



- **Compound Interest:** The addition of interest to the principal sum each year or some fixed time period is known as compounding. So, the compound interest is the interest on interest. The interest of each year or some fixed time period is added to the principal sum and the new amount becomes the principal for the next year and interest is calculated on the increased amount for the next year.
- In case of compound interest, when interest is compounded annually, the amount is given by:
 - Amount (A) = $P \left(1 + \frac{r}{100} \right)^t$
 - Where, P= Principal
- r = Rate of interest
- t = time/number of years
- And, Compound Interest = Amount (A) - Principal (P)
- Compound Interest (C.I) = $P \left(1 + \frac{r}{100} \right)^t - P$

- $= P \left[\left(1 + \frac{r}{100} \right)^{t-1} \right]$

- And, Rate of interest (r) = $\left[\left(\frac{A}{P} \right)^{1/t-1} \right] \% \text{ p.a.}$

- Compound interest can be compounded half-yearly and quarterly, etc. Accordingly, the formula varies;

- When interest is compounded half-yearly:

- Amount (A) = $P \left(1 + \frac{r/2}{100} \right)^{2t}$

- Compound Interest (C.I.) = $P \left[\left(1 + \frac{r/2}{100} \right)^{2t} - 1 \right]$

- And, Rate of interest (r) = $2 * 100 \left[\left(\frac{A}{P} \right)^{\frac{1}{t} * 2} - 1 \right] \% \text{ p.a.}$

- When interest is compounded quarterly:

- Amount (A) = $P \left(1 + \frac{r/4}{100} \right)^{4t}$

- Compound Interest (C.I.) = $P \left[\left(1 + \frac{r/4}{100} \right)^{4t} - 1 \right]$

- And, Rate of interest (r) = $4 * 100 \left[\left(\frac{A}{P} \right)^{\frac{1}{t} * 4} - 1 \right] \% \text{ p.a.}$

- When interest is compounded annually but time is in fraction i.e. $3\frac{2}{3}$ years, then;

- Amount = $P \left(1 + \frac{r}{100} \right)^3 * \left(1 + \frac{(\frac{2}{3})r}{100} \right)$

- So, in general, if the interest is compounded n times a year;

- Amount (A) = $P \left(1 + \frac{r/n}{100} \right)^{n*t}$

- Compound interest (CI): $P \left[\left(1 + \frac{r/n}{100} \right)^{n \times t} - 1 \right]$
- And, Rate of interest (r) = $n \times 100 \left[\left(\frac{A}{P} \right)^{\frac{1}{t}} - 1 \right]$ % p.a.
- When there are different rates of interest for different years e.g., r1%, r2%, r3% for 1st, 2nd and 3rd year respectively, then;
- Amount = $P \left(1 + \frac{r1}{100} \right) \left(1 + \frac{r2}{100} \right) \left(1 + \frac{r3}{100} \right)$

Some quicker methods:

- If a sum becomes P times in t years, the rate of compound interest r is = $100 [(P)^{1/t} - 1]$
- If C.I. is given, we can find the S.I. by the formula;
- Simple Interest = $\frac{rt}{100[(1 + \frac{r}{100})^t - 1]}$ * Compound interest
- If C.I. and S.I. are given in the question, we can find the rate of interest by the formula;
- Rate of interest = $2 * \frac{\text{Difference in C.I. and S.I.}}{\text{S.I.}} * 100$
- If the difference between the C.I. and S.I. on a certain sum of money for 2 years at r% is Rs. X, the sum is given by;
- Sum = $X \left(\frac{100}{r} \right)^2$
- If the difference between C.I. and S.I. on a certain sum for 3 years at r% is Rs. X, the sum is given by;
- Sum = $\frac{X * (100)^3}{r^2 (300 + r)}$
- If a certain sum of money grows to Rs. X in n years and Rs. Y in (n+1) years, the rate of interest is given by;

- Rate of interest: $\frac{(Y-X) * 100}{X}$
- If a sum of money X becomes Y in t_1 years at compound rate of interest, after t_2 years it will become;

$$\frac{(Y)^{t_2/t_1}}{(X)^{t_2/t_1} - 1}$$
- Rs. $\frac{(Y)^{t_2/t_1}}{(X)^{t_2/t_1} - 1}$
- If a loan of Rs. X at r% rate of interest C.I. is to be paid back in n equal yearly instalments; the value of each instalment is given by;

$$= \frac{P}{\left(\frac{100}{100+r}\right) + \left(\frac{100}{100+r}\right)^2 + \dots + \left(\frac{100}{100+r}\right)^n}$$