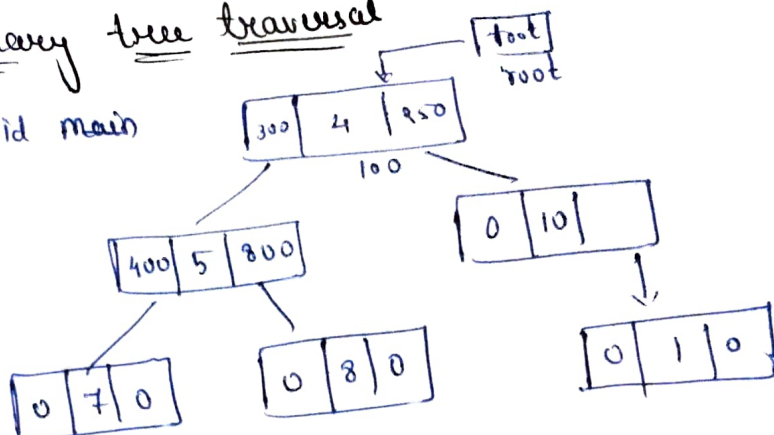


# Binary tree traversal

Void main



Inorder: left root right

7, 5, 8, 4, 10, 1

Preorder: root left right

4, 5, 7, 8, 10, 1

Postorder: left right root

7, 8, 5, 1, 10, 4

Void main(){

structnode \*root;

printf("Preorder is : \n");

preorder (root);

Inorder (root);

Postorder (root);

}

Void preorder (structnode \*root);

if (root == '0'){

return ;

else{

printf(" %d", root->data);

preorder (root->left);

preorder (root->right);

}

void main

```

void inorder ( structnode * root ) {
    if ( root == '0' ) {
        return 0;
    }
    else {
        inorder ( root → left );
        printf ( " %d", root → data );
        inorder ( root → right );
    }
}

```

```

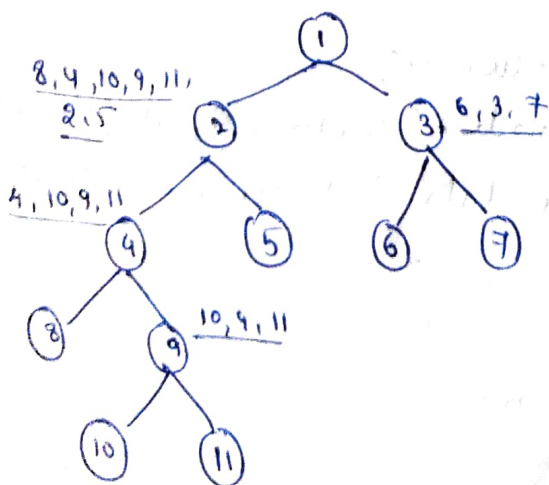
void postorder ( structnode * root ) {
    if ( root == '0' ) {
        return;
    }
    else {
        postorder ( root → left );
        postorder ( root → right );
        printf ( " %d", root → data );
    }
}

```

Construct a B.T from pre order & Inorder

Preorder :- 1, 2, 4, 8, 9, 10, 11, 5, 3, 6, 7

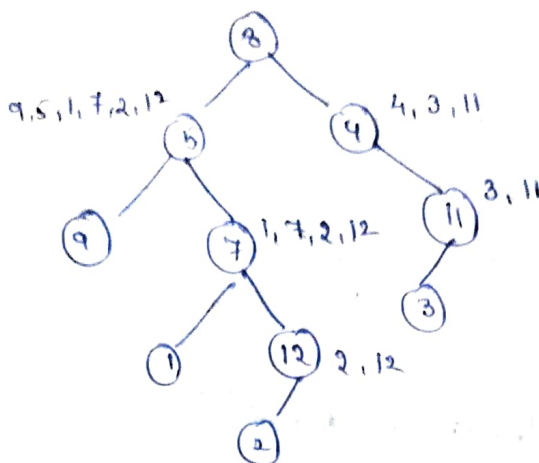
Inorder :- 8, 4, 10, 9, 11, 2, 5, 1, 6, 3, 7



