EE324 Problem Sheet 7

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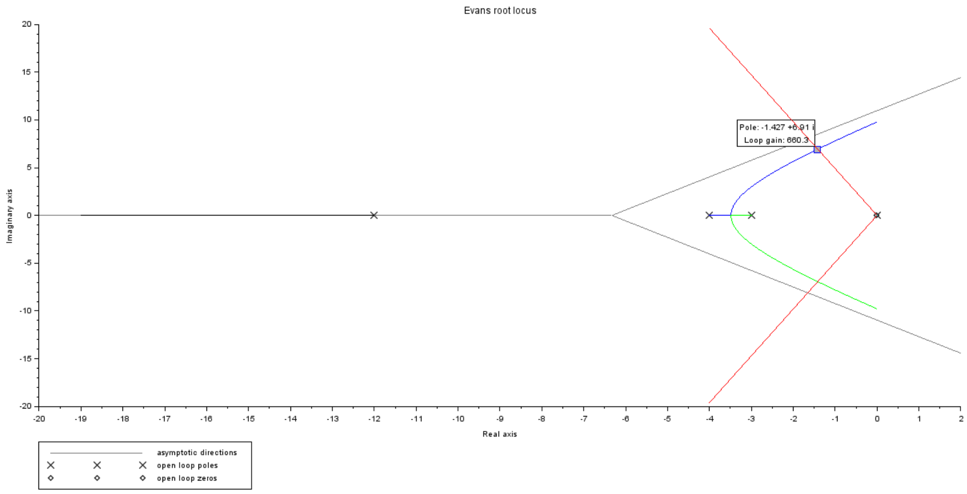
Contents

