EE324 Problem Sheet 10

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Question 1

The required matrices are chosen to be as follows:
A = $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 8\end{bmatrix}$ B = $\begin{bmatrix}2\\5\\10\end{bmatrix}$ C = $\begin{bmatrix}1 & 4 & 7\end{bmatrix}$ T = $\begin{bmatrix}1 & 2 & 1\\2 & 1 & 2\\3 & 4 & 5\end{bmatrix}$ D = 5 The various values were computed as shown below:

```
G(s) before the transformation :

                2    3
   -3 - 56s + 22s + 5s
   --------------------
                2   3
   -3 - 24s - 14s + s
Eigenvalues of A :

   15.555283
  -1.4194088
  -0.1358738
Poles of A :

   15.555283
  -1.4194088
  -0.1358738
G(s) after the transformation :

                2    3
   -3 - 56s + 22s + 5s
   --------------------
                2   3
   -3 - 24s - 14s + s
```

G(s) is indeed the same before and after the given transformations, and the eigenvalues are indeed the poles of the transfer function.
The proper and improper functions were chosen to be:

$$G_1(s) = \frac{s^2 + 3s + 2}{s^2 + 9s + 20} \quad G_2(s) = \frac{s + 1}{s^2 + 9s + 20}$$

The parameters for both were obtained as shown below:

```
Proper function
A
  -6.24  -0.32
   8.68  -2.76
B
  -3.9191836
   2.9393877
C
   1.5309311   0.
D
   1.
Strictly Proper function
A
  -8.7058824  -2.8235294
   6.1764706  -0.2941176
B
  -1.3719887
   0.3429972
C
  -0.728869   0.
D
   0.
```

D is 0 in case we have a strictly proper system, which means there isn't a direct component of the input signal in the output, which is the case in case D has degree of numerator = denominator.

Question 2

$$G_1(s) = \frac{s+3}{s^2+5s+4} \quad G_2(s) = \frac{s+1}{s^2+5s+4}$$

For part 2, a small offset from 1 was given to compute the approximate 2 x 2 matrix as this prevents pole zero cancellation but at the same time doesn't affect the actual matrices much. Hence, we use s+1.001 in the numenerator

```
First function
A
  -1.5384615   0.3076923
   4.3076923  -3.4615385
B
  -1.1094004
   1.6641006
C
  -0.9013878   0.
D
   0.
```

Figure 1: Part (a)

```
Second function
A
  -3.9983998   0.0011991
   4.0011991  -1.0016002
B
  -1.2649742
   0.6331196
C
  -0.79053   0.
D
   0.
```

Figure 2: Part (b)

Question 3

The required matrices are chosen to be as follows:

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 7 \end{bmatrix}$ The results obtained are as follows:

```
G(s) :

                  2
  -32 + 22s + 5s
  ---------------
                2
    6 - 7s + s
Eigenvalues of A :

  1.
  4.
  6.
Poles of A :

  6.
  1.
```

A is a diagonal matrix, so therefore

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s-a_1} & 0..... & \\ 0 & \frac{1}{s-a_2} & ..... \\ ... & ... & ... \end{bmatrix}$$

If $c_x = 0$ or $b_x = 0$, then the corresponding term has a weight of 0 in the final term hence that pole doesn't appear. The second element of C was chosen to be 0 hence the second pole i.e. -4 doesn't appear in the final result. Similar results were obtained on changing other values in C and B to 0.

Question 4

The matrices are as follows:

$$\text{A} = \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & a_3 & a_4 \\ 0 & 0 & a_5 \end{bmatrix} \ \text{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \ \text{C} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}$$

G(s) is given by

$$G(s) = C(sI - A)^{-1}B$$

$$G(s) = G_1(s) + G_2(s)$$

$$G_1(s) = \frac{c_1(b_1(s-a_3)(s-a_5) + b_2a_2(s-a_5) + a_2a_4b_3)}{(s-a_1)(s-a_2)(s-a_3)}$$

$$G_2(s) = \frac{c_2(b_2(s-a_1)(s-a_5) + b_3a_4(s-a_1)) + c_3b_3(s-a3)(s-a_1)}{(s-a_1)(s-a_2)(s-a_3)}$$

