# Employing Markov Matrices as a Generic Agent-based Modeling Pattern

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## Introduction

This paper describes an ongoing effort to treat agent interactions as an abstract algebraic structure. Much work has been done on agent-based modeling (ABM) over the past couple of decades, but it has been focused on capturing specific phenomena through the use of such models. In contrast, our project, inspired by the work of Stepanov and Rose (2015), has sought to find a generic paradigm that can capture the essence of a wide variety of typical agent-based models. In doing so, we have not sought to model brand-new phenomena with ABMs, but instead, to find a generic basis for some of the most common ABMs in use.

The primary motivations for this attempt are to make it easier to capture a wide range of phenomena in ABMs, to increase code reuse, and ultimately to enable the creation of ABMs through the filling in of forms, choosing among various operations and chaining them together to produce customized agent behavior. Thus far, the results have been promising, although there is much work to be done.

After many months of searching, we have found Markov matrices to be a useful mechanism for unifying the action phase of a wide variety of ABMs. The remained of the paper will first look at generic programming, then describe our search for a fruitful generic paradigm for ABMs, and finally describe how Markov matrices have, so far, appeared as the most likely candidate for a generalized ABM “interaction engine.”

## I. Putting the Focus on Generic Programming

## II. Searching for the Right Paradigm

### The “Prehension” as an Abstract Entity in Our Models

Following Whitehead (2014), we call the elements of our structure prehensions. A prehension can be roughly understood as a state of affairs in the world as seen from a particular point of view. (In this case the world is the world of our model [see Morgan 2012], but Whitehead views this as a useful metaphysics for understanding the actual world.)

A module over a ring.

### 

### The Meaning of Prehensions

An agent’s prehension of its environment is its view of its surroundings.

A null prehension could arise, e.g., from the environment when an agent has no neighbors.

We note that this structure essentially means that the combination of an agent state with a prehension implements Aristotle’s notion of the “practical syllogism” in code. (See Aristotle, 350 BCE.)

In our system, the major premise of Aristotle's practical syllogism is the agent's understanding of its own condition, e.g., for a fashion follower in Adam Smith’s fashion model (to be described in more detail later; see Smith 1759):

**Major Premise:** I want the fashion I wear be the same as the fashion of the trendiest people. (My own state vector.)

The minor premise is the agent's understanding of its environment:

**Minor Premise:** The trendy people I see around me are wearing blue. (Enviroment prehension)

The conclusion of an Aristotelian practical syllogism is not a proposition, but an action. Thus, given the major and minor premises above, the "conclusion" of the practical syllogism is:

**Conclusion:** I change my garb to blue. (Agent state vector resulting from combining the major and minor premises.)

Similarly, in Schelling's segregation model (to be described in more detail later; see Schelling 2006), we have:

**Major Premise:** I wish to live in a neighborhood where at least X% of the people are like me. (My own state vector.)

**Minor Premise:** My current neighborhood contains only (X - Y)% of people like me. (Enviroment prehension)

**Conclusion:** I move to a new neighborhood. (New state vector resulting from combining the major and minor premises.)

## III. Markov Matrices as the Basis for Agent Actions?

### Forest Fire Model

### Abelian Sandpile Model

### Adam Smith’s Fashion Model

### Thomas Schelling’s Segregation Model

### A Herding Model

## Conclusion

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