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Comparative Analysis of various Direction Of Arrival Estimation Algorithms

Minor Project Presentation

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Presentation Outline

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3. Problem Statement
4. Motivation
5. DOA Algorithms
6. Beamforming Algorithms
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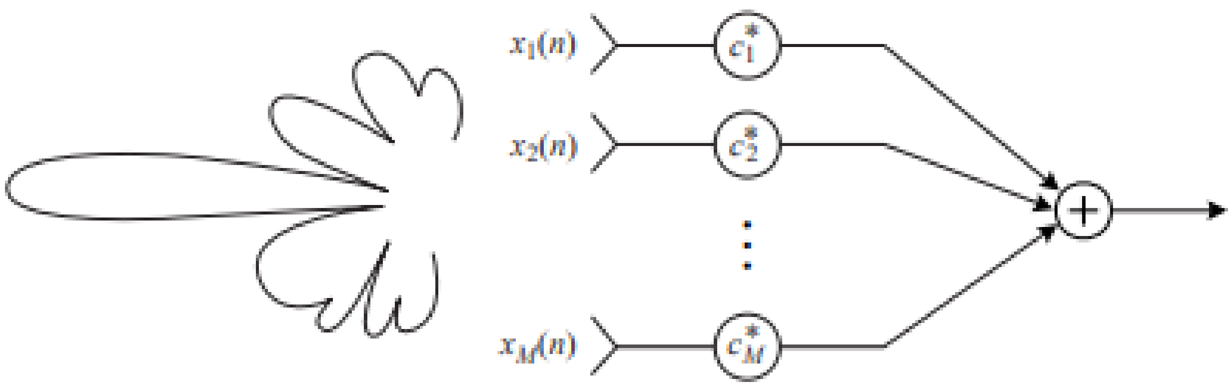
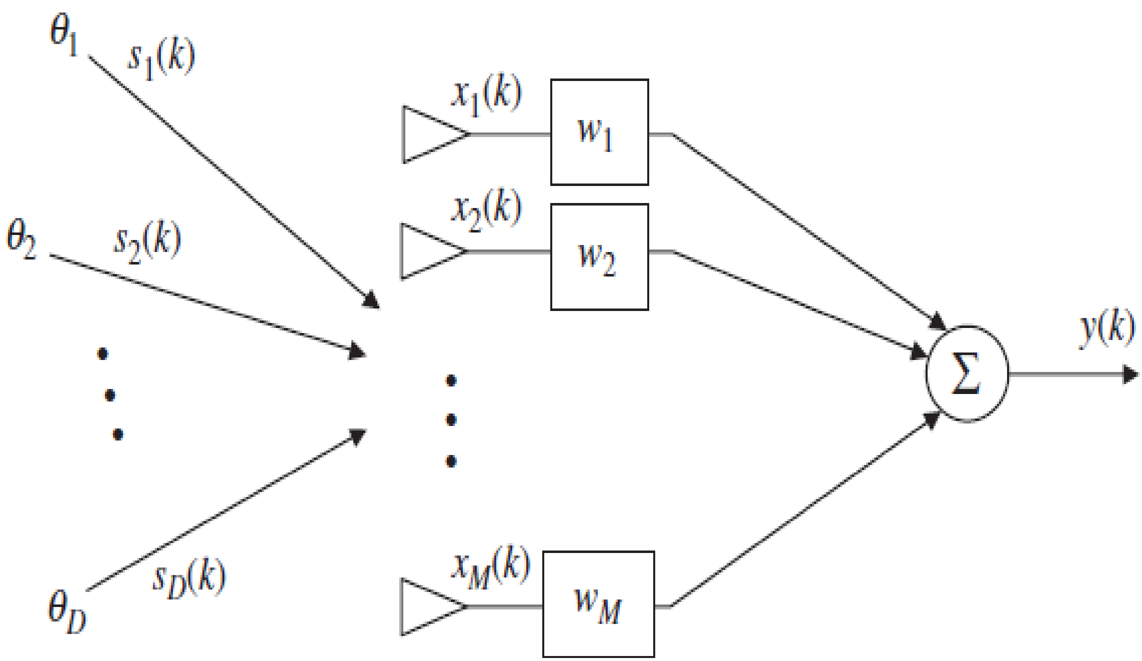
Introduction

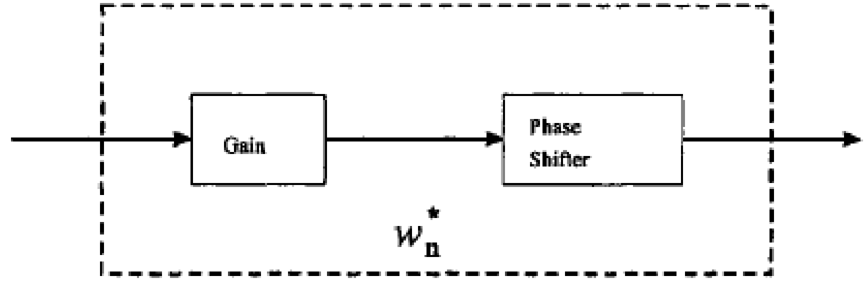
- Array processing deals with techniques for the analysis and processing of signals collected by a group of sensors.
- The collection of sensors makes up the array, and the manner in which the signals from the sensors are combined and handled constitutes the processing.
- The type of processing is dictated by the needs of the particular application.
- Array processing has found widespread application in a large number of areas, including radar, sonar, communications, seismology, geophysical prospecting for oil and natural gas, diagnostic ultrasound, and multichannel audio systems.

Beamforming

- Generally, an array receives spatially propagating signals and processes them to emphasize signals arriving from a certain direction; that is, it acts as a spatially discriminating filter.
- This spatial filtering operation is known as beamforming, because essentially it emulates the function of a mechanically steered antenna.
- An array processor steers a beam to a particular direction by computing a properly weighted sum of the individual sensor signals.







Principle of DOA estimation

Literature Survey

Ref No.	Paper Title	Journal / Conference	Major Observations
1.	Smoothing Periodograms from Time Series with Continuous Spectra	Nature, 161:686- 87, 1948	Description of Bartlett's method for spectral estimation
2.	High-resolution frequency-wavenumber spectrum analysis	Proceedings of the IEEE, vol. 57, no. 8, pp. 1408–1418, 1969	MVDR Beamformer for conventional beamforming and DOA estimation
3	ESPRIT-Estimation of Signal Parameters via Rotational Invariance Techniques	IEEE Trans on Acoustics Speech and Signal Processing. July 1989. Vol. 37. No.7 pp 984~995.	An algorithm which exploits the rotational invariance of signal subspace induced by sensor array

Literature Survey

Ref No.	Paper Title	Journal / Conference	Major Observations
4.	Multiple Emitter Location and signal Parameter Estimation	IEEE Trans. On Antennas and Propagation, March 1986. Vol. 34. No. 3. pp 276-280.	Introduction and description of MUSIC algorithm for DOA
5.	Two decades of array signal processing research: The parametric approach	IEEE Signal Processing Magazine, vol. 13, no. 4, pp. 67–94, Jul. 1996.	Brief Overview of Current trends in Array Signal Processing

Problem Statement

Performance Analysis of various Direction Of Arrival algorithms :

- Bartlett’s Method
- MVDR
- MUSIC
- ESPRIT

Simulation of beamforming using conventional and adaptive algorithms

Motivation

- DOA estimation is a key research area with various engineering applications, such as wireless communications, radar, tracking of various objects, earthquake, medicine etc.

- Positioning and Location-Awareness are key objectives in networks of the future

- One way to accomplish this is via array signal processing with algorithms which estimate the direction-of-arrival (DOA) of the received waves from various sources.

Objectives

1. Study and analysis of various DOA algorithms using MATLAB

   (Specifically Bartlett, MVDR, MUSIC, ESPRIT for linear arrays)

2. Study and simulation of planar arrays and DOA using MVDR and MUSIC algorithm.

3. Study and simulation of beamformers using conventional and adaptive algorithms

   (Specifically conventional, optimal MVDR, SMI)

Methodology

1 • Simulation of various DOA algorithms on MATLAB

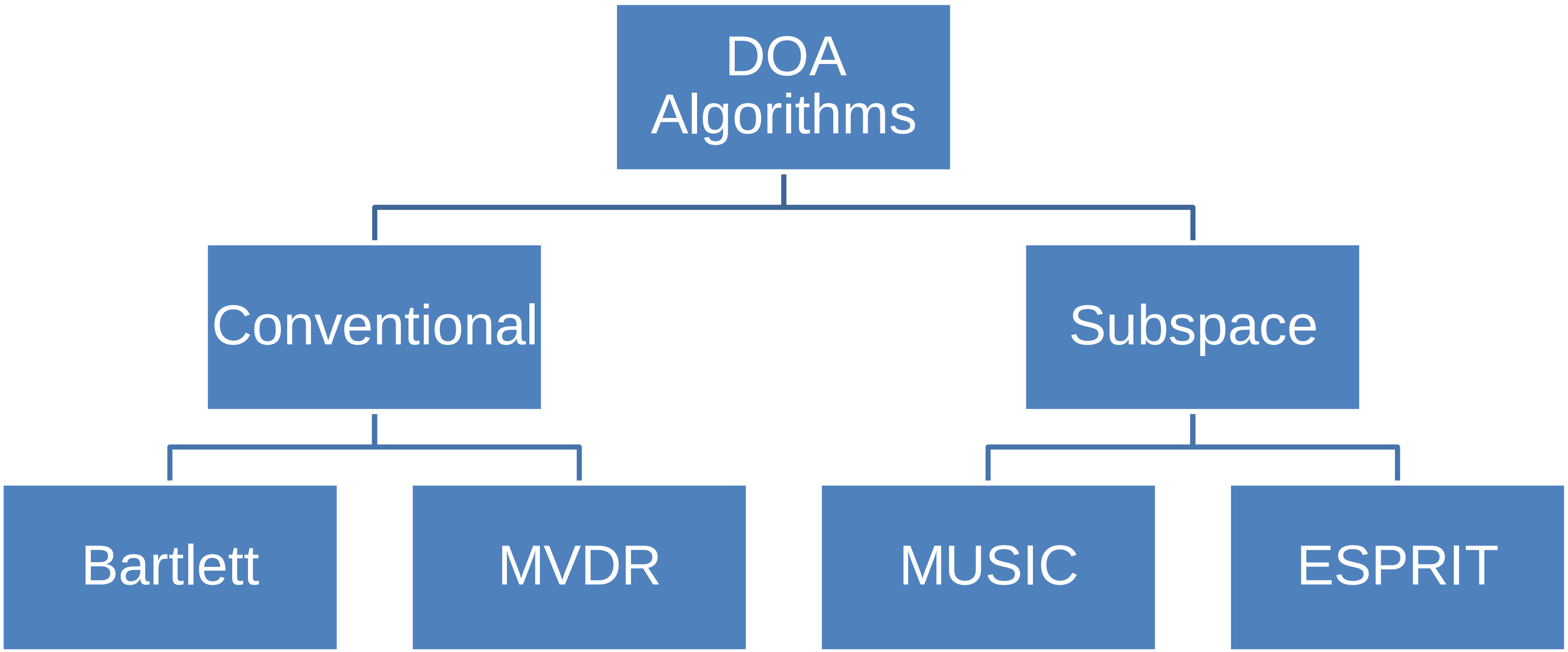
2 • Performance analysis of algorithms on MATLAB based on parameters like SNR, No. of Snapshots and Resolution.

3 • Modifying the standard MUSIC for improved resolution and source coherence issue

4 • Extending linear array to a planar array and implementing planar MVDR and planar MUSIC

5 • Simulating Adaptive Beamformers based on conventional, optimal and adaptive algorithms

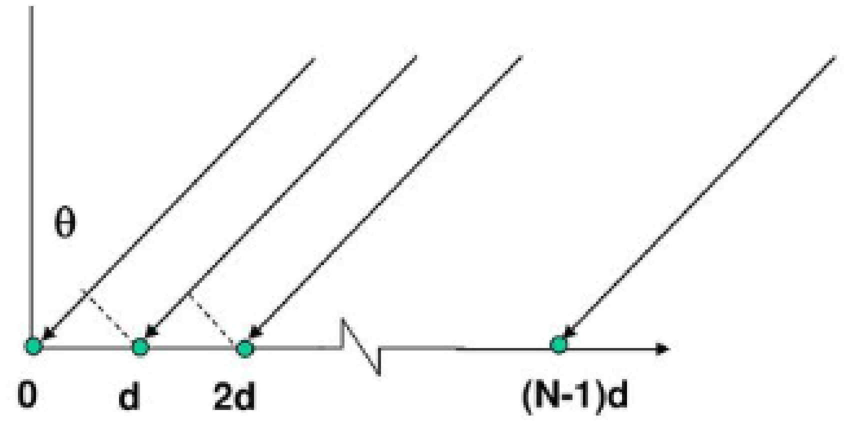
DOA Algorithms



Array Signal Model

- Path Difference=$dsin\theta$
- Phase Difference=ψ(θ)

$$\psi(\theta)=\frac{2\pi dsin\theta}{\lambda}$$



- For N element array, we define array steering vector A or a(θ) as

$$A= [e^{-j\Psi*0} \quad e^{-j\Psi*1} \ldots \quad e^{-j\Psi*(N-1)}]$$

- Received signal $x(t) = a(\theta_0) * s(t) + n(t)$

$$X = A * S + N$$

- Received Correlation Matrix: $R_{xx} = X * X^H$

Bartlett's Algorithm

- It is also referred as conventional or delay sum model
- It assumes the channel to be noiseless.
- It magnifies the signal from certain direction by compensating phase shift due to array antenna
- Message signal Y $= W^H X = W^H * (A * S + N)$
- If we do not consider noise, then

$$W^H = A \; for \; Y = S$$

- Power of signal $P = E[Y] = YY^H = AHXX^HA = AHRx_{xA}$
- Power spectrum $P_{Bartlett}(\theta) = a(\theta)^H R_{xx} a(\theta)$

Limitations

- Primitive method.
- Very low resolution

Transmitte r → Channel → Receiver → Spectral Estimation → Plotting Power Spectrum

Transmitte r	Channel	Receiver	Spectral Estimation	Plotting Power Spectrum
$A*S(t)$	$A*S(t) + N$	$Y = W^H *(A*S(t) + N)$	$E(Y)$	$P_{Bartlett}(\theta) = a(\theta)^H R_{xx} a(\theta)$

Where $W^H = A$

MVDR

- Also called Capon's method.
- It stands for minimum variance distortion-less response
- It maintains signal from desired direction (distortion-less $\Rightarrow W^H A = I$) and minimise signal from other directions/noise (minimum variance of noise $\Rightarrow min\{W^H R_{xx} W\}$
- Solving the constraints we get $W = \frac{{R_{xx}}^{-1} A^H}{A{R_{xx}}^{-1} A^H}$

- Power of signal $P = E[Y] = YY^H = W^H XX^H W$

$$= W^H R_{xx} W = \frac{1}{A R_{xx}{}^{-1} A^H}$$

- Power spectrum $P_{MVDR}(\theta) = \frac{1}{a(\theta) R_{xx}{}^{-1} a(\theta)^H}$

Limitations

- Resolution is significantly enhanced compared to Bartlett but still not good enough
- Subspace approaches yield better results

Transmitter → Channel → Receiver → Spectral Estimation → Plotting Power Spectrum

*A*S(t)*

*A*S(t) + N*

$Y = W^H * (A*S(t) + N)$
where,

$$W = \frac{R_{xx}{}^{-1} A^H}{A R_{xx}{}^{-1} A^H}$$

E(Y)

$$P_{MVDR}(\theta) = \frac{1}{a(\theta) R_{xx}{}^{-1} a(\theta)^H}$$

MUSIC

- MUSIC stands for **Mu**ltiple **Si**gnal **C**lassification.
- MUSIC was proposed by R.O.Schmidt.
- MUSIC conducts characteristic decomposition for the covariance matrix of any array output data, resulting in both signal subspace and noise subspace.
- MUSIC detects DOA by a spectral peak search of these subspaces

- Consider Received correlation matrix
- $R_{xx} = E[XX^H] = E[ASS^HA^H] + E[NN^H]$

  $= AE[SS^H]A^H + \sigma^2 I$

  $= AR_{ss}A^H + \sigma^2 I$
- When R_{xx} is decomposed into eigen values. It contains N eigen vectors of which M belong to signal space and (M-N) belong to noise space.
- $E[A^HQ_N]$ is random distribution but when $\theta = \theta_0$, it drops drastically as signal space vectors dominate at Direction of Arrival θ_0.

- Hence the plot of $P = \frac{1}{A^H Q_N {Q_N}^H A}$ gives peaks at θ_0
- Power spectrum $P_{MUSIC}(\theta) = \frac{1}{a(\theta)^H Q_N {Q_N}^H a(\theta)}$

Advantage

- MUSIC has high resolution, accuracy and stability to detect DOA signals.

Transmitte r → Channel → Receiver → Find Noise subspace of R_{xx} → Plotting Power Spectrum

*A*S(t)*

*A*S(t) + N*

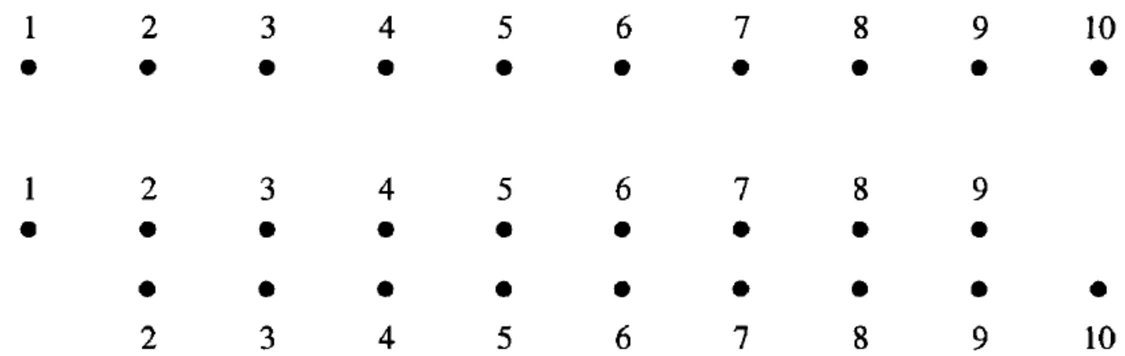
$R_{xx} = AR_{ss}A^H + \sigma^2 I$

$E[A^H Q_N]$

$P_{MUSIC}(\theta) = \frac{1}{a(\theta)^H Q_N Q_N{}^H a(\theta)}$

ESPRIT

- It stands for Estimation of Signal Parameters via Rotational Invariance Techniques
- We assume by rotational invariance property of ULA that array is composed of 2 sub arrays with array manifold vectors A_1 & A_2



- Received signals can be written as
- $X_1 = A_1 S + N_1 \qquad\qquad X_2 = A_2 S + N_2$
- Let $A2 = A1\Phi$
- Where $\Phi = \begin{bmatrix} e^{j\psi_0} & 0 & 0 & 0 \\ 0 & e^{j\psi_1} & 0 & 0 \\ 0 & 0 & e^{j\psi_{(M-1)}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\psi_i = \frac{2\pi d}{\lambda} \sin \theta_i$$

- Now we have 2 received correlation matrix
- $R_{xx1} = A_1 R_{ss1} {A_1}^H + \sigma^2 I$
- $R_{xx2} = A_2 R_{ss2} {A_2}^H + \sigma^2 I$
- ESPRIT implied that $R_{ss1}\Psi = R_{ss2}$
- Where Eigen values of Ψ (λ_i's)=diagonal elements of Φ
- Hence $\theta_i = \sin^{-1}\left(\frac{\arg(\lambda_i)}{2\pi d/\lambda}\right)$

Beamformers

- Beamformers
  - Conventional
  - Optimal- MVDR
  - Adaptive - SMI

Conventional Beamformer (CBR)

- The CBR is optimal to white noise only.
- It does not take into account other signals(interference) present in some directions.

- Constraint: $w^H a(\theta_s) = 1$

- Solution: $w_{CBF} = \dfrac{a(\theta_s)}{a^H(\theta_s)a(\theta_s)}$

Optimal MVDR

- Optimal Beamformers take into account other signals (interferes).
- It tends to place a null towards interference signals
- MVDR is one of the optimal Beamformers.
- Constraint: $min\{W^H C W\}$ $subject\ to$

  $W^H a(\theta_s) = I$
- Solution: $W_{MVDR} = \dfrac{C^{-1} a(\theta_s)}{a^H(\theta_s) C^{-1} a(\theta_s)}$

Adaptive SMI

- Adaptive beamformer is improvisation of Optimal Beamformer.
- Optimal beamformers consider the interference and channel characteristics as fixed statistics.
- But in practice, this is not the case.
- Hence we require Adaptive Beamformers which change their weights adaptively based on interference characteristics

Details of Software /Hardware

- MATLAB

Results

I. Conducted Simulation of:

- Bartlett
- MVDR
- MUSIC

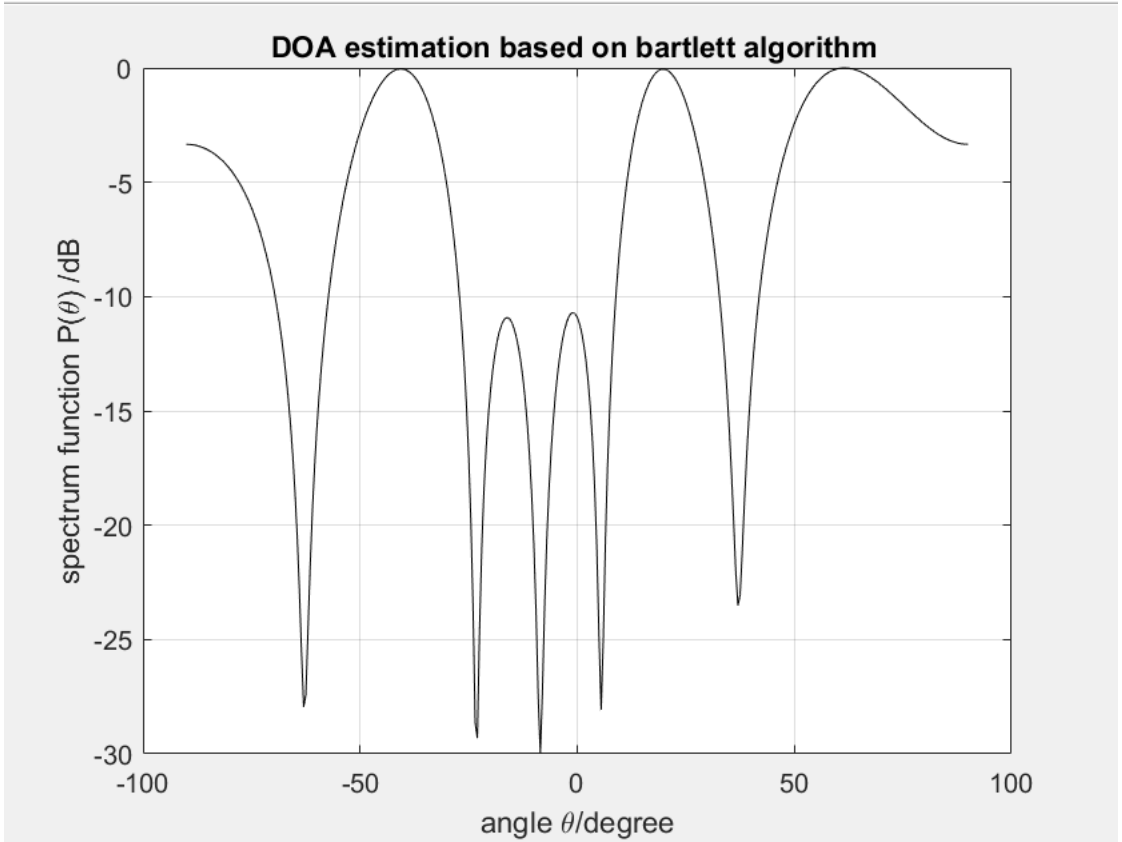
Static Parameters:

