

Solving Nonlinear Equations of One Variable

by

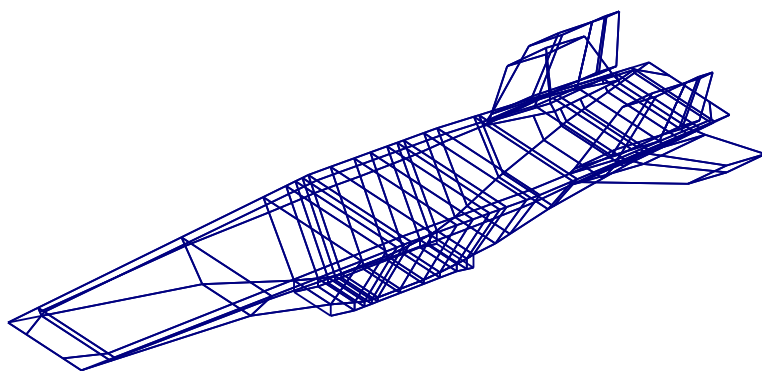
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Doctoral Committee:

Professor James F. Driscoll, Chair

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2012

This dissertation is in honor of Adlai Stevenson and William Jennings Bryan, who were both twice the Democratic nominee for President of the United States without winning either time. As an interesting side note, Adlai Stevenson's grandfather, Adlai E. Stevenson I, was William Jennings Bryan's running mate in the 1900 election.

A C K N O W L E D G M E N T S

It is imperative that I thank all of the authors of the dissertation template from the Department of Atmospheric, Oceanic and Space Sciences, which is available online at <http://aoss.engin.umich.edu/>. To my knowledge, the authors of that template include Jin Ji, Roque D. Oliveira, and Jason Gilbert. I also must thank Sara Spangelo for suggesting that this template be ready by the end of April 2011.

Preface

The text of this document is of course mainly meant to show how the template works. The topic is thus a basic problem which has been solved in a great number of ways. This sample topic, which is solving equations of one variable using iterative techniques, allows us to use sample equations and figures so that we can see how they will look in this template. In addition to this subject, the text also serves as a very unusual users' manual. Chapter 2, which does not match the other chapters at all, gives instructions on the actual commands that are used with this template.

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LIST OF ABBREVIATIONS

CFD Computational Fluid Dynamics

ABSTRACT

Solving Nonlinear Equations of One Variable

by

Derek J. Dalle

Chair: James F. Driscoll

We show that it is possible to get approximate solutions to analytically intractable equations using iterative methods. Thus we show that the author could pass an undergraduate class in numerical analysis. In addition, a unique extension to Brent's method is proposed that results in slight improvements in convergence.

CHAPTER 1

Introduction

The first part. For example in Computational Fluid Dynamics (CFD), which gives me a nice example of an abbreviation to demonstrate, the equation of state cannot be solved analytically when a perfect gas is not assumed.

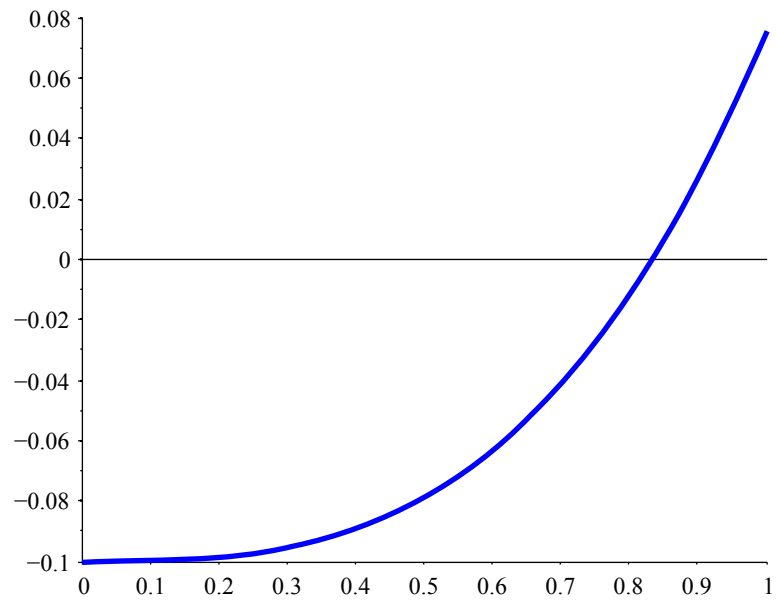


Figure 1.1: An example of a search function

And this should at least continue onto a second page. There are many texts that have a section on the subject, for instance [1].

