NCERT DISCRETE 11.9.2 Q10

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Question 11.9.2.10:If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.
Solution:

Parameter	Description	Value
a_0	first term	none
d	common difference	none
$x(n)$	n^{th} term	$a_0 + nd$
$y(n)$	Sum of n terms	$\frac{n+1}{2}[2a_0 + nd]$
$y(p-1)$	sum of first p terms	$\frac{p}{2}[2a_0 + (p-1)d]$
$y(q-1)$	sum of first q terms	$\frac{q}{2}[2a_0 + (q-1)d]$
$y(p+q-1)$	sum of first p+q terms	$\frac{p+q}{2}[2a_0 + (p+q-1)d]$

TABLE I
GIVEN PARAMETERS LIST

Let $x(n) = a_0 + nd$
Now let's find the z transform of the $x(n)$ using the linearity property.

$$X(z) = \frac{a_0}{1-z^{-1}} + d\frac{z^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$y(n) = x(n) * u(n) \tag{2}$$

Now apply z transform on both sides

$$Y(z) = X(z)U(z) \tag{3}$$

$$Y(z) = \frac{a_0}{(1-z^{-1})^2} + d\frac{z^{-1}}{(1-z^{-1})^3} \tag{4}$$

by comparison of the above equations:

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1-z^{-1}} \tag{5}$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{6}$$

$$n(n-1)u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-1}}{(1-z^{-1})^3} \tag{7}$$

referencing the equations from 4,5 and 6, we obtain the inverse z transform:

$$y(n) = a_0(n) + \frac{d}{2}(n)(n-1) \tag{8}$$

as we considered n=0 as our first term, we have to replace n by (n+1)
Sum of first n terms is given as:

$$y(n) = a_0(n+1) + \frac{d}{2}(n+1)(n) \tag{9}$$

now as per the question sum of first p terms is same as sum of first q terms y(p-1) = y(q-1)[as sequence starts from n=0 first p terms are from 0 to p-1, same for q and p+q.]

$$a_0(p) + \frac{d}{2}(p-1)(p) = a_0(q) + \frac{d}{2}(q-1)(q) \tag{10}$$

$$d = (-)\frac{2a_0}{p+q-1} \tag{11}$$

now for first p+q terms:

$$y(p+q-1) = a_0(p+q) + \frac{d}{2}(p+q-1)(p+q) \tag{12}$$

substitue d in this

$$y(p+q-1) = a_0(p+q) - \frac{a_0}{p+q-1}(p+q-1)(p+q) \tag{13}$$

$$y(p+q-1) = a_0(p+q) - a_0(p+q) \tag{14}$$

$$y(p+q-1) = 0. \tag{15}$$