CSE 431/531: Algorithm Analysis and Design — Fall 2022

Homework 1

Instructor: Shi Li — **Deadline: 9/25/2022**

Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	Total
Max. Score	20	30	30	80
Your Score				

Problem 1. Asymptotic Notations.

(1a) For each pair of functions $f(n)$ and $g(n)$ in the following table, indicate whether $f(n) = O(g(n)), f(n) = \Omega(g(n))$ and $f(n) = \Theta(g(n))$ respectively.

$f(n)$	$g(n)$	O	Ω	Θ
$\log_2(n^3)$	$10\log_2(\sqrt{n})$			
$5n^2 + n$	$n \log n$			
$10n^2 + n + 10$	n^3			
e^n	2^{2n}			

(1b) Prove $\lceil 10n\sqrt{n}\,\rceil = O(n\sqrt{n})$ using the definition of the O-notation.

In the following two problems, we assume every vertex is incident to at least one edge. So we have $n = O(m)$. Then the running time $O(n+m)$ on the slides becomes $O(m)$.

Problem 2: Cycle detection in (undirected) graphs A cycle in an *undirected* graph $G = (V, E)$ is a sequence of $t \geq 3$ *different* vertices $v_1, v_2, \cdots, v_t$ such that $(v_i, v_{i+1}) \in E$ for every $i = 1, 2, \cdots, t-1$ and $(v_t, v_1) \in E$. Given the linked-list representation of an (undirected) graph $G = (V, E)$, design an $O(m)$-time algorithm to decide if G contains a cycle or not; if it contains a cycle, output one (you only need to output one cycle). To output the cycle, you can just output $v_1, v_2, \cdots, v_t$.

If the correctness of the algorithm is easy to see from your pseudo-code, then there is no need to prove the correctness separately. However, you should briefly mention why the algorithm runs in time $O(m)$.

Problem 3: Cycle detection in directed graphs A cycle in a *directed* graph $G = (V, E)$ is a sequence of $t \geq 2$ *different* vertices $v_1, v_2, \cdots, v_t$ such that $(v_i, v_{i+1}) \in E$ for every $i = 1, 2, \cdots, t-1$ and $(v_t, v_1) \in E$. Given the linked-list representation of a directed graph $G = (V, E)$, design an $O(m)$-time algorithm to decide if G contains a cycle or not; if it contains a cycle, output one (you only need to output one cycle). To output the cycle, you can just output $v_1, v_2, \cdots, v_t$.

If the correctness of the algorithm is easy to see from your pseudo-code, then there is no need to prove the correctness separately. However, you should briefly mention why the algorithm runs in time $O(m)$.

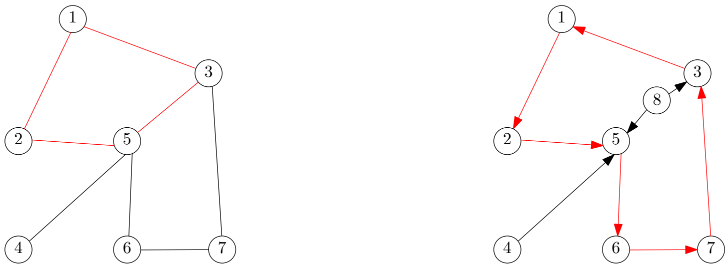

Figure 1: Cycles in undirected and directed graphs are denoted as red edges. (1, 2, 5, 3) is a cycle in the undirected graph. (1, 2, 5, 6, 7, 3) is a cycle in the directed graph. However, (1, 2, 5, 8, 3) is not a cycle in the directed graph.

Remark In a cycle of a directed graph, the directions of the edges have to be consistent. See Figure 1. So, converting a directed graph to a undirected graph and then using algorithm for Problem 2 does not give you a correct algorithm for Problem 3.