**Consumer Utility**

Our model assumes that consumers have some general amount of utility by which they favor mom-and-pop stores, although moment-to-moment changes in preferences might override this. To account for both this preference and variability, the utility gained from shopping at a mom-and-pop is a positive constant plus some random factor:

U = (pM + rM[0,1))[[1]](#footnote-1)

Consumers gain utility from shopping at the big-box store as well. Thus, the consumer’s overall utility gain is:

U = max{(pM + rM[0,1)), rB[0,1)}

The overall utility in the market is expressed as the sum of these utilities over the population:

Σi Ui

To rephrase our assumptions: consumers gain the most utility when both mom-and-pops and big-boxes are around, they do second best when they have available only mom-and-pops, and worst when they have only big-boxes. But it turns out that even in cases of high preference for mom-and-pop store, the consumers’ least preferred outcome can arise.

**Big-box Statistics**

**Runs**

The following graphs represent the average run time of models where consumer “added” preference for mom-and-pops is 0.0, 0.2, 0.4, 0.6, and 0.8 respectively. All other variables beside preference are held constant.[[2]](#footnote-2) For each preference level, we ran seventy-five runs of seventy-five periods each. Note that in all experiments the big-box store appears in period 20.

**[Graph 1.0] [Graph 1.2]**

**[Graph 1.4] [Graph 1.6]**

**[Graph 1.8]**

Clearly the trend is for environments whose consumers have a “low enough" preference to lose their mom-and-pop stores. In [Graphs 1.0, 1.2, 1.4. and 1.6], the consumer utility was raised by the presence of the big-box store; yet in the end, its presence drove out the mom-and-pops, leaving it in the cases where consumers preferred mom-and-pops to big-boxes where the end result was a net loss in utility.

**Mathematical Analysis**

We’ll denote the utility at step t by

( Σi Ui ) t

Our thesis is that the ultimate behavior in this sequence of steps is for the utility gain to be the expected value of rB[0,1), that is, 0.5. In other words:

limt→∞ ( Σi Ui ) t = 0.5.

Consumers begin with their 2nd preference: having the mom-and-pop stores without big-boxes. Once the big-box appears, there will be both kinds of store, and consumers will have the outcome of the highest utility. It is sufficient to show that positive consumer preference *for* mom-and-pops is expected to result in the disappearance of mom-and-pops, leaving consumers with their least desired outcome.

In our model, if a retailer’s capital falls below zero, it is out of the game. To keep track of funds, we shall represent income and expense by *I* and *E*. In symbols, therefore, before the *big-box* appears, the *mom-and-pop* should be either making due, or making profit:

*IM ≥ E*

The condition our model concerns is where the *B* enters the market, both the *big-boxes* and the *mom-and-pops* operate at a net loss until the *mom-and-pop*s’ run out of funds; namely

(a) *E* *≥ IB ≥ IM*

or

(b) *E* *≥ IM > IB*

Since the *big-box* has an initial storehouse of funds larger than that of the *mom-and-pop*s, condition (a) clearly will lead to the *mom-and-pop*s’ demise and the *big-box*’s success. Condition (b) has this result if and only if the *big-box* doesn’t lose funds too quickly.

Generally speaking, we are looking at the relations

(c) *IB ~E ~ IM*

where the tildes are placeholders for inequity signs.

Let’s understand how our model’s key players factor into the retailers’ incomes: We let *N* stand in place for the number of consumers who shop at retailers, *D* the dollar amount consumers give each store each turn, *G* the number of types of goods (hence the number of *mom-and-pop*s), and *c* the probability a consumer will shop at a *mom-and-pop*. The probability *c* is a function of pM (higher pM is directly proportional to the higher mom-and-pop shopping chances).[[3]](#footnote-3)

Initially, when there are no *big-boxe*s, the expected value for each *mom-and-pop*’s income is the money gotten from all consumers who decide to shop for their good that day. Since the total number of customers for a particular store is divided amongst the *G* goods, a *mom-and-pop*’s expected income is

*IM*= *D* \* (*N / G*)

When a *big-box* appears, however, preference scales income down:

*IM*= *D* \* (*N / G*) \* *c*

The equation for the expected income of the *big-boxes* is simpler. Since they sell all goods, its number of goods is sold is not a factor in its income, and the chance a customer will shop with them is the chance they won’t shop at *mom-and-pops*, namely (1-*c*).

