NCERT DISCRETE 11.9.2 Q10

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Question 11.9.2.10:If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.
Solution: Now let's find the z transform of the given in question y(p-1)=y(q-1)

Parameter	Description	Value
a_0	first term	none
d	common difference	none
$x(n)$	n^{th} term	a_0+nd
$y(n)$	Sum of n terms	$\frac{n+1}{2}[2a_0+nd]$
$y(p-1)$	sum of first p terms	$\frac{p}{2}[2a_0+(p-1)d]$
$y(q-1)$	sum of first q terms	$\frac{q}{2}[2a_0+(q-1)d]$
$y(p+q-1)$	sum of first p+q terms	$\frac{p+q}{2}[2a_0+(p+q-1)d]$

TABLE I
GIVEN PARAMETERS LIST

$x(n)$ using the linearity property.

$$X(z)=\frac{a_0}{1-z^{-1}}+d\frac{z^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$y(n)=x(n)*u(n) \tag{2}$$

Now apply z transform on both sides

$$Y(z)=X(z)U(z) \tag{3}$$

$$Y(z)=\frac{a_0}{(1-z^{-1})^2}+d\frac{z^{-1}}{(1-z^{-1})^3} \tag{4}$$

by comparison of the above equations:
(**??**)
the inverse z transform:

$$y(n)=a_0(n)+\frac{d}{2}(n)(n-1) \tag{5}$$

as we considered n=0 as our first term, we have to replace n by (n+1)
Sum of first n terms is given as:

$$y(n)=a_0(n+1)+\frac{d}{2}(n+1)(n) \tag{6}$$

$$a_0(p)+\frac{d}{2}(p-1)(p)=a_0(q)+\frac{d}{2}(q-1)(q) \tag{7}$$

$$d=(-)\frac{2a_0}{p+q-1} \tag{8}$$

now for first p+q terms:

$$y(p+q-1)=a_0(p+q)+\frac{d}{2}(p+q-1)(p+q) \tag{9}$$

substitue d in this

$$y(p+q-1)=a_0(p+q)-\frac{a_0}{p+q-1}(p+q-1)(p+q) \tag{10}$$

$$y(p+q-1)=a_0(p+q)-a_0(p+q) \tag{11}$$

$$y(p+q-1)=0. \tag{12}$$