GATE 2023-EE Q49

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Question 49: The period of the discrete-time signal x[n] described by the equation below is N = (Round off to the nearest integer).

$$x[n] = 1 + 3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

Solution:

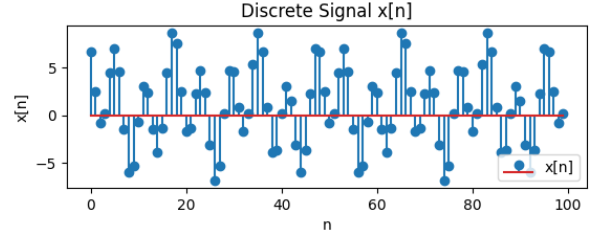| Parameter	Description	Value
f_1	Sinusoid1 Frequency	15/16
f_2	Sinusoid2 Frequency	6

TABLE 1
GIVEN PARAMETERS LIST

Discrete Signal x[n]

Fig. 1.

The time period must be an integer for a discrete time signal.

$$T_1 = \frac{1}{f_1} = \frac{16}{15} \tag{1}$$

$$T_2 = \frac{1}{f_2} = 6 \tag{2}$$

$$N = \text{LCM}(T_1, T_2) = 48 \tag{3}$$

$$\tag{4}$$

The Time Period of the signal is

$$N = 48$$

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Let's find the z-transform of x[n] by using the linearity property is:

$$X(z) = Z(1) + Z\left(3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right)\right) \tag{5}$$

$$- Z\left(5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)\right) \tag{6}$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{3\sin\left(\frac{3\pi}{4}\right)z}{z^2 - 2z\cos\left(\frac{15\pi}{8}\right) + 1} \tag{7}$$

$$- \frac{5\sin\left(-\frac{\pi}{4}\right)z}{z^2 - 2z\cos\left(\frac{\pi}{3}\right) + 1} \tag{8}$$

$$X(z) = \frac{1}{1 - 3\cos\left(\frac{15\pi}{8}\right)z^{-1} + z^{-2}} \tag{9}$$

$$+ \frac{5}{1 - 2\cos\left(\frac{\pi}{3}\right)z^{-1} + z^{-2}} \tag{10}$$