GATE 2023-EE Q49

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QUESTION:The period of the discrete-time signal x[n] described by the equation below is N = (Round off to the nearest integer).

$$x[n] = 1 + 3\sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5\sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

SOLUTION:
The signal can be expressed as the sum of two sinusoids:
Sinusoid 1: Frequency

$$(f_1) = \frac{15\pi}{8\pi} = \frac{15}{16}$$

Sinusoid 2: Frequency

$$(f_2) = \frac{\pi}{6\pi} = \frac{1}{6}$$

Therefore, the frequency components of $x[n]$ are:

$$f_1 = \frac{15}{16} \quad \text{and} \quad f_2 = \frac{1}{6}$$

$$T_i = \frac{1}{f_i}$$

The time period must be an integer for a discrete time signal.
Therefore, we need to find the smallest integer N that is a multiple of both T_1 and T_2:

$$T_1 = \frac{1}{f_1} = \frac{16}{15}$$

(not an integer)

$$T_2 = \frac{1}{f_2} = 6$$

(an integer)
Since T_1 is not an integer. However, T_2 is an integer,and it is a factor of $\frac{16}{15}$ (15 divides into 16 exactly once).
Therefore, the smallest N that satisfies the periodicity condition is:

$$N = \text{LCM}(T_1, T_2) = \text{LCM}(16, 6) = 48$$

The frequency components of the signal are $f_1 = \frac{15}{16}$ and $f_2 = \frac{1}{6}$. The time period of the signal is

$$N = 48$$

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