GATE 2021-CS Q34

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Question 34:Let G be a group of order 6, and let H be a subgroup of G such that $1 < |H| < 6$. Which one of the following options is correct?

1) Both G and H are always cyclic.
2) G may not be cyclic, but H is always cyclic.
3) G is always cyclic, but H may not be cyclic.
4) Both G and H may not be cyclic.

Solution: Given: $|G| = 6$
Since H is a subgroup, by Lagrange's theorem:

$$|H| = 1, 2, 3, \text{ or } 6 \text{ (Divisors of } 6)$$

Now, it is given that $1 < |H| < 6$, so $|H| = 2$ or 3.
Since 2 and 3 are both prime, and every group of prime order is cyclic, H is surely cyclic. However, the order of $|G| = 6$ is not prime.
So, G may or may not be cyclic.
Therefore, G may not be cyclic, but H is always cyclic.