CHAPTER 2

Using This Template

This chapter is stuck among the others as a brief users' manual for this template. The approach to this template is to result in \LaTeX source code files (*i.e.* .tex files) that are as simple as possible. It also tries to do as much as possible automatically so that the user does not have to spend a lot of effort trying to match the confusing and arbitrary guidelines from Rackham. This is particularly useful for the first few pages, for example the title page, dedication, and abstract page, which are difficult to make in \LaTeX and are supposed to go in a certain order.

In addition to the description in this chapter, anyone may, of course, also look at the source code for this file, thesis-sample.tex. That file contains all of the source for this .pdf in a single file, but it will work just as well with multiple input files combined with the `\input` command.

The final generic comment about this template is that it has been updated to take advantage of \LaTeX 's capabilities to create documents with links in them. Provided you are using a modern PDF viewer to view this document, you may have already noticed this. It creates a list of bookmarks, which can be used to quickly navigate what may be a long document. It also turns references within the text into links. The best examples in this file are the entries in the Table of Contents. Although the chapter and section names are shown in black (in accordance with the Rackham guidelines), clicking on them does navigate to the start of the chapter, section, *etc.*

2.1 General Usage

The way to invoke usage of this template is to put

```
\documentclass{thesis-umich.cls}
```

at the beginning of your preamble. This can also work if the thesis-umich.cls file is not in the same directory as your .tex file. To do so, just give the relative path.

```
\documentclass{./tex/thesis-umich.cls}
```

Much like a usual article or report in \LaTeX , the user specifies the primary information about the document in the preamble with commands like

```
\author{Derek J. Dalle}  
\chair{James F. Driscoll}
```

At the beginning of the document, *i.e.* wherever the user types `\begin{document}`, the title page will automatically be created and inserted at the beginning of the document. If you forget to declare any of the required fields, it will generate a title page with a message such as “Insert an author!”

However, the template does a lot more in the preamble than just create a title page. The preamble (that is, whatever comes before `\begin{document}` in the primary `.tex` file) is also the place for the user to specify a dedication, any acknowledgments, a foreword, *etc.* This is done in a manner very similar manner to declaring the author, title, and so on. Suppose that someone wants to have a simple dedication “To Mom” like the one in the Rackham guidelines, the following command is all that is needed.

```
\dedication{To Mom}
```

This will cause the document to have a dedication page with the corresponding text. If the `\dedication` command is not present, there will not be a dedication page. All the work of either having or not having a dedication has been compressed into a single command! Things other than simple text *are* allowed in the dedication, so feel free to put equations or whatever inside there. There are a few more commands that can be used to customize the appearance of the dedication page, and also for the other preamble text pages, but that is left to Section 2.2.3.

2.2 Front Matter

The \LaTeX term “frontmatter” refers to all of the pages that occur before the beginning of the first chapter. It is usually made clear to the reader because the pages in the front matter are numbered with lower-case Roman numerals instead of Arabic numerals.

The present template, `thesis-umich.cls` attempts to remove as much work associated with the front matter as possible. The template inserts all of the front matter pages automatically, so that there is not even a need to use a command like `\maketitle`. The first thing after `\begin{document}` should be the start of the first chapter.

