11.16.3.8.4

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
A three coins are tossed once, what is the probability of getting atmost 2 heads?
Solution:
Define a discrete random variable X = number of heads
We will assume our random variable as a sum of outcomes of three bernoulli random variables

$$X = X_1 + X_2 + X_3 \tag{0.1}$$

Where

$$X_i = \begin{cases} 1, & \text{Outcome in Heads} \\ 0, & \text{Outcome in Tails} \end{cases} \tag{0.2}$$

$$p_{X_i}(n) = \begin{cases} 1-p, & n = 0 \\ p, & n = 1 \end{cases} \tag{0.3}$$

Where $p = \frac{1}{2}$
Using properties of Z-Transform of PMF

$$M_X(z) = M_{X_1}(z) M_{X_2}(z) M_{X_3}(z) \tag{0.4}$$

$$M_{X_1}(z) = \sum_{n=-\infty}^{\infty} p_{X_1}(n) z^{-n} = (1-p) + pz^{-1} \tag{0.5}$$

$$M_{X_2}(z) = \sum_{n=-\infty}^{\infty} p_{X_2}(n) z^{-n} = (1-p) + pz^{-1} \tag{0.6}$$

$$M_{X_3}(z) = \sum_{n=-\infty}^{\infty} p_{X_3}(n) z^{-n} = (1-p) + pz^{-1} \tag{0.7}$$

$$M_X(z) = ((1-p) + pz^{-1})^3 \tag{0.8}$$

$$= \sum_{n=-\infty}^{\infty} {}^3C_n (1-p)^{3-n} p^n z^{-n} \tag{0.9}$$

$$p_X(n) = {}^3C_n p^n (1-p)^{3-n} \tag{0.10}$$

$$p_X(n) = \frac{{}^3C_n}{8} \tag{0.11}$$

The Probability Mass Function (PMF) for the given random variable is

$$p_X(n) = \begin{cases} \frac{1}{8}, & n = 0 \\ \frac{3}{8}, & n = 1 \\ \frac{3}{8}, & n = 2 \\ \frac{1}{8}, & n = 3 \end{cases} \tag{0.12}$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(n) = \sum_{k=-\infty}^{n} {}^3C_k \left(\frac{1}{2}\right)^3 = \begin{cases} 0 & x < 0 \\ {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{1}{8} & 0 \le x < 1 \\ {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{4}{8} & 1 \le x < 2 \\ {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{7}{8} & 2 \le x < 3 \\ {}^3C_3 \left(\frac{1}{2}\right)^3 + {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = 1 & 3 \le x \end{cases} \tag{0.13}$$

The probability of getting atmost 2 heads is

$$F_X(2) = \frac{7}{8} \tag{0.14}$$

Simulation:
To run a simulation we need to generate random numbers with uniform probability, which is done as shown below:

1) Generate a random number by calling rand(). It generates a random number between 0 and RANDMAX
2) Divide the generated number by RANDMAX so that it becomes a real number in the range [0, 1)
3) If the number is less than p, take it as event happened, else the event did not happen

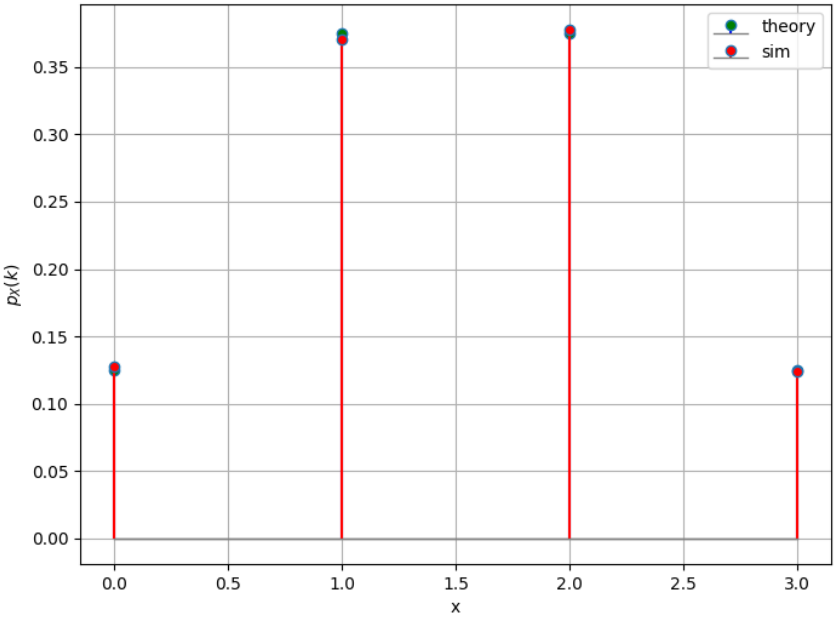

Fig. 3.1: Probability Mass Function of given Random variable

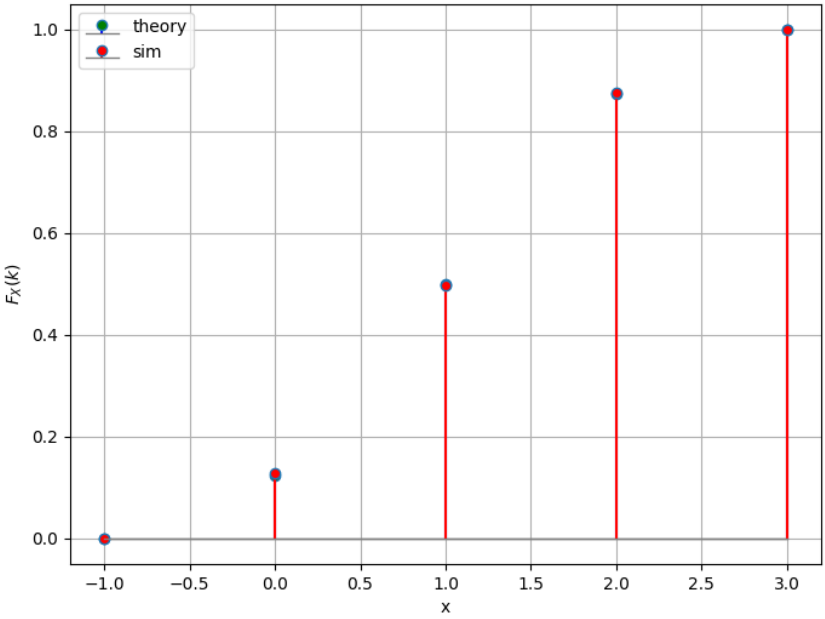

Fig. 3.2: Cumulative Distribution Function of given Random variable