# Employing Markov Chains as a Generic Agent-based Modeling Pattern

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## Introduction

This paper describes an ongoing effort to treat agent interactions as an abstract algebraic structure. Much work has been done on agent-based modeling (ABM) over the past couple of decades, but it has been focused on capturing specific phenomena through the use of such models. In contrast, our project, inspired by the work of Stepanov and Rose (2015), has sought to find a generic paradigm that can capture the essence of a wide variety of typical agent-based models. In doing so, we have not sought to model brand-new phenomena with ABMs, but instead, to find a generic basis for some of the most common ABMs in use.

The primary motivations for this attempt are to make it easier to capture a wide range of phenomena in ABMs, to increase code reuse, and ultimately to enable the creation of ABMs through the filling in of forms, choosing among various operations and chaining them together to produce customized agent behavior. Thus far, the results have been promising, although there is much work to be done.

After many months of searching, we have found Markov chains to be a useful mechanism for unifying the action phase of a wide variety of ABMs. The remained of the paper will first look at generic programming, then describe our search for a fruitful generic paradigm for ABMs, and finally describe how Markov chains have, so far, appeared as the most likely candidate for a generalized ABM “interaction engine.”

## I. Putting the Focus on Generic Programming

## II. Searching for the Right Paradigm

### The “Prehension” as an Abstract Entity in Our Models

Following Whitehead (2014), we call the elements of our structure prehensions. A prehension can be roughly understood as a state of affairs in the world as seen from a particular point of view. (In this case the world is the world of our model [see Morgan 2012], but Whitehead views this as a useful metaphysics for the actual world.)

The operation ⊕, which we will call “prehend”, accepts two prehensions as arguments and produces a third prehension.

Axioms:

Closure: Every prehending involving two prehensions will produce a prehension.

Associativity: (a ⊕ b) ⊕ c = a ⊕ (b ⊕ c)

In a typical agent model, this will mean that we must ensure that, say, a neighborhood can interact with a neighborhood (b ⊕ c), and then with an agent (a ⊕ (b ⊕ c)). Furthermore, this must produce an identical prehension to that produced by an agent interacting with one neighborhood and then another one ((a ⊕ b) ⊕ c).

Identity: Any prehension prehending the null prehension produces itself.

Invertibility: For any prehension, there is another prehension that combines with it to produce the null prehension.

The operation ⊗, which we will call “intensify” (although it may also de-intensify) accepts an element of R and an element of G (a prehension), and produces an element of G.

Axioms:

a, b ∈ G:

(a ⊗ b)x = ax ⊗ bx

a(x ⊗ y) = ax ⊗ ay

### The Meaning of Prehensions

An agent’s prehension of itself is its view of its own internal state.

An agent’s prehension of its environment is its view of its surroundings.

But from the point of view of the prehension module, these prehensions are interchangeable.

A null prehension could arise, e.g., from the environment when an agent has no neighbors. It could arise internally when an agent has “no opinion” on the relevant parameters, e.g., a color-blind agent in our fashion model.

Invertibility may occur, for instance, when an agent has some internal tendency to act in some way (e.g., to move to a new neighborhood or switch fashions) but some force in the environment exactly offsets that tendency (e.g., that “authorities” establish come penalty for so acting).

An intensification of a prehension leaves the elements of the prehension in the same internal relationship, but they are scaled up or down relative to other prehensions. This is useful for capturing situations like the gradual dissipation of an attitude, or increasing fanaticism over time.

We note that this interpretation of prehensions essentially means that the combination of two prehensions to produce a third implements Aristotle’s notion of the “practical syllogism” in code. (See Aristotle, 350 BCE.)

In our system, the major premise of Aristotle's practical syllogism is the agent's understanding of its own condition, e.g., for a fashion follower in Adam Smith’s fashion model (to be described in more detail later; see Smith 1759):

**Major Premise:** I want the fashion I wear be the same as the fashion of the trendiest people. (Self prehension)

The minor premise is the agent's understanding of its environment:

**Minor Premise:** The trendy people I see around me are wearing blue. (Enviroment prehension)

The conclusion of an Aristotelian practical syllogism is not a proposition, but an action. Thus, given the major and minor premises above, the "conclusion" of the practical syllogism is:

**Conclusion:** I change my garb to blue. (Prehension resulting from combining the major and minor premises.)

Similarly, in Schelling's segregation model (to be described in more detail later; see Schelling 2006), we have:

**Major Premise:** I wish to live in a neighborhood where at least X% of the people are like me. (Self prehension)

**Minor Premise:** My current neighborhood contains only (X - Y)% of people like me. (Enviroment prehension)

**Conclusion:** I move to a new neighborhood. (Prehension resulting from combining the major and minor premises.)

## III. Markov Chains as the Basis for Agent Actions?

### Forest Fire Model

### Abelian Sandpile Model

### Adam Smith’s Fashion Model

### Thomas Schelling’s Segregation Model

## Conclusion

## Bibliography

Aristotle, and W. D. Ross (trans.). “Nicomachean Ethics,” 350 BCE. http://classics.mit.edu/Aristotle/nicomachaen.html.

Knuth, Donald E. 1992. *Literate Programming*. Stanford, California: Center for the Study of Language and Information.

Moeller, Thomas L., and Jaime Milstein. 1996a. “Algebraic Representations for Finite-State Machines. I. Monoid-Ring Formulation.” *Linear Algebra and Its Applications* 239 (May 1, 1996): 109–26. doi:10.1016/S0024-3795(96)90006-3.

Moeller, Thomas L., and Jaime Milstein. 1996b. “Algebraic Representations for Finite-State Machines. II. Module Formulation.” *Linear Algebra and Its Applications* 247 (November 1, 1996): 133–50. doi:10.1016/0024-3795(95)00110-7.

Moeller, Thomas L., and Jaime Milstein. 1998. “Generalized Algebraic Structures for the Representation of Discrete Systems.” *Linear Algebra and Its Applications* 274, no. 1 (April 15, 1998): 161–91. doi:10.1016/S0024-3795(97)00327-3.

Morgan, Mary S. 2012. *The World in the Model: How Economists Work and Think*. Cambridge; New York: Cambridge University Press.

Schelling, Thomas C. 2006. *Micromotives and Macrobehavior*. New York: Norton.

Smith, Adam. 1759. *The Theory of Moral Sentiments*. http://www.econlib.org/library/Smith/smMS.html.

Stepanov, Alexander A, and Daniel E Rose. 2015. *From Mathematics to Generic Programming*. Upper Saddle River, NJ [u.a.]: Addison-Wesley.

Whitehead, Alfred North. 2014. *Process and Reality*. [S.l.]: Free Press. http://rbdigital.oneclickdigital.com.