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Treating Agent-Based Models as vector spaces

# The nature of the vector space

To be a vector space an algebraic structure must contain a primary set that is an additive group, G, satisfying four group axioms: closure, associativity, identity and invertibility. There is an operator ⊕ which takes two elements of the group and yields a third element and its operation satisfies these axioms.

In addition, a vector space contains a secondary set, R, a ring of coefficients, with a second operation, ⊗, which takes an element of R and an element of G and produces and element of G.

## An Analysis of a Generic ABM Interaction as a Vector Space

### Elements: Following Whitehead (2014), we call the elements of our group *prehensions*. A prehension can be roughly understood as a state of affairs in the world *as seen from a particular point of view*. (In this case the world is the world of our model [see Morgan 2012], but Whitehead views this as a useful metaphysics for the actual world.)

### The operation ⊕, which we will call “prehend”, accepts two prehensions as arguments and produces a third prehension.

#### Axioms:

##### **Closure:** Every prehend involving two prehensions will produce a third prehension.

##### **Associativity:** (a ⊕ b) ⊕ c = a ⊕ (b ⊕ c) In a typical agent model, this will mean that we must ensure that, say, a neighborhood can interact with a neighborhood (b ⊕ c), and then with an agent (a ⊕ (b ⊕ c)). Furthermore, this must produce an identical prehension to that produced by an agent interacting with one neighborhood and then another one ((a ⊕ b) ⊕ c).

##### **Identity:** Any prehension prehending the null prehension remains unchanged.

##### **Invertibility:** For any prehension, there is another prehension that combines with it to produce the null prehension.

### The operation ⊗, which we will call “intensify” (although it may also de-intensify) accepts an element of R and an element of G (a prehension), and produces an element of G.

#### Axioms:

##### a, b ∈ G:

###### (a ⊕ b)x = ax ⊕ bx

###### a(x ⊕ y) = ax ⊕ ay

### Meaning:

#### An agent’s prehension of itself is its view of its own internal state.

#### An agent’s prehension of its environment is its view of its surroundings.

#### But from the point of view of the prehension module, these prehensions are interchangeable.

#### A null prehension could arise, e.g., from the environment when an agent has no neighbors. It could arise internally when an agent has “no opinion” on the relevant parameters, e.g., a color-blind agent in our fashion model.

# The advantages of employing this abstraction

## Why bother?

### We achieve a uniform template for all models. Programming new models becomes much easier.

### We will have taken a huge step towards enabling “fill-in-the-template” style programming of ABMs.

# a sketch of the usual action pattern

An agent gathers together a prehension of its environment, and then combines that with how it presents itself to the environment (how it is prehended) to produce a new prehension, which it adopts as its own. The new prehension may simply be adopted, or it may trigger some further step, such as a move on the part of the agent.

However, while the above may be typical, our model allows the reverse: in some models (e.g., Forest Fire), it may be the environment that adopts the new prehension.

# Bibliography

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