1.1.8.22

EE24BTECH11024 - G.Abhimanyu Koushik

Question:
If point $\mathbf{A}\begin{pmatrix}2\\-4\end{pmatrix}$ is equidistant from $\mathbf{P}\begin{pmatrix}3\\8\end{pmatrix}$ and $\mathbf{Q}\begin{pmatrix}-10\\y\end{pmatrix}$, find the values of y. Also find distance PQ.
Solution:

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix}2\\-4\end{pmatrix}$	First point
$\mathbf{P}$	$\begin{pmatrix}3\\8\end{pmatrix}$	Second point
$\mathbf{Q}$	$\begin{pmatrix}-10\\y\end{pmatrix}$	Point such that $\mathbf{A}$ is equidistant from $\mathbf{P}$ and $\mathbf{Q}$

TABLE 0: Variables Used

$$\|\mathbf{A}-\mathbf{P}\|^2 = \|\mathbf{A}-\mathbf{Q}\|^2 \tag{0.1}$$
$$(\mathbf{A}-\mathbf{P})^\top(\mathbf{A}-\mathbf{P}) = (\mathbf{A}-\mathbf{Q})^\top(\mathbf{A}-\mathbf{Q}) \tag{0.2}$$
$$\left(\mathbf{A}^\top\right)(\mathbf{A}) + \left(\mathbf{P}^\top\right)(\mathbf{P}) - 2\left(\mathbf{A}^\top\right)(\mathbf{P}) = \left(\mathbf{A}^\top\right)(\mathbf{A}) + \left(\mathbf{Q}^\top\right)(\mathbf{Q}) - 2\left(\mathbf{A}^\top\right)(\mathbf{Q}) \tag{0.3}$$
$$\left(\mathbf{P}^\top\right)(\mathbf{P}) - 2\left(\mathbf{A}^\top\right)(\mathbf{P}) = \left(\mathbf{Q}^\top\right)(\mathbf{Q}) - 2\left(\mathbf{A}^\top\right)(\mathbf{Q}) \tag{0.4}$$
$$\begin{pmatrix}3 & 8\end{pmatrix}\begin{pmatrix}3\\8\end{pmatrix} - 2\begin{pmatrix}2 & -4\end{pmatrix}\begin{pmatrix}3\\8\end{pmatrix} = \begin{pmatrix}-10 & y\end{pmatrix}\begin{pmatrix}-10\\y\end{pmatrix} - 2\begin{pmatrix}2 & -4\end{pmatrix}\begin{pmatrix}-10\\y\end{pmatrix} \tag{0.5}$$
$$73 - 2(-26) = 100 + y^2 - 2(-4y - 20) \tag{0.6}$$
$$y^2 + 8y + 15 = 0 \tag{0.7}$$
$$y^2 + 8y + 16 = 1 \tag{0.8}$$
$$y = -3, -5 \tag{0.9}$$

The value of y is -3 or -5.
The Distance d between P and Q is

$$d = \|P - Q\| \tag{0.10}$$
$$d = \sqrt{(\mathbf{P}-\mathbf{Q})^\top(\mathbf{P}-\mathbf{Q})} \tag{0.11}$$
$$\tag{0.12}$$

$$d = \sqrt{\begin{pmatrix}13 & 11\end{pmatrix}\begin{pmatrix}13\\11\end{pmatrix}} \text{ or } d = \sqrt{\begin{pmatrix}13 & 13\end{pmatrix}\begin{pmatrix}13\\13\end{pmatrix}} \tag{0.13}$$

$$d = \sqrt{290} \text{ or } d = 13\sqrt{2} \tag{0.14}$$

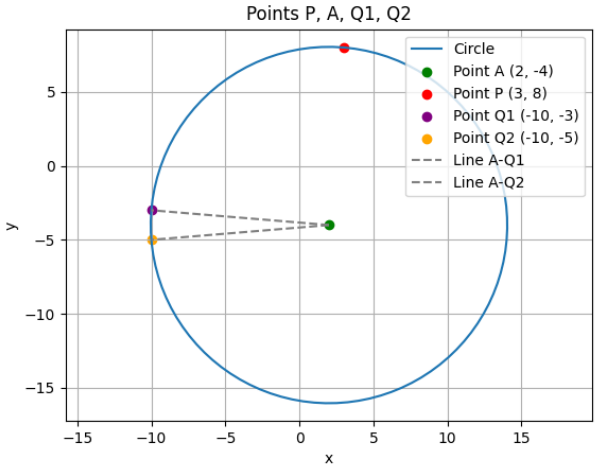

Fig. 0.1: Plot of the given points and the bisector