1 Question 1

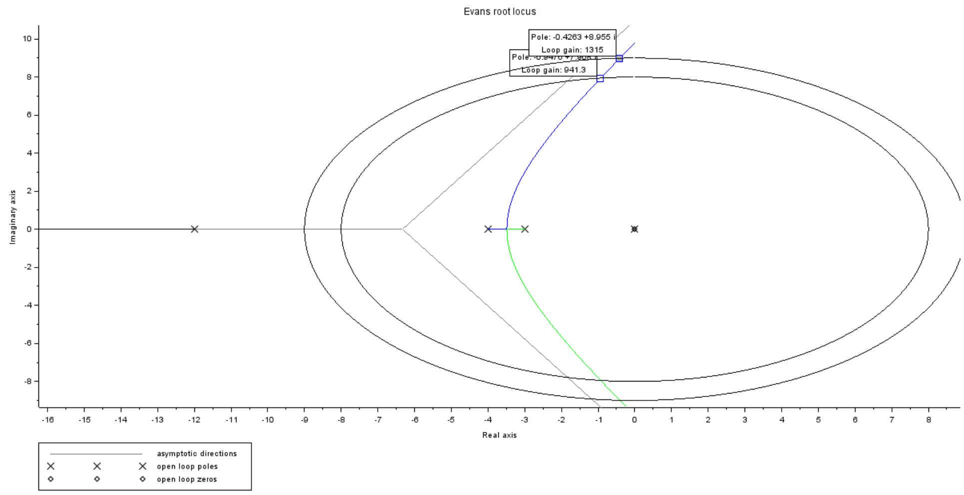
1.1 Part a

k = 660.3, for z = .01 and zeta = .2



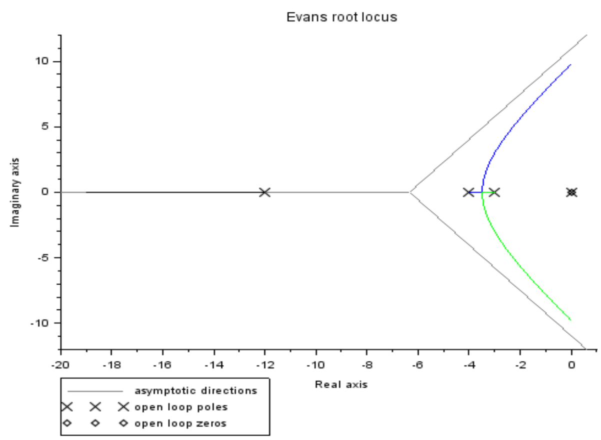
1.2 Part b

k = 941.3, for undamped freq of 8
k = 1315, for frq of 9

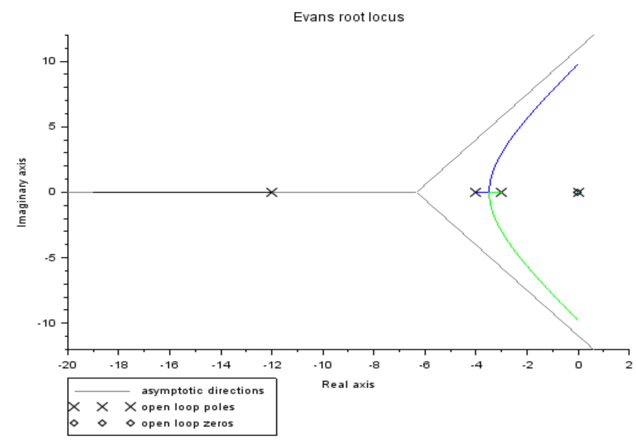


1.2.1 Part c

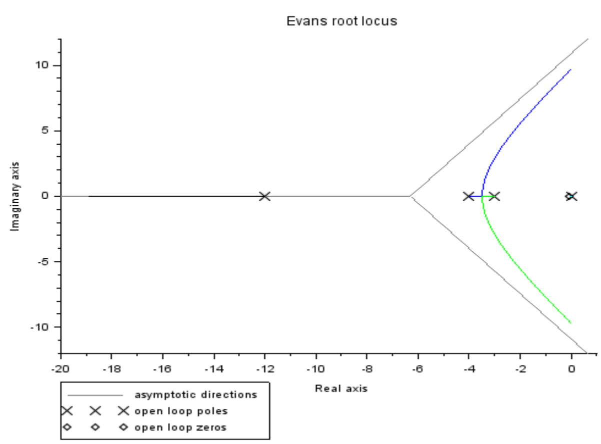
The RLocus plots for various values of z are



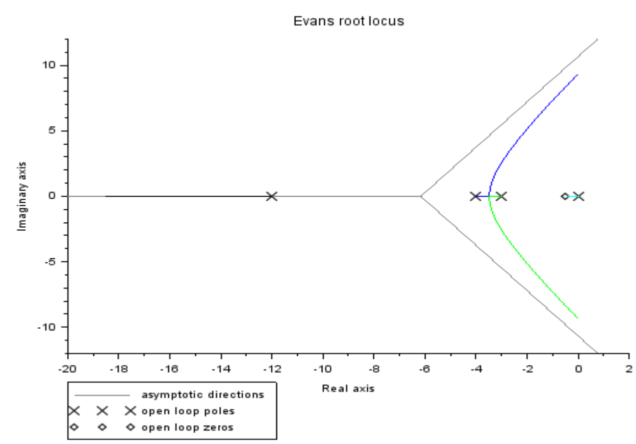
(a) $z = -.001$



(b) $z = -.01$



(a) $z = -.1$



(b) $z = -.5$

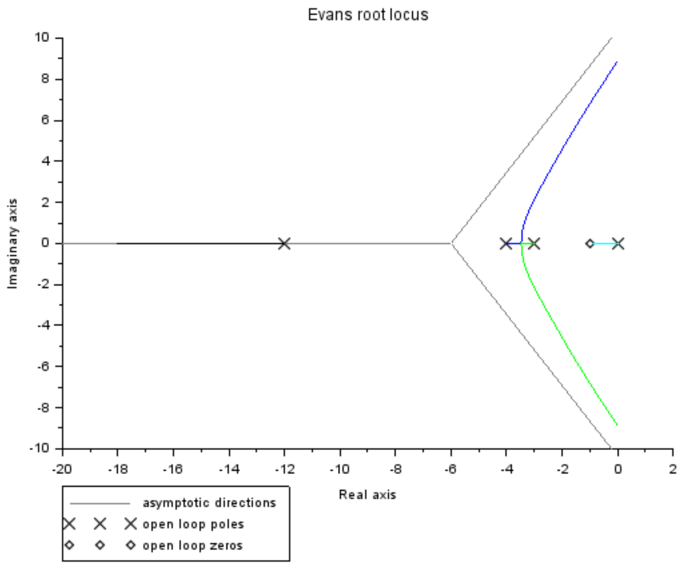


Figure 3: $z = -1$

2 Code

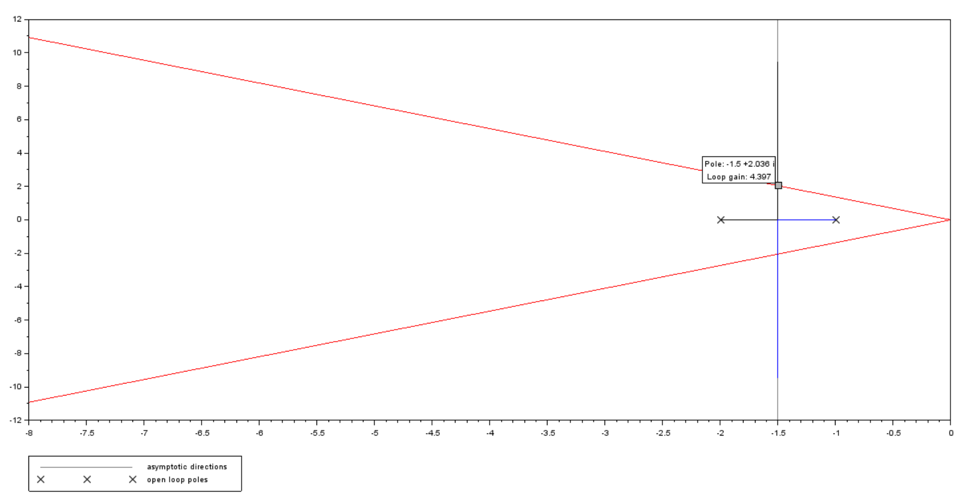
```
s = poly(0,'s')

G = 1/((s+4)*(s+3)*(s+12));

// part a
scf(0);
G1 = (s+.01)/s*G;
evans(G1, kpure(G1))

zeta = .2
m = sqrt(1-zeta^2)/zeta;
x = -4:.01:0;
os_line_1 = m.*x;
os_line_2 = -1*m.*x;
plot(x,os_line_1,'r');
plot(x,os_line_2,'r');                    // k = 660.3

// part b
scf(1);
evans(G1, kpure(G1));
w1 = 8;
w2 = 9;
t = 0:.01:2*%pi;

x1 = w1*sin(t);
```

```
y1 = w1*cos(t);                    // k = 941.3

x2 = w2*sin(t);
y2 = w2*cos(t);                  // k = 1315
plot2d(x1,y1);
plot2d(x2,y2);

// part c
z_range = [.001, .01, .1,.5, 1]
i = 2;
for z = z_range
    G1 = (s+z)/s*G;
    scf(i);
    evans(G1, kpure(G1));
    xs2png(i, 'q1_c_'+string(z*1000));
    i = i+1;
end
```