Construct a binary tree from Postorder & Inorder

Postorder : 9, 1, 2, 12, 7, 5, 3, 11, 4, 8

Inorder : 9, 5, 1, 7, 2, 12, 3, 4, 8, 11



Implementation of B.T

```
struct node {
    int data;
    struct node * left, * right;
};
```

```
struct node * create();
```

```
void main() {
    struct node * root;
    root = NULL;
    root = create();
}
```

```
struct node * create() {
```

```
    int x;
```

```
    struct node * newnode;
```

```
    newnode = (struct node *) malloc(sizeof(struct node));
```

```
    printf("Enter data (-1 for no node):");
```

```
    scanf("%d", &x);
```

```
    if (x == -1)
```

```
        return 0;
```

```
    }
```

```
    else {
```

```
        newnode->data = x;
```

```
        printf("Enter left child of %d", x);
```

```
        newnode->left = create();
```

```
struct node * create() {
```

```
    int i;
```

```
    new = (struct node *) malloc (size of (struct node));
```

```
    new → left = NULL;
```

```
    new → right = NULL;
```

```
    printf("enter the data");
```

```
    scanf("%d", &i);
```

```
    if (i == -1)
```

```
        return NULL;
```

```
    new → data = i;
```

```
    printf("enter the data for left child %d", i);
```

```
    new → left = create();
```

```
    new → right = create();
```

```
    return new;
```

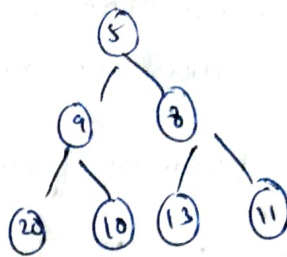
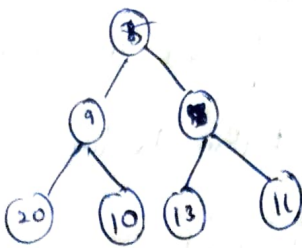
```
}
```

Heap : Heap is also a tree data structure in this we heap is divided into 2 types. It is complete binary tree.

- (i) min heap
- (ii) max heap

(i) min heap : The element at root must be less than or equal to its children. It is recursively true for all its sub trees.

|   |   |    |    |    |    |   |   |
|---|---|----|----|----|----|---|---|
| 8 | 9 | 11 | 20 | 10 | 13 | 5 | 2 |
|---|---|----|----|----|----|---|---|

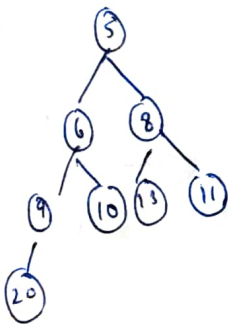


|   |   |    |    |    |    |   |   |
|---|---|----|----|----|----|---|---|
| 8 | 9 | 11 | 20 | 10 | 13 | 5 | 6 |
|---|---|----|----|----|----|---|---|

before inserting

|   |   |   |   |    |    |    |    |
|---|---|---|---|----|----|----|----|
| 5 | 6 | 9 | 9 | 10 | 13 | 11 | 20 |
|---|---|---|---|----|----|----|----|

After inserting

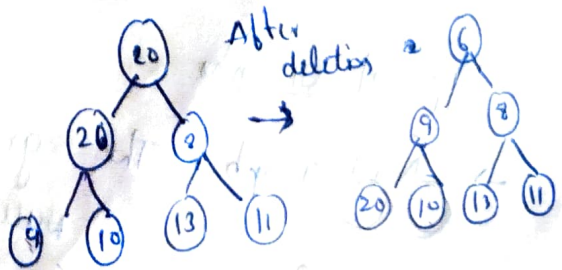
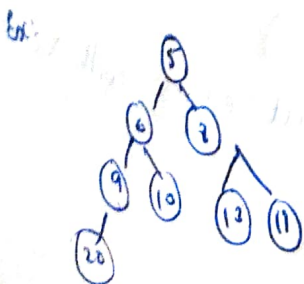


= In Insertion we need to traverse from bottom to top

= In deletion we need to traverse from top to bottom

= while deleting a node we delete root node and insert last level node at root & heapify algorithm (checking heap property).

|   |   |   |   |    |    |    |    |
|---|---|---|---|----|----|----|----|
| 5 | 6 | 8 | 9 | 10 | 13 | 11 | 20 |
|---|---|---|---|----|----|----|----|