Table 2.1: List of all identifier commands

| ITEM | USAGE | COMMENT |
|------------|-------------------------------|---|
| Author | <code>\author{...}</code> | Works as in standard \LaTeX |
| Chair | <code>\chair{...}</code> | Name of chair <i>without</i> any title or affiliation. This appears only on the abstract page, and only if there is no co-chair. |
| Co-chair | <code>\cochair{...}</code> | Names of all co-chairs <i>without</i> any titles or affiliations. This appears only on the abstract page. Note that by convention, it is not chair <i>and</i> co-chair, but just two co-chairs. |
| Committee | <code>\committee{...}</code> | Formatted names of committee members <i>with</i> the appropriate titles and university names. This will appear only on the title page. |
| Department | <code>\department{...}</code> | Title of department of student |
| Title | <code>\title{...}</code> | Works as in standard \LaTeX |
| Year | <code>\year=2012</code> | Year that dissertation will be <i>completed</i> |

2.2.1 Identifiers

The template is not able to read minds, of course, so there needs to be some way of inputting the relevant information. This section covers how to specify the author, title, and so on. For the most part, this works just like any other \LaTeX document, but a dissertation has a few more identifiers than most documents (How many books or reports have a committee?). So there are a few extra commands provided by this template, and they work *almost* exactly like the standard commands.

A full list of the identifiers is given in Table 2.1. All of these commands are required except for `\chair` and `\cochair`. Those are only used if there is an in-dissertation abstract page, and in that case only one of the two commands needs to be used, depending on whether or not you have co-chairs. If the `\cochair` command is invoked, the chair will be ignored, and the co-chairs will be inserted on the abstract page.

The only command that is somewhat unusual to use is the `\committee` command. It requires the author to separate the different committee members manually using line-ending commands. In general, this will look something like

```
\committee{
  Professor 1 \\
  Professor 2, Other School \\
  Professor 3}
```

If any of the required fields are not specified, the compilation does not crash, but rather a

reminder message (such as “Insert a Title!”) will be placed on the title page in the place of whatever identifier is missing.

The last comment in this section is on the ability to refer to the fields of the commands in Table 2.1 automatically throughout the text. This can be done using commands like `\insertauthor`, `\insertyear`, *etc.* This is not Earth-shattering, but it may be convenient if, for example, you are not sure if you will finish your dissertation in December or January.

2.2.2 Frontispiece and Copyright

Two optional pages occur after the title page and before page i. The first of these is called the frontispiece, which is usually a picture that goes along with the theme of your document. In this sample document, it is a schematic of a conceptual design of a scramjet vehicle. To get a frontispiece in your dissertation, use a command of the following form.

```
\frontispiece{\includegraphics{...}}
```

The other page has the copyright information. By default, the template assumes that there should be a copyright page, and the copyright holder is the author. To prevent the copyright page from appearing, use the command `\hidecopyright`. To assign the copyright to someone other than the author, use the following command.

```
\copyrightholder{Someone Else}
```

2.2.3 Text Pages

The handling of the first few pages after the title page is one of the best features of this template. The pages that occur between the copyright page and the abstract page all consist of short pieces of text that are usually a single paragraph. The list of such pages that exist in this template (in the order they would appear if all used) is dedication, acknowledgments, preface, prologue, and foreword. Of course, it would be unusual to use more than one of the last three. The text for each of these pages is set up using a command of the same name.

```
\foreword{This is going to be the best dissertation ever.}
```

Usually the contents of each of these pages will be longer than a single sentence, and thus it should be noted that each of these commands allows most types of \LaTeX input. For example, the following is perfectly acceptable input—at least as far as the template is concerned.

Table 2.2: List of styles for frontmatter text pages

| STYLE | DESCRIPTION |
|-------|---|
| 1 | Justified text with no header or lines |
| 2 | Justified text with bold italic header and no lines |
| 3 | Justified text, capitalized header, no lines |
| 4 | Justified text with lines and no header |
| 5 | Justified text with bold italic header and lines |
| 6 | Justified text with capitalized header and lines |
| other | Centered text with no header or lines |

```
\foreword{This is going to be the \emph{best}. \\
\begin{center} Really, really. \end{center}}
```

As I mentioned before, a given page will appear in the document if and only if the corresponding command is used. The order in which the pages appear does not depend on the order the commands are used in the preamble. You can also prevent the pages from appearing by using commands like `\hideforeword`.

