

# NCERT DISCRETE 11.9.2 Q10

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**Question 11.9.2.10:** If the sum of first  $p$  terms of an A.P. is equal to the sum of the first  $q$  terms, then find the sum of the first  $(p+q)$  terms.

Solution: Now let's find the z transform of the

sequence starts from  $n=0$  first  $p$  terms are from 0 to  $p-1$ , same for  $q$  and  $p+q$ .]

Parameter	Description	Value
$a_0$	first term	none
$d$	common difference	none
$x(n)$	$n^{th}$ term	$a_0 + nd$
$y(n)$	Sum of $n$ terms	$\frac{n+1}{2}[2a_0 + nd]$
$y(p-1)$	sum of first $p$ terms	$\frac{p}{2}[2a_0 + (p-1)d]$
$y(q-1)$	sum of first $q$ terms	$\frac{q}{2}[2a_0 + (q-1)d]$
$y(p+q-1)$	sum of first $p+q$ terms	$\frac{p+q}{2}[2a_0 + (p+q-1)d]$

TABLE I

GIVEN PARAMETERS LIST

$$a_0(p) + \frac{d}{2}(p-1)(p) = a_0(q) + \frac{d}{2}(q-1)(q) \quad (7)$$

$$d = (-) \frac{2a_0}{p+q-1} \quad (8)$$

now for first  $p+q$  terms:

$$y(p+q-1) = a_0(p+q) + \frac{d}{2}(p+q-1)(p+q) \quad (9)$$

substitutue d in this

$x(n)$  using the linearity property.

$$X(z) = \frac{a_0}{1-z^{-1}} + d \frac{z^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$y(n) = x(n) * u(n) \quad (2)$$

Now apply z transform on both sides

$$Y(z) = X(z)U(z) \quad (3)$$

$$Y(z) = \frac{a_0}{(1-z^{-1})^2} + d \frac{z^{-1}}{(1-z^{-1})^3} \quad (4)$$

by comparison of the above equations:

(??)

referencing the equations from 5,6 and 7, we obtain the inverse z transform:

$$y(n) = a_0(n) + \frac{d}{2}(n)(n-1) \quad (5)$$

as we considered  $n=0$  as our first term, we have to replace  $n$  by  $(n+1)$

Sum of first  $n$  terms is given as:

$$y(n) = a_0(n+1) + \frac{d}{2}(n+1)(n) \quad (6)$$

now as per the question sum of first  $p$  terms is same as sum of first  $q$  terms  $y(p-1) = y(q-1)$  [as

$$y(p+q-1) = a_0(p+q) - \frac{a_0}{p+q-1}(p+q-1)(p+q) \quad (10)$$

$$y(p+q-1) = a_0(p+q) - a_0(p+q) \quad (11)$$

$$y(p+q-1) = 0. \quad (12)$$