

Assignment for Research and Development / AI

1) **Understanding the model**

- The given curve depends on three parameters:
- - θ : rotation of the main linear component,
- - M : exponential scaling of the sinusoidal oscillation,
- - X : horizontal shift (translation).

2) **Handling missing parameter t**

- The dataset `xy_data.csv` provided only (x, y) points.
- Since the question mentioned $(6 < t < 60)$, t was assumed to be **uniformly distributed** between 6 and 60, corresponding to each data point.

3) **Model formulation**

- The model equations were coded as:
- ```
python
```
- $x = t \cdot \cos(\theta) - \exp(M \cdot t) \cdot \sin(0.3 \cdot t) \cdot \sin(\theta) + X$
- $y = 42 + t \cdot \sin(\theta) + \exp(M \cdot t) \cdot \sin(0.3 \cdot t) \cdot \cos(\theta)$

### 4) **Optimization approach**

- Defined residuals as differences between predicted and actual  $(x, y)$  points.
- Used `scipy.optimize.least_squares` to minimize the sum of squared residuals.
- Applied parameter bounds directly from the question.
- To ensure the optimizer did not get stuck in local minima, a grid search over  $\theta$  ( $1^\circ$  to  $49^\circ$ ) was performed with fine optimization for  $M$  and  $X$  at each step.

### 5) **Result obtained**

After fitting:

$\theta = 29.5827^\circ, M = -0.05, X = 55.0136$

The parameter  $M$  reached its lower bound ( $-0.05$ ), implying a slightly decaying oscillation along the curve.

### 6) **Validation**

- Computed the **L1 distance** between the predicted and observed points:  
**L1 total = 28268.38**
- Generated a visual plot comparing the dataset and fitted curve.  
The model successfully follows the overall shape and periodic trend of the data.

Libraries used: `pip install numpy pandas scipy matplotlib`

## Results

| Parameter            | Symbol   | Value          |
|----------------------|----------|----------------|
| Angle                | $\theta$ | 29.58268733°   |
| Exponential factor   | M        | -0.0500        |
| Translation constant | X        | 55.01357359    |
| L1 Distance          | —        | 28268.37724916 |

$x(t) = (t \cdot \cos(0.516315) - \exp(-0.050000 \cdot t) \cdot \sin(0.3 \cdot t) \cdot \sin(0.516315) + 55.013574)$

$y(t) = (42 + t \cdot \sin(0.516315) + \exp(-0.050000 \cdot t) \cdot \sin(0.3 \cdot t) \cdot \cos(0.516315))$

$6 \leq t \leq 60$

## Visualization

