

NCERT DISCRETE 11.9.2 Q10

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Question 11.9.2.10: If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.

Solution: Now let's find the z transform of the

given in question $y(p-1)=y(q-1)$

$$a_0(p) + \frac{d}{2}(p-1)(p) = a_0(q) + \frac{d}{2}(q-1)(q) \quad (7)$$

$$d = (-) \frac{2a_0}{p+q-1} \quad (8)$$

now for first $p+q$ terms:

$$y(p+q-1) = a_0(p+q) + \frac{d}{2}(p+q-1)(p+q) \quad (9)$$

Parameter	Description	Value
a_0	first term	none
d	common difference	none
$x(n)$	n^{th} term	$a_0 + nd$
$y(n)$	Sum of n terms	$\frac{n+1}{2}[2a_0 + nd]$
$y(p-1)$	sum of first p terms	$\frac{p}{2}[2a_0 + (p-1)d]$
$y(q-1)$	sum of first q terms	$\frac{q}{2}[2a_0 + (q-1)d]$
$y(p+q-1)$	sum of first p+q terms	$\frac{p+q}{2}[2a_0 + (p+q-1)d]$

TABLE I

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$x(n)$ using the linearity property.

$$y(p+q-1) = a_0(p+q) - \frac{a_0}{p+q-1}(p+q-1)(p+q) \quad (10)$$

$$X(z) = \frac{a_0}{1-z^{-1}} + d \frac{z^{-1}}{(1-z^{-1})^2} \quad (1) \quad y(p+q-1) = a_0(p+q) - a_0(p+q) \quad (11)$$

$$y(n) = x(n) * u(n) \quad (2) \quad y(p+q-1) = 0. \quad (12)$$

Now apply z transform on both sides

$$Y(z) = X(z)U(z) \quad (3)$$

$$Y(z) = \frac{a_0}{(1-z^{-1})^2} + d \frac{z^{-1}}{(1-z^{-1})^3} \quad (4)$$

by comparison of the above equations:

using equations from appendix (??)

the inverse z transform:

$$y(n) = a_0(n) + \frac{d}{2}(n)(n-1) \quad (5)$$

as we considered $n=0$ as our first term, we have to replace n by $(n+1)$

Sum of first n terms is given as:

$$y(n) = a_0(n+1) + \frac{d}{2}(n+1)(n) \quad (6)$$