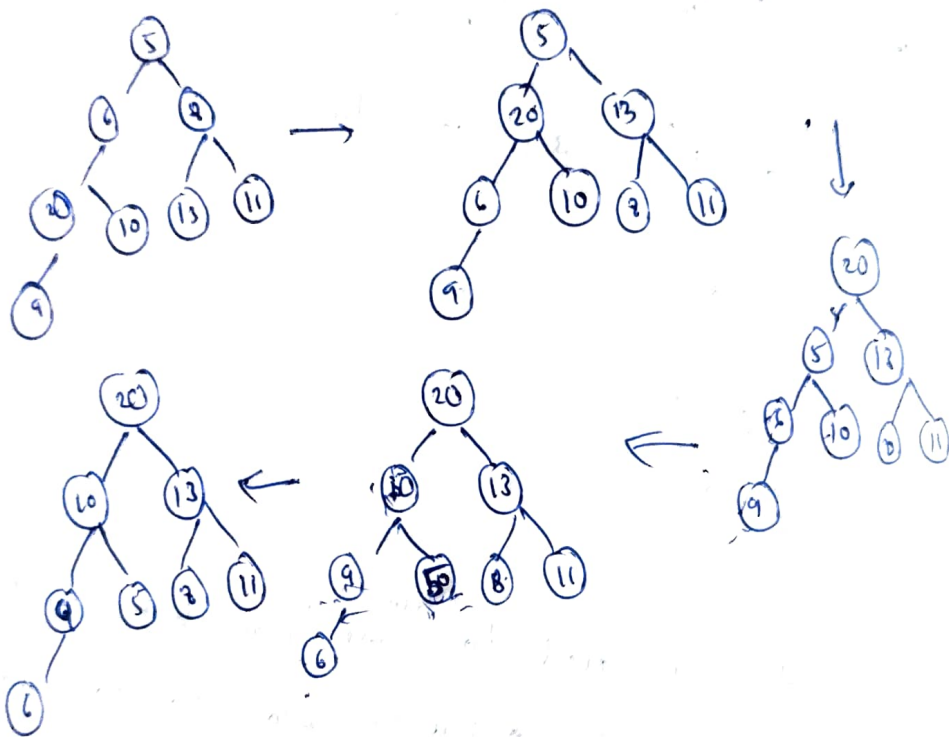




Max heap : Max heap - The element at root must be greater than or equal to its children. It is recursively true for all its sub trees.

Insertion : (traverse from bottom to top)

|   |   |   |   |    |    |    |    |
|---|---|---|---|----|----|----|----|
| 5 | 6 | 8 | 9 | 10 | 13 | 11 | 20 |
|---|---|---|---|----|----|----|----|



Graph - Graph is a non-linear data structure. It consists of set of vertices & edges. If a graph  $G$  consists of  $E$  edges,  $V$  vertices, we can represent the graph as  $G(E, V)$ .

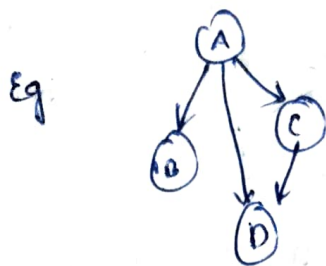
Types of Graphs

→ Null graph : The graph without edges, with vertices is null graph.

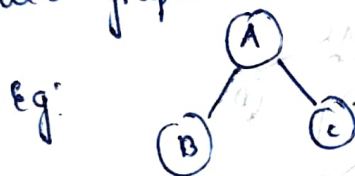
Eg: (A) (B) (C)

→ Trivial graph: A graph with only 1 Vertex is known as trivial graph.

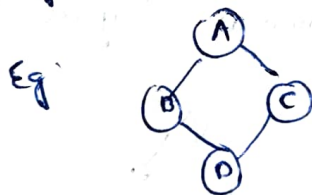
→ Directed graph (Di-graph): A graph with a particular direction from one <sup>vertex</sup> node to another vertex is directed graph



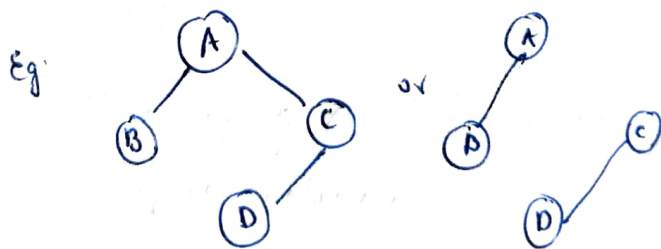
→ Undirected graph: A graph without direction is Undirected graph.



