10.4.1.1.8

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
Find the roots of the equation $x^3 - 4x^2 - x + 1 = (x-2)^3$
Solution:
Theoritical solution:
The equation can be simplified to

$$x^3 - 4x^2 - x + 1 = (x-2)^3 \tag{0.1}$$

$$x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8 \tag{0.2}$$

$$2x^2 - 13x + 9 = 0 \tag{0.3}$$

Applying quadratic formula gives solution as

$$x_1 = \frac{13 - \sqrt{97}}{4} \tag{0.4}$$

$$x_2 = \frac{13 + \sqrt{97}}{4} \tag{0.5}$$

Computational solution:
Two methods to find solution of a quadratic equation are:
Matrix-Based Method:
For a polynomial equation of form $x_n + b_{n-1}x^{n-1} + \cdots + b_2x^2 + b_1x + b_0 = 0$ we construct a matrix called companion matrix of form

$$\Lambda = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \\ -b_0 & -b_1 & -b_2 & \dots & -b_{n-1} \end{pmatrix} \tag{0.6}$$

The eigenvalues of this matrix are the roots of the given polynomial equation.
The solution given by the code is

$$x_1 = 0.7878 \tag{0.7}$$

$$x_2 = 5.7122 \tag{0.8}$$

Newton-Raphson Method:
Start with an initial guess x_0, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.9}$$

where,

$$f(x) = 2x^2 - 13x + 9 \tag{0.10}$$

$$f'(x) = 4x - 13 \tag{0.11}$$

The problem with this method is if the roots are complex but the coeffcients are real, x_n either converges to an extrema or grows continuously without any bound. To get the complex solutions, however , we can just take the initial guess point to be a random complex number.

The output of a program written to find roots is shown below:

$$r_1 = 0.7878 \tag{0.12}$$

$$r_2 = 5.7122 \tag{0.13}$$