3 Question 2

3.1 Part a

k = 4.397, for os = .1

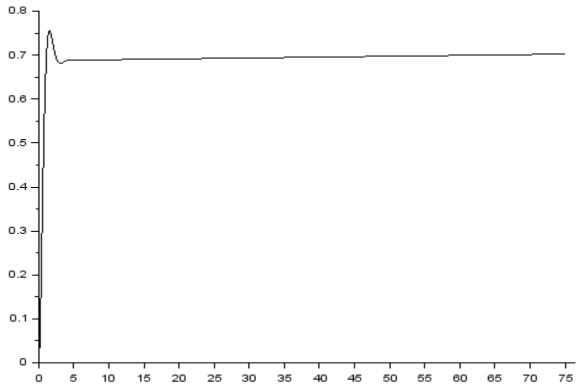


3.2 Part b

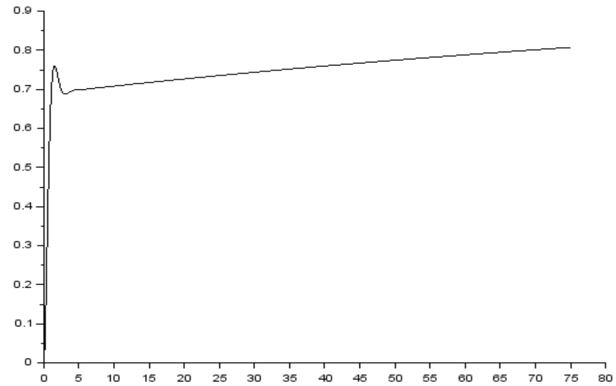
sse for that k is .313 and after using lag compensator it reduces to .022

3.3 Part c

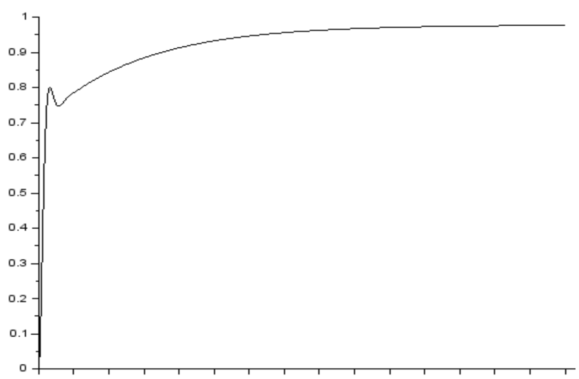
Step responses for different poles and zeros while maintaining ratio



