

# **Solving Nonlinear Equations of One Variable**

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A Dissertation submitted to

The Faculty of  
The School of Engineering and Applied Science  
of The George Washington University  
in partial satisfaction of the requirements  
for the degree of Doctor of Philosophy

May 28, 2015

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## **Solving Nonlinear Equations of One Variable**

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*to Christine*

## Acknowledgments

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## Abstract

### Solving Nonlinear Equations of One Variable

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## List of Abbreviations

**CRTBP** Circular Restricted Three Body Problem

## Preface

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## Foreword

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## Prologue

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# CHAPTER 1

## Introduction

Here's an acronym Circular Restricted Three Body Problem (CRTBP)

### 1.1 Float environments

There are many possible float environments, and this section will serve as an introduction and demonstration of each of them. In addition, it offers the ability to ensure that this template actually follows the guidelines.

#### 1.1.1 Figures

Here is a figure as shown in Figure 1.1.

#### 1.1.2 Tables

here's a table in Table 1.1

### 1.2 References and Citation

#### 1.2.1 Clever referencing

$\LaTeX$  offers the powerful ability to automatically handle references using `\label` and a corresponding `\ref`. While Chapter 1 has more detail on some good practices for  $\LaTeX$  that I've picked up.

#### 1.2.2 References

We can cite lots of useful papers [1, 2].

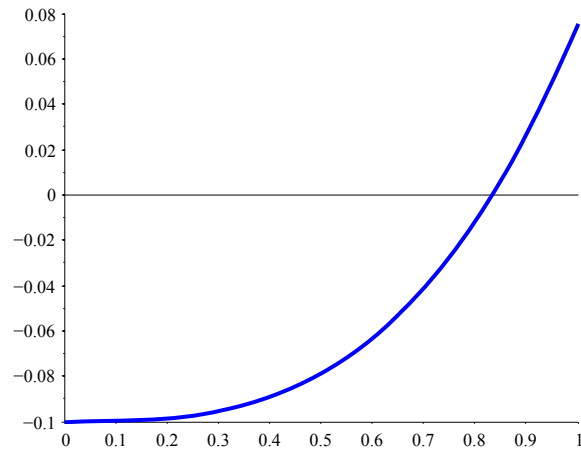


Figure 1.1: Long caption to appear in text

Day	Min Temp	Max Temp	Summary
Monday	11C	22C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9C	19C	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.
Wednesday	10C	21C	Rain will still linger for the morning. Conditions will improve by early afternoon and continue throughout the evening.

Table 1.1: Long caption for text

## 1.3 Math

Here are some nice equations Equations (1.1) and (1.2)

$$\min_{s \subset W} J(s) = \sum_{i=1}^{l-1} H(s_j, s_{j+1}) \quad (1.1)$$

$$\max_{s \subset W} P_{tr}(s) = \prod_{i=1}^{l-1} P_{tr}(s_j, s_{j+1})$$

$$\min_{s \subset W} J(s) = \sum_{i=1}^{l-1} H(s_j, s_{j+1}) \quad (1.2)$$

subject to  $P_{tr}(s) > \epsilon_{tr}$



## CHAPTER 2

### Setting

The next chapter has the good stuff.

#### 2.1 Convergence Criteria

Actually, it might have the worst stuff. But it is slightly easier to write than the material in Chapter 1.

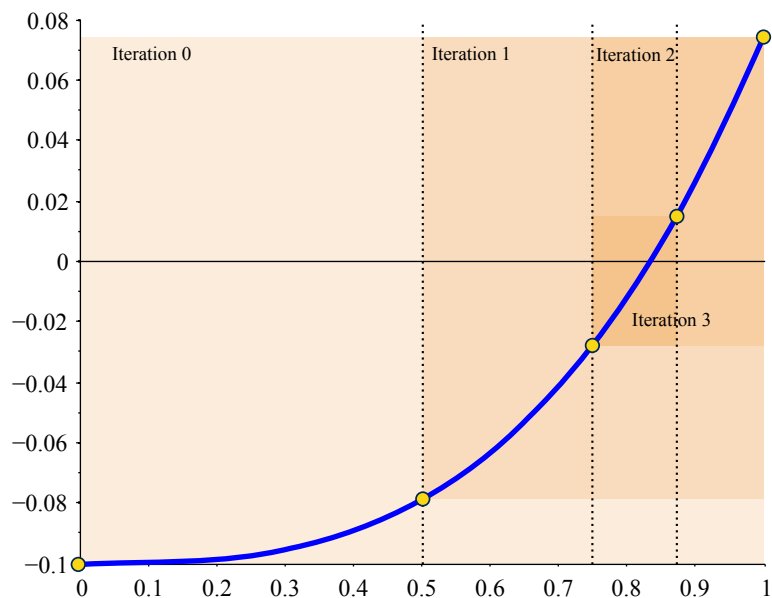


Figure 2.1: Illustration of  $x$ - and  $y$ -tolerances for bisection iterations

It takes very little text to fill a page in this format, but there is even less text on most of these sample pages.

## 2.2 Why we are doing it

It is usually a good idea to give reasons for your research. If you do not, the people who paid you to waste all that time will feel really bad about it, and then they will not provide the same opportunity to future students.

I need this page to see what even-numbered pages look like.

## Bibliography

- [1] Sanjay P Bhat and Dennis S Bernstein. A topological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon. *Systems & Control Letters*, 2000.
- [2] Nalin Chaturvedi, Amit K Sanyal, N Harris McClamroch, et al. Rigid-body attitude control. *Control Systems, IEEE*, 31(3):30–51, 2011.

# Appendix A

## Methods

Here is how to implement the methods.

### A.1 Bisection

The easiest method.

$$x_k = \frac{a_k + b_k}{2} \tag{A.1}$$

### A.2 False Position

The next one.

## Appendix B

# Using Appendices

This appendix contains the portion of the users' manual that describes how to use appendices with this template. It is put in this appendix rather than in Chapter ?? simply so that there are two appendices, so that a list of appendices can appear earlier in the document.

### B.1 Starting the Appendices

Actually, using appendices is quite simple. Immediately after the end of the last chapter and before the start of the first appendix, simply enter the command `\appendix`. This will tell L<sup>A</sup>T<sub>E</sub>X to change how it interprets the commands `\chapter`, `\section`, *etc.*

Each appendix is actually a chapter, so once the `\appendix` command has been called, start a new appendix by simply using the `\chapter` command.

Note that the `\appendix` command should be called only once—not before the start of each appendix.

### B.2 Lists Including the Appendices

As mentioned in Section ??, the command

must appear in the preamble if there are more than one appendices. For some reason, Rackham does not want the individual appendices and their sections to appear in the Table of Contents, so a special List of Appendices page (which must occur in the Table of Contents!) is required as a sort of extension to the Table of Contents.

This does not require a user of this template to do anything, but it is so silly that I felt it was worth explaining. Also, there is nowhere for the sections or subsections of appendices to show up in the Table of Contents or any of the lists, but they do still create navigation tabs in a modern PDF viewer.