### Consumer Utility

Our model assumes that consumers have some general amount of utility by which they favor mom-and-pop stores, although moment-to-moment changes in preferences might override this. To account for both this preference and variability, for the $k^{th}$ consumer, the utility gained from shopping at a mom-and-pop is a positive constant plus some random factor:

$$U_k = (p_M + r_{kM}[0,1])$$[1]

Consumers gain utility from shopping at the big-box store as well. Thus, the consumer's overall utility gain is:

$$U_k = \max\{(p_{kM} + r_{kM}[0,1]), r_{kB}[0,1]\}$$

The overall utility in the market is expressed as the sum of these utilities over the population:

$$\Sigma_k U_k$$

To rephrase our assumptions: consumers gain the most utility when both mom-and-pops and big-boxes are around, they do second best when they have available only mom-and-pops, and worst when they have only big-boxes. But it turns out that even in cases of high preference for mom-and-pop store, the consumers' least preferred outcome can arise.

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[1] The variables $p_M$ and $r_M[0,1]$ are between 0 and 1 inclusive. $p_M$ is consumer preference for mom-and-pops, and $r_X$ is a random number. We abbreviate "mom-and-pop" by *M* and "big-box" by *B*. The subscript indicates the variable belongs to the type of retailer.

**Mathematical Analysis**

We'll denote the utility at step t by

$( \Sigma_i U_i )_t$

We give the expected utility in the three periods of our model.

1. When there are only mom-and-pops, $E( ( \Sigma_i U_i )_t ) = E ( \Sigma_i (p_M + r_{iM}[0,1])) = ( \Sigma_i (E\ p_M + E\ r_{iM}[0,1])) = ( \Sigma_i ( p_M + 0.5) ) = N( p_M + 0.5)$ by linearity.
2. When there are big-boxes as well as mom-and-pops, $E( ( \Sigma_i U_i )_t ) = E ( \Sigma_i (\max \{ p_M + r_{iM}[0,1), r_{iB}[0,1) \} )) = \Sigma_i E \max \{ p_M + r_{iM}[0,1), r_{iB}[0,1) \} = N((2/3) + (1/2)p_M+(1/2)p_M\char`^2-(1/6)p_M\char`^3)$.
3. When there are only big-boxes, $E( ( \Sigma_i U_i )_t ) = E ( \Sigma_i r_{iB}[0,1)) = N(0.5)$

We note that for preferences between 0 and 1, the expected value in 2. is greater than that of 1. which in turn is greater than that of 3.

**Big-box Statistics**

**Runs**

The following graphs represent the average run time of models where consumer "added" preference for mom-and-pops is 0.0, 0.2, 0.4, 0.6, and 0.8 respectively. All other variables beside preference are held constant.[2] For each preference level, we ran seventy-five runs of seventy-five periods each. Note that in all experiments the big-box store appears in period 20.

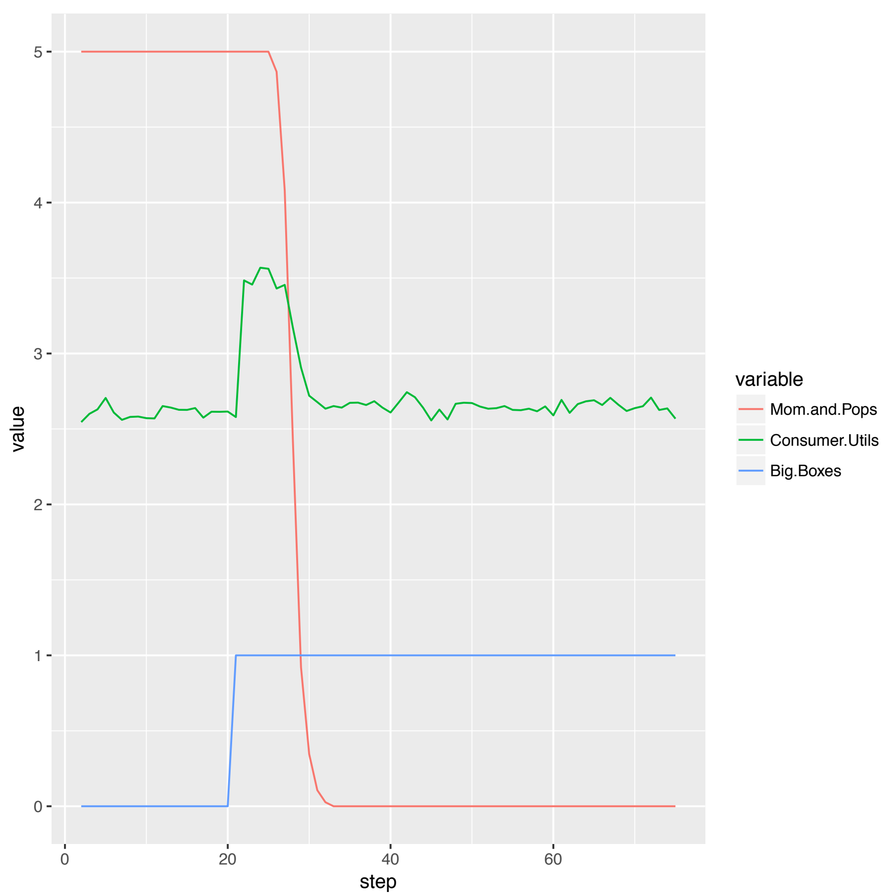
In each experiment we give the expected utilities for (1) having only mom-and-pops, (2) having both kinds of store, and (3) having only big-box stores.

