Attempts at Making The Fashion Model Markov Based

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6 September 2016

Currently conceived as a deterministic process, Indra's Fashion model can be conceived as a Markov process. Below, I discuss several possibilities, their merits, and their demerits.

1 Introduction

To make the models' underlying machinery as common-to-all as possible, in these considerations we follow the Forest Fire Model by describing all our agent state information by one state vector, namely

$$\begin{pmatrix} HR & HB & FR & FB \end{pmatrix}$$

where HR denotes the probability a hipster agent will wear red, HB; a hipster, blue; FR a follower, red; and FB a follower, blue.

This understanding of our agent's state information necessarily implies that our transition matrix surveyed from the population be stored in the matrix

$$\begin{pmatrix} p^H_{RR} & p^H_{RB} & 0 & 0 \\ p^H_{BR} & p^H_{BB} & 0 & 0 \\ 0 & 0 & p^F_{RR} & p^F_{RB} \\ 0 & 0 & p^F_{BR} & p^F_{BB} \end{pmatrix}$$

for this is the only way (under our state vector) we can guarantee hipsters do not end up being a follower and vice versa.

(Since this is a block diagonal matrix, we may consider simply making two 2×2 matrices where the state vectors of the hipsters and followers are distinct 1×2 vectors. But for this reason, it is likely irrelevant if we were to separately discuss these matrices. The ideas would probably be essentially the same. A possible benefit of containing all the information in one matrix is that all the information is in one place, but perhaps we can organize multiple transition matrices in a non-clumsy, elegant way.)

2 Survey Results in Cumulative Matrix

My first consideration was to survey the environment where we cumulate transition matrices multiplying by P_O or P_S, each time we encounter an agent of the opposite type wearing, respectively, the opposite or same fashion as the agent who currently occupies our cell.

$$P_O = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0.1 & 0.9 \end{pmatrix} P_S = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0.1 & 0.9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

There are, perhaps, too great of downsides to this approach for it to be desirable.

1. If we were to interpret the matrix multiplication in the way of Markov processes, then the entry of, say, the first row first column of P_S^2 is $p_{BB}p_{BB} + p_{BR}p_{RB}$ that is, the probability of the blue wearing hipster retaining his fashion in spite of seeing two followers of his color (blue). But the way we have to read things is strange. Let's take the second term of the sum, $p_{BR}p_{RB}$. This represents the chance the hipster sees one blue follower and changes his mind to wear red; and then he sees another blue(!) wearing follower and changes his mind to wear blue again. But since his frame of mind is "I should wear red" when he sees the second blue wearing follower, does it make since to say he has some chance to follow the follower in wearing blue? This seems prohibitively damning of this interpretation.

2. We still have to filter for agents of the opposite type. Our matrix cannot filter both wether or not the other agent is of the opposite type and of

the opposite fashion. So this process is complicated by several layers of filters of different types, one a matrix, the other an if-statement.

3. As $\lim_{n\to\infty} P_O^n$ we tend toward the matrix,

$$P_O = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

   But this is not what we want! This means that even though a follower sees a bazillion hipsters of the opposite type, as he evaluates his environment, will only have a %50 change of changing type.

   We may try to remedy this problem by treating 0.5 as meaning certain change of clothing, where normally we'd consider 1 to be certainty. This would function similarly to the vector model, where a vector %50 between $\begin{pmatrix} 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \end{pmatrix}$. We'd have to reinterpret how to change the prob_to_state method in the markov class, but this is where I believe we'd include the agent's variability, using it to change the point at which agent will break down, and must change to a certain fashion.

3 Surveying Environment Directly Tells us What Belongs in the Trans Matrix

Another way we could go about surveying the environment to see what our cell's transition matrix could be is this: we take a census of the neighboring agents, and we use it to calculate precisely what the entries of the cell's transition matrix ought to be. For example, let's have $\Box$ be the probability a follower will have a hipster change his fashion (because he's a square), and $\Diamond$ be the probability the hipsters will change the follower's fashion (because hipsters are seen as valuable as diamonds). Our (single!) transition matrix would be,

$$T = \begin{pmatrix} 1-\Box & \Box & 0 & 0 \\ \Box & 1-\Box & 0 & 0 \\ 0 & 0 & \Diamond & 1-\Diamond \\ 0 & 0 & 1-\Diamond & \Diamond \end{pmatrix}$$

Now all we have to do is calculate the value for $\Box$ and for $\Diamond$. It could be, for example

$$\Box = \frac{number\ of\ same\ color\ followers}{total\ agents\ in\ square\ view}$$

$$\Diamond = \frac{number\ of\ opposite\ color\ followers}{total\ agents\ in\ square\ view}$$

where we may include the agent within the cell in the total agents in square view to not divide by zero. Or we can use an if statement ...

We may include the agents variability in this way:

$$T = \begin{pmatrix} 1-(var)\Box & (var)\Box & 0 & 0 \\ (var)\Box & 1-(var)\Box & 0 & 0 \\ 0 & 0 & (var)\Diamond & 1-(var)\Diamond \\ 0 & 0 & 1-(var)\Diamond & (var)\Diamond \end{pmatrix}$$

This way the higher the var, the more readily say an agent will change his mind after seeing just one agent of the opposite type.