

GATE 2023-EE Q49

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Question 49: The period of the discrete-time signal $x[n]$ described by the equation below is N = (Round off to the nearest integer).

$$x[n] = 1 + 3 \sin \left(\frac{15\pi}{8}n + \frac{3\pi}{4} \right) - 5 \sin \left(\frac{\pi}{3}n - \frac{\pi}{4} \right)$$

Solution:

Parameter	Description	Value
f_1	Sinusoid1 Frequency	15/16
f_2	Sinusoid2 Frequency	6

TABLE I

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$$X(z) = Z(1) + Z \left(3 \sin \left(\frac{15\pi}{8}n + \frac{3\pi}{4} \right) \right) \quad (4)$$

$$- Z \left(5 \sin \left(\frac{\pi}{3}n - \frac{\pi}{4} \right) \right) \quad (5)$$

$$X(z) = \sum_{n=-\infty}^{\infty} 1 \cdot z^{-n} \quad (6)$$

$$+ \sum_{n=-\infty}^{\infty} \left(3 \sin \left(\frac{15\pi}{8}n + \frac{3\pi}{4} \right) \right) \cdot z^{-n} \quad (7)$$

$$+ \sum_{n=-\infty}^{\infty} \left(5 \sin \left(\frac{\pi}{3}n - \frac{\pi}{4} \right) \right) \cdot z^{-n} \quad (8)$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{3 \sin \left(\frac{3\pi}{4} \right) z}{z^2 - 2z \cos \left(\frac{15\pi}{8} \right) + 1} \quad (9)$$

$$- \frac{5 \sin \left(-\frac{\pi}{4} \right) z}{z^2 - 2z \cos \left(\frac{\pi}{3} \right) + 1} \quad (10)$$

$$X(z) = \frac{1}{1 - 3 \cos \left(\frac{15\pi}{8} \right) z^{-1} + z^{-2}} \quad (11)$$

The time period must be an integer for a discrete-time signal.

$$T_1 = \frac{1}{f_1} = \frac{16}{15} \quad (1)$$

$$T_2 = \frac{1}{f_2} = 6 \quad (2)$$

$$N = \text{LCM}(T_1, T_2) = 48 \quad (3)$$

The Time Period of the signal is $N = 48$.

Let's find the Z-transform of $x[n]$ by using the linearity property:

