

9.3.11

EE24BTECH11024 - G. Abhimanyu Koushik

Question:

Solve the differential equation $\frac{d^2y}{dx^2} + 1 = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 0$

Solution:

Variable	Description
c_1	First Integration constant
c_2	Second Integration constant
n	Order of given differential equation
a_i	Coefficient of i th derivative of the function in the equation
c	constant in the equation

TABLE 0: Variables Used

Theoretical Solution:

$$\frac{d^2y}{dx^2} + 1 = 0 \quad (0.1)$$

$$\frac{d^2y}{dx^2} = -1 \quad (0.2)$$

$$\int \frac{d^2y}{dx^2} dx = \int -1 dx \quad (0.3)$$

$$\frac{dy}{dx} = -x + c_1 \quad (0.4)$$

$$\int \frac{dy}{dx} dx = \int (-x + c_1) dx \quad (0.5)$$

$$y = \frac{-x^2}{2} + c_1x + c_2 \quad (0.6)$$

$$(0.7)$$

Substituting the initial conditions gives

$$y(0) = 0 \implies \frac{-0^2}{2} + c_1 \cdot 0 + c_2 = 0 \quad (0.8)$$

$$c_2 = 0 \quad (0.9)$$

$$y'(0) = 0 \implies -0 + c_1 = 0 \quad (0.10)$$

$$c_1 = 0 \quad (0.11)$$

The theoretical solution is $f(x) = \frac{-x^2}{2}$

Computational Solution:

Consider the given linear differential equation

$$a_n y^n + a_{n-1} y^{n-1} + \dots + a_1 y' + a_0 y + c = 0 \quad (0.12)$$

Where y^i is the i th derivative of the function then

$$y'(t) = \lim_{h \rightarrow 0} \frac{y(t+h) - y(t)}{h} \quad (0.13)$$

$$y(t+h) = y(t) + h y'(t) \quad (0.14)$$

$$(0.15)$$

Similarly

$$y^i(t+h) = y^i(t) + h y^{i+1}(t) \quad (0.16)$$

$$y^{n-1}(t+h) = y^{n-1}(t) + h y^n(t) \quad (0.17)$$

$$y^{n-1}(t+h) = y^{n-1}(t) + h \left(-\frac{a_{n-1}}{a_n} y^{n-1} - \frac{a_{n-2}}{a_n} y^{n-2} - \dots - \frac{a_0}{a_n} y - \frac{c}{a_n} \right) \quad (0.18)$$

$$(0.19)$$

Where i ranges from 0 to $n-1$

Discretizing the steps gives us

$$y_{k+1} = y_k + h y'_k \quad (0.20)$$

$$y'_{k+1} = y'_k + h y''_k \quad (0.21)$$

$$\vdots \quad (0.22)$$

$$y^{n-2}_{k+1} = y^{n-2}_k + h y^{n-1}_k \quad (0.23)$$

$$y^{n-1}(t+h) = y^{n-1}(t) + h \left(-\frac{a_{n-1}}{a_n} y^{n-1} - \frac{a_{n-2}}{a_n} y^{n-2} - \dots - \frac{a_0}{a_n} y - \frac{c}{a_n} \right) \quad (0.24)$$

where k ranges from 0 to number of data points with y_0^i being the given initial condition.
For the given question

$$y(t+h) = y(t) + h y'(t) \quad (0.25)$$

$$y'(t+h) = y'(t) + h y''(t) \quad (0.26)$$

$$y_{k+1} = y_k + h y'_k \quad (0.27)$$

$$y'_{k+1} = y'_k + h y''_k \quad (0.28)$$

$$y'_{k+1} = y'_k - h \quad (0.29)$$

Record the y_k for $x_k = \text{lowerbound} + kh$ and then plot the graph. The result will be as given below.

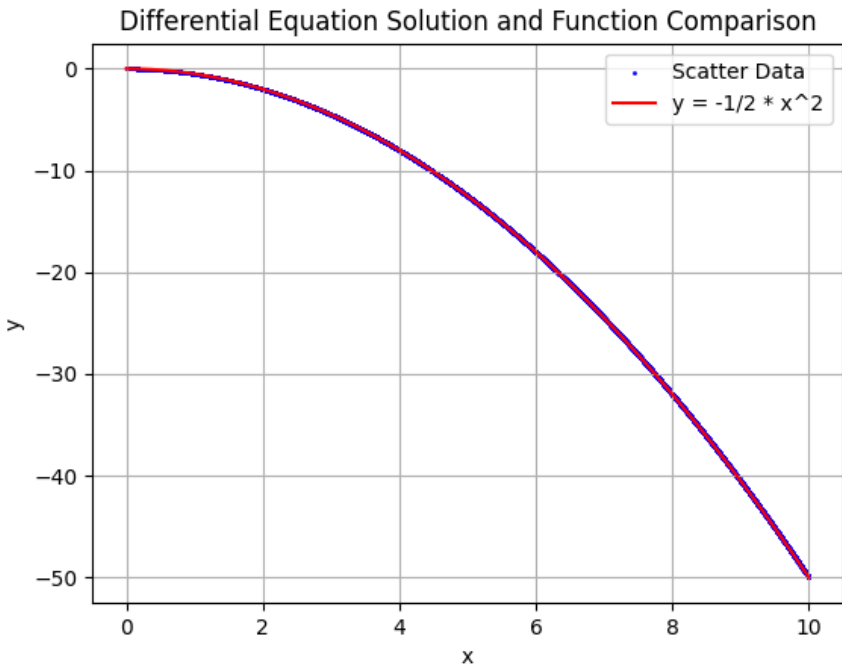


Fig. 0.1: Comparison between the Theoretical solution and Computational solution