# EE324: Problem Sheet 3

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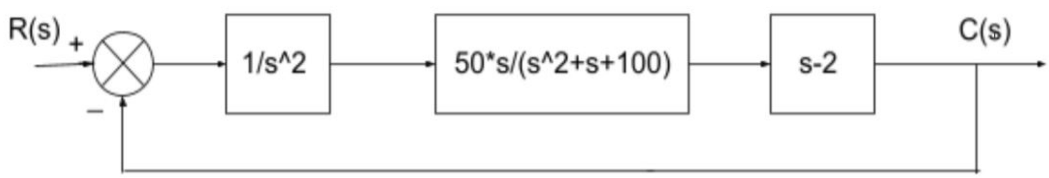
## 1 Question 1

### 1.1 Part a



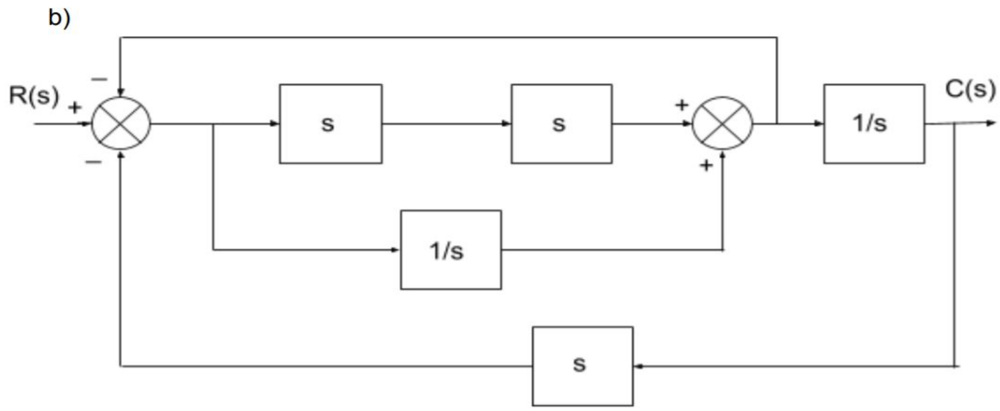

Considering,

$$h1 = \frac{1}{s^2}, \quad h2 = \frac{50s}{s^2+s+100}, \quad h3 = s-2$$

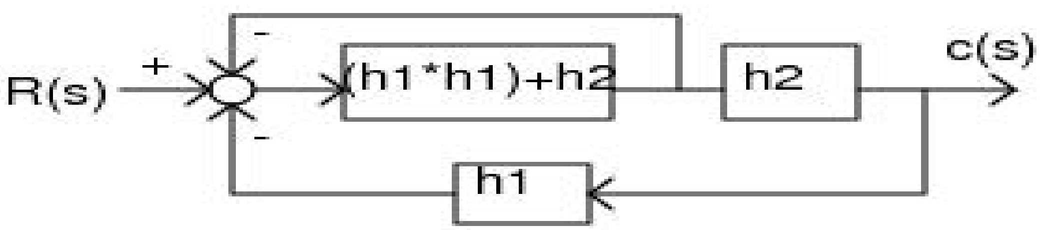
and using simple block reduction technique, we get

$$H_{eq} = \frac{h4}{1+h4} \quad where \quad h4 = h3 * h2 * h1$$

## 1.2 Part b




Considering, $h1 = s$, $h2 = 1/s$
we reduce the blocks as per following:

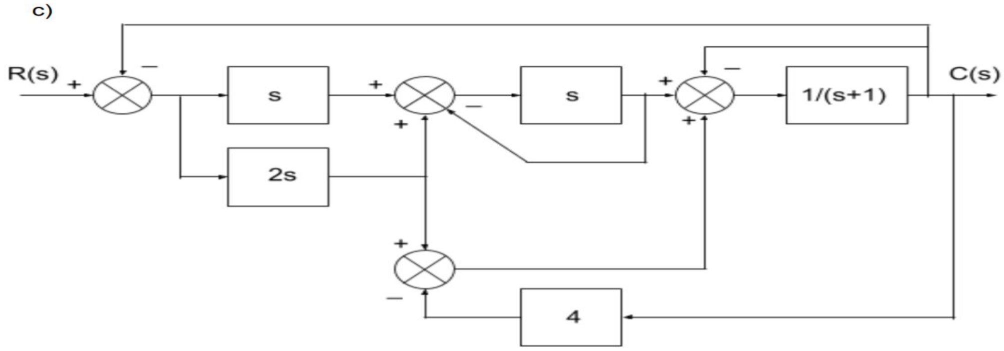



Considering, $h3 = (h1 * h1) + h2$ , $h4 = \frac{h3}{1+h3} * h2$

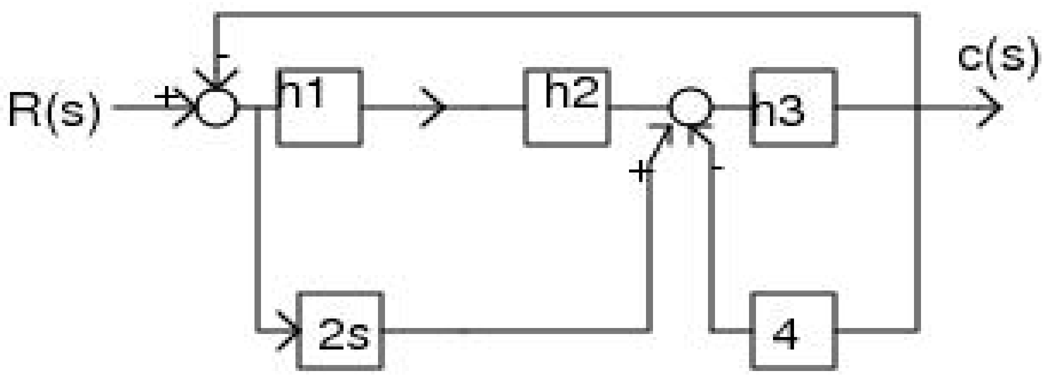
$$H_{eq} = \frac{h1 * h4}{1 + h1 * h4}$$

## 1.3 Part c

c)




Considering, $h1 = s + 2s, \quad h2 = \frac{s}{1+s}, \quad h3 = \frac{1}{s+2}$
We get :




Now, applying simple block reduction techniques and considering,

$$h4 = (h1 * h2) + 2s \ , \quad h5 = \frac{4 * h3}{1 + 4 * h3}, \quad h6 = h4 * h5$$

We get :

$$H_{eq} = \frac{h6}{1 + h6}$$

# 2 Question 2

## 2.1 Part a

Given $G(s)$, a proportionality gain $K$ has been put in the forward path in series with the plant and then the feedback loop has been closed with unity negative feedback.
Hence, the final transfer function becomes :

$$H_{eq} = \frac{k * G(s)}{1 + k * G(s)} = \frac{10 * k}{s^3 + 6s^2 + 8s + 10 * k}$$

## 2.2 Part b

Plot of loci of the closed loop poles as K varies from 0 to 100 in steps of 0.1:


## 2.3 Part c

Critical value of $k$ from the plot, after which if it is increased then the system becomes unstable is :

$$k = 4.8$$

## 2.4 Part d

For any general $k$, we have the poles as the roots of the polynomial

$$s^3 + 6s^2 + 8s + 10k = 0$$

The Routh table table for this polynomial is :

| | | | |
|---|---|---|---|
| $s^3$ | 1 | 8 | 0 |
| $s^2$ | 6 | 10k | 0 |
| $s^1$ | (48-10k)/6 | 0 | 0 |
| $s^0$ | 10k | 0 | 0 |

Here, for $k < 4.8$, the expression $(48 - 10k)/6$ is positive and there are no sign swaps in the $1^{st}$ column of the table. Hence, the system is stable.
For $k > 4.8$, the expression $(48 - 10k)/6$ is negative and there are two sign swaps in the $1^{st}$ column of the table. Hence, we encounter two poles in the Open right half complex plane which leads to system instability.

# 3 Question 3

## 3.1 Part a

```
--> rta
 rta  =
    1.          5.   1.
    3.          4.   3.
    3.6666667   0.   0.
    4.          3.   0.
  -2.75         0.   0.
    3.          0.   0.
```

## 3.2 Part b

```
--> rtb
 rtb  =


                1                   6      8
                -                   -      -
                1                   1      1

               eps                  5      20
               ---                  -      --
                1                   1      1

             -5 +6eps           -20 +8eps  0
             --------           ---------  -
               eps                 eps     1

        -25 +50eps -8eps²          20      0
        -----------------          --      -
            -5 +6eps               1       1

    -2.274D-13 -160eps -64eps²      0      0
    --------------------------      -      -
        -25 +50eps -8eps²           1      1
```

```
               20                   0      0
               --                   -      -
               1                    1      1
```

### 3.3 Part c

```
--> rtc
 rtc  =

   1.           3.    2.
  -2.          -6.   -4.
  -8.          -12.   0.
  -3.          -4.    0.
  -1.3333333    0.    0.
  -4.           0.    0.
```

### 3.4 Part d

```
--> rtd
 rtd  =


    1    -6   1   -6
    -    --   -   --
    1    1    1   1

    1    0    1   0
    -    -    -   -
    1    1    1   1

    -6   0    -6  0
    --   -    --  -
    1    1    1   1

   -24   0    0   0
   ---   -    -   -
    1    1    1   1
```

```
eps    -6  0    0
---    --  -    -
 1     1   1    1

-144  0    0    0
----  -    -    -
eps   1    1    1

864   0    0    0
----  -    -    -
-144  1    1    1
```

# 4 Question 4

## 4.1 Part a

Consider the polynomial :

$$p(s) = 4s^6 + s^5 + 6s^4 + s^3 + 6s^2 + s + 2$$