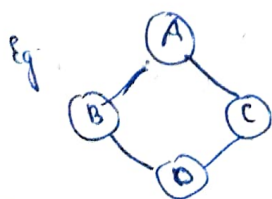
→ Connected graph: In a connected graph, from one node you can visit any other node is known as connected graph



→ Disconnected graph Atleast one node is not reachable from any node

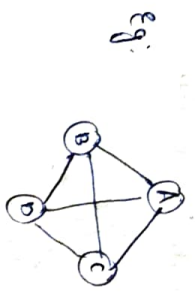


→ Regular graph: In regular graph degree (no. of Edges connected) of each node is equal



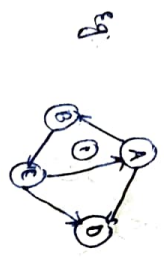
$$\left. \begin{array}{l} D(A) = 2 \\ D(B) = 2 \\ D(C) = 2 \\ D(D) = 2 \end{array} \right\} \text{Same}$$

Complete graph: The graph in which each vertex is connected with every other vertex. It is known as complete graph.  $T.N. \text{ of vertices} = 4, n-1 = 3$

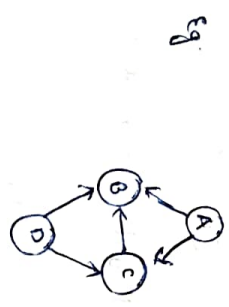


Eg:  
 $d(A) = 3$   
 $d(B) = 3$   
 $d(C) = 3$   
 $d(D) = 3$

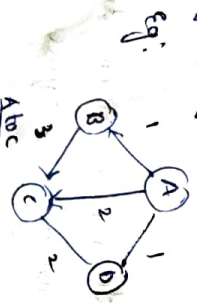
Cyclic graph: A graph with (closed loop) or at least one cycle is known as cyclic graph.



Ayclic graph: A graph without a cycle (closed path) is known as ayclic graph.



Weighted graph: A graph with weights is known as weighted graph.



Eg:  
 $A \rightarrow B = 1$   
 $A \rightarrow C = 2$   
 $B \rightarrow D = 3$   
 $C \rightarrow D = 2$

BFS: (Breadth First Search).

→ BFS follows Queue (FIFO)

1. First we need to select any node as source node.
2. Insert into queue.
3. Mark it as visited.
4. Remove source node & print, increment front.
5. Insert into queue. Visit adjacent vertices & remove it & print and mark it as visited.

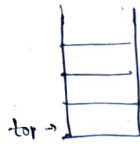
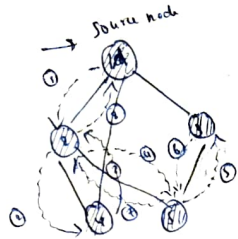
DFS: Depth First Search

→ DFS follows Stack (LIFO)

Algorithm

1. Select a node as source node.
2. Mark it as visited, and display.
3. Check for any unvisited adjacent nodes following a particular path.
4. Insert the adjacent node if it is unvisited into the stack.
5. If there is no unvisited adjacent nodes backtracking will be done (removing the node from stack) and print it. & then (pop out).
6. Repeat the top (top →).
7. Check again that for any unvisited adjacent nodes, mark it as visited, if not visited else visited pop out the node.
8. Follow the same process until stack is empty.





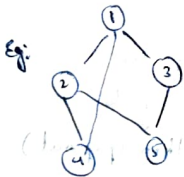
1 2 4 5 3 visited

print: 4 3 5 2 1

graph Representation (1. Adjacency & Matrix & Adjacency list)

Adjacency matrix:- In adjacency matrix boolean values are considered. '0' indicates no edge between nodes  
 '1' indicates edge between nodes

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 1 |
| 3 | 1 | 0 | 0 | 0 | 1 |
| 4 | 1 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 | 0 |



2) Adjacency list:

|         |   |   |   |   |      |
|---------|---|---|---|---|------|
| edge 1: | 1 | 2 | 3 | 4 | Null |
| edge 2: | 2 | 1 | 4 | 5 | 0    |
| edge 3: | 3 | 1 | 5 | 0 | 0    |
| 4:      | 2 | 1 | 0 | 0 | 0    |
| 5:      | 3 | 2 | 0 | 0 | 0    |

Assignment BFS & DFS work with Adjacency Matrix.

DFS: with Adjacency list

Min & Max heap & Insert

Implement DFS & BFS with Adjacency matrix

Construct min & Max heap & perform Insertion & deletion operation (array with array)

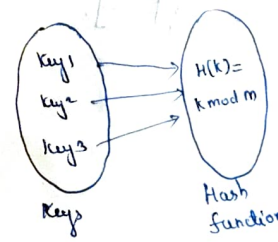
Hashing: Hashing is a technique or a process which is keys with the values in a hash table.  
 => with the help of hashing we can efficiently access the elements from the hash table.  
 => The efficiency depends upon the type of Hash function is used.

