

$$E(\max(r_1 + p_M, r_2)) = \int_{-\infty}^{\infty} x f(x) dx$$

where $f(x) = F'(x)$

F being the cumulative distribution function

$$F(x) = \int_0^x f(x) dx \quad (1)$$

$$= \text{prob}\{\max\{r_1 + p_M, r_2\} < x\} \quad (2)$$

$$= \text{prob}\{r_1 + p_M < x\} * \text{prob}\{r_2 < x\} \quad (3)$$

$$= \begin{cases} (x^2 - p_M x), & p_M < x < 1 \\ (x - p_M), & 1 < x < 1 + p_M \\ 1, & x > 1 + p_M \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Thus,

$$f(x) = \begin{cases} 2x - p_M, & p_M < x < 1 \\ 1, & 1 < x < 1 + p_M \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

And so,

$$\int_{-\infty}^{\infty} x f(x) dx \quad (6)$$

$$= \int_{p_M}^{1+p_M} 2x^2 - p_M x dx + \int_1^{1+p_M} x dx \quad (7)$$

$$= \frac{2}{3} + \frac{1}{2}p_M + \frac{1}{2}p_M^2 - \frac{1}{6}p_M^3 \quad (8)$$