GATE 2023-EE Q49

EE23BTECH11052 - Abhilash Rapolu

Question 49: The period of the discrete-time signal $x[n]$ described by the equation below is $N =$ (Round off to the nearest integer).

$$x[n] = 1 + 3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

Solution:

Parameter	Description	Value
f_1	Sinusoid1 Frequency	15/16
f_2	Sinusoid2 Frequency	6

TABLE I
GIVEN PARAMETERS LIST

The time period must be an integer for a discrete-time signal.

$$T_1 = \frac{1}{f_1} = \frac{16}{15} \tag{1}$$

$$T_2 = \frac{1}{f_2} = 6 \tag{2}$$

$$N = \text{LCM}(T_1, T_2) = 48 \tag{3}$$

The Time Period of the signal is $N = 48$.

Let's find the Z-transform of $x[n]$ by using the linearity property:

$$X(z) = Z(1) + Z\left(3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right)\right) \tag{4}$$

$$- Z\left(5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)\right) \tag{5}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 1 \cdot z^{-n} \tag{6}$$

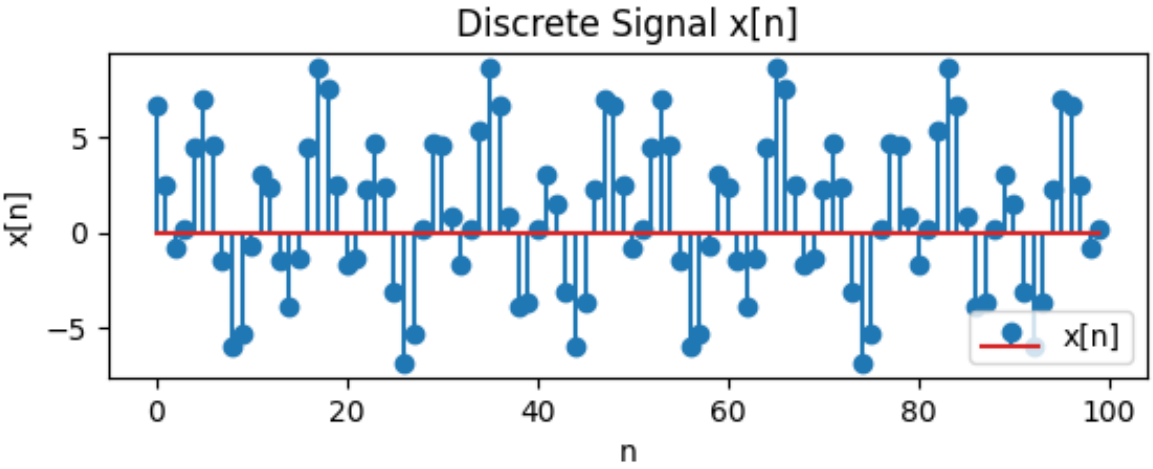
$$+ \sum_{n=-\infty}^{\infty} (3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) \cdot z^{-n} \tag{7}$$

$$+ \sum_{n=-\infty}^{\infty} (5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right) \cdot z^{-n} \tag{8}$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{3\sin\left(\frac{3\pi}{4}\right) z}{z^2 - 2z\cos\left(\frac{15\pi}{8}\right) + 1} \tag{9}$$

$$- \frac{5\sin\left(-\frac{\pi}{4}\right) z}{z^2 - 2z\cos\left(\frac{\pi}{3}\right) + 1} \tag{10}$$

$$\boxed{X(z) = \frac{1}{1 - 3\cos\left(\frac{15\pi}{8}\right) z^{-1} + z^{-2}}} \tag{11}$$


Discrete Signal x[n]
x[n]
5
0
−5
0
20
40
60
80
100
n
x[n]