Assignment 4

EE24Btech11024 - G. Abhimanyu Koushik

1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{-1, 1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2-1}$. Then the function $f \circ g$ is:

(Jun 2022)

   a) one-one but not onto function
   b) onto but not one-one function
   c) both one-one and onto function
   d) neither one-one nor onto function

2) If the system of equations $\alpha x + y + z = 5,\ x + 2y + 3z = 4,\ x + 3y + 5z = \beta$, has infinitely many solutions, then the ordered pair (α, β) is equal to:

(Jun 2022)

   a) $(1, -3)$ b) $(-1, 3)$ c) $(1, 3)$ d) $(-1, -3)$

3) If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then $\frac{A}{B}$ is equal to:

(Jun 2022)

   a) $\frac{11}{9}$ b) 1 c) $-\frac{11}{9}$ d) $-\frac{11}{3}$

4) $\lim_{x\to 0} \frac{\cos(\sin x)-\cos x}{x^4}$ is equal to:

(Jun 2022)

   a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$ d) $\frac{1}{12}$

5) Let $f(x) = \min\{1, 1 + x\sin x\}$, $0 \leq x \leq 2\pi$. If m is the number of points where f is not differentiable and n is the number of points where f is not continuous, then the ordered pair (m, n) is equal to

(Jun 2022)

   a) $(2, 0)$ b) $(1, 0)$ c) $(1, 1)$ d) $(2, 1)$

6) Consider a cuboid of sides $2x$, $4x$ and $5x$ and a closed hemisphere of radius r. If the sum of their surface areas is constant k, then the ratio $x : r$, for which the sum of their volumes is maximum is

(Jun 2022)

   a) $2 : 5$ b) $19 : 45$ c) $3 : 8$ d) $19 : 15$

7) The area of region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to:

(Jun 2022)

   a) $\frac{32}{3}$ b) $\frac{40}{3}$ c) 16 d) 19

8) If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c,\ g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to:

(Jun 2022)

a) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$ b) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$ c) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$ d) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

9) If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx}+2y = xe^x$, $y(1) = 0$ then the local maximum value of the function $z(x) = x^2y(x) - e^x,\ x \in \mathbb{R}$ is:

(Jun 2022)

a) $1-e$ b) 0 c) $\frac{1}{2}$ d) $\frac{4}{e}-e$

10) If the solution to the differential equation $\frac{dy}{dx}+e^x\left(x^2-2\right)y = \left(x^2-2x\right)\left(x^2-2\right)e^{2x}$ satisfies $y(0) = 0$, then the value of $y(2)$ is ____.

(Jun 2022)

a) -1 b) 1 c) 0 d) e

11) If m is the slope of a common tangent to curves $\frac{x^2}{16}+\frac{y^2}{9} = 1$ and $x^2+y^2 = 12$, then $12m^2$ is equal to:

(Jun 2022)

a) 6 b) 9 c) 10 d) 12

12) The locus of the mid point of the line segment joining the point $(4, 3)$ and the points on the ellipse $x^2+2y^2 = 4$ is an ellipse with eccentricity:

(Jun 2022)

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{2\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{2}$

13) The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9} = 1$ at the point $\left(8, 3\sqrt{3}\right)$ on it passes through the point:

(Jun 2022)

a) $\left(15, -2\sqrt{3}\right)$ b) $\left(9, 2\sqrt{3}\right)$ c) $\left(-1, 9\sqrt{3}\right)$ d) $\left(-1, 6\sqrt{3}\right)$

14) If the plane $2x+y-5z = 0$ is rotated about its line of intersection with the plane $3x-y+4z-7 = 0$ by an angle $\frac{\pi}{2}$. then the plane after the rotation passes through the point:

(Jun 2022)

a) $(2, -2, 0)$ b) $(-2, 2, 0)$ c) $(1, 0, 2)$ d) $(-1, 0, -2)$

15) If the lines $\mathbf{r} = \left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{j}-\hat{k}\right)$ and $\mathbf{r} = \left(\alpha\hat{i}-\hat{j}\right)+\mu\left(2\hat{i}-3\hat{k}\right)$ are co-planar, then the distance of the plane contaioning these two lines from the point $(\alpha, 0, 0)$ is:

(Jun 2022)

a) $\frac{2}{9}$ b) $\frac{2}{11}$ c) $\frac{4}{11}$ d) 2