Assignment 1

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D: Single Correct

1) Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are) $(JEEAdv.2013)$
   (a) $x^2 + y^2 - 6x + 8y + 9 = 0$
   (b) $x^2 + y^2 - 6x + 7y + 9 = 0$
   (c) $x^2 + y^2 - 6x - 8y + 9 = 0$
   (d) $x^2 + y^2 - 6x - 7y + 9 = 0$
2) A circle **S** passes through the point $(0, 1)$ and is orthogonal to the circle $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then $(JEEAdv.2014)$
   (a) Radius of **S** is 8
   (b) Radius of **S** is 7
   (c) Centre of **S** is $(-7, 1)$
   (d) Centre of **S** is $(-8, 1)$
3) Let **RS** be the diameter of the Circle $x^2+y^2 = 1$, where **S** is the point $(1, 0)$. Let **P** be a variable point (other than R and S) on the circle and tangents to the circle at **S** and **P** meet at the point **Q**. The normal to the circle at **P** intersects a line drawn through **Q** parallel to **RS** at point **E**. Then the locus of **E** passes through the point (s) $(JEEAdv.2016)$
   (a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$
   (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
   (c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$
   (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$
4) Let **T** be a line passing through the points $\mathbf{P}(-2, 7)$ and $\mathbf{Q}(2, -5)$. Let $\mathbf{F_1}$ be the set of all pairs of circles $(\mathbf{S_1}, \mathbf{S_2})$ such that **T** is tangent to $\mathbf{S_1}$ at **P** and tangent to $\mathbf{S_2}$ at **Q**, and also such that $\mathbf{S_1}$ and $\mathbf{S_2}$ touch each other at a point, say **M**. Let $\mathbf{E_1}$ be the set representing the locus of **M** as the pair $(\mathbf{S_1}, \mathbf{S_2})$ varies in $\mathbf{F_1}$. Let the set of all straight line segments joining a pair of distinct points of $\mathbf{E_1}$) and passing through the point $R(1, 1)$ be $\mathbf{F_2}$. Then which of the following statement(s) is (are) TRUE? $(JEEAdv.2018)$
   a) The point $(-2, 7)$ lies on $\mathbf{E_1}$
   b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie on $\mathbf{E_2}$
   c) The point $\left(\frac{1}{3}, 1\right)$ lies on $\mathbf{E_2}$
   d) The point $\left(0, \frac{3}{2}\right)$ does not lie on $\mathbf{E_1}$

E: Subjective

1) Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point $(5, 5)$ (1978)
2) Let **A** be the centre of circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points $\mathbf{B}(1, 7)$ and **CD** $(4, -2)$ on the circle meet at point **C**. Find the area of the quadrilateral $ABCD$. $(1981 - 4Marks)$
3) Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$ $(1981 - 4Marks)$
4) Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted is $x^2 + y^2 = hx + ky$ $(1983 - 5Marks)$
5) The abscissa of two points **A** and **B** are roots of the equation $x^2+2ax-b^2 = 0$ and their ordinates are roots of the equation $x^2+2px-q^2 = 0$. Find the equation and the radius of the circle with **AB** as diameter. $(1984 - 4Marks)$
6) Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a Circle $\mathbf{C_1}$ of diameter 6. If the centre of $\mathbf{C_1}$ lies in the first quadrant, find the equation of circle $\mathbf{C_2}$ which is concentric with $\mathbf{C_1}$ and cuts intecepts of length 8 on these lines $(1986 - 5Marks)$
7) Let a given Line $\mathbf{L_1}$ intersects the x and y axes at **P** and **Q** respectively. Let another line $\mathbf{L_2}$, perpendicular to $\mathbf{L_1}$, cut the x and y axes at **R** and **S**, respectively. Show that the locus of the point of intersection of **PS** and **QR** is a circle passing through origin. $(1987 - 3Marks)$
8) The circle $x^2+y^2-4x-y+4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of

the triangle is $x + y - xy + k(x^2 + y^2)^{1/2}$. Find **k**. $(1987 - 4 Marks)$

9) If $\left(m_i, \frac{1}{m_i}\right), m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$ $(1989 - 2 Marks)$

10) A circle touches the line $y = x$ at a point **P** such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of circle. $(1990 - 5 Marks)$

11) Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles. $(1991 - 4 Marks)$