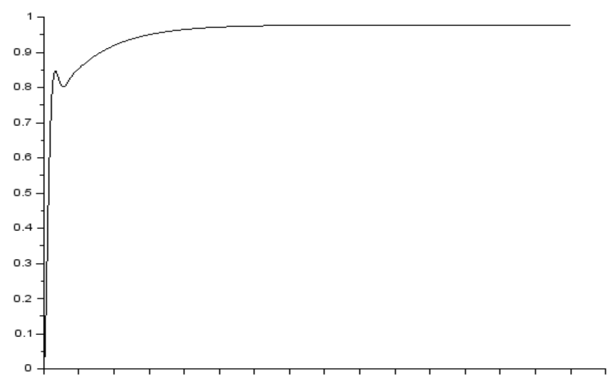
(a) $z = -.001$



(b) $z = -.01$



(a) $z = -.1$



(b) $z = -.2$

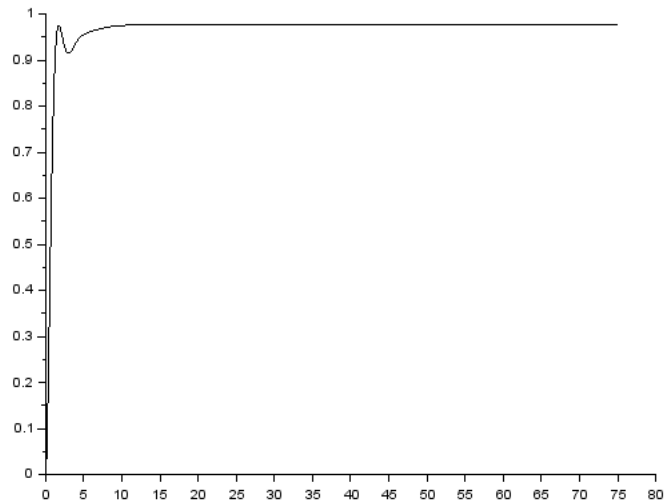


Figure 6: $z = -.5$

4 Code

```
s = poly(0,'s');

G = 1/(s^2+3*s+2);

// part a
scf(0);
os = .1;
m = 1/(-1*log(os)/(%pi));
x = -8:.01:0;
os_line_1 = m.*x;
os_line_2 = -1*m.*x;
plot(x,os_line_1,'r');
plot(x,os_line_2,'r');
evans(G, 90);                          // k =4.397

// part b
k = 4.397;
sse_original = 1/(1+k/2);               // .313
sse_with_lag_compensator = 1/(1+20*k/2);                    // .0222

// part c
zero_ = [.001, .01, .1, .2,.5];
i =1;
t = 0:.01:75
for z = zero_
```

```
    scf(i);
    p = z/20;
    G_open = G*(s+z)/(s+p)*k;
    G1 = G_open/(1+G_open);
    G1  = syslin('c', G1);
    plot2d(t, csim('step', t, G1));
    xs2png(i, 'q2_c_'+string(1000*z));
    i = i+1;
end
```

5 Question 3

Given the Open loop transfer function as:

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

We intend to design a PD Controller and a lead compensator such that the settling time is half as the time in $Q2$ $Part\ a$ and the $\%$ $overshoot$ is same as in $Q2$

5.1 Part b

In this Part, we design the PD controller
As settling time is inversely proportional to the absolute value of real part of the root, in $Q2$, all the roots after breakaway point lied on the $s = -1.5$ line.

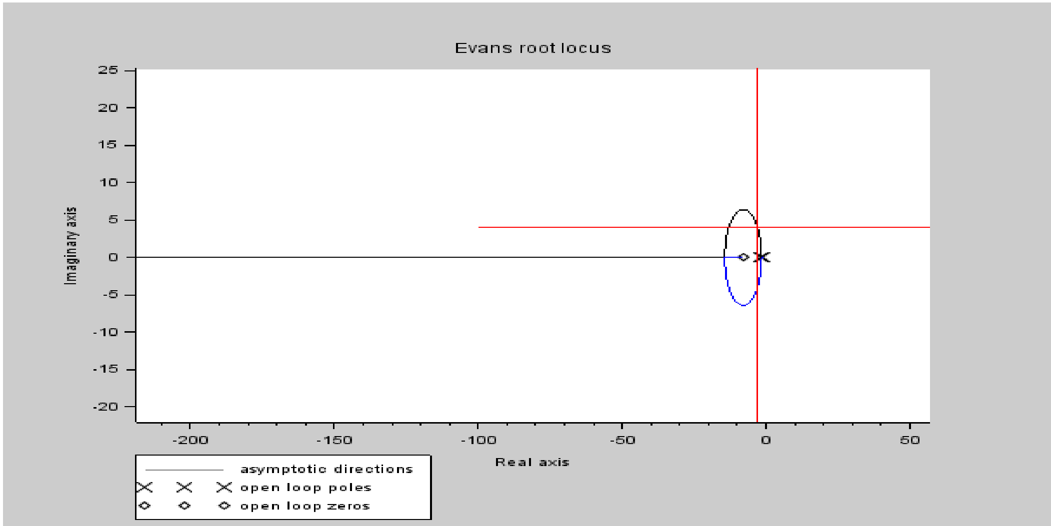
1. Hence, to halve the settling time, the root locus must intersect the $s = -3$ line.
2. To maintain the % overshoot, the root locus must also intersect the line which makes an angle θ with the negative real axis such that $cos(\theta) = 0.591$
3. Hence, the root locus must pass through the point of intersection of $s = -3$ and the line described above in $point$ 2. After calculating, the point is found to be $s = -3 + j4.0934$
4. We add a zero on the negative real axis so that the root locus passes through the point. After, calculation, the location of the zero to be added is found to be $s = -7.9187$

Now, the open loop transfer function becomes :

$$G_{PD}(s) = (s + 7.9187)\frac{1}{s^2 + 3s + 2}$$

The value of Gain k such that the above conditions are met is

$$k = 3$$



Hence, the PD controller required to satisfy above conditions is as follows :

$$k_d s + k_p = 3s + 23.756$$

The Root locus by cascading this PD Controller block is given by:

