Assignment 1

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1) If $\int \frac{\cos x}{\sin^3 x\left(1+\sin^6 x\right)^{\frac{2}{3}}} dx = f(x)\left(1+\sin^6 x\right)^{\frac{1}{\lambda}} + c$, where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to:

(Jan 2020)

a) $-\frac{9}{8}$ b) $\frac{9}{8}$ c) 2 d) -2

2) Let $y = f(x)$ be a solution to the differential equation $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0,\ |x| < 1$ If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(-\frac{1}{\sqrt{2}}\right)$ is equal to

(Jan 2020)

a) $-\frac{1}{\sqrt{2}}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{\sqrt{3}}{2}$

3) $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(Jan 2020)

a) e b) $\frac{1}{e^2}$ c) $\frac{1}{e}$ d) e^2

4) In a bag there are 5 red balls, 3 white balls, 4 black balls. Four balls are drawn from the bag. Find the number of ways in which at most 3 red balls are selected

(Jan 2020)

a) 450 b) 360 c) 490 d) 510

5) Let $f(x) = \left(\sin\left(\tan^{-1} x\right) + \sin\left(\cot^{-1} x\right)\right)^2 - 1$ where $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1} f(x)\right)$ and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to:

(Jan 2020)

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $-\frac{\pi}{6}$ d) $\frac{5\pi}{6}$

6) Let $f : \mathbb{R} \to \mathbb{R}$ be such that for all $x \in \mathbb{R}$, $\left(2^{1+x} + 2^{1-x}\right)$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P, then the minimum value of $f(x)$ is:

(Jan 2020)

a) 0 b) 4 c) 3 d) 2

7) Which of the following is a tautology?

(Jan 2020)

a) $(P \wedge (P \to Q)) \to Q$ b) $P \wedge (P \vee Q)$ c) $(Q \to (\wedge (P \to Q)))$ d) $P \vee (P \wedge Q)$

8) A is 3×3 matrix whose elements are from the set $\{-1, 0, 1\}$. Find the number of matrices A such that $tr(AA^\top) = 3$. Where $tr(A)$ is sum of diagonal elements of matrix A.

(Jan 2020)

a) 572 b) 612 c) 672 d) 682

9) The mean and standard deviation of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and standard deviation become half of their original values, then q is equal to:

(Jan 2020)

a) -20 b) -5 c) 10 d) -10

10) If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$, ${}^{21}C_r$ respectively, then:

(Jan 2020)

a) $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{42}\right)$ b) $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{42}\right)$ c) $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{21}\right)$ d) $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{21}\right)$

11) Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then which of the following is **TRUE**?

(Jan 2020)

a) $P\left(\frac{A}{A \cup B}\right) = \frac{1}{4}$ b) $P\left(\frac{A}{B'}\right) = \frac{1}{3}$ c) $P\left(\frac{A}{B}\right) = \frac{2}{3}$ d) $P\left(\frac{A'}{B'}\right) = \frac{1}{3}$

12) The inverse of the function $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ is

(Jan 2020)

a) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$ b) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$ c) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$ d) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

13) If the equation, $x^2 + bx + 45 = 0\,(b \in \mathbb{R})$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then:

(Jan 2020)

a) $b^2 + b = 12$ b) $b^2 - b = 42$