list = {11, 12, 13, 14, 15}

size = 10

$$H(\text{key}) = \text{key} \bmod \text{size} \quad \text{or} \quad H(k) = k \bmod m$$

$$\begin{aligned} H(11) &= 11 \bmod 10 = 1 \\ H(12) &= 12 \bmod 10 = 2 \\ H(13) &= 13 \bmod 10 = 3 \\ H(14) &= 14 \bmod 10 = 4 \\ H(15) &= 15 \bmod 10 = 5 \end{aligned}$$



| Key  | Value |
|------|-------|
| Key1 | 0     |
| Key2 | 1     |
| Key3 | 2     |
| ...  | ...   |

Hash table

| Value | keys |
|-------|------|
| 0     |      |
| 1     | 11   |
| 2     | 12   |
| 3     | 13   |
| 4     | 14   |
| 5     | 15   |
| 6     |      |
| 7     |      |
| 8     |      |
| 9     |      |

## Hashing Terminology

Hash key, Hash function, Hash table  
something which

Hash Key, Hash function, which is given as

Key: Key is Integer or a string  
Input to hash function.

Hash function:- Hash function is a mathematical function that uses key as a input & it will generate value or index. where we can store keys, based on the value generated by hash function.

Hash Table. Hash table is a type of data structure which is used to map.

## Types of Hash functions

- i) Division Method
- ii) mid-square method
- iii) Multiplication method
- iv) Digit-folding method

$\Rightarrow$  Division Method:  $H(\text{key}) = \text{key} \bmod m$

$\downarrow$                        $\downarrow$

key value          hash table size

$$\text{list} = \{10, 11, 13, 17\}, \quad m=10.$$
$$H(10) = 10 \pmod{10}$$
$$H(11) = 11 \bmod 10$$
$$h(13) = 13 \bmod 10$$
$$H(17) = 17 \bmod 10$$

|   |    |
|---|----|
| 0 | 10 |
| 1 | 11 |
| 2 |    |
| 3 | 13 |
| 4 |    |
| 5 |    |
| 6 |    |
| 7 | 17 |
| 8 |    |
| 9 |    |

ii) Mid-Square Method:

you need to find out the square of key and then find middle or digits.

list = { 10, 11, 13 }

$$h(k) = k^2$$

$$h(k^2) = \{ \underline{100}, \underline{121}, \underline{169} \}$$

|   |    |
|---|----|
| 0 | 10 |
| 1 |    |
| 2 | 11 |
| 3 |    |
| 4 |    |
| 5 |    |
| 6 | 13 |
| 7 |    |

100

721

$$1769$$

This only works for large  $n$  numbers (i.e. after 2 or more digit numbers)

### iii) Multiplication Method

$$H(k) = \text{floor} \left( \frac{m(kA \bmod 1)}{k} \right)$$

size <sup>valley</sup> constant

$$A = 0.68103$$

$$A = 0.618033$$

$0 < A < 1$

iv) Digit folding Method:-

$$H(K) = H(k_1, k_2, \dots, k_n)$$

$$H(\underbrace{123}_{k_1} \underbrace{45}_{k_2})$$

$$S = k_1 + k_2 + \dots + k_n$$

$$= 12 + 34 + 5$$
$$= 51$$

A hand-drawn number line from 0 to 100. A vertical dashed line is drawn at the 50 mark. To the right of this line is a place value chart with two rows and two columns. The top row is labeled 'Tens' and the bottom row is labeled 'Ones'. The number 51 is written on the number line to the left of the dashed line, and the number 12345 is written in the 'Tens' column of the chart.

## Collisions :

collision is a problem when two or more keys are mapped with the same location in the hash table

$$H(k) = k \bmod M$$

$$\text{list} = \{20, 10, 11, 21, 33\}, m = 10$$

$$H(20) = 20 \bmod 10 = 0$$

$$H(10) = 10 \bmod 10 = 0$$

$$H(1) = 1 \bmod 6 = 1$$

$$h(21) = 21 \bmod 10 = 1$$

$$H(33) = 33 \bmod 10 = 3$$

|   |    |
|---|----|
| 0 | 20 |
| 1 | 11 |
| 2 |    |
| 3 | 33 |
| 4 |    |
| 5 |    |
| 6 |    |
| 7 |    |
| 8 |    |
| 9 |    |

← but (21, 8, 10) (which is collision)

## Collision resolving Techniques

- Separate chaining (open hashing)
- Linked list is used in separate chaining
- Open Addressing (closed hashing)
  - linear probing
  - quadratic probing
  - double-hashing

