the hands start from the same position, they will coincide after $65\frac{5}{11}$ minutes. In this position, they are 0° apart.

Notes:

a) The hands of a clock coincide only once between 11 O' clock and 1 O' clock, so they coincide 11 times in 12 hours and 22 times in 24 hours or one day.

b) The clock is too fast if the hands of the clock coincide in less than $65\frac{5}{11}$ minutes.

c) The clock is too slow if the hands of the clock take more than $65\frac{5}{11}$ minutes to coincide.

7) In every hour, the hands of the clock are straight and point in opposite directions once. In this position, they are thirty-minute divisions apart.

Notes:

a) In 12 hours, the hands are straight and in opposite direction 11 times, i.e. between 5 and 7, they will be straight and in opposite direction only once at 6 O' clock.

b) In 12 hours, the hands are in straight line 22 times including both the positions, i.e., when they coincide and when they are in opposite directions.

8) In every hour, the hands are twice at the right angle and they are 15-minute divisions apart in this position.

9) In every minute, the minute hand moves through $6^\circ \left(\frac{360^\circ}{60} \right)$ and the hour hand moves through $0.5^\circ \left(\frac{30^\circ}{60} \right)$. So the minute hand is 12 times as fast as the hour hand and gains 5.5° over the hour hand in one minute.

10) If a clock indicates 6:20, when the correct time is 6:00, it is said to be 20 minutes too fast. And, if it indicates 5:40, when the correct time is 6:00, it is said to be 20 minutes too slow.

Some quicker methods:

1) Hands of a clock will coincide between H and (H+1) O' clock at $\left(\frac{60H}{11} \right)$ minutes past H O' clock.

2) The angle between the hands of a clock will be 90° between H and (H+1) O'clock at $(5H \pm 15) \frac{12}{11}$ minutes past H O' clock.

3) The hands of a clock will be in straight line and point in opposite directions between H and (H+1) O' clock at $(5H - 30) \frac{12}{11}$ minutes past H, when $H > 6$ and $(5H+30) \frac{12}{11}$ minutes past H, when $H < 6$.

4) When the minute hand is behind the hour hand, the angle between the hands of a clock at M minutes past H O' clock is given by;

$$= 30 \left(H - \frac{M}{5} \right) + \frac{M}{2} \text{ degrees.}$$

5) When the minute hand is ahead of the hour hand, the angle between the hands of a clock at M minutes past H O' clock is given by;

$$= 30 \left(\frac{M}{5} - H \right) - \frac{M}{2} \text{ degrees.}$$

6) Angle between the hands of a clock at X hours Y minutes:

To find the angle, consider time from 12 O' clock when the hands of a clock are together.

Now, total time from 12 O' clock to X hour Y minutes = $(60X + Y)$ minutes

Angle traced by hour hand in $(60X + Y)$ minutes = $\frac{(60X+Y)}{2}$ degrees

Angle moved by minute hand is $6Y$ (from 12 O' clock position)

\therefore Angel between the hands = $6Y - \frac{(60X+Y)}{2}$

$$= \frac{11Y - 60X}{2}$$