The style of each page can also be set by the user. By default, each page will appear with a bold, italic heading corresponding to the name of the page. However, there are five other formats, which can be controlled using an optional argument. For example, the following command creates a dedication page with no heading (*i.e.* it does not say “Dedication” on the page) but with lines above and below the dedication text.

```
\dedication[4]{To Mom}
```

A complete list of the available styles is given in Table 2.2. The style of each page can be set independently, but it is also possible to change which style is used by default.

```
\frontpagestyle{6}
```

This would make all of the commands that were called without optional inputs to create pages using style 6. In this document, the dedication uses style 1, the acknowledgments use style 6, and the preface uses style 2. The heading is the reason that there is the ability to declare a prologue, a foreword, and a preface. The only difference between each pair of commands is the resulting heading.

The final capability of the front page commands is to change the width of the paragraph for each page. By default each page except for the preface and prologue will create a text region that is 66% of the maximum width allowed by the Rackham margin guidelines (The other two are set to 75% because they may have more text.), but these numbers can be

changed. Suppose that for whatever reason the dedication page would look better if the text area were quite a bit wider.

```
\dedicationwidth{0.8}
```

Using this command would set the area to 80% of the maximum allowable width. This command exists for each of the text pages, and the argument should be a single number between 0 and 1.

2.2.4 Lists of Things

For whatever reason, the Rackham guidelines say more about the Table of Contents than any other page. There are lots of comments about what **must** and **must not** go in the Table of Contents. Of course, that is a relatively difficult thing to control in L^AT_EX, which is why it is critical to have a template that takes care of all that automatically.

Suffice it to say that this template handles the Table of Contents appropriately, but this section is also meant to address the List of Figures, List of Tables, *etc.* According to the guidelines, a corresponding list must appear if there is more than one figure, table, map, program, illustration, or appendix. How to insert a map, program, or illustration is discussed in Section 2.3.1. This brings us to one of the things that the template is not able to do automatically. There is no way to determine if the document will have more than one of each type of float, so the user must manually inform the template which lists to create. For example, if the document has more than one map in it, the command

```
\showlistofmaps
```

must occur somewhere in the preamble. Note that this also applies to the list of appendices, which for some reason are not supposed to appear as simply specially-numbered chapters in the Table of Contents. For more information about appendices with this template, see Appendix B. The template assumes that the dissertation will contain at least two figures and tables. If, for example, there is only one figure,

```
\hidelistoffigures
```

must be put in the preamble.

The other type of list that can occur is for abbreviations of various types. This is a somewhat convenient feature, particularly there are a lot of acronyms in the dissertation. This template utilizes the acronym package. In the preamble put a command like the following.

```

\abbreviations{
  \acro{CFD}{Computational Fluid Dynamics}
  \acro{LOA}{List of Abbreviations}
  \acro{H2O}[$\mathrm{H_2O}$]{water}}

```

This will define a bunch of abbreviations that can be used. When you want to use one of the acronyms within the text, simply use the `\ac` command to refer to the abbreviation you want. This will automatically spell out what the abbreviation stands for on the first use and only print out the abbreviation on subsequent uses. For example, when the definitions from the previous example are used, `\ac{H2O}` will result in “H₂O (water)” the first time it is used and just “H₂O” all subsequent times. In addition to the `\abbreviations` command, the template also provides `\acronyms` and `\symbols`. Each of these works in the same way, and if they are called, a page will appear between the list of appendices and the abstract with the corresponding header. As a final note, the abbreviations will only appear in the list if they are used at least once in the text of the document.

2.2.5 Abstract Page

The abstract page can optionally occur within the document. To add an abstract to a dissertation with this template, one can simply use the `\abstract` command. In other words, this works almost exactly like the `\dedication`, `\acknowledgments`, *etc.* commands. However, it will create a page with slightly more information, and there is no optional argument that determines the style of the page.