**Pref. $p_M$ = 0.0**

(1) 2.6

(2) 3.5

(3) 2.6



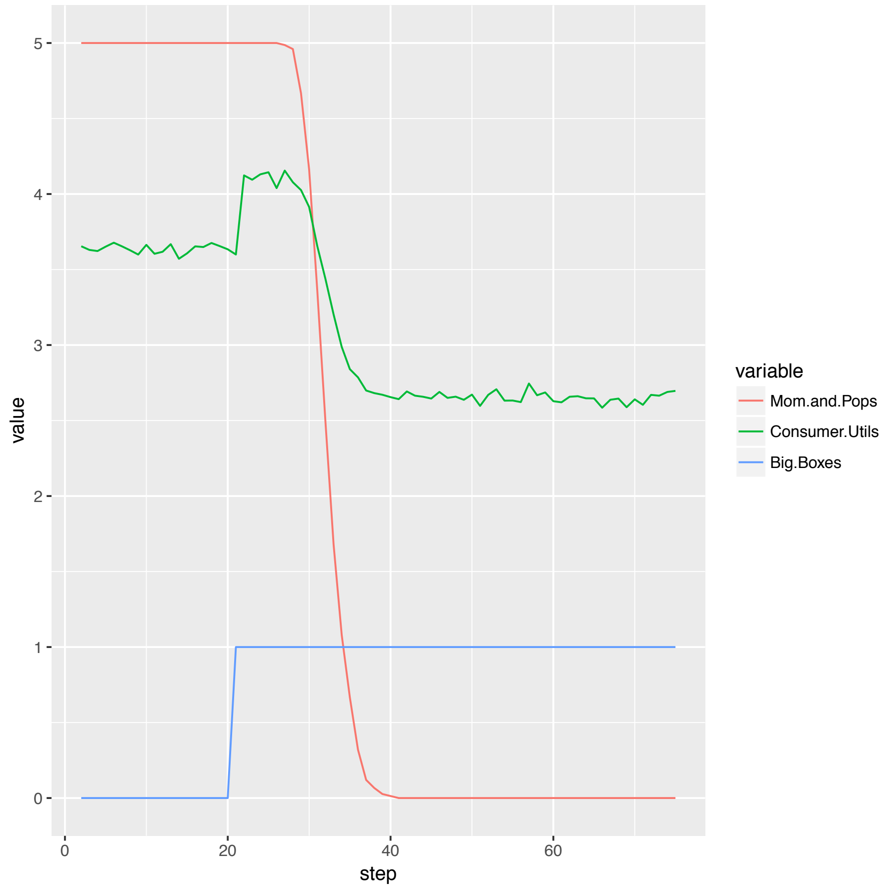

[2] There are 26 consumers, 5 mom-and-pop stores, each of which have an initial endowment of $30, gain $2 per purchase, and loose $10 per step. The big-box appears on period 20. It has an initial endowment 1000 times that of the big-box-store, and it pays five times the rent.