- $a_1 = a_3$ **:** cancellation when $b_2(a_3 - a_5) + b_3a_4$ or $a_2 = 0$
- $a_1 = a_5$ **:** cancellation when $a_2 = 0$ or $a_4 = 0$
- $a_3 = a_5$ **:** cancellation when $c_2(a_3 - a_1) + c_1a_2 = 0$ or $a_4 = 0$

Examples for each case:

$$A_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \ A_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix} \ A_3 = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

The results are obtained as follows:

```
G(s) for a1 = a3 :

              2
  -5 - s + 5s
  ------------
             2
  4 - 5s + s
Eigenvalues of A :

  1.
  1.
  4.
Poles of A :

  4.
  1.
```

```
G(s) for a1 = a5 :

               2
  -66 + s + 5s
  -------------
              2
   4 - 5s + s
Eigenvalues of A :

  1.
  4.
  1.
Poles of A :

  4.
  1.
```

```
G(s) for a3 = a5 :

                 2
  -3 - 14s + 5s
  --------------
               2
   4 - 5s + s
Eigenvalues of A :

  4.
  1.
  1.
Poles of A :

  4.
  1.
```

2 poles are present in the output for each hence we've effectively cancelled out a pole in each case.

Codes

Code for Question 1

```
clear;
s = %s;
A = [1, 2, 3; 4, 5, 6; 7, 8, 8];
B = [2; 5; 10];
C = [1, 4, 7];
T = [1, 2, 1; 2, 1, 2; 3, 4, 5];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) before the transformation :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);
A = inv(T) * A * T;
B = inv(T) * B;
C = C * T;
G = D + C * inv(s*I - A) * B;
printf("G(s) after the transformation : \n");
disp(G);

printf("Proper function\n")
G = (s^2 + 3*s + 2)/(s^2 + 9*s + 20);
[A, B, C, D] = abcd(G);
printf("A"); disp(A); printf("B"); disp(B); printf("C"); disp(C); printf("
    D"); disp(D);

printf("Strictly Proper function\n");
G = (s + 1)/(s^2 + 9*s + 20);
[A, B, C, D] = abcd(G);
printf("A"); disp(A); printf("B"); disp(B); printf("C"); disp(C); printf("
    D"); disp(D);
```

Code for Question 2

```
clear;
s= %s;
G = (s+3)/((s+1)*(s+4));
printf("First function \n")
[A, B, C, D] = abcd(G);
printf("A"); disp(A); printf("B"); disp(B); printf("C"); disp(C); printf("
    D"); disp(D);
```

```

G = (s+1.001)/(s^2 + 5*s + 4);
printf("Second function \n")
[A, B, C, D] = abcd(G);
printf("A"); disp(A); printf("B"); disp(B); printf("C"); disp(C); printf("
    D"); disp(D);
```

Code for Question 3

```
clear;
s = %s;
A = [1, 0, 0; 0, 4, 0; 0, 0, 6];
B = [1; 5; 8];
C = [1, 0, 7];
T = [1, 2, 1; 2, 1, 2; 3, 4, 5];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);
```

Code for Question 4

```
clear;
s = %s;
A = [1, 1, 0; 0, 1, 2; 0, 0, 4];
B = [1; 2; 3];
C = [1, 1, 7];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) for a1 = a3 :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);

A = [1, 0, 0; 0, 4, 2; 0, 0, 1];
B = [1; 4; 3];
```

```
C = [1, 1, 7];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) for a1 = a5 :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);

A = [1, 1, 0; 0, 1, 2; 0, 0, 4];
B = [1; 2; 3];
C = [1, 1, 7];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) for a3 = a5 :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);

A = [4, 1, 0; 0, 1, 2; 0, 0, 1];
B = [1; 2; 3];
C = [3, 1, 2];
I = eye(3, 3);
D = 5;
G = D + C * inv(s*I - A) * B;
printf("G(s) for a3 = a5 :\n");
disp(G);
eig_values = spec(A);
printf("Eigenvalues of A : \n");
disp(eig_values);
poles = roots(G.den);
printf("Poles of A : \n");
disp(poles);
```