

### Question 11.9.2.10

If the sum of first  $p$  terms of an A.P. is equal to the sum of the first  $q$  terms, then find the sum of the first  $(p + q)$  terms?

### Solution:

The sum of first  $p$  terms of an arithmetic progression (A.P) is given by  
Let  $a_1$  be the first term which is  $a_0 + d$

$$s_p = \frac{q}{2}[2a_1 + (q - 1)d]$$

$$s_p = \frac{q}{2}[2a_0 + (q + 1)d]$$

If  $s_p = s_q$ , then:

$$\frac{p}{2}[2a_0 + (p + 1)d] = \frac{q}{2}[2a_0 + (q + 1)d]$$

simplifying the equation we get:

$$(p) * (2a_0 + pd + d) = (q) * (2a_0 + qd + d) \quad (1)$$

$$2a_0p + (p^2) * d + pd = 2a_0q + (q^2) * d + qd \quad (2)$$

$$2a_0(p - q) + (p - q)(p + q) * d + (p - q) * d = 0 \quad (3)$$

$$(p - q)[2a_0 + (p + q) * d + d] = 0 \quad (4)$$

since  $p$  and  $q$  are not equal. We can eliminate the term  $(p - q)$

$$2a_0 + (p + q) * d + d = 0 \quad (5)$$

Sum of the first  $p + q$

$$S_{p+q} = \frac{p+q}{2}[2a_0 + (p+q+1) * d]$$

As we have seen in the equation (5)  $2a_0 + (p + q) * d + d = 0$  is 0. Therefore  $S_{p+q}$  is 0.