5.2 Part a

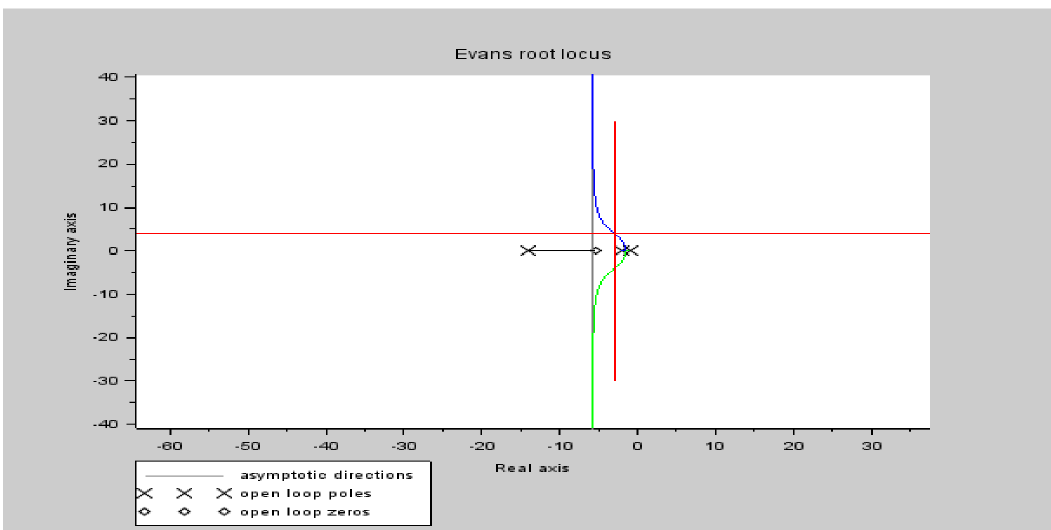
In this part, we first calculate the angle contributed by the extra zero in *Part b* to the pole $s = -3 + j4.0934$. This angles comes out to be 39.768^o
We need to add a pole zero pair on the negative real axis such that

$$\theta_z - \theta_p = 39.768^o$$

where θ_z and θ_p are the angle contributions from the respective pole and zero.
I choose $\theta_z = 60^o$ and hence $\theta_p = 20.232^o$
And subsequently, the location of the extra zero and pole to be added comes out to be $s = -5.3633$ and $s = -14.1063$ repsectively.
Hence, the lead compensator block is given by

$$k\left(\frac{s + 5.3633}{s + 14.1063}\right)$$

The Root locus by cascading this lead compensator block is given by:

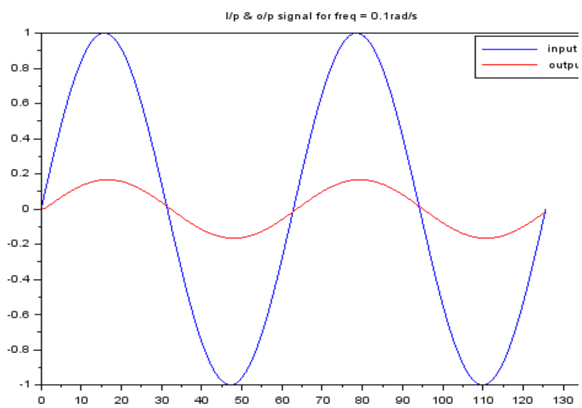


The value of the Gain k so that one of the closed loop pole is at $s = -3 + j4.0934$ is $k = 48.076$

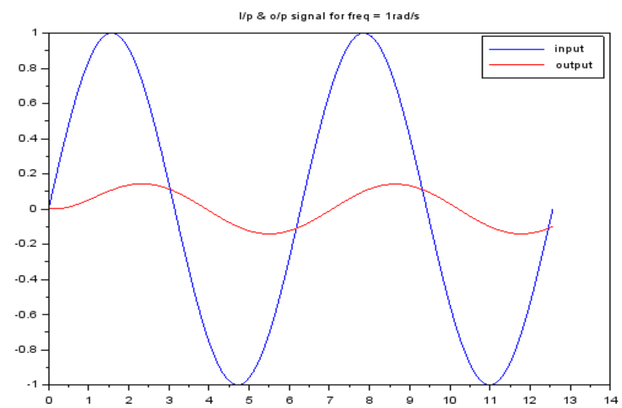
6 Question 4

6.1 Part a

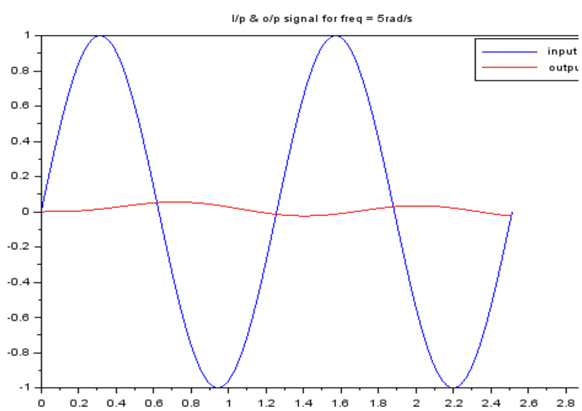
The input and output responses for various frequencies