*IB*= *D* \* *N \** (1-*c*)

Therefore, by substitution, (c) becomes

(c’) *D* \* *N \** (1 - *c*) *~ I ~ D* \* (*N / G*) \* *c*

And this expression we may massage[[4]](#footnote-4) (dividing out common terms) into something easy to comprehend:

(1 - *c*) *~ I / (N \* D) ~* (1 */ G*) \* *c*

We may call the expression in the middle *“P”* meaning “profit threshold.”

(d) (1 - *c*) *~ P ~* (1 */ G*) \* *c*

Let both ( 1 - *c* ) and (1 */ G*) \* *c* be less than *P.* Then *big-boxes* and *mom-and-pops* are in either situation (a) or (b). We may now make an interesting conclusion about our model:

( i ) If ( 1 - *c* ) > ( 1 / *G* ) \* *c*, *big-boxes* have higher income than *mom-and-pops* regardless of population size, and income per visitor. This corresponds to situation (a).

When *c* > 0.5, as our model assumes, then consumers have at least a slight preference for *mom-and-pops*, and the only factor giving *big-boxe*s relative success over *mom-and-pops* is the number of goods sold. For instance, a good mom-and-pop-friendly town might have shop at *mom-and-pops* 75% of the time. However if there are as few as four different types of good, our inequality

(1 - *c*) > ( 1 / *G* ) \* *c*

becomes

0.25 > 0.1875

which is bad news for the mom-and-pop store because there is plenty of space for *P* to fit between those two numbers: say, *E* = $4, *N* = 20, *M* = $1. With these constants, the *mom-and-pops* would have survived without the *big-boxes*, but the *big-boxes* puts too much pressure on the *mom-and-pops*, and the *mom-and-pops* will eventually be run out of business.

We also may conclude that even in extreme cases, where there is a high preference for *mom-and-pops*, *B*s will win the day if too many kinds of goods exist in the economy. Let’s have *c* = 0.99, and *G* = 100. Then the *big-boxes* still have the upper hand:

0.01 > 0.0099

And there is still an interval where *P* can come in and ruin the *mom-and-pops* chance, leaving the *big-boxes* as the only surviving retailers.

What of case (b), where *E* *≥ IM > IB* ? We want to know under what conditions the *mom-and-pops* will run out of funds quicker than the *big-boxes*. Formally, let CX be the capital of X, then our result occurs if and only if the zero of

C*B* + t(I*B* - E) = 0 t = -C*B*/(I*B* - E)

is less than

C*M* +t(I*M* - E) = 0 t = -C*M*/(I*M* - E)

Expressed without t:

-C*B*/(I*B* - E) < -C*M*/(I*M* - E).

Massaging it a bit:

C*B*/C*M* > (I*B* - E)/(I*M* - E) = O*B* / O*M*

which basically means our condition is met whenever the relative capital of *big-boxes* to *mom-and-pops* is greater than their relative operating costs. Examples are easy to fabricate:

CB = 5

CM = 1

OB = 4

OM = 1

That is when the capital of the *big-box* is five times that of the *mom-and-pops* and its operating cost is 4 times that of the *mom-and-pops*, it will survive the *mom-and-pops*.

1. The variables pM and rM(0,1] are between 0 and 1 inclusive. pM is consumer preference for mom-and-pops, and rX is a random number. We abbreviate “mom-and-pop” by *M* and “big-box” by *B.* The subscript indicates the variable belongs to the type of retailer. [↑](#footnote-ref-1)
2. There are 26 consumers, 5 mom-and-pop stores, each of which have an initial endowment of $30, gain $2 per purchase, and loose $10 per step. The big-box appears on period 20. It has an initial endowment 1000 times that of the big-box-store, and it pays five times the rent. [↑](#footnote-ref-2)
3. More specifically c = (0.5) \* pM + 0.5 [↑](#footnote-ref-3)
4. Since all terms are positive, we need not worry about the inequalities’ reversing. [↑](#footnote-ref-4)