### • Separate chaining

list = {20, 10, 11, 21, 33, 44}

$$H(10) = 10 \bmod 10 = 0$$

$$H(20) = 20 \bmod 10 = 0$$

$$H(11) = 11 \bmod 10 = 1$$

$$H(21) = 21 \bmod 10 = 1$$

$$H(33) = 33 \bmod 10 = 3$$

$$H(44) = 44 \bmod 10 = 4$$

|   |    |    |    |
|---|----|----|----|
| 0 | 20 | 10 | 30 |
| 1 | 11 | 21 |    |
| 2 |    |    |    |
| 3 | 33 |    |    |
| 4 | 44 |    |    |
| 5 |    |    |    |
| 6 |    |    |    |
| 7 |    |    |    |
| 8 |    |    |    |
| 9 |    |    |    |

## Open addressing

linear probing

$$(u+1) \% m$$

$q$  — 0 to  $m-1$

$u$  — collision location

quadratic

$$(u+i^2) \% m$$

Double hashing

$$(u+vi)$$

### ① Linear probing

$$h(k) = k \% m$$

② If you have free space in hash table at a particular location then we have to store the key value in the hash table.

③ If you don't have empty space, we have to find empty space based on  $(u+i) \% m$ .

$u$  → collision location in hash table

$i$  → 0 to  $m-1$  where  $m$  is the size

④ Repeat 2 & 3 Until (size-1) i.e.  $(m-1)$  and store.

Given

5, 10, 4, 21, 7, 9

$$m=10, \quad h(k) = 2k+1$$

$$h(5) = (2(5)+1) \% 10 = 1$$

$$h(10) = (2(10)+1) \% 10 = 1$$

$$h(4) = (2(4)+1) \% 10 = 9$$

$$h(2) = 5$$

$$h(7) = 5$$

$$\text{Gk } h(9) = 9$$

|   |    |
|---|----|
| 0 | 9  |
| 1 | 5  |
| 2 | 10 |
| 3 |    |
| 4 |    |
| 5 | 2  |
| 6 | 7  |
| 7 |    |
| 8 |    |
| 9 | 4  |



| key | location | Probes | for 10 $(u+i) \% m$                             |
|-----|----------|--------|---|
| 5   | 1        | 1      | for 2 $\{(1+0) \% 10 = 1$<br>$(1+1) \% 10 = 2$  |
| 10  | 1 (coll) | 2      |   |
| 4   | 9        | 1      | for 7 $2 \{(5+0) \% m = 5$<br>$(5+1) \% 10 = 6$ |
| 2   | 5        | 1      |   |
| 7   | 5        | 1      | for 3. $(9+10) \% 10 = 9$<br>$(9+1) \% 10 = 0$  |
| 9   | 9 (coll) | 2      |   |

Note: The element k will be stored at first free location from  $(u+i) \% m$  in has

| key | location | Probes | for 10 $(u+i) \% m$ |
|-----|----------|--------|---------------------|
| 3   | 6        | 1      |                     |
| 6   | 9        | 1      |                     |
| 11  | 4        | 1      |                     |
| 4   | 7        | 1      |                     |
| 7   | 1        | 1      |                     |
| 9   | 9 (coll) | 2      |                     |

$m = 10$   
 $h(k) = (2k+3)$   
 $h(3) = 9$   
 $h(6) = 5$   
 $h(9) = 1$   
 $h(11) = 5 \rightarrow 6$   
 $h(4) = 1 \rightarrow 3$   
 $h(7) = 5 \rightarrow 4$   
 $h(1) = 5 \rightarrow 9$

| key | location | Probes | for 10 $(u+i) \% m$ |
|-----|----------|--------|---------------------|
| 3   | 6        | 1      |                     |
| 6   | 9        | 1      |                     |
| 11  | 4        | 1      |                     |
| 4   | 7        | 1      |                     |
| 7   | 1        | 1      |                     |
| 9   | 9 (coll) | 2      |                     |

$5+0 = 5$   
 $5+1 = 6$   
 $1+0 = 1$   
 $1+1 = 2$

### Quadratic Probing

3, 6, 9, 11, 4, 7  
 $m = 10$ ,  $h(k) = 2k + 3$ ,  $(u+i^2) \% m$

$h(3) = (6+3) \% 10 = 9$   
 $h(6) = (12+3) \% 10 = 5$   
 $h(9) = (18+3) \% 10 = 1$   
 $h(11) = (22+3) \% 10 = 5$   
 $h(4) = (8+3) \% 10 = 1$   
 $h(7) = (14+3) \% 10 = 7$   
 $h(1) = (2+3) \% 10 = 5$

| key | location | Probes | for 10 $(u+i) \% m$ |
|-----|----------|--------|---------------------|
| 3   | 6        | 1      |                     |
| 6   | 9        | 1      |                     |
| 11  | 4        | 1      |                     |
| 4   | 7        | 1      |                     |
| 7   | 1        | 1      |                     |
| 9   | 9 (coll) | 2      |                     |

| key | loc      | Probes | for 10 $(u+i) \% m$ |
|-----|----------|--------|---------------------|
| 3   | 6        | 1      |                     |
| 6   | 9        | 1      |                     |
| 11  | 4        | 1      |                     |
| 4   | 7        | 1      |                     |
| 7   | 1        | 1      |                     |
| 9   | 9 (coll) | 2      |                     |

