11.16.3.8.4 Presentation

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Problem Statement

A three coins are tossed each once, what is the probability of getting atmost 2 heads?

Solution

Define a discrete random variable X = number of heads
We will take our random variable as a sum of outcomes of three bernoulli random variables

$$X = X_1 + X_2 + X_3 \tag{3.1}$$

Where

$$X_i = \begin{cases} 1, & \text{Outcome in Heads} \\ 0, & \text{Outcome in Tails} \end{cases} \tag{3.2}$$

$$p_{X_i}(n) = \begin{cases} 1 - p, & n = 0 \\ p, & n = 1 \end{cases} \tag{3.3}$$

Where $p = \frac{1}{2}$

Using properties of Z-Transform of PMF

$$M_X(z) = M_{X_1}(z)M_{X_2}(z)M_{X_3}(z) \tag{3.4}$$

$$M_{X_1}(z) = \sum_{n=-\infty}^{\infty} p_{X_1}(n)z^{-n} = (1-p) + pz^{-1} \tag{3.5}$$

$$M_{X_2}(z) = \sum_{n=-\infty}^{\infty} p_{X_2}(n)z^{-n} = (1-p) + pz^{-1} \tag{3.6}$$

$$M_{X_3}(z) = \sum_{n=-\infty}^{\infty} p_{X_3}(n)z^{-n} = (1-p) + pz^{-1} \tag{3.7}$$

$$M_X(z) = ((1-p) + pz^{-1})^3 \tag{3.8}$$

$$= \sum_{n=-\infty}^{\infty} {}^3C_n(1-p)^{3-n}p^n z^{-n} \tag{3.9}$$

$$p_X(n) = {}^3C_n p^n (1-p)^{3-n} \tag{3.10}$$

$$p_X(n) = \frac{{}^3C_n}{8} \tag{3.11}$$

The Probability Mass Function (PMF) for the given random variable is

$$p_X(n) = \begin{cases} \frac{1}{8}, & n = 0 \\ \frac{3}{8}, & n = 1 \\ \frac{3}{8}, & n = 2 \\ \frac{1}{8}, & n = 3 \end{cases} \tag{3.12}$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(n) = \sum_{k=-\infty}^{n} {}^3C_k \left(\frac{1}{2}\right)^3 \tag{3.13}$$

$$= \begin{cases} 0 & , x < 0 \\ {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{1}{8} & , 0 \le x < 1 \\ {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{4}{8} & , 1 \le x < 2 \\ {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = \frac{7}{8} & , 2 \le x < 3 \\ {}^3C_3 \left(\frac{1}{2}\right)^3 + {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_0 \left(\frac{1}{2}\right)^3 = 1 & , 3 \le x \end{cases} \tag{3.14}$$

The probability of getting atmost 2 heads is

$$F_X(2) = \frac{7}{8} \tag{3.15}$$

Plot

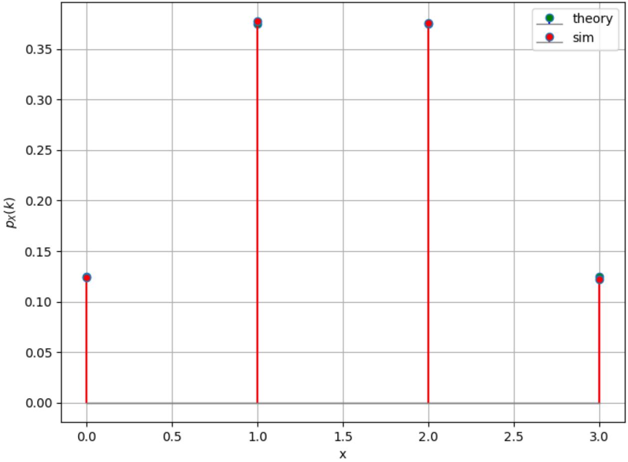

Figure: PMF of the Random variable

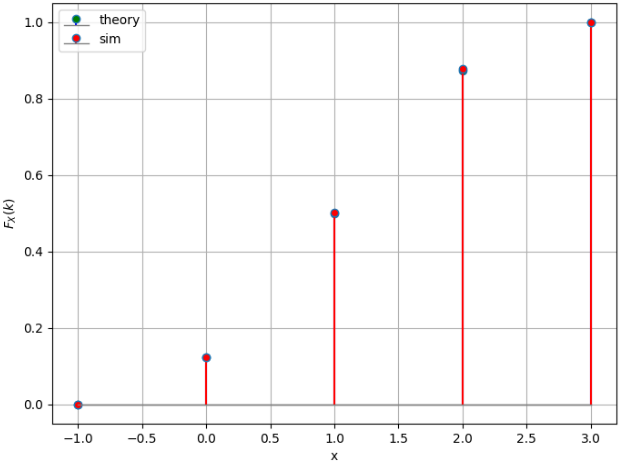

Figure: CDF of the Random variable

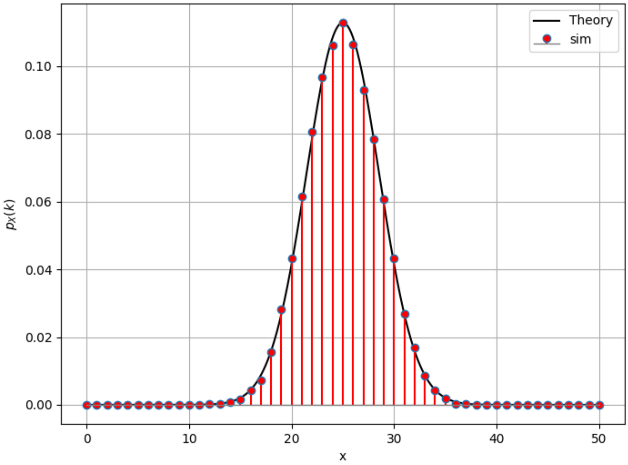

Figure: PMF of Binomial distribution with normal curve, n = 50, p = 0.5

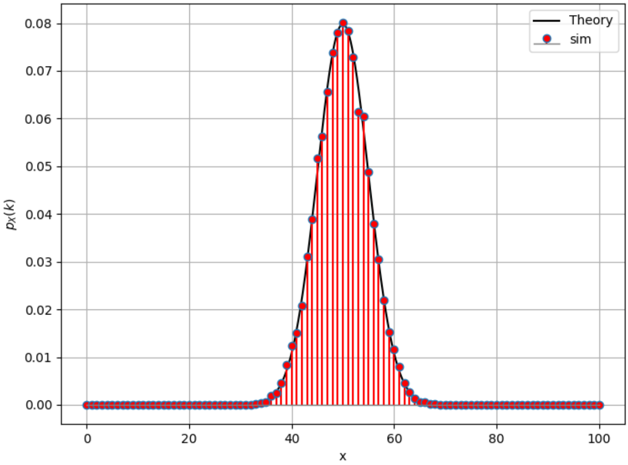

Figure: PMF of Binomial distribution with normal curve, n = 100, p = 0.5

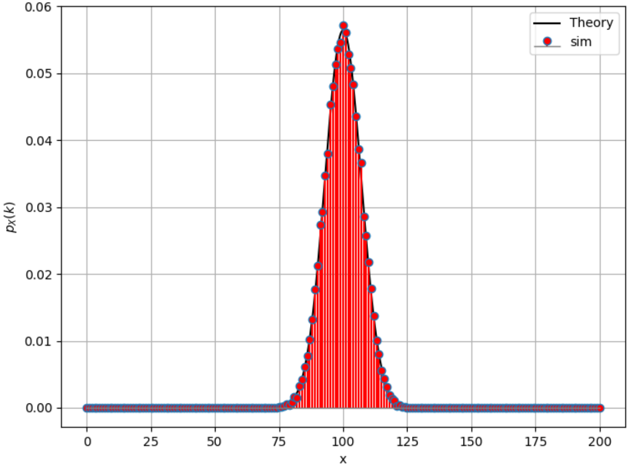

Figure: PMF of Binomial distribution with normal curve, n = 200, p = 0.5