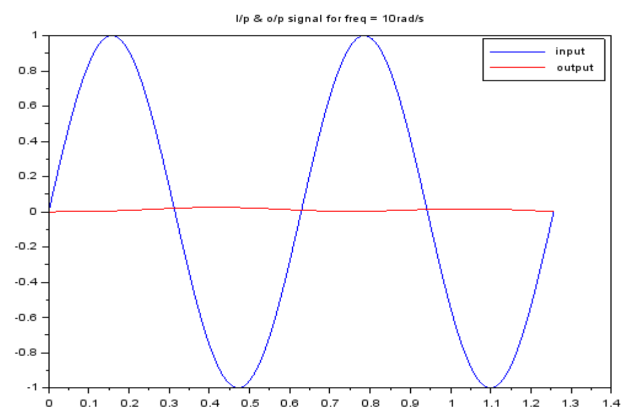
(a) $w = .1$



(b) $z = 1$



(a) $z = 5$



(b) $z = 10$

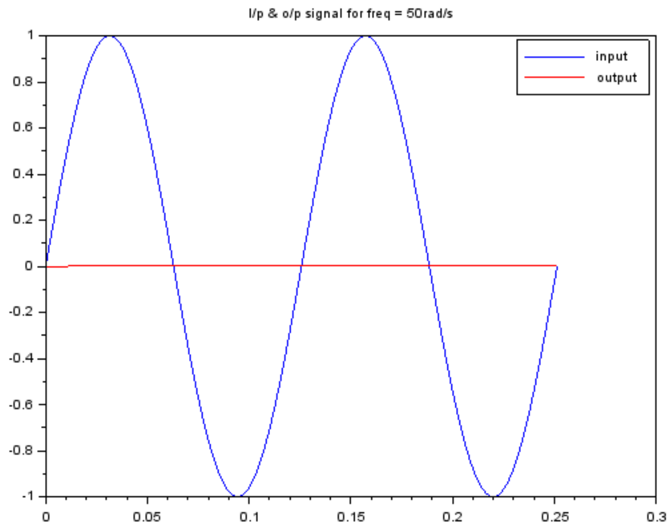


Figure 9: $z = 50$

6.2 Part c

the bode plot for this is

```
s = poly(0,'s');
G = 1/(s^2+5*s+6);
G = syslin('c',G);

// part a
w_range = [.1, 1, 5, 10, 50];
output_amp = zeros(1,5);
phase = zeros(1,5);
i = 1;

for w = w_range
    scf(i-1);
    t = 0:%pi/(100*w):4*(%pi)/w;
    x = sin(w*t);
    y = csim(x,t,G);
    output_amp(i) = max(y);
    t1 = t(y == max(y))(1);
    t2 = t(x == max(x))(1);
    phase(i) = 180*w*(t1-t2)/(%pi);
    plot(t,x, 'b');
    plot(t,y, 'r');
    title('I/p & o/p signal for freq = '+string(w)+'rad/s');
    legend('input','output')
    xs2png(i-1, 'q4_a'+ string(10*w));
```