Collision at 5  
 $(5+0^2) \% 10 = 5$   
 $(5+1^2) \% 10 = 6$  free  
 $(5+4^2) \% 10 = 9$   
 $(5+9^2) \% 10 = 4$  free

Collision at 1  
 $(1+10^2) \% 10 = 1$   
 $(1+1^2) \% 10 = 2$   
 $(1+2^2) \% 10 = 5$  free



# Double Hashing

3, 9, 4, 7, 8, 2

$$h_1(k) = 2k + 3, \quad h_2(k) = 3k + 1, \quad m = 10$$

$$(u + v * f) \% m$$

$$1) 3 \Rightarrow (2 \times 3 + 3) \% 10 = 9$$

$$= 9$$

$$2) 9 \Rightarrow (2 \times 9 + 3) \% 10 = 1$$

$$= 1$$

$$3) 4 \Rightarrow (2 \times 4 + 3) \% 10 = 11 \text{ but}$$

collision occurs  
apply second function

$$V = (3 \times 4 + 1) \% 10$$

$$= (13) \% 10$$

$$= 3$$

$$(u + v * f) \% m$$

$$\rightarrow (1 + 3 \times 0) \% 10 = 1$$

$$(1 + 3 \times 1) \% 10 = 4$$

$$h(3) = (2(7) + 3) \% 10 = 7$$

$$h(13) = (2(8) + 3) \% 10 = 9$$

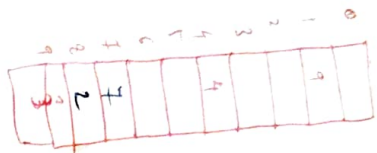
$$V = (3(8) + 1) \% 10 = 23$$

$$u = 1$$

$$h(2) = (2(2) + 3) \% 10 = 7$$

$$V = 3k + 1$$

$$= 3(2) + 1 = 7$$



key location Prob

3 9 1 1

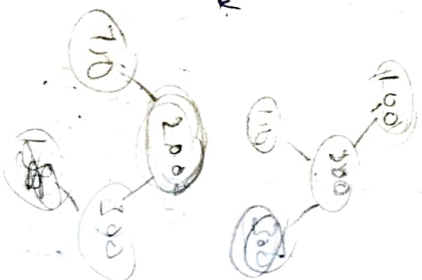
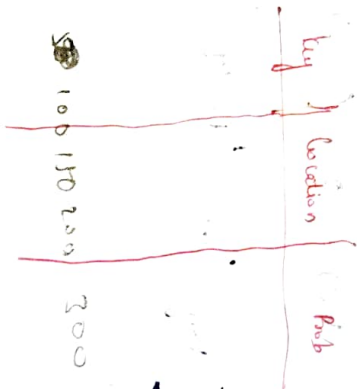
4 1 1 2

4 1 1 2

$$V = 3$$

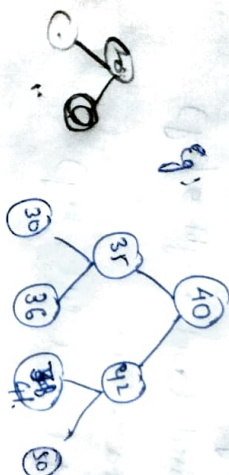
u = starting collision

$$V =$$



## BST

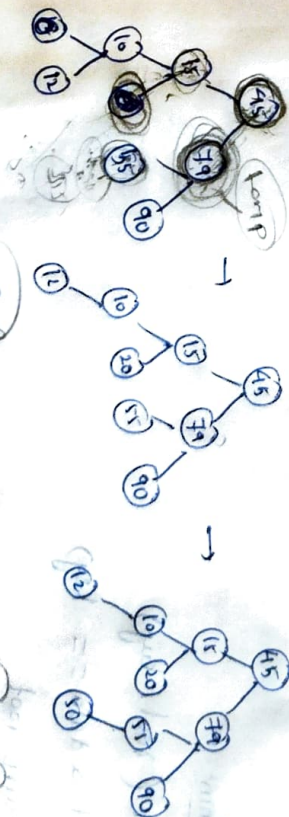
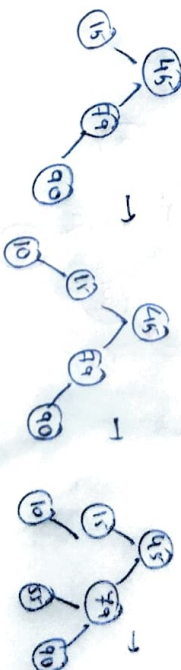
Binary search tree is a non-linear data structure in this left side element must be less than its parent  $\rightarrow$  right side element must be greater than its parent. This condition should be applied for its subtree recursively.



