Consumer Utility

Our model assumes that consumers have some general amount of utility by which they favor mom-and-pop stores, although moment-to-moment changes in preferences might override this. To account for both this preference and variability, for the k^{th} consumer, the utility gained from shopping at a mom-and-pop is a positive constant plus some random factor:

$U_k = (p_M + r_{kM}[0,1))$[1]

Consumers gain utility from shopping at the big-box store as well. Thus, the consumer's overall utility gain is:

$U_k = \max\{(p_{kM} + r_{kM}[0,1)), r_{kB}[0,1)\}$

The overall utility in the market is expressed as the sum of these utilities over the population:

$\Sigma_k U_k$

To rephrase our assumptions: consumers gain the most utility when both mom-and-pops and big-boxes are around, they do second best when they have available only mom-and-pops, and worst when they have only big-boxes. But it turns out that even in cases of high preference for mom-and-pop store, the consumers' least preferred outcome can arise.

[1] The variables p_M and $r_M(0,1]$ are between 0 and 1 inclusive. p_M is consumer preference for mom-and-pops, and r_X is a random number. We abbreviate "mom-and-pop" by *M* and "big-box" by *B*. The subscript indicates the variable belongs to the type of retailer.

Big-box Statistics

Runs

The following graphs represent the average run time of models where consumer "added" preference for mom-and-pops is 0.0, 0.2, 0.4, 0.6, and 0.8 respectively. All other variables beside preference are held constant.[2] For each preference level, we ran seventy-five runs of seventy-five periods each. Note that in all experiments the big-box store appears in period 20.

[Graph 1.0] **[Graph 1.2]**
[Graph 1.4] **[Graph 1.6]**
[Graph 1.8]

Clearly the trend is for environments whose consumers have a "low enough" preference to lose their mom-and-pop stores. In [Graphs 1.0, 1.2, 1.4. and 1.6], the consumer utility was raised by the presence of the big-box store; yet in the end, its presence drove out the mom-and-pops, leaving it in the cases where consumers preferred mom-and-pops to big-boxes where the end result was a net loss in utility.

[2] There are 26 consumers, 5 mom-and-pop stores, each of which have an initial endowment of $30, gain $2 per purchase, and loose $10 per step. The big-box appears on period 20. It has an initial endowment 1000 times that of the big-box-store, and it pays five times the rent.

Mathematical Analysis

We'll denote the utility at step t by

$(\Sigma_i U_i)_t$

We give the expected utility in the three periods of our model.

1. When there are only mom-and-pops, $E((\Sigma_i U_i)_t) = E (\Sigma_i (p_M + r_{iM}[0,1))) = (\Sigma_i (E\, p_M + E\, r_{iM}[0,1))) = (\Sigma_i (p_M + 0.5)) = N(p_M + 0.5)$ by linearity.
2. When there are big-boxes as well as mom-and-pops, $E((\Sigma_i U_i)_t) = E (\Sigma_i (\max \{ p_M + r_{iM}[0,1), r_{iB}[0,1) \})) = \Sigma_i E \max \{ p_M + r_{iM}[0,1), r_{iB}[0,1) \} = (1/3)(1-p_M)^3 + ((2p_M-1)/4)(1-p_M)^2+(7/12) = (1/12) (8+2 p_M - p_M^2+2 p_M^3)$.[3]

[Compute Specific cases]

3. When there are only big-boxes, $E((\Sigma_i U_i)_t) = E (\Sigma_i r_{iB}[0,1)) = N(0.5)$

[Do mathematical analysis of specific cases.]

[3] TODO: Write out calculation here.