GATE EE Q.17

EE23BTECH11203 - Adarsh A*

Question : A continuous-time system that is initially at rest is described by,

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$

where $x(t)$ is the input voltage and $y(t)$ is the output voltage.
The impulse response of the system is?

(A) $3e^{-2t}$

(B) $\frac{1}{3}e^{-2t}u(t)$

(C) $2e^{-3t}u(t)$

(D) $2e^{-3t}$ (GATE 2023 EE)

Solution:

Parameter	Value	Description
$x(t)$	-	Input voltage
$y(t)$	-	Output voltage
$h(t)$	$\frac{y(t)}{x(t)}$	Impulse response
$X(s)$	-	Input voltage in s-domain
$Y(s)$	-	Output voltage in s-domain
$H(s)$	$\frac{Y(s)}{X(s)}$	Impulse response in s-domain

Input Table

Given equation is,

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t) \tag{1}$$

Applying Laplace transform,

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \tag{2}$$

$$y(t) \xleftrightarrow{\mathcal{L}} Y(s) \tag{3}$$

From the differentiation property,

$$\frac{dy(t)}{dt} \xleftrightarrow{\mathcal{L}} sY(s) \tag{4}$$

The equation becomes,

$$sY(s) + 3Y(s) = 2X(s) \tag{5}$$

$$Y(s)(s+3) = 2X(s) \tag{6}$$

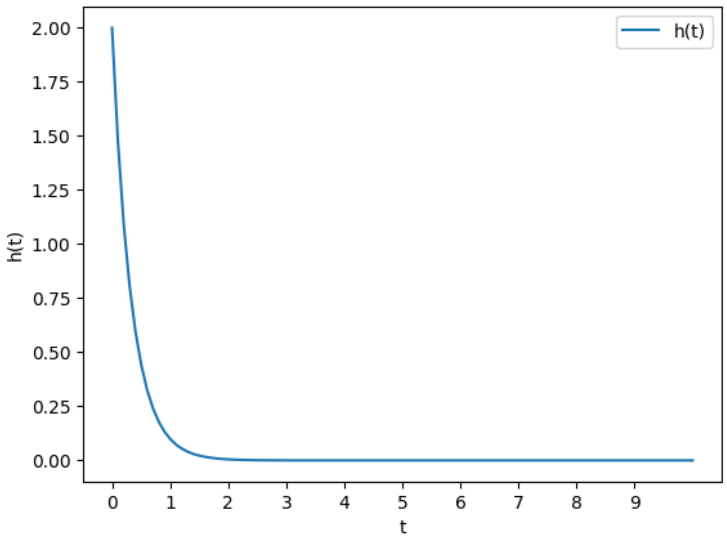
$$H(s) = \frac{Y(s)}{X(s)} \tag{7}$$

$$H(s) = \frac{2}{s+3} \tag{8}$$

$$\frac{1}{s+a} \xleftrightarrow{\mathcal{L}} e^{-at}u(t) \tag{9}$$

Using these results,

$$h(t) = 2e^{-3t}u(t) \tag{10}$$



(a) Plot of $h(t)$ *vs* t