According to the Rackham Guidelines, the abstract page **must** have a page number. However, the abstract page created by this template meets the guidelines for the required abstract that is needed separately from the dissertation, which **must not** have a page number. So that’s pretty silly. Anyway, it would be pretty convenient to just use the appropriate page from the dissertation as the abstract page, so there is a command to hide the page number on the abstract page.

```

\hideabstractpagenumber

```

Once the abstract page has been printed, remember to get rid of this command so that the abstract in the dissertation (which is not required, by the way) will have the appropriate page number.

2.3 Text Portion

The template also provides several tools that can be used in the main text part of the document, as well. One of the features of this template is that it loads most of the common packages automatically. This alleviates the need for an annoying list of `\usepackage` commands in the preamble. This does not mean, however that other packages cannot be used.

2.3.1 Alternative Float Types

The Rackham guidelines refer to objects such as maps and illustrations, which behave much like a figure, but may be numbered and organized separately. Correspondingly, template provides environments for maps, programs, and illustrations. They work exactly like a figure, as shown in the following example.

```
\begin{map}  
  \centering  
  \includegraphics{...}  
  \caption{...}  
\end{map}
```

The only difference between this and a figure is that the caption will say “Map ...” instead of “Figure ...” As mentioned in Section 2.2.4, this is also tied to a List of Maps, although the user must have the command

```
\showlistofmaps
```

if there are at least two maps in the document.

2.3.2 Additional Text Formatting

The template contains several commands which can be used to format special types of text. These work exactly like built-in \LaTeX commands such as `\textbf`, but they combine multiple formatting features. A complete list with examples is in Table 2.3.

Another capability that is provided by this template is the short code snippets that have been interspersed throughout this chapter. This is done using the `code` environment, and an example is shown in Program 2.1. The only differences between this environment and the `verbatim` environment are that there is an indentation to set it off from the rest of the text, and the resulting text is not enormous compared to the rest of the text.

Table 2.3: List of text function from this template

| COMMAND | EXAMPLE | POSSIBLE USE |
|-------------------------|---|-----------------------------------|
| <code>\tfile</code> | thesis-umich.cls | name of a file |
| <code>\tfunction</code> | sqrt | name of functions in programs |
| <code>\tmenu</code> | <div style="border: 1px solid black; padding: 2px; display: inline-block;">Format</div> | a button or menu entry |
| <code>\tstring</code> | 'off' | contents of a string in a program |
| <code>\tvar</code> | n_apples | name of variable in programs |

```
\begin{code}
Stuff to appear exactly as you type it...
\end{code}
```

Program 2.1: Entering small code snippets

2.3.3 Bibliography

The bibliography does not require any special additional guidelines. It will work best with Bib_TE_X, and the usual

```
\bibliography{./ref/references}
```

type of command will result in a correctly formatted bibliography. The references within the text will automatically generate links to the part of the bibliography page where the reference information is present.

There is an additional capability that puts links to the page numbers on which each reference was cited. As an example. suppose you cited reference 1 on pages 24, 36, and 40. Then the bibliography will have the numbers 24, 36, and 40 immediately after the first reference, and these numbers will be links to the corresponding pages. In order to activate this behavior, use the `backref` option in the very first line.

```
\documentclass[backref]{thesis-umich.cls}
```

2.3.4 Document Format

The template has a capability to change the format of the entire document by simply changing

```
\documentclass{thesis-umich.cls}
```

to

```
\documentclass[report]{thesis-umich.cls}
```

This can also be combined with other options.

```
\documentclass[report,backref]{thesis-umich.cls}
```

The `report` format will change the base font size from 12pt to 10pt, decrease the size of the margins, and change the main text into two-column format. It also puts headers on pages that are not the first page of a chapter. The purpose of this option is to compress the dissertation into a tighter, more journal-like format for personal records. However, it will *not* ensure that all of your figures, tables, and equations will fit with the new format.