Number of elements : 8

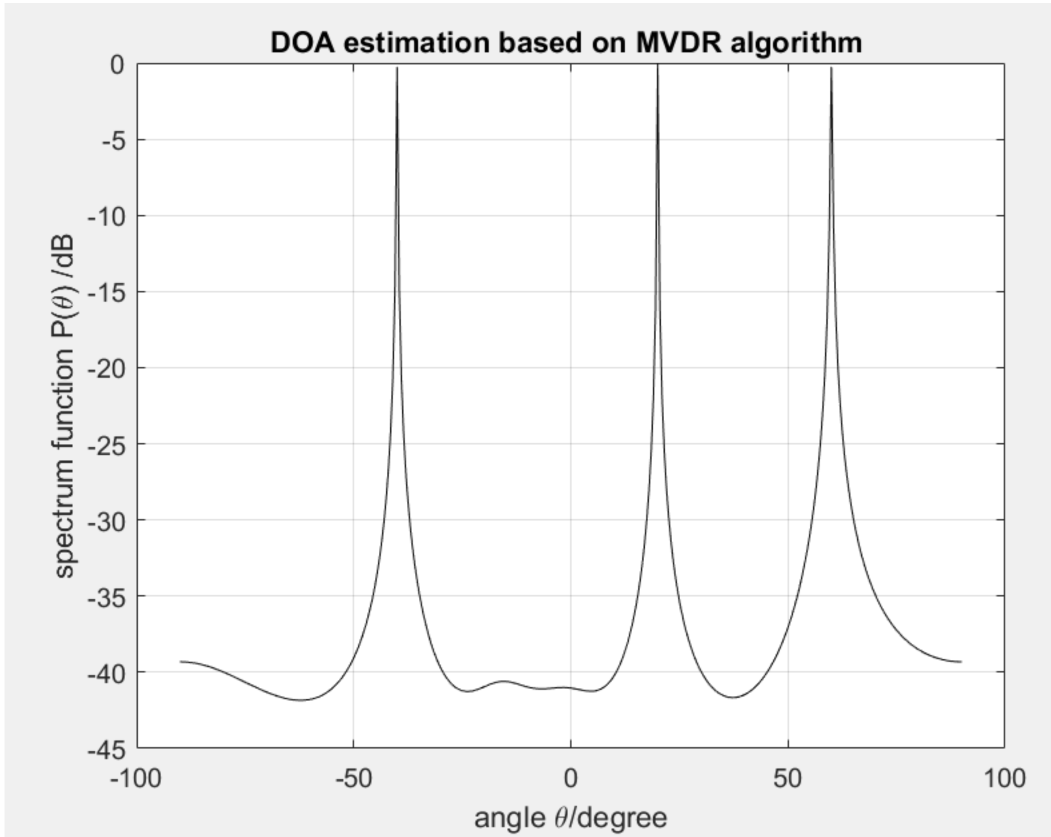
Snapshots : 200

SNR : 20db

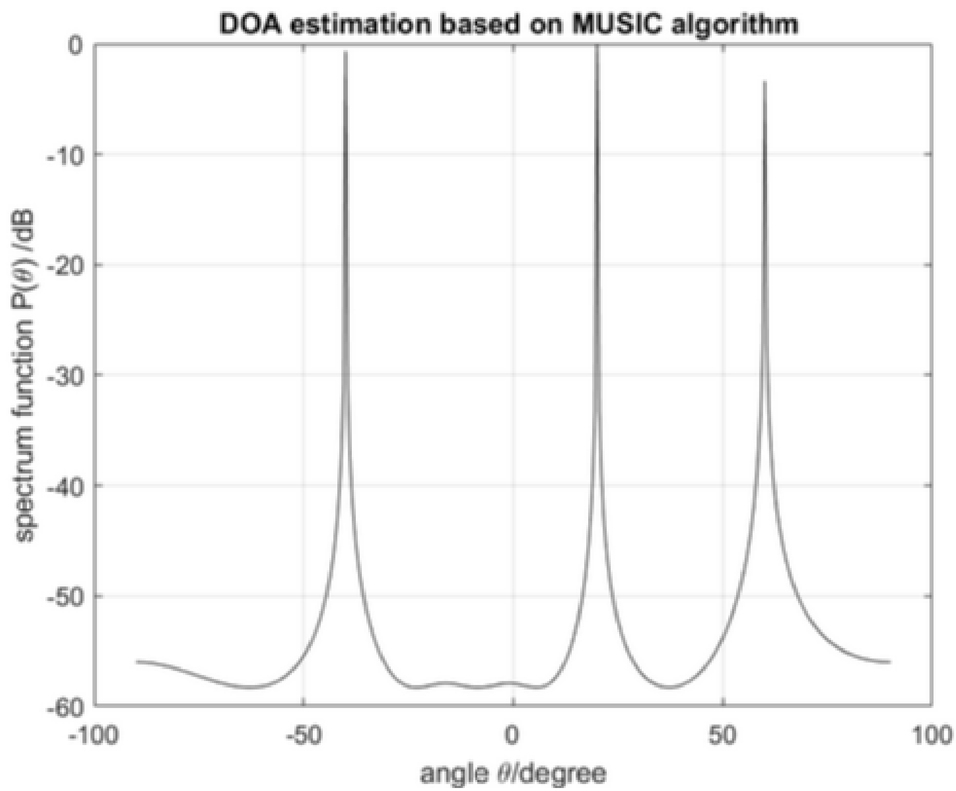
Source : -40^{o} 20^{o} 60^{o}



Bartlett's Algorithm


DOA estimation based on MVDR algorithm
spectrum function P(θ) /dB
angle θ/degree
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-100
-50
0
50
100

MVDR


DOA estimation based on MUSIC algorithm
spectrum function P(θ) /dB
angle θ/degree
0
-10
-20
-30
-40
-50
-60
-100
-50
0
50
100

MUSIC

Results (cont)

II. Conducted performance analysis of:

- Bartlett
- MVDR
- MUSIC

Following parameters were varied:

- SNR
- Resolution

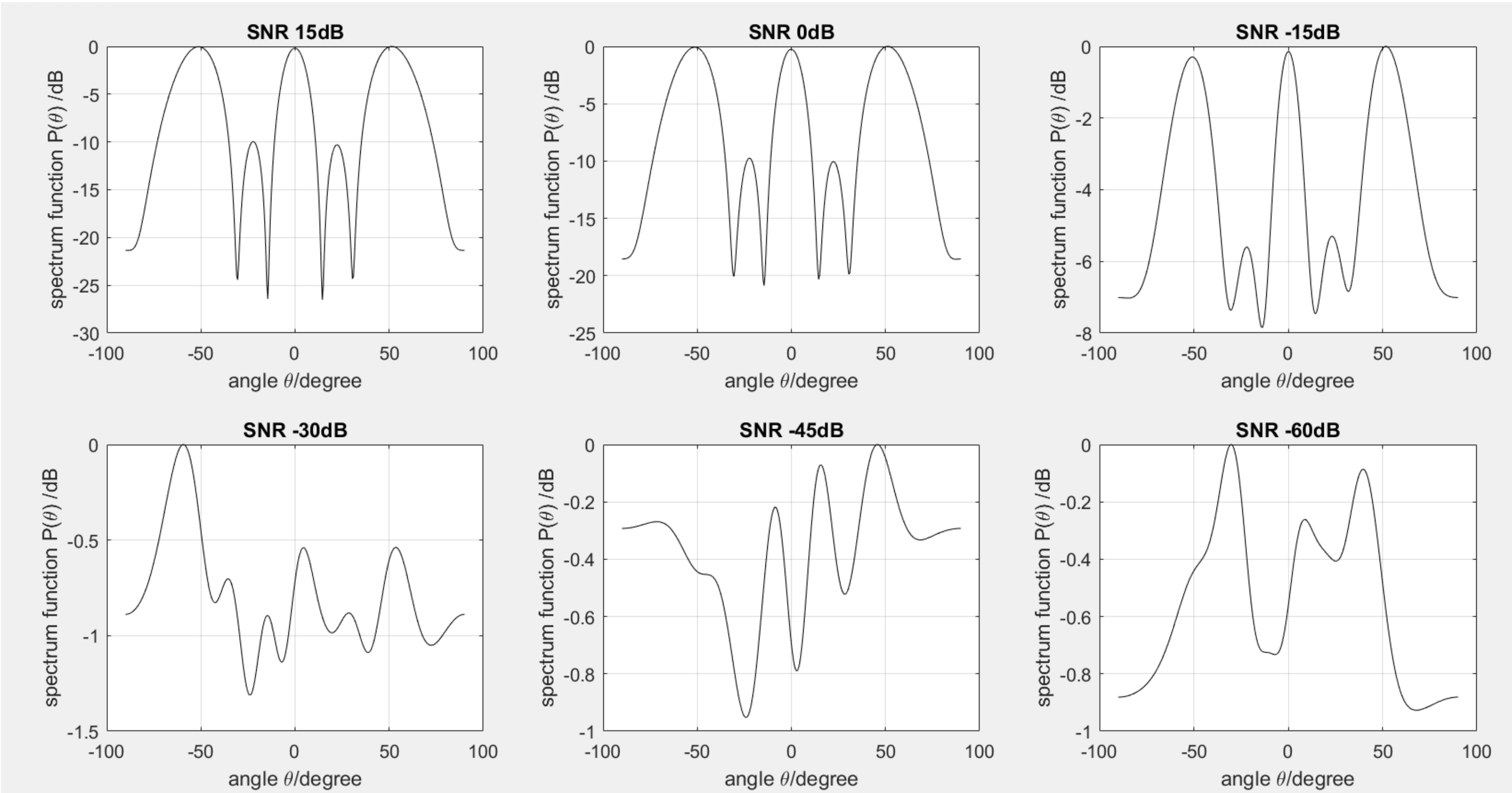
Measurement of SNR

Static Parameters:

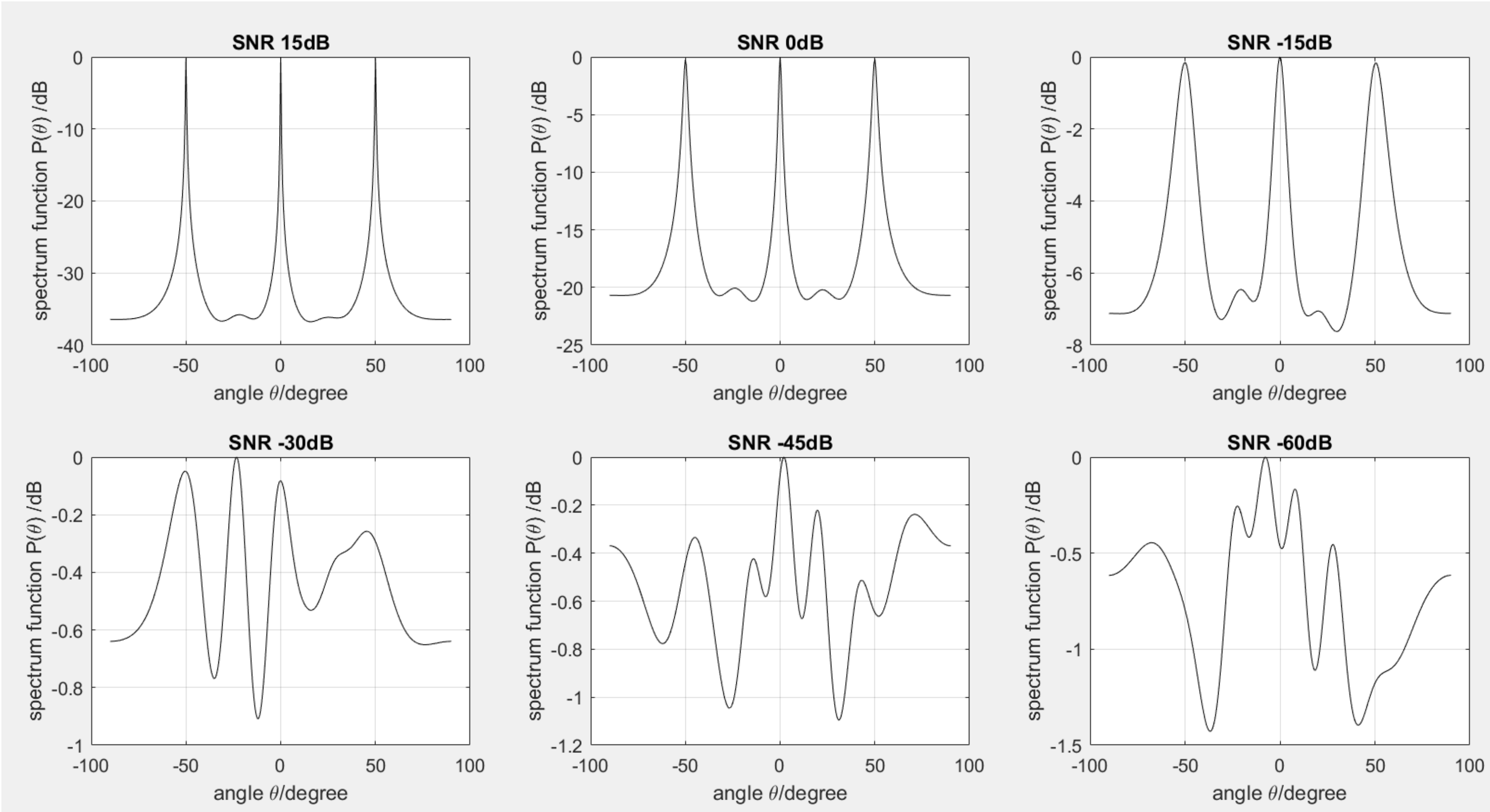
Number of elements : 8

Snapshots : 200

Source : -50^{o} 0^{o} 50^{o}

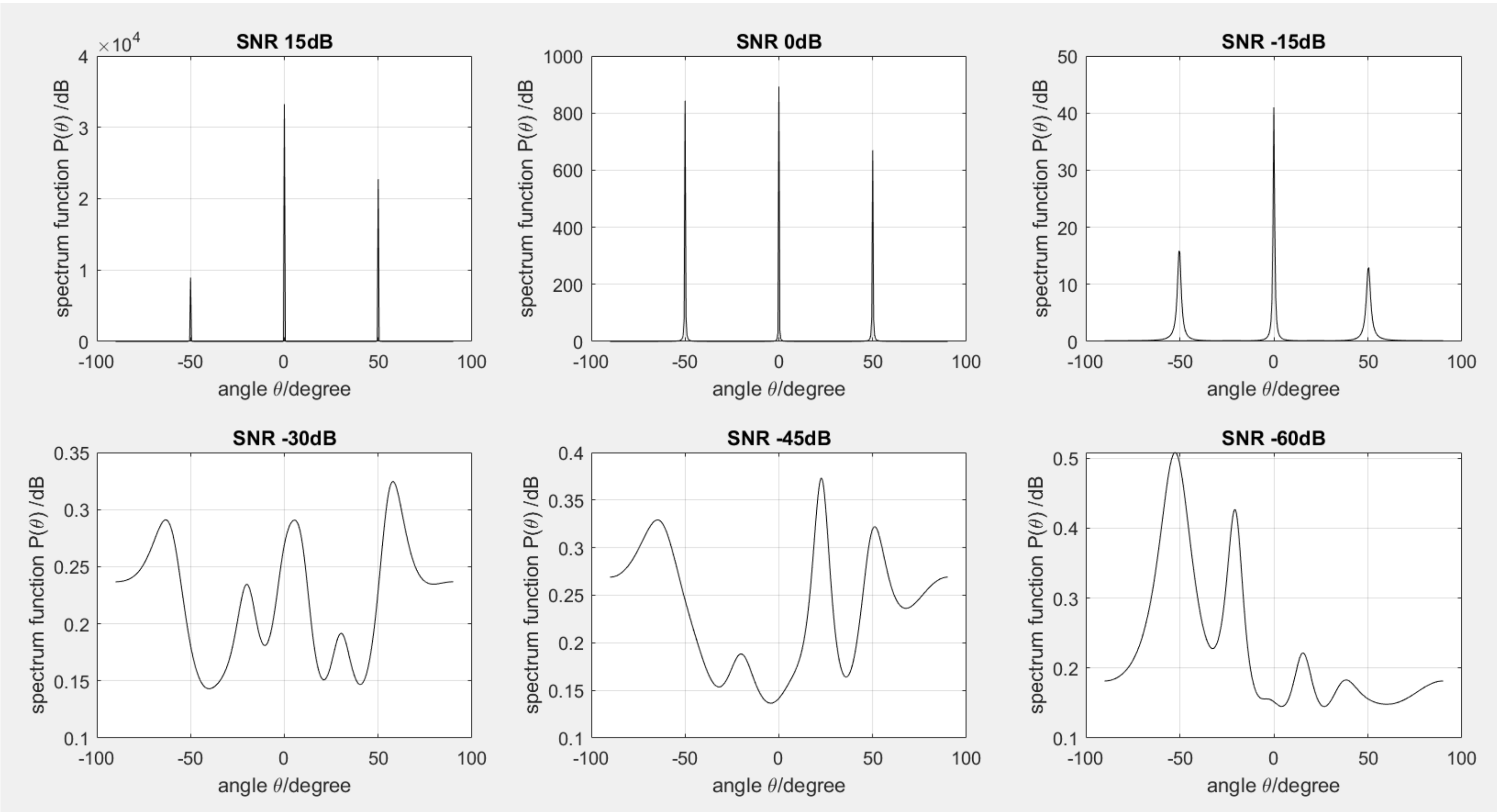


Bartlett's Algorithm



SNR 15dB
SNR 0dB
SNR -15dB
SNR -30dB
SNR -45dB
SNR -60dB
spectrum function P(θ) /dB
angle θ/degree

MVDR


SNR 15dB
SNR 0dB
SNR -15dB
SNR -30dB
SNR -45dB
SNR -60dB
spectrum function P(θ) /dB
angle θ/degree

MUSIC

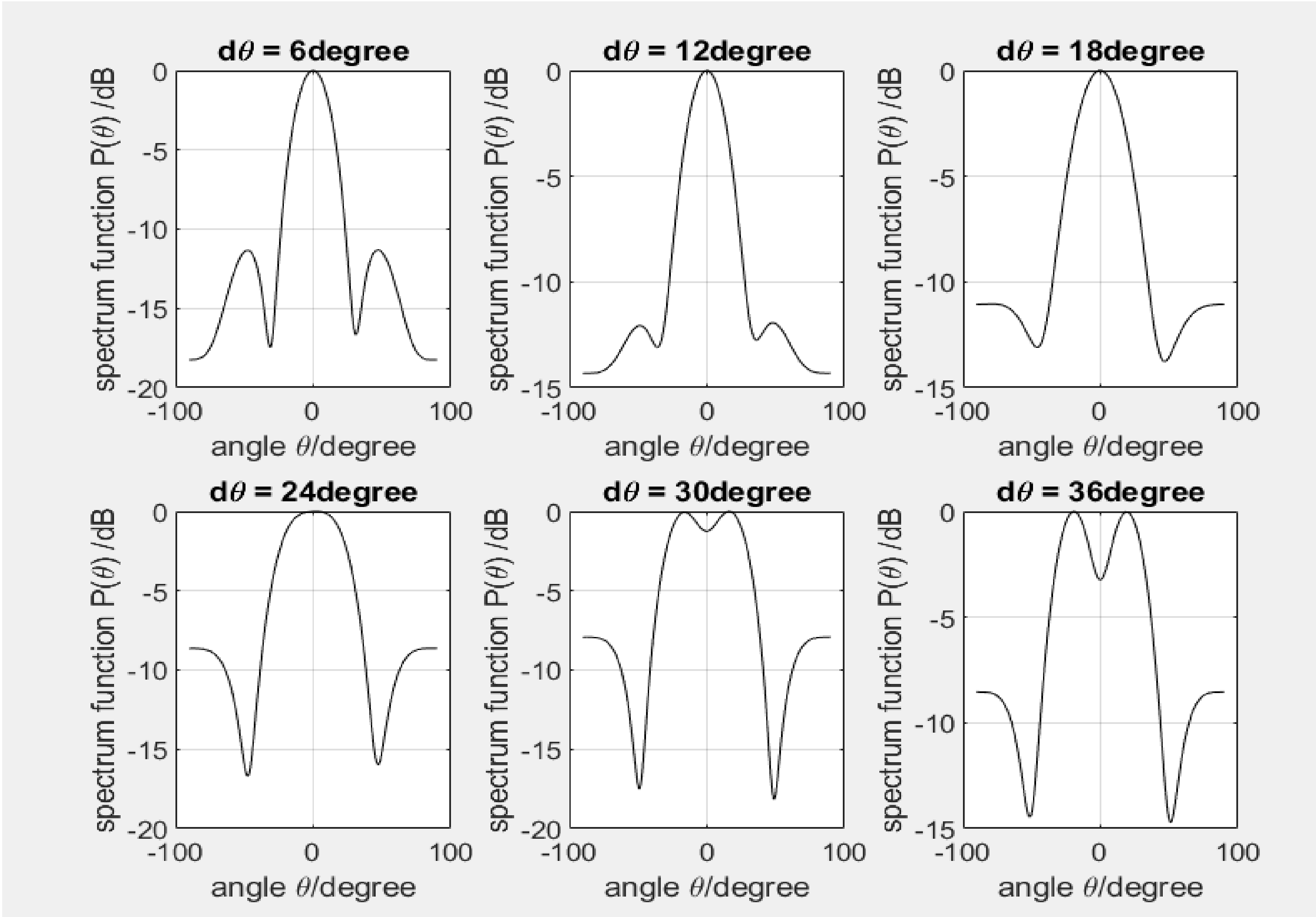
Measurement of Resolution

Static Parameters:

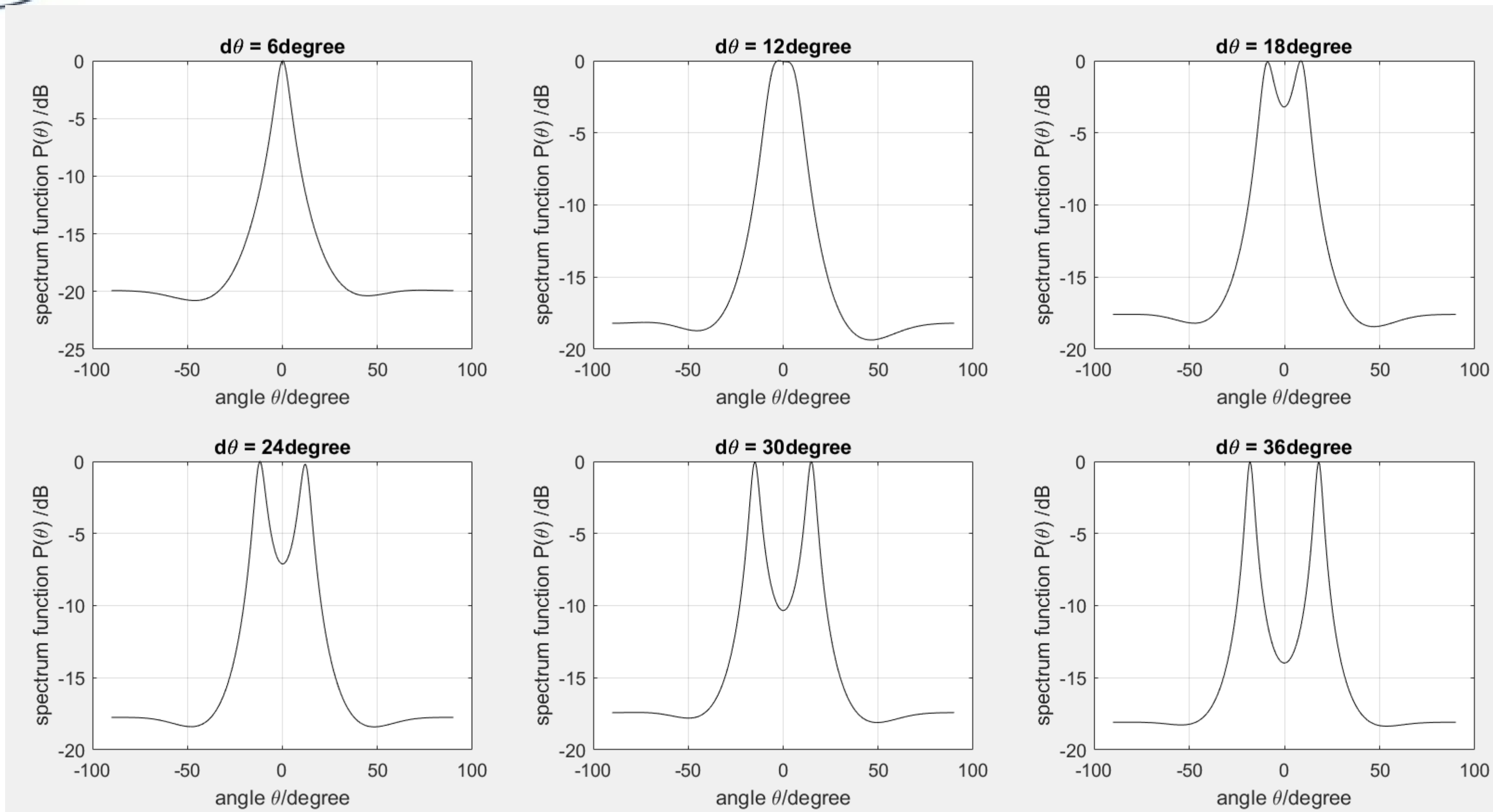
Number of elements : 8

Snapshots : 200

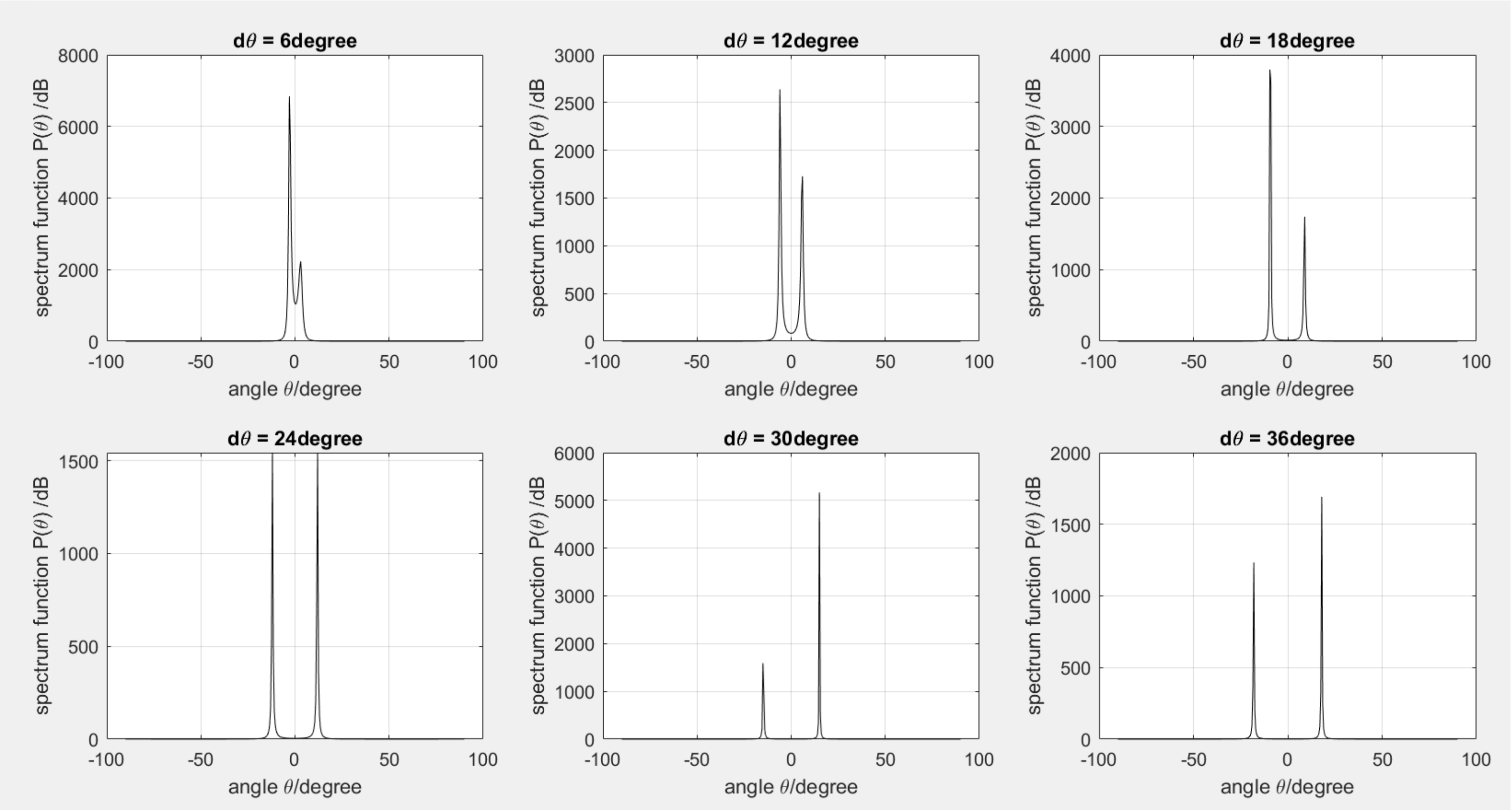
SNR : 0db



Bartlett’s Algorithm


dθ = 6degree
dθ = 12degree
dθ = 18degree
dθ = 24degree
dθ = 30degree
dθ = 36degree
spectrum function P(θ) /dB
angle θ/degree

MVDR


dθ = 6degree
dθ = 12degree
dθ = 18degree
dθ = 24degree
dθ = 30degree
dθ = 36degree
spectrum function P(θ) /dB
angle θ/degree

MUSIC

Comparison of Different Algorithm

Algorithm	SNR (Lowest distinguishable)	Resolution
Bartlett	-15dB	24^{o}
MVDR	-15dB (Higher Accuracy than Bartlett)	18^{o}
MUSIC	-30dB	6^{o}

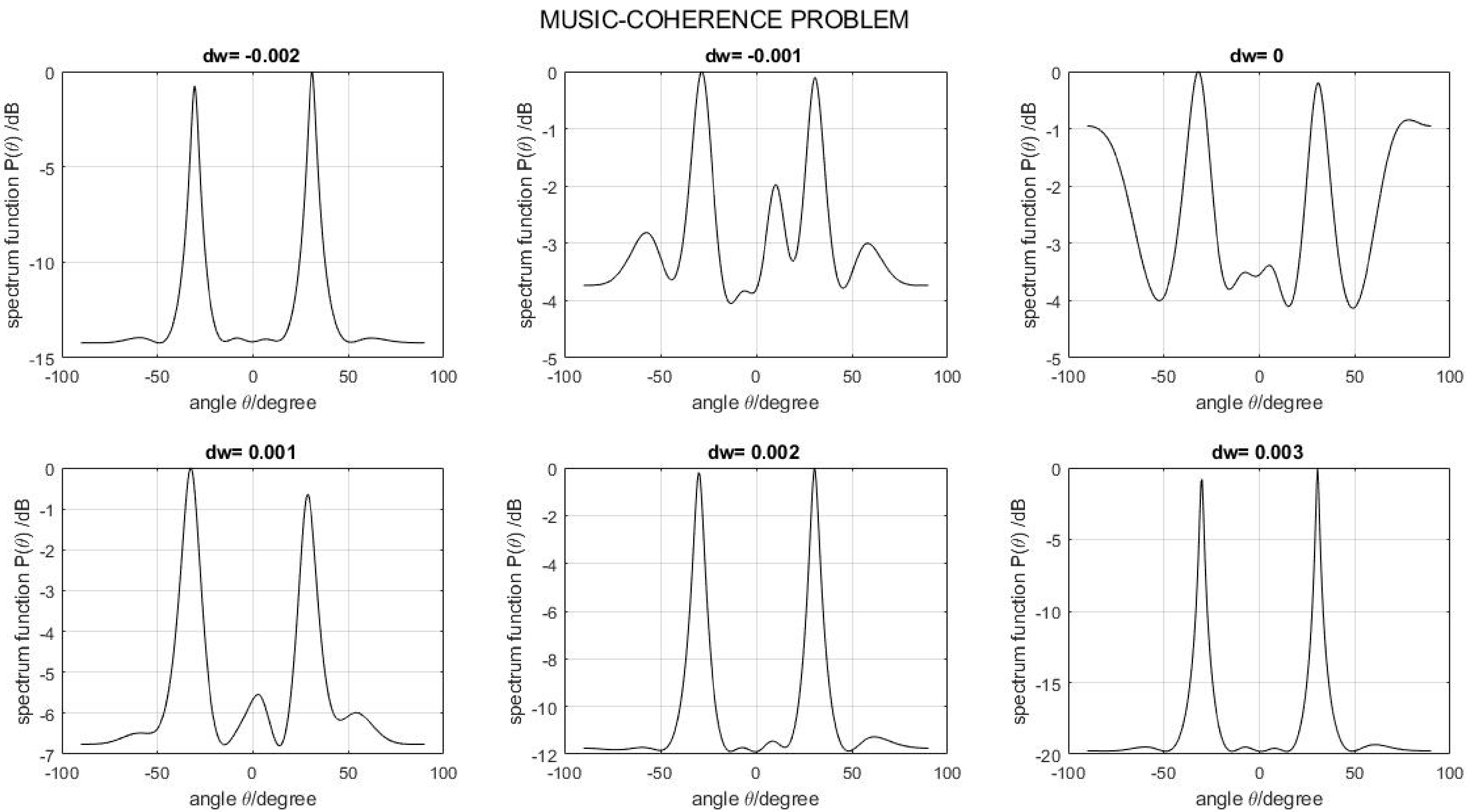
Results (cont)

III. Conducted detailed performance analysis of:

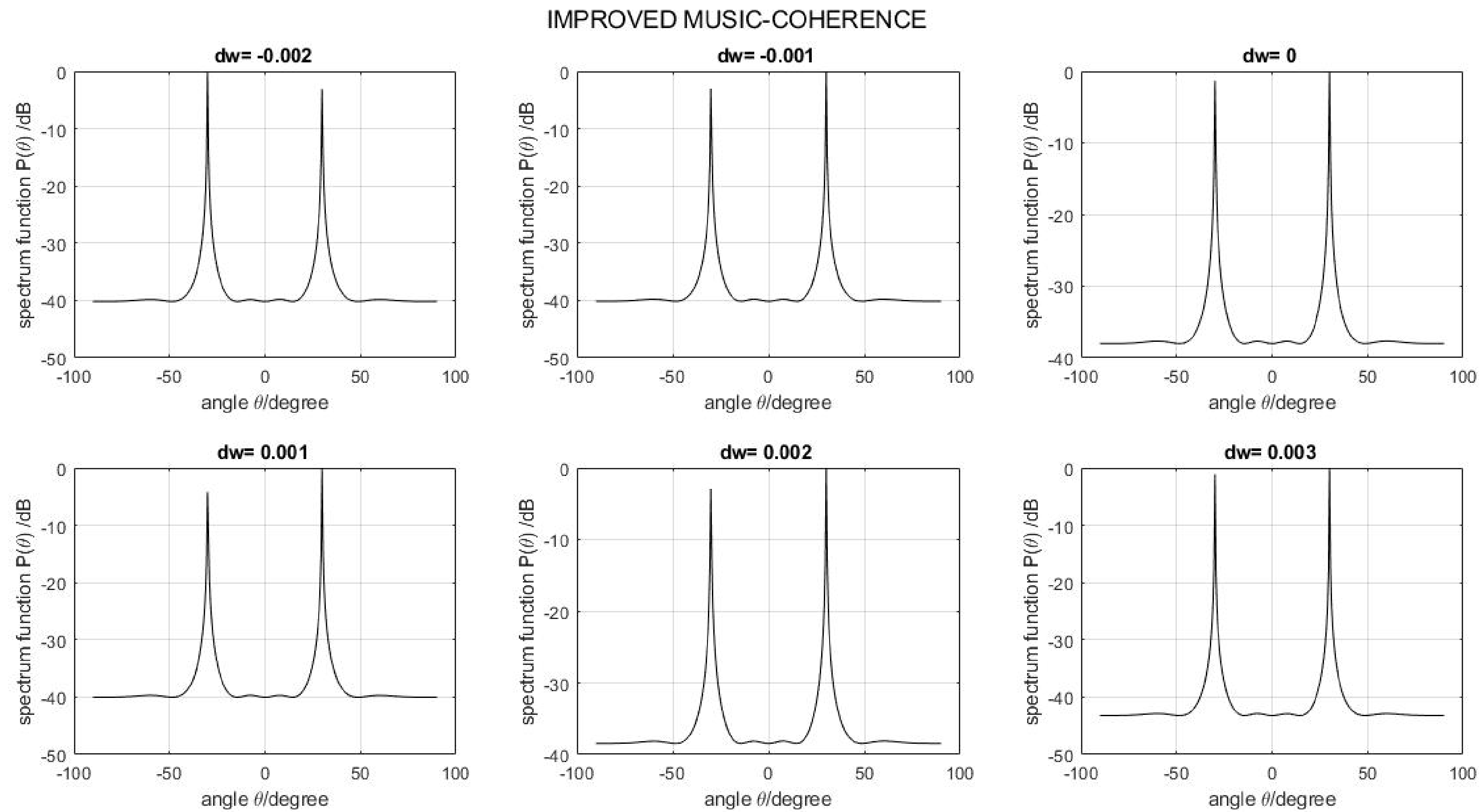
- MUSIC

Following parameters were varied:

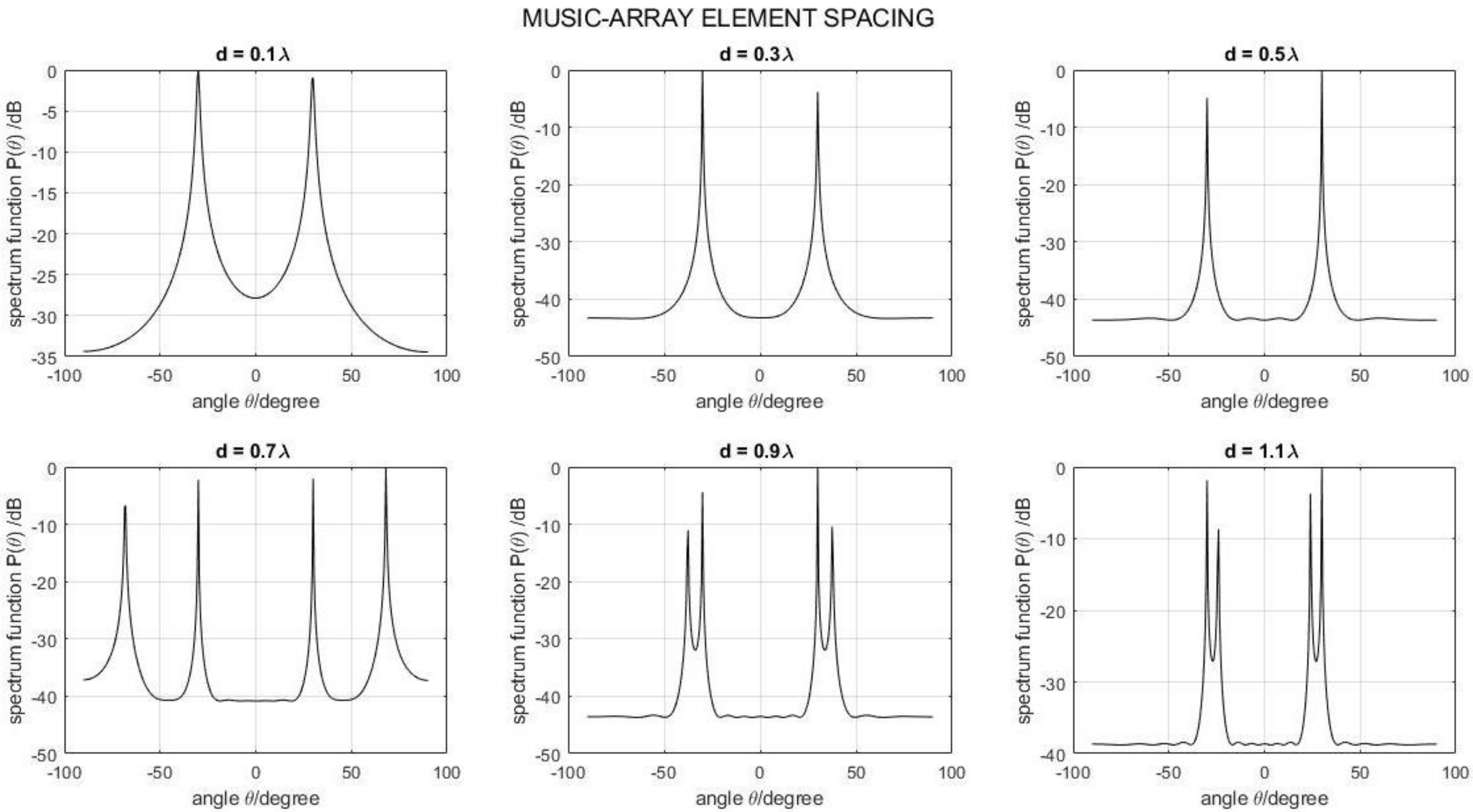
- SNR
- Resolution
- Number of Snapshots
- Number of Array elements
- Distance between array elements
- Coherence



