

GATE 2023-EE Q49

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Question 49: The period of the discrete-time signal $x[n]$ described by the equation below is $N =$ (Round off to the nearest integer).

$$x[n] = 1 + 3 \sin \left(\frac{15\pi}{8}n + \frac{3\pi}{4} \right) - 5 \sin \left(\frac{\pi}{3}n - \frac{\pi}{4} \right)$$

Solution:

| Parameter | Description | Value |
|-----------|---------------------|-------|
| f_1 | Sinusoid1 Frequency | 15/16 |
| f_2 | Sinusoid2 Frequency | 6 |

TABLE I

GIVEN PARAMETERS LIST

The time period must be an integer for a discrete-time signal.

$$T_1 = \frac{1}{f_1} = \frac{16}{15} \quad (1)$$

$$T_2 = \frac{1}{f_2} = 6 \quad (2)$$

$$N = \text{LCM}(T_1, T_2) = 48 \quad (3)$$

The Time Period of the signal is $N = 48$.

Let's find the Discrete Fourier Transform ($X[k]$):

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn} \quad (4)$$

$$X[k] = \sum_{n=0}^{47} \left(1 + 3 \sin \left(\frac{15\pi}{8}n + \frac{3\pi}{4} \right) - 5 \sin \left(\frac{\pi}{3}n - \frac{\pi}{4} \right) \right) \cdot e^{-j \frac{2\pi}{48} kn} \quad (5)$$

$$X[k] = \begin{cases} 48 & \text{if } k = 0 \\ 50.9117 - 50.9117j & \text{if } k = 3 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