CHAPTER 3

Setting

The next chapter has the good stuff.

3.1 Convergence Criteria

Actually, it might have the worst stuff. But it is slightly easier to write than the material in Chapter 1.

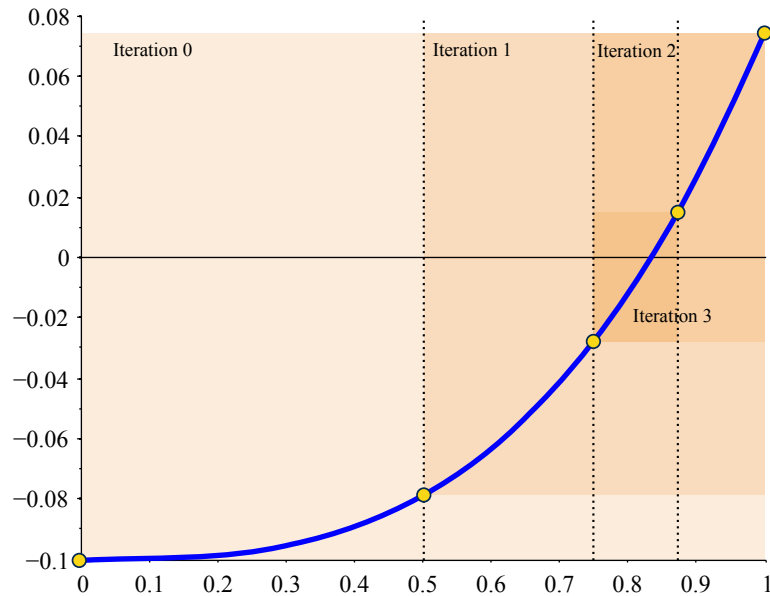


Figure 3.1: Illustration of x - and y -tolerances for bisection iterations

It takes very little text to fill a page in this format, but there is even less text on most of these sample pages.

3.2 Why we are doing it

It is usually a good idea to give reasons for your research. If you do not, the people who paid you to waste all that time will feel really bad about it, and then they will not provide the same opportunity to future students.

I need this page to see what even-numbered pages look like.

APPENDIX A

Methods

Here is how to implement the methods.

A.1 Bisection

The easiest method.

$$x_k = \frac{a_k + b_k}{2} \tag{A.1}$$

A.2 False Position

The next one.

(A map of the United States)

Program A.1: Map of the United States

APPENDIX B

Using Appendices

This appendix contains the portion of the users' manual that describes how to use appendices with this template. It is put in this appendix rather than in Chapter 2 simply so that there are two appendices, so that a list of appendices can appear earlier in the document.

B.1 Starting the Appendices

Actually, using appendices is quite simple. Immediately after the end of the last chapter and before the start of the first appendix, simply enter the command `\appendix`. This will tell \LaTeX to change how it interprets the commands `\chapter`, `\section`, *etc.*

Each appendix is actually a chapter, so once the `\appendix` command has been called, start a new appendix by simply using the `\chapter` command.

```
\appendix
\chapter{The First Appendix}
...

\chapter{The Boring Part That Is Not in the Chapters}
```

Note that the `\appendix` command should be called only once—not before the start of each appendix.

B.2 Lists Including the Appendices

As mentioned in Section 2.2.4, the command

```
\showlistofappendices
```

must appear in the preamble if there are more than one appendices. For some reason, Rackham does not want the individual appendices and their sections to appear in the Table of Contents, so a special List of Appendices page (which must occur in the Table of Contents!) is required as a sort of extension to the Table of Contents.

This does not require a user of this template to do anything, but it is so silly that I felt it was worth explaining. Also, there is nowhere for the sections or subsections of appendices to show up in the Table of Contents or any of the lists, but they do still create navigation tabs in a modern PDF viewer.

BIBLIOGRAPHY

- [1] Chapra, S. C. and Canale, R. P., *Numerical Methods for Engineers*, McGraw-Hill, 4th ed., 2002.