Left right child must be less than or greater than its

- i) Searching
- ii) Insertion
- iii) Deletion
- iv) Traversal

45, 15, 79, 90, 10, 55, 12, 20, 50



## Algorithm:

insert (root, key)

1) if (root == null)

return root;

2) if (root → data < key)

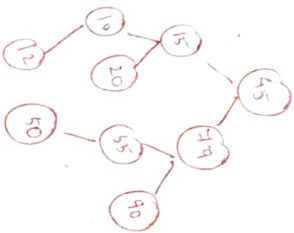
root → right = insert (root → right, key)

3) else if (root → data > key)

root left = insert (root left, key)

4) return root.

## searching



## Algorithm:

search (root, key)

1) if (root → data == key)

return root

2) else if (root → data < key)

return search (root → right, key)

3) else

return search (root → left, key).

## Solution:

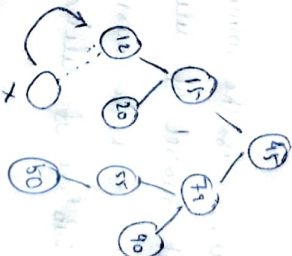
1) The node to be deleted is a leaf node (without any children).

2) The node to be deleted is having one child

3) The node to be deleted is having two children.

② Node with one child:-

Replace its child with node with its child.  
remove its child.



if (root → left == null) {

temp = root → right;

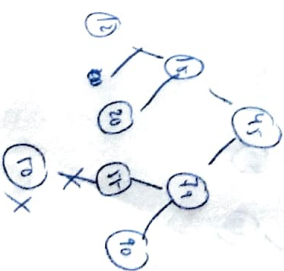
root → data = temp → data;

free (temp);

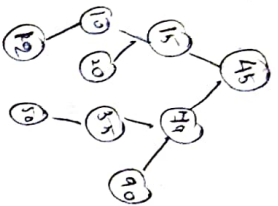
① if (root → left == null && root → right == null)

free (root)

return null;

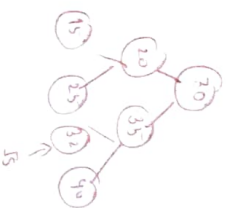
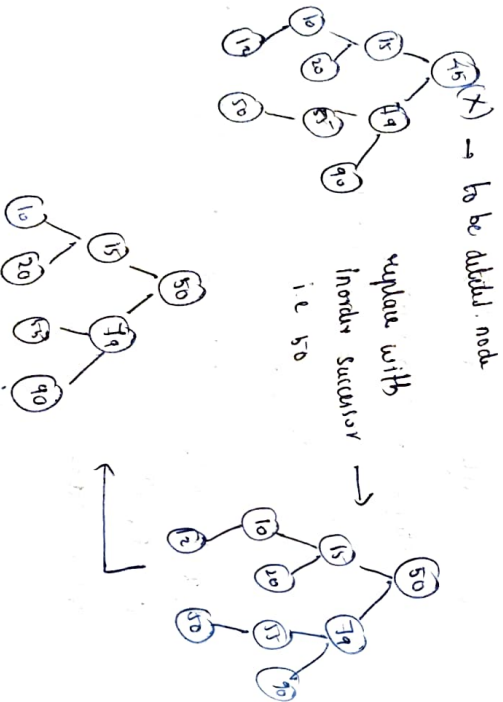


③ A node with 2 children



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- i) Find out inorder successor or inorder predecessor
- ii) replace with the deleted node with inorder successor
- iii) remove inorder successor (i.e. the minimum element in a right subtree of deleted node)  
(or) inorder predecessor (i.e. the maximum element in left subtree of deleted node)



int min = inordermin(root -> right);  
root -> data = min;  
delete(root, min);

```

inordermin (struct node *root) {
    if (root == NULL) return 0;
    while (root -> left != NULL)
        root = root -> left;
    return (root -> data);
}

```