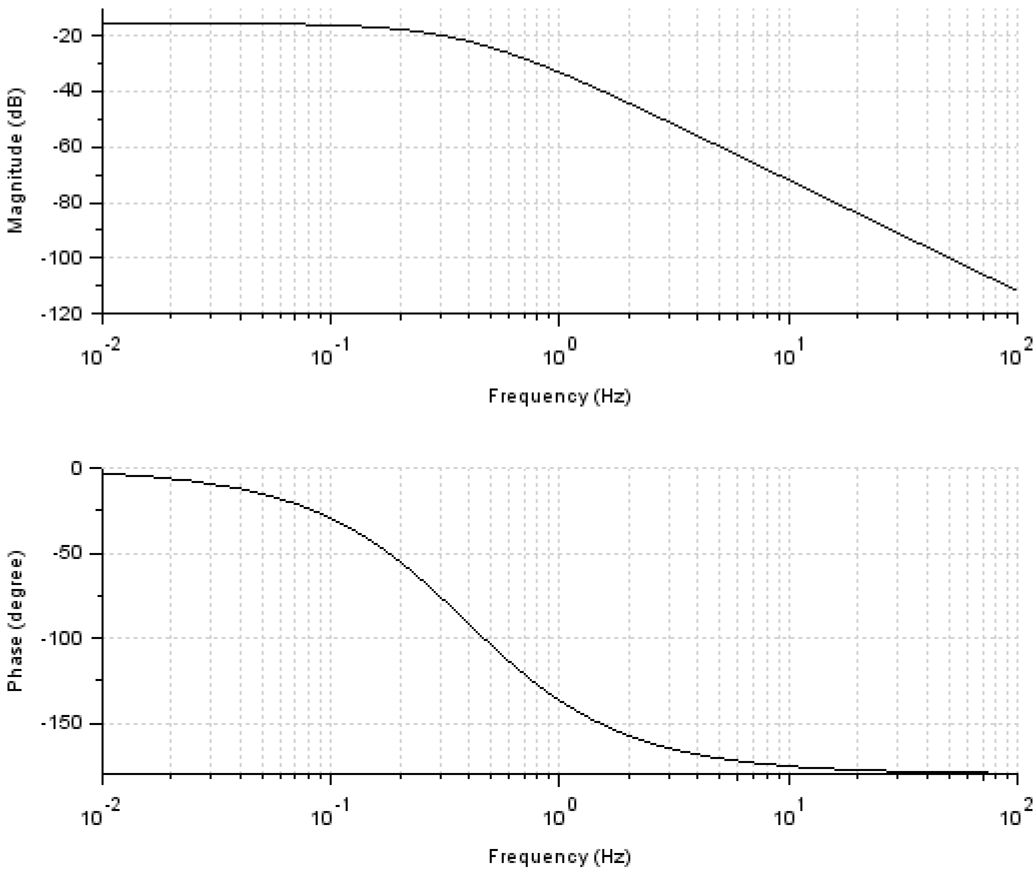
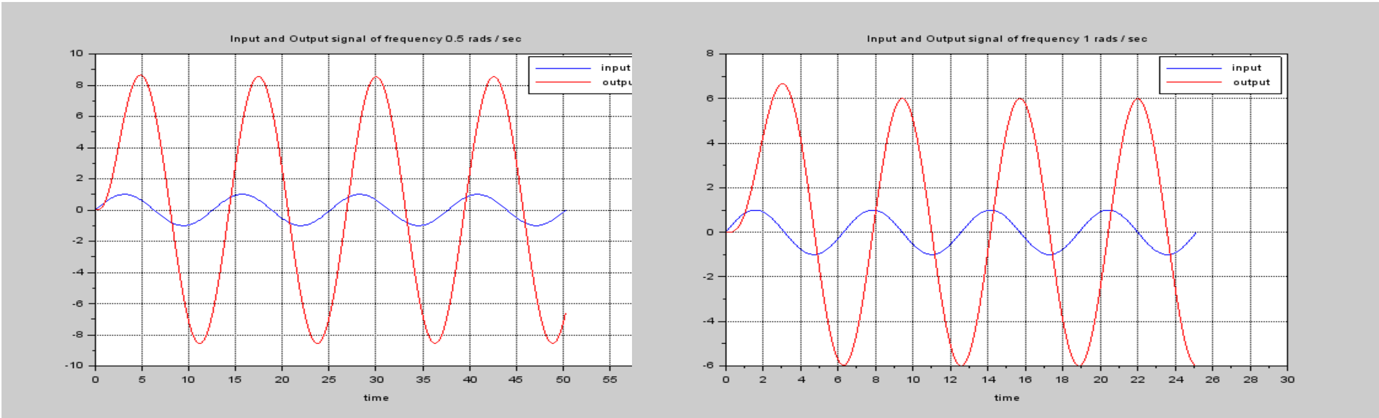
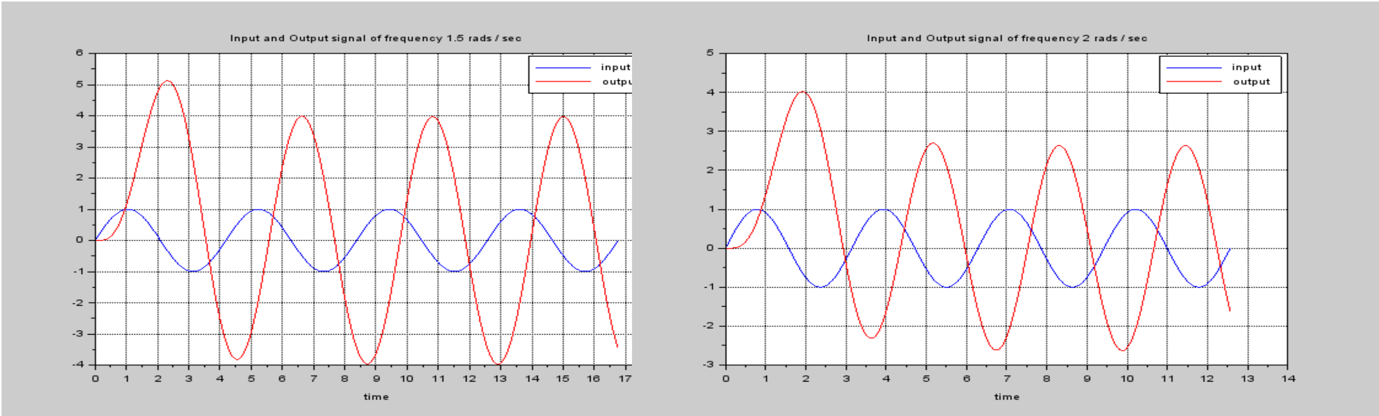
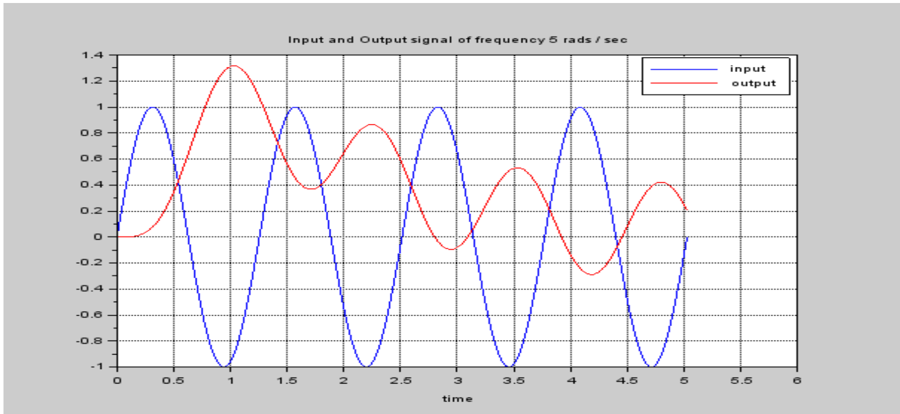


