

Assignment for Research and Development / AI

1) **Understanding the model**

- The given curve depends on three parameters:
- - θ : rotation of the main linear component,
- - M : exponential scaling of the sinusoidal oscillation,
- - X : horizontal shift (translation).

2) **Handling missing parameter t **

- The dataset `xy_data.csv` provided only (x, y) points.
- Since the question mentioned $(6 < t < 60)$, t was assumed to be **uniformly distributed** between 6 and 60, corresponding to each data point.

3) **Model formulation**

- The model equations were coded as:
- ``python
- ```
x = t*np.cos(theta) - np.exp(M*t)*np.sin(0.3*t)*np.sin(theta) + X
```
- ```
y = 42 + t*np.sin(theta) + np.exp(M*t)*np.sin(0.3*t)*np.cos(theta)
```

4) Optimization approach

- Defined residuals as differences between predicted and actual (x, y) points.
- Used `scipy.optimize.least_squares` to minimize the sum of squared residuals.
- Applied parameter bounds directly from the question.
- To ensure the optimizer did not get stuck in local minima, a grid search over θ (1° to 49°) was performed with fine optimization for M and X at each step.

5) Result obtained

After fitting:

$\theta=29.5827^\circ, M=-0.05, X=55.0136$

The parameter M reached its lower bound (-0.05), implying a slightly decaying oscillation along the curve.

6) Validation

- Computed the **L1 distance** between the predicted and observed points:
L1 total = 28268.38
- Generated a visual plot comparing the dataset and fitted curve.
The model successfully follows the overall shape and periodic trend of the data.

Libraries used: `pip install numpy pandas scipy matplotlib`

Results

Parameter	Symbol	Value
Angle	θ	29.58268733°
Exponential factor	M	-0.0500
Translation constant	X	55.01357359
L1 Distance	—	28268.37724916

$$x(t) = (t \cos(0.516315) - \exp(-0.050000*t) \sin(0.3*t) \sin(0.516315) + 55.013574)$$

$$y(t) = (42 + t \sin(0.516315) + \exp(-0.050000*t) \sin(0.3*t) \cos(0.516315))$$

$$6 \leq t \leq 60$$

Visualization

