NCERT DISCRETE 11.9.2 Q10

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Question 11.9.2.10:If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p + q)$ terms.
Solution:
Let $x(n) = a_0 + nd$
Now let's find the z transform of the $x(n)$ using the linearity property.

$$X(z) = \frac{a_0}{1 - z^{-1}} + d\frac{z^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

Now apply z transform on both sides

$$Y(z) = X(z)U(z) \quad (3)$$

$$Y(z) = \frac{a_0}{(1 - z^{-1})^2} + d\frac{z^{-1}}{(1 - z^{-1})^3} \quad (4)$$

by comparison of the above equations:

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1 - z^{-1}} \quad (5)$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2} \quad (6)$$

$$n(n - 1)u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-1}}{(1 - z^{-1})^3} \quad (7)$$

by solving the above equations, we obtain the inverse z transform:

$$y(n) = a_0(n - 1) + \frac{d}{2}(n - 1)(n - 2) \quad (8)$$

as we considered n=0 as our first term, we have to replace n by (n+1)
Sum of first n terms is given as:

$$y(n) = a_0(n) + \frac{d}{2}(n - 1)(n) \quad (9)$$

now as per the question y(p) = y(q)

$$a_0(p) + \frac{d}{2}(p - 1)(p) = a_0(q) + \frac{d}{2}(q - 1)(q) \quad (10)$$

$$d = (-)\frac{2a_0}{p + q - 1} \quad (11)$$

now for p+q terms:

$$y(p + q) = a_0(p + q) + \frac{d}{2}(p + q - 1)(p + q) \quad (12)$$

substitue d in this

$$y(p + q) = a_0(p + q) - \frac{a_0}{p + q - 1}(p + q - 1)(p + q) \quad (13)$$

$$y(p + q) = a_0(p + q) - a_0(p + q) \quad (14)$$

$$y(p + q) = 0. \quad (15)$$