Assignment 6

EE24Btech11024 - G. Abhimanyu Koushik

A. Multiple Choice

1) Let $9 = x_1 < x_2 < \cdots < x_7$ be in an A.P with common difference d. If the standard deviation of x_1, $x_2 \ldots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to:

(Jan 2023)

a) $18\left(1 + \frac{1}{\sqrt{3}}\right)$ b) 34 c) $2\left(9 + \frac{8}{\sqrt{7}}\right)$ d) 25

2) For the system of linear equations $\alpha x + y + z = 1$, $x + \alpha y + z = 1$, $x + y + \alpha z = \beta$, which of the following is **NOT** correct?

(Jan 2023)

a) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$
b) It has no solution if $\alpha = -2$ and $\beta = 1$
c) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$
d) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

3) Let $\mathbf{a} = 5\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ be two vectors. Then which one of the following statements is **TRUE**?

(Jan 2023)

a) The projection of $\mathbf{a}$ on $\mathbf{b}$ is $-\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\mathbf{b}$
b) The projection of $\mathbf{a}$ on $\mathbf{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\mathbf{b}$
c) The projection of $\mathbf{a}$ on $\mathbf{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\mathbf{b}$
d) The projection of $\mathbf{a}$ on $\mathbf{b}$ is $-\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\mathbf{b}$

4) Let $P(x_0, y_0)$ be the point on hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to: (Jan 2023)

a) -3 b) 9 c) -9 d) 3

5) If $y(x) = x^x$, $x > 0$, then $y''(2 - 2y'(2))$ is equal to:

(Jan 2023)

a) $8\log_e 2 - 2$ b) $4\log_e 2 + 2$ c) $4(\log_e 2)^2 - 2$ d) $4(\log_e 2)^2 + 2$

B. Numericals

1) The total number of six digit numbers formed using digits 4, 6, 9 only and divisible by 6 is _____.

(Jan 2023)

2) The number of integral solutions to the equation $x + y + z = 21$, where $x \geq 1$, $y \geq 3$, $z \geq 4$ is equal to _____.

(Jan 2023)

3) The line $x = 8$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point $\mathbf{P}$ in the first quadrant passes through the point $\left(0, 4\sqrt{3}\right)$ and intersects the x-axis at $\mathbf{Q}$, then $(3PQ)^2$ is equal to _____.

(Jan 2023)

4) If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to _____.

(Jan 2023)

5) If $\int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x \cos 3x+\cos^2 x+\cos^3 x \cos 3x\right)}{1+5^{\cos x}} dx = \frac{k\pi}{16}$, then k is equal to _____.

(Jan 2023)

6) Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P, then the sum of the squares of all possible values of x is _____.

(Jan 2023)

7) If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to _____.

(Jan 2023)

8) The sum of the common terms of the following three arithmetic progressions.
3, 7, 11, 15, ..., 399
2, 5, 8, 11, ..., 359
2, 7, 12, 17, ..., 197 is equal to _____.

(Jan 2023)

9) Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point $(3, -2, 5)$ and perpendicular to line joining the points $(1, 2, 3)$ and $(-2, 3, 5)$. Then the value of $\alpha\beta\gamma$ is equal to _____.

(Jan 2023)

10) The point of intersection **C** of the plane $8x + y + 2z = 0$ and the line joining points **A** $(-3, -6, 1)$ and **B** $(2, 4, -3)$ divides the line segment AB internally in the ratio $k : 1$. If a, b, c ($|a|$, $|b|$, $|c|$ are coprimes) are the direction ratios of the perpendicular from the point **C** on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to _____.

(Jan 2023)