Discrete

EE1205 : Signals and Systems
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I. Question 11.9.5 (18)

If a and b are the roots of $x^2 - 3x + p = 0$ and c , d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that $(q + p) : (q - p) = 17:15$.

II. Solution

Parameter	Value	Description
$x_1(n)$	-	G.P. Sequence
$x_1(0)$	a	First term of G.P.
$x_1(1)$	b	Second term of G.P.
$x_1(2)$	c	Third term of G.P.
$x_1(3)$	d	Fourth term of G.P.
r	$\frac{b}{a}$	Common ratio

TABLE I
Given Parameters

Given $x_1(0)$ and $x_1(1)$ are the roots of $x^2-3x+p = 0$ So, we have :

$$a + b = 3 \tag{1}$$
$$ab = p \tag{2}$$

Also, $x_1(2)$ and $x_1(3)$ are the roots of $x^2-12x+q = 0$, so,

$$c + d = 12 \tag{3}$$
$$cd = q \tag{4}$$

From 1 and 3 , we get ,

$$a(1 + r) = 3 \tag{5}$$

And ,

$$ar^2(1 + r) = 12 \tag{6}$$

On dividing eq. 5 and eq. 6, we get

$$\frac{ar^2(1 + r)}{a(1 + r)} = \frac{12}{3} \tag{7}$$
$$r^2 = 4 \tag{8}$$
$$r = \pm 2 \tag{9}$$

When r = 2, a = 1
When r = -2, a= -3
Case 1 : When $r = 2$ and $a = 1$

$$p = ab \tag{10}$$
$$p = 2 \tag{11}$$
$$q = cd \tag{12}$$
$$q = 32 \tag{13}$$
$$\frac{q + p}{q - p} = \frac{32 + 2}{32 - 2} \tag{14}$$
$$= \frac{17}{15} \tag{15}$$

Case 2 : When $r = -2$ and $a = -3$

$$p = ab \tag{16}$$
$$p = -18 \tag{17}$$
$$q = cd \tag{18}$$
$$q = 288 \tag{19}$$
$$\frac{q + p}{q - p} = \frac{288 - 18}{288 + 18} \tag{20}$$
$$= \frac{135}{153} \tag{21}$$

Hence , case 1 satisfies the condition .

$$x_1(n) \longleftrightarrow X(z)$$

$$ar^n u(n) \longleftrightarrow \frac{a}{1 - rz^{-1}} \ ; \ |z| > |r| \tag{22}$$

$$\therefore \ X(z) = \frac{1}{1 - 2z^{-1}} \ ; \ (|z| > 2) \tag{23}$$