Figure 10: Caption






```
    i = i+1;
end

scf(5)
bode(G, 0.01, 100);


// part b
H = 60/(s^3+6*s^2+11*s+6)
G = syslin('c',H)
figure;
clf();
evans(G);

w_arr = [0.5 1 1.5 2 5]
mg = []
phase =[]
for w = w_arr
    t = 0: %pi /(50* w) :8* %pi / w;
    ip = sin(w*t)
    op = csim(ip,t,G)
    mg = [mg;max(op)]
```




```
    to = t(op == max (op))(1)
    ti = t(ip == max (ip))(1)
    phase = [phase;(to-ti)*w*180/%pi]
    figure
    clf()
    xgrid()
    plot (t ,ip ,'b')
    plot (t ,op ,'r')
    title ('Input and Output signal of frequency ' + string (w) + ' rads /
    sec ')
    xlabel ('time')
    legend ("input","output")
end


scf(6)
bode(G, 0.01, 100);
```

H]

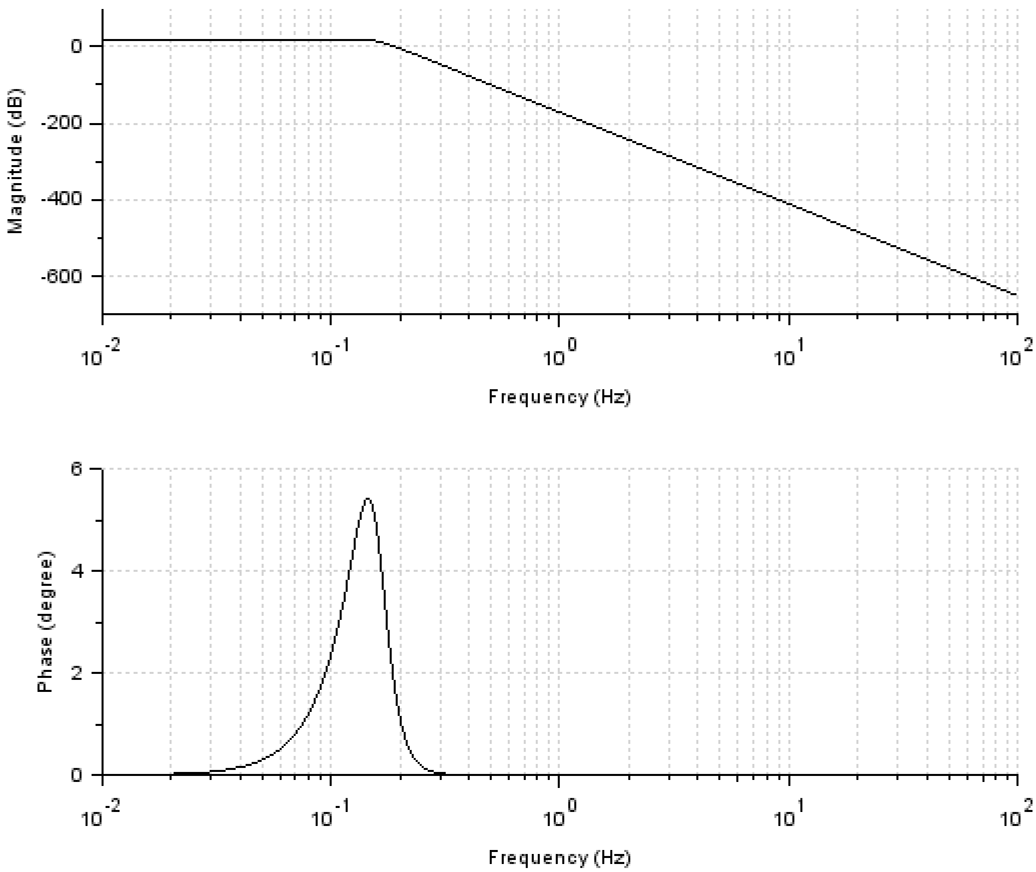


Figure 13: Caption