**Pref. $p_M$ = 0.2**

(1) 3.6

(2) 4.1

(3) 2.6

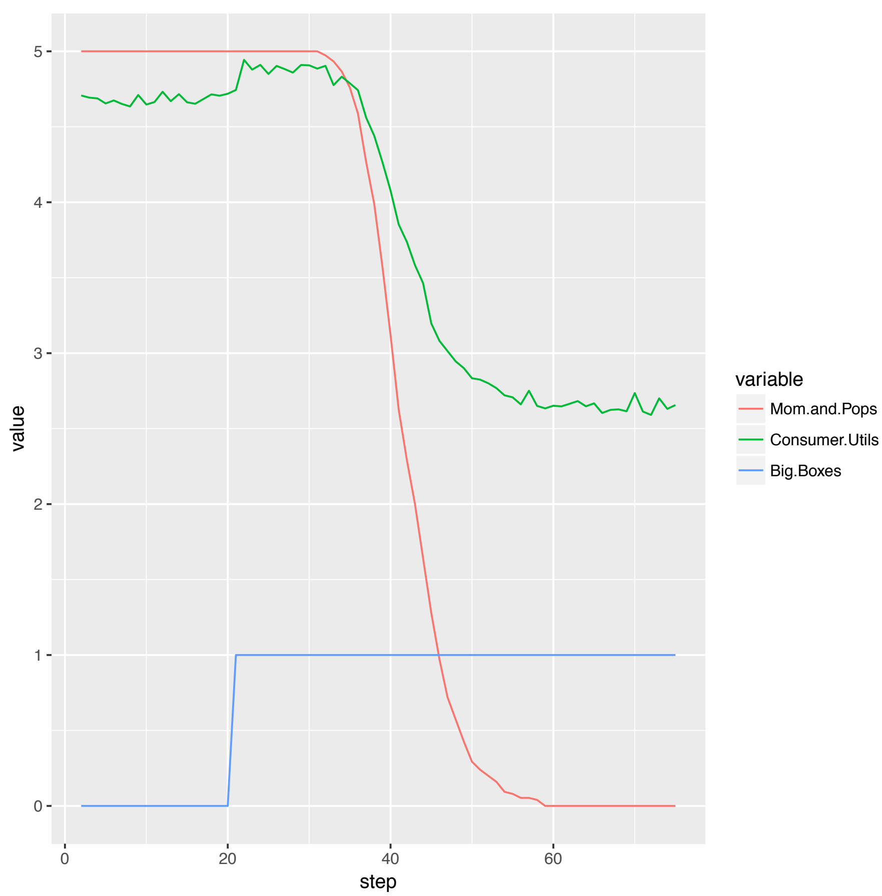



**Pref. $p_M$ = 0.4**

(1) 4.6

(2) 4.8

(3) 2.6

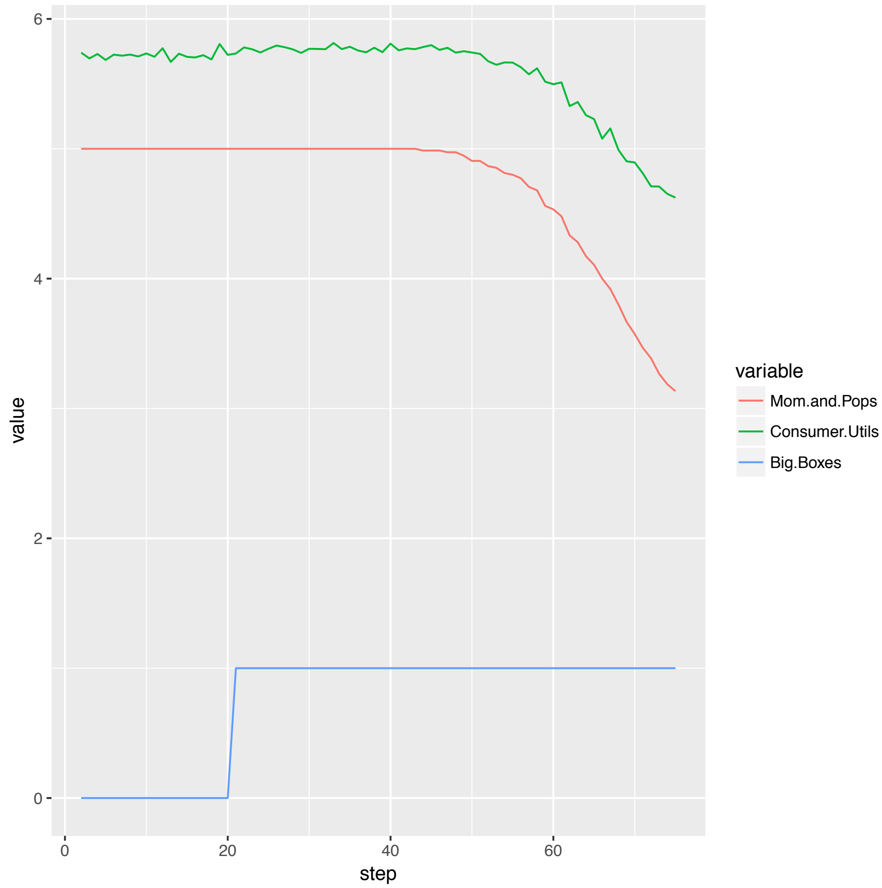


**Pref. $p_M = 0.6$**

(1) 5.7

(2) 5.8

(3) 2.6




**Pref. $p_M = 0.8$**

(1) 6.8

(2) 6.8

(3) 2.6