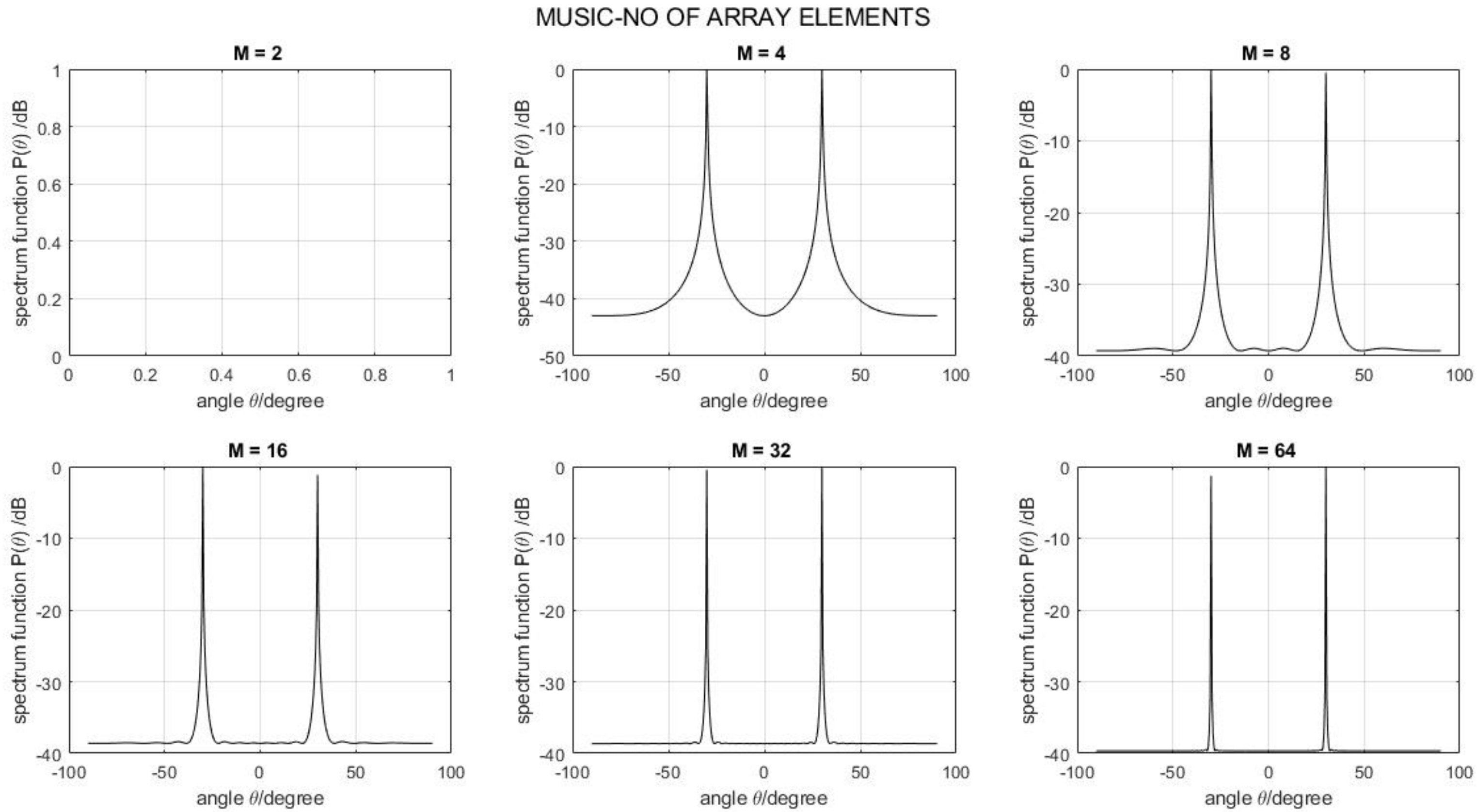
MUSIC – Coherence Problem



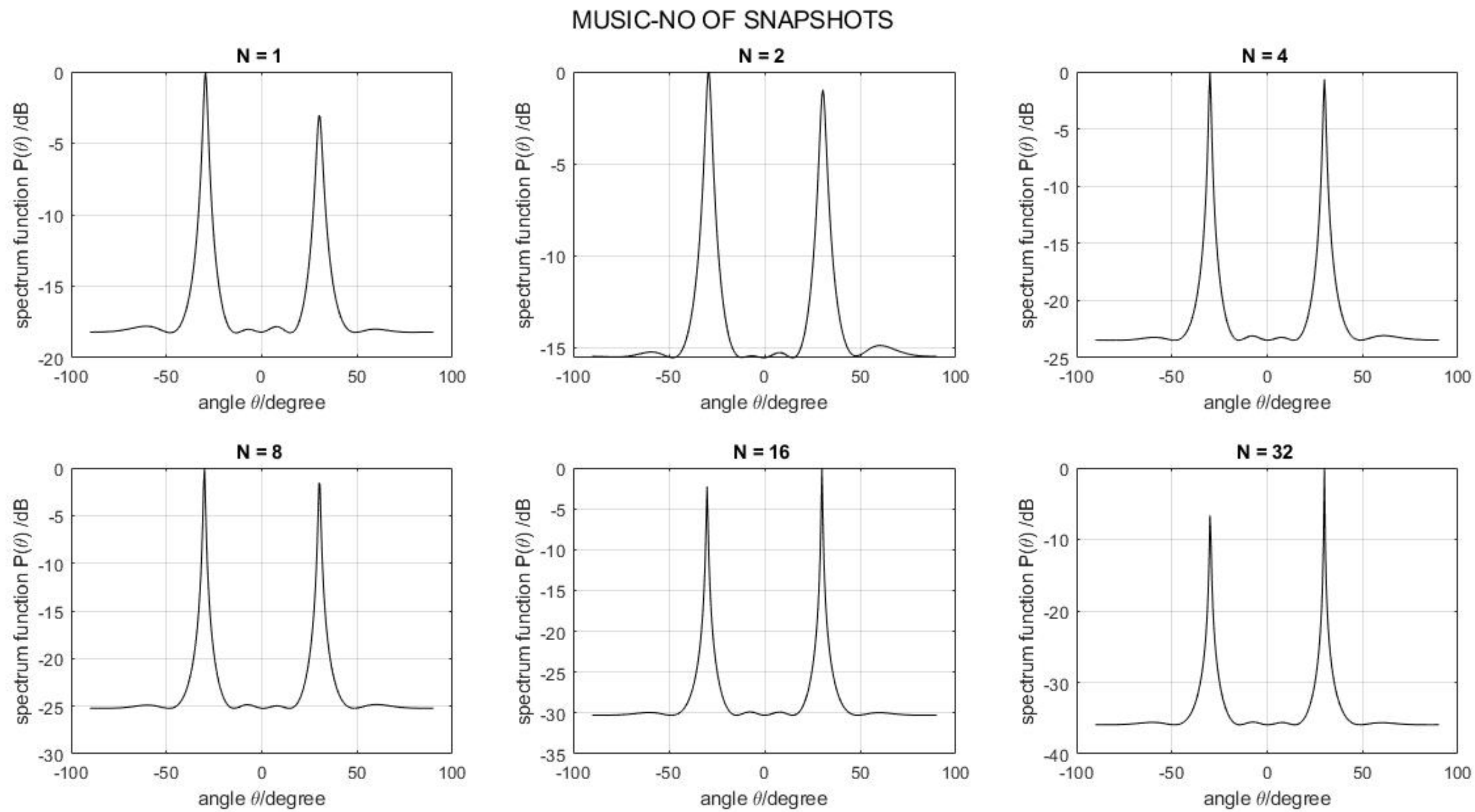
MODIFIED MUSIC – Coherence



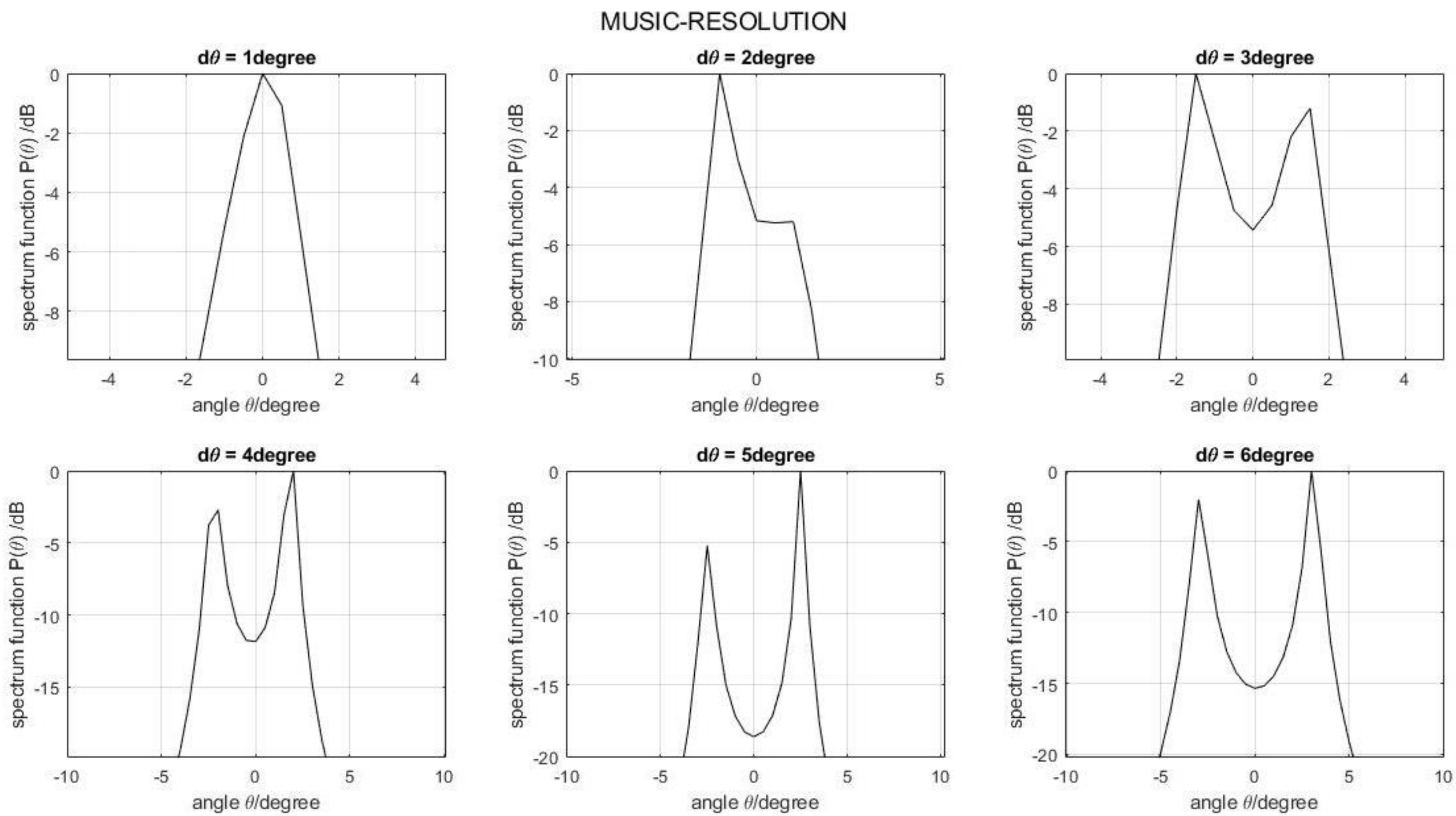
MUSIC – Array element spacing



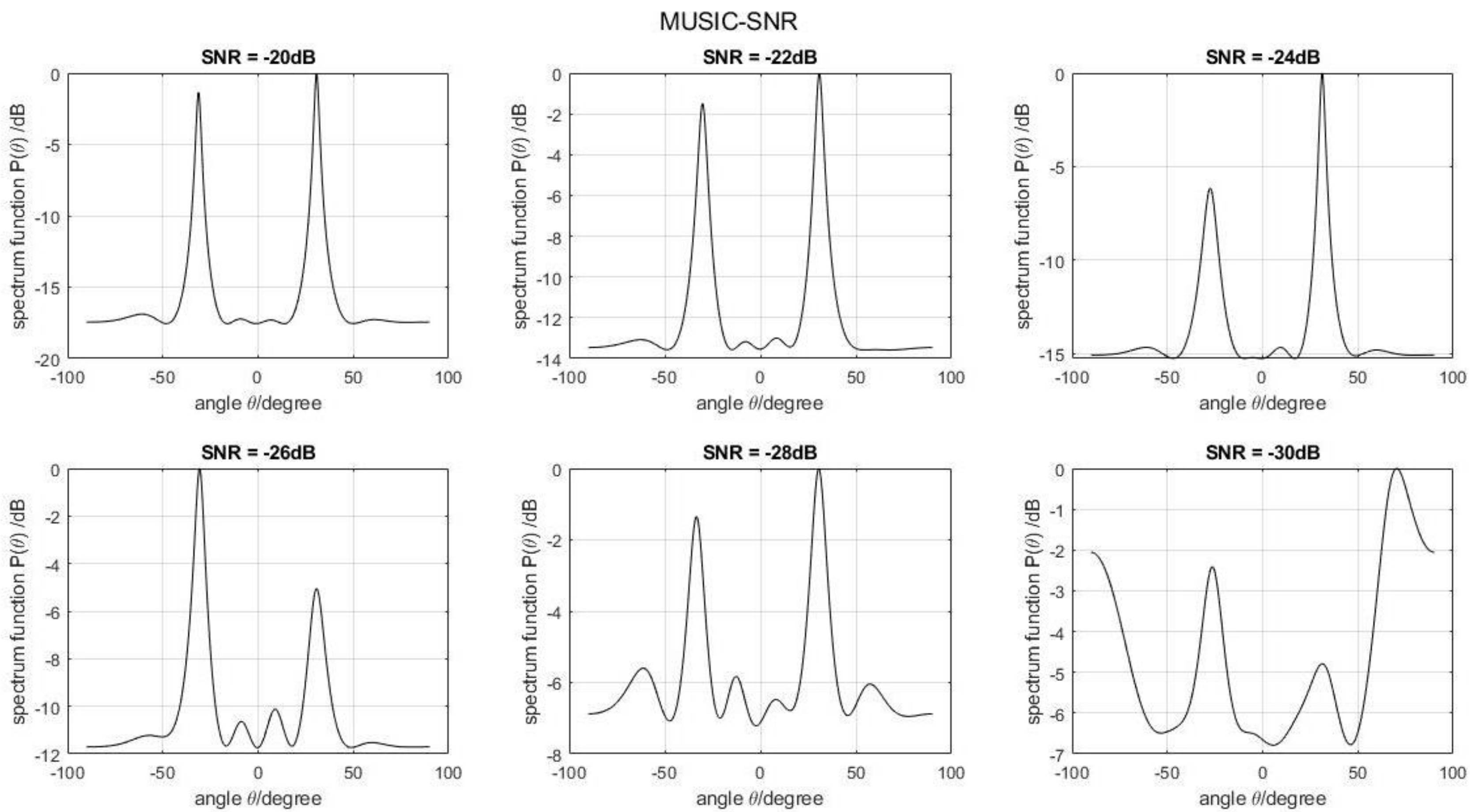
MUSIC – No of Array Elements



MUSIC – No of Snapshots



MUSIC – Resolution


MUSIC-SNR
SNR = -20dB
SNR = -22dB
SNR = -24dB
SNR = -26dB
SNR = -28dB
SNR = -30dB
spectrum function P(θ) /dB
angle θ/degree

MUSIC – SNR

Results (cont)

III. Conducted performance analysis of:

- ESPRIT

Following parameters were varied:

- SNR
- Number of Snapshots

SNR	DOA angles	
0	-30	45
-3	-30	45
-6	-30	45
-9	-30	46
-12	-29	45
-15	-28	45
