

10.3.2.4.4 Presentation

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1 Problem

2 Solution

3 Plot

Problem Statement

Check if the pair of linear equations $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$.
And if consistent, obtain the solution

Solution

Given

$$2x - 2y - 2 = 0 \quad (3.1)$$

$$4x - 4y - 5 = 0 \quad (3.2)$$

Simplifying and using matrix notation,

$$\begin{pmatrix} 1 & -1 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (3.3)$$

The matrix A can be decomposed into:

$$A = L \cdot U, \quad (3.4)$$

where:

$$L = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}, \quad (3.5)$$

$$U = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}. \quad (3.6)$$

Factorization of LU:

Given a matrix **A** of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

1. Start by initializing **L** as the identity matrix $\mathbf{L} = \mathbf{I}$ and **U** as a copy of **A**.

2. For each column $j \geq k$, the entries of *U* in the *k*-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j} \quad \forall \quad j \geq k \quad (3.7)$$

3. For each row $i > k$, the entries of *L* in the *k*-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right) \quad \forall \quad i > k \quad (3.8)$$

The system $A\mathbf{x} = \mathbf{b}$ is transformed into $L \cdot U \cdot \mathbf{x} = \mathbf{b}$. Let \mathbf{y} satisfy $L\mathbf{y} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}. \quad (3.9)$$

Using forward substitution:

$$y_1 = 1 \quad (3.10)$$

$$4y_1 + y_2 = 5 \quad (3.11)$$

$$y_2 = 1 \quad (3.12)$$

Thus:

$$\mathbf{y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (3.13)$$

Next, solve $U\mathbf{x} = \mathbf{y}$:

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (3.14)$$

Using back substitution:

$$0x + 0y = 1 \quad (3.15)$$

As it is not possible for any value of x and y , the solution does not exist.

Plot

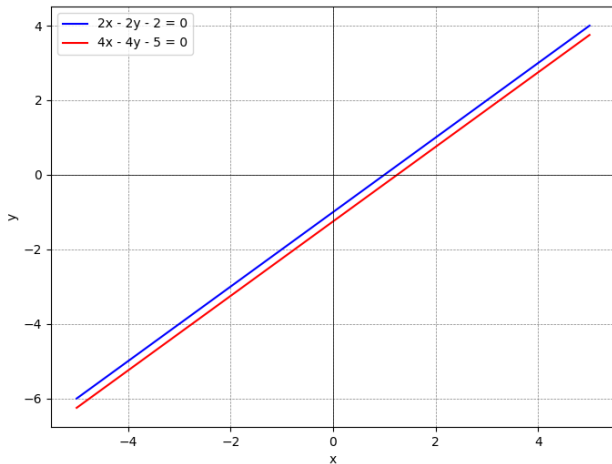


Figure: Graph of the given lines