## **Consumer Utility**

Our model assumes that consumers have some general amount of utility by which they favor mom-and-pop stores, although moment-to-moment changes in preferences might override this. To account for both this preference and variability, for the  $k^{th}$  consumer, the utility gained from shopping at a mom-and-pop is a positive constant plus some random factor:

$$U_k = (p_M + r_{kM}[0,1])^1$$

Consumers gain utility from shopping at the big-box store as well. Thus, the consumer's overall utility gain is:

$$U_k = max\{(p_{kM} + r_{kM}[0,1]), r_{kB}[0,1]\}$$

The overall utility in the market is expressed as the sum of these utilities over the population:

$$\Sigma_k \, U_k$$

To rephrase our assumptions: consumers gain the most utility when both mom-and-pops and big-boxes are around, they do second best when they have available only mom-and-pops, and worst when they have only big-boxes. But it turns out that even in cases of high preference for mom-and-pop store, the consumers' least preferred outcome can arise.

 $<sup>^1</sup>$  The variables  $p_M$  and  $r_M[0,1]$  are between 0 and 1 inclusive.  $p_M$  is consumer preference for mom-and-pops, and  $r_X$  is a random number. We abbreviate "mom-and-pop" by M and "big-box" by B. The subscript indicates the variable belongs to the type of retailer.

### **Mathematical Analysis**

We give the expected utility in the three periods of our model.

- (1) When there are only mom-and-pops,  $E(\Sigma_i U_i) = E(\Sigma_i (p_M + r_{iM}[0,1])) = (\Sigma_i (E p_M + E r_{iM}[0,1])) = (\Sigma_i (p_M + 0.5)) = N(p_M + 0.5)$  by linearity.
- (2) When there are big-boxes as well as mom-and-pops,  $E((\Sigma_i U_i)) = E(\Sigma_i (max \{ p_M + r_{iM}[0,1], r_{iB}[0,1] \})) = \Sigma_i E max \{ p_M + r_{iM}[0,1], r_{iB}[0,1] \} = N((2/3) + (1/2)p_M+(1/2)p_M^2-(1/6)p_M^3).$
- (3) When there are only big-boxes, E (  $\Sigma_i~U_i$  ) = E (  $\Sigma_i~r_{iB}[0,1])$  = (  $\Sigma_i~E~r_{iB}[0,1])$  = N( 0.5 )

We note that for preferences in [0,1], the expected value in (2) is greater than that of (1) which in turn is greater than that of (3).

#### **Big-box Statistics**

#### Runs

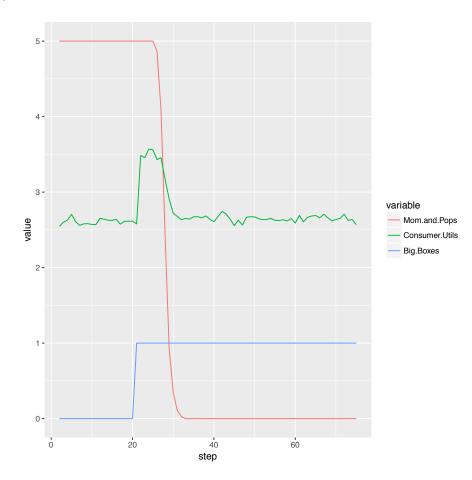
The following graphs represent the average run time of models where consumer "added" preference for mom-and-pops is 0.0, 0.2, 0.4, 0.6, and 0.8 respectively. All other variables beside preference are held constant.<sup>2</sup> For each preference level, we ran seventy-five runs of seventy-five periods each. Note that in all experiments the big-box store appears in period 20.

In each experiment we give the expected utilities for (1) having only mom-and-pops, (2) having both kinds of store, and (3) having only big-box stores. We divided the utility by 5 in both the calculation and the model for sake of the data's presentation.

In the experiments where the preference for momand-pops is 0.0, 0.2, 0.4, and 0.6, mom-and-pops tend to vanish leaving the result where consumers end up with the situation of least utility.

## Pref. $p_M = 0.0$

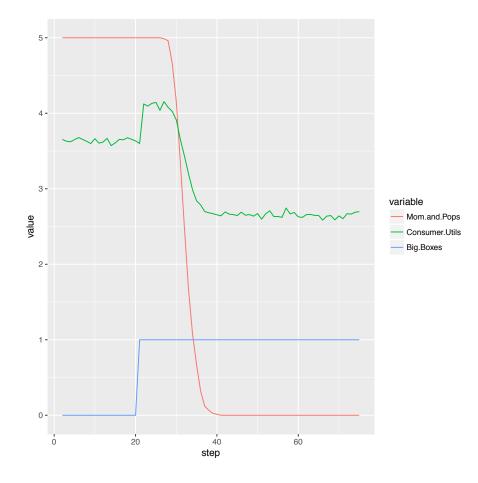
- (1) 2.6
- $(2)\ 3.5$
- (3) 2.6



<sup>&</sup>lt;sup>2</sup> There are 26 consumers, 5 mom-and-pop stores, each of which have an initial endowment of \$30, gain \$2 per purchase, and loose \$10 per step. The big-box appears on period 20. It has an initial endowment 1000 times that of the big-box-store, and it pays five times the rent.

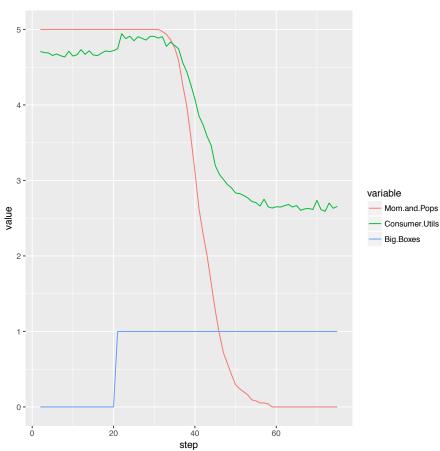
# Pref. $p_M = 0.2$

- (1) 3.6
- (2) 4.1
- (3) 2.6



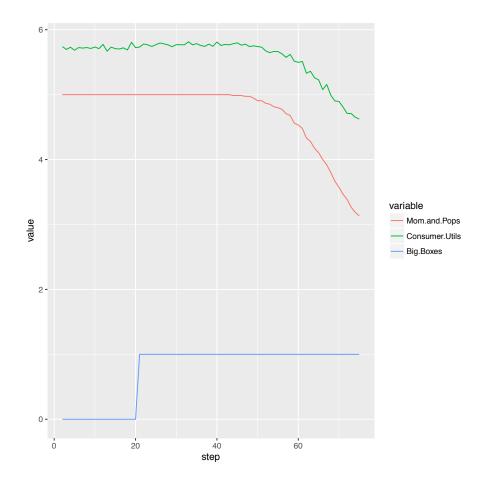
# Pref. $p_M = 0.4$

- (1) 4.6
- (2) 4.8
- (3) 2.6



# Pref. $p_M = 0.6$

- (1) 5.7
- (2) 5.8
- (3) 2.6



# Pref. $p_M = 0.8$

- (1) 6.8
- (2) 6.8
- (3) 2.6