-18	-29	47
-21	-34	41

Snapshots	DOA angles	
1	-34	46
2	-28	46
3	-30	44
4	-29	44
5	-31	44
6	-30	44
7	-30	43
8	-30	45
9	-30	45
10	-31	45

Results (cont)

IV. Simulation of planar DOA estimation algorithms:

- Planar MVDR
- Planar MUSIC

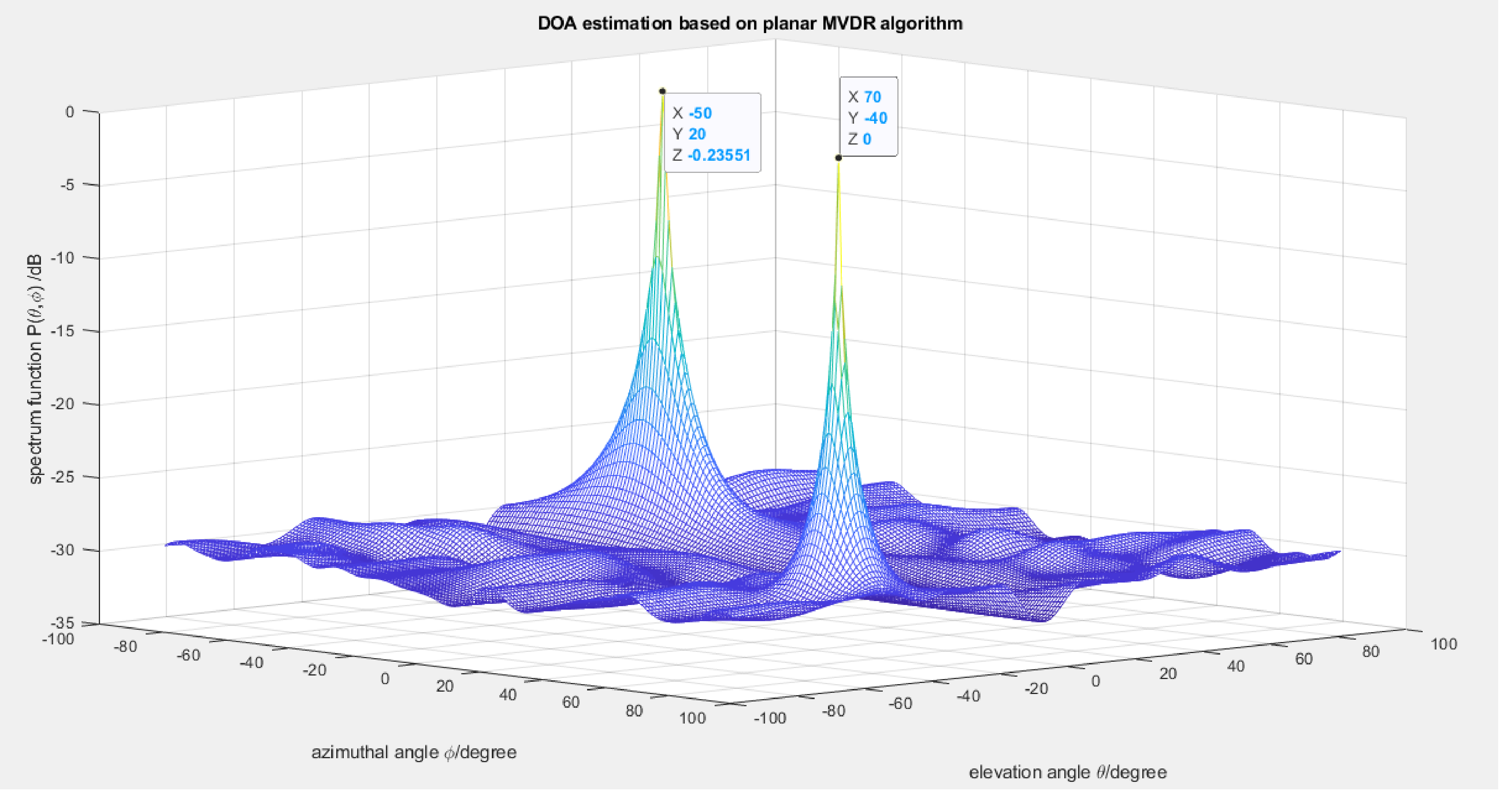
Static Parameters:

Number of elements : 8*8

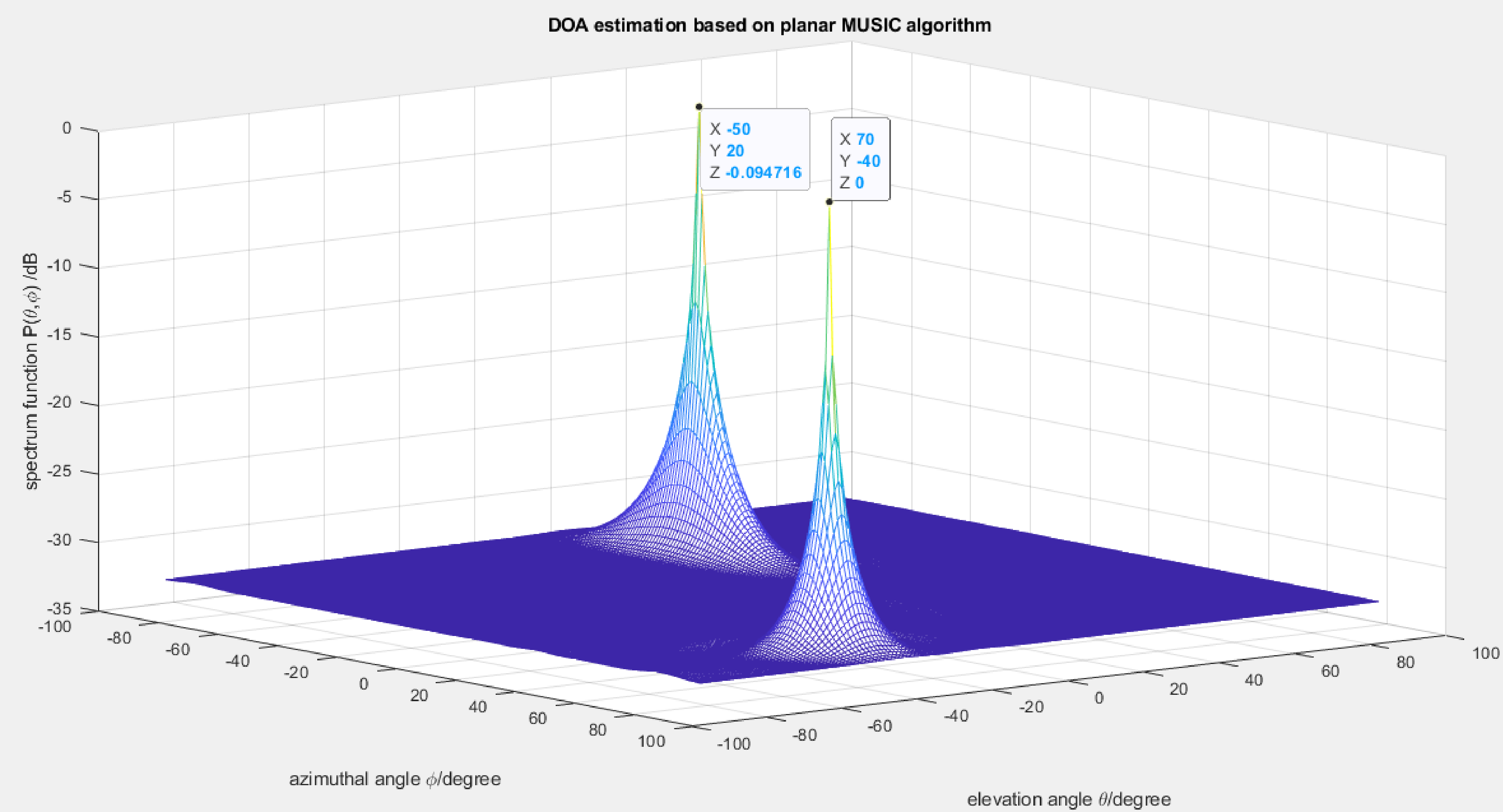
Snapshots : 100

SNR : 0db

Planar MVDR


DOA estimation based on planar MVDR algorithm
X -50
Y 20
Z -0.23551
X 70
Y -40
Z 0
spectrum function P(θ,φ) /dB
0
-5
-10
-15
-20
-25
-30
-35
-100
-80
-60
-40
-20
0
20
40
60
80
100
azimuthal angle φ/degree
-100
-80
-60
-40
-20
0
20
40
60
80
100
elevation angle θ/degree

