Assignment 1

EE24Btech11024 - G. Abhimanyu Koushik

1) For which of the following ordered pairs (μ, δ) the system of linear equations
$x + 2y + 3z = 1$
$3x + 4y + 5z = \mu$
$4x + 4y + 4z = \delta$
is inconsistent?

a) $(4, 6)$ b) $(3, 4)$ c) $(1, 0)$ d) $(4, 3)$

2) Let $y = f(x)$ be a solution to the differential equation
$\sqrt{1 - x^2}\frac{dy}{dx} + \sqrt{1 - y^2} = 0, \ |x| < 1$
If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(-\frac{1}{\sqrt{2}}\right)$ is equal to

a) $-\frac{1}{\sqrt{2}}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{\sqrt{3}}{2}$

3) If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$, ${}^{21}C_r$ respectively, then:

a) $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{42}\right)$ b) $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{42}\right)$ c) $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{21}\right)$ d) $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{21}\right)$

4) Which of the following is a tautology?

a) $(P \wedge (P \to Q)) \to Q$ b) $P \wedge (P \vee Q)$ c) $(Q \to (\wedge (P \to Q)))$ d) $P \vee (P \wedge Q)$

5) Let $f : \mathbb{R} \to \mathbb{R}$ be such that for all $x \in \mathbb{R}$, $\left(2^{1+x} + 2^{1-x}\right)$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P, then the minimum value of $f(x)$ is:

a) 0 b) 4 c) 3 d) 2

6) The locus of a point which divides the line segment joining the point $(0, -1)$ and a point on parabola, $x^2 = 4y$, internally in the ration $1 : 2$ is:

a) $9x^2 - 12y = 8$ b) $4x^2 - 3y = 2$ c) $x^2 - 3y = 2$ d) $9x^2 - 3y = 2$

7) For $a > 0$, let the curves C_1:$y^2 = ax$ and C_2:$x^2 = ay$ intersect at origin **O** and a point **P**. Let the line $x = b\,(0 < b < a)$ intersect the chord OP and the x-axis at points **Q** and **R**, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then a satisfies the equation

a) $x^6 - 12x^3 + 4 = 0$ b) $x^6 - 12x^3 - 4 = 0$ c) $x^6 + 6x^3 - 4 = 0$ d) $x^6 - 6x^3 + 4 = 0$

8) The inverse of the function $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ is

a) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$ b) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$ c) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$ d) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

9) $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

a) e b) $\frac{1}{e^2}$ c) $\frac{1}{e}$ d) e^2

10) Let $f(x) = \left(\sin\left(\tan^{-1} x\right) + \sin\left(\cot^{-1} x\right)\right)^2 - 1$ where $|x| > 1$. If
$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1} f(x)\right)$
and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to:

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $-\frac{\pi}{6}$ d) $\frac{5\pi}{6}$

11) If the equation, $x^2 + bx + 45 = 0\,(b \in \mathbb{R})$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then:

a) $b^2 + b = 12$ b) $b^2 - b = 42$ c) $b^2 - b = 30$ d) $b^2 + b = 72$

12) The mean and standard deviation of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and standard deviation become half of their original values, then q is equal to:

a) -20 b) -5 c) 10 d) -10

13) If $\int \frac{\cos x}{\sin^3 x\left(1+\sin^6 x\right)^{\frac{2}{3}}} dx = f(x)\left(1 + \sin^6 x\right)^{\frac{1}{\lambda}} + c$, where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to:

a) $-\frac{9}{8}$ b) $\frac{9}{8}$ c) 2 d) -2

14) Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then which of the following is **TRUE**?

a) $P\left(\frac{A}{A\cup B}\right) = \frac{1}{4}$ b) $P\left(\frac{A}{B'}\right) = \frac{1}{3}$ c) $P\left(\frac{A}{B}\right) = \frac{2}{3}$ d) $P\left(\frac{A'}{B'}\right) = \frac{1}{3}$

15) If volume of a parallelepiped whose coterminous edges are given by
$\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$,
$\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and
$\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$
be 1cu.unit. If θ is the angle between the edges $\vec{u}$ and $\vec{w}$ then, $\cos\theta$ will be:

a) $\frac{7}{6\sqrt{6}}$ b) $\frac{5}{7}$ c) $\frac{7}{6\sqrt{3}}$ d) $\frac{5}{3\sqrt{3}}$