Planar MUSIC



Results (cont)

V. Simulation of following beamformers:

- Conventional
- Optimal MVDR
- Adaptive SMI

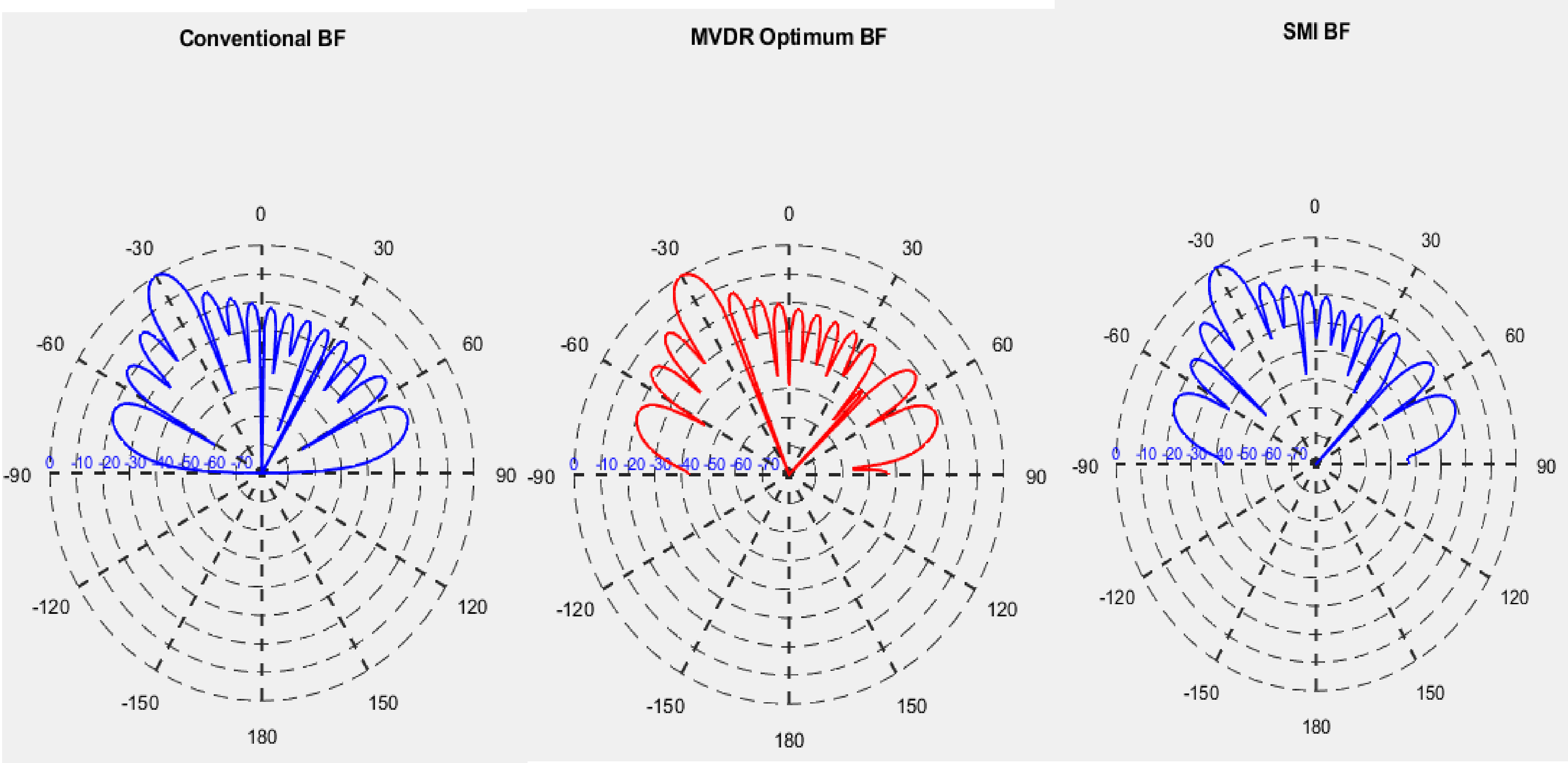
Static Parameters:

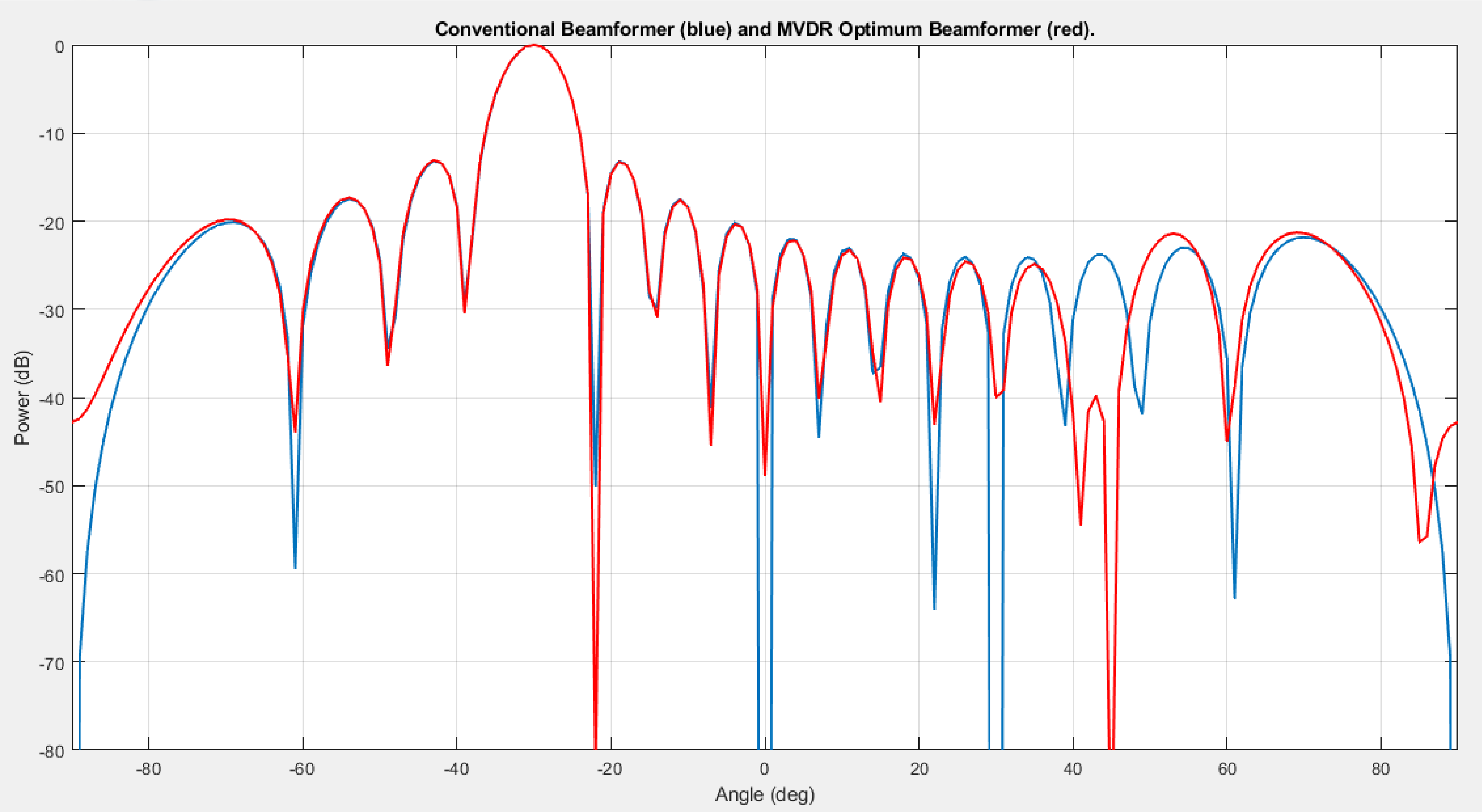
Number of elements : 8

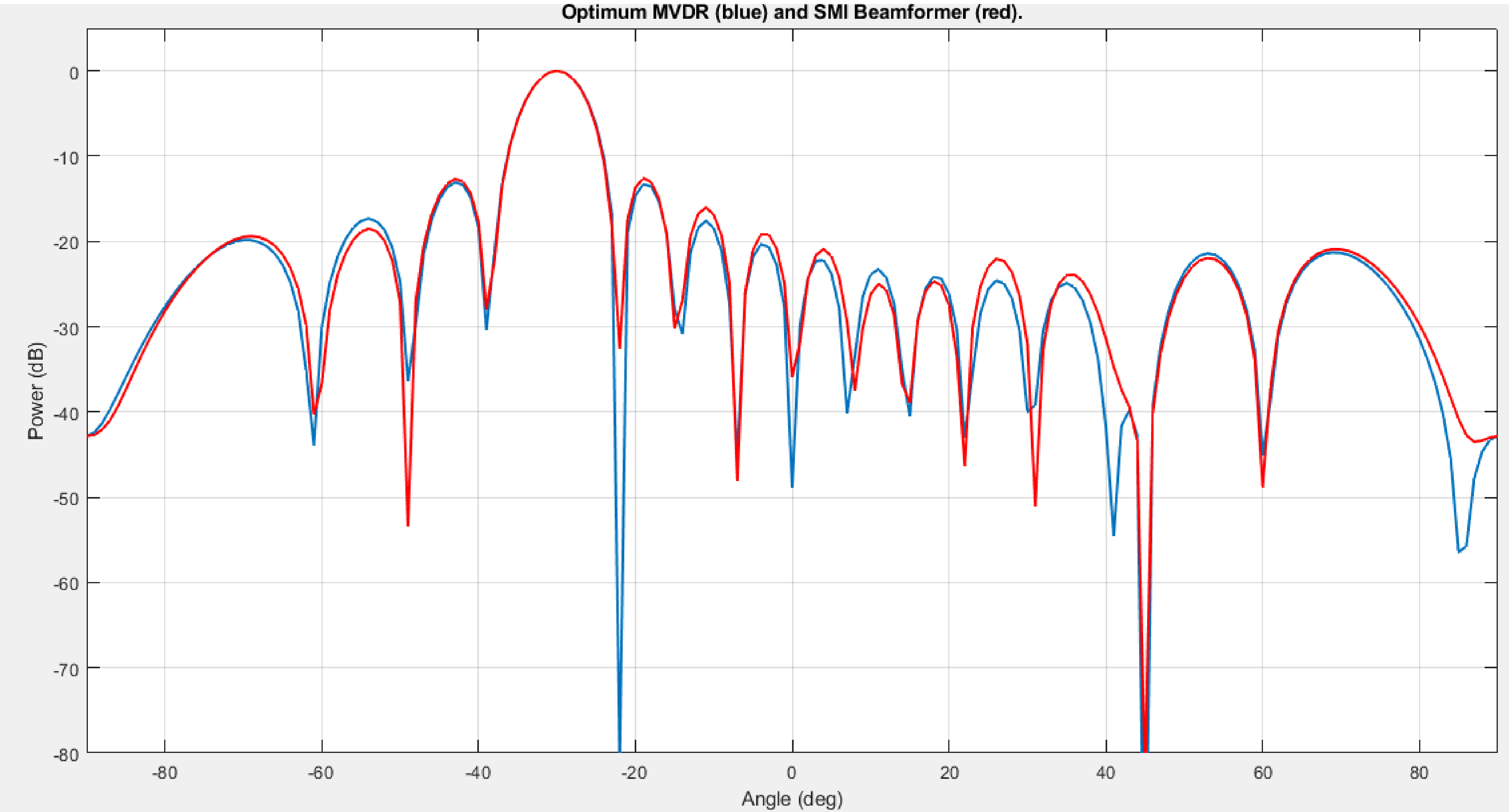
Signal Directions: -30^0

Interfere Direction: 45^0

Beam patterns




Conventional Beamformer (blue) and MVDR Optimum Beamformer (red).
Power (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
-80
-60
-40
-20
0
20
40
60
80
Angle (deg)


Optimum MVDR (blue) and SMI Beamformer (red).
Power (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
-80
-60
-40
-20
0
20
40
60
80
Angle (deg)

Conclusion

1. Simulated and analysed various DOA algorithms using MATLAB for linear arrays.
2. Detailed study of MUSIC algorithm and modification of MUSIC was made to mitigate frequency coherence issue.
3. Extended linear array approach to planar array DOA and observed functionality only in one half of the azimuth plane.
4. Simulated beamformers: conventional, optimal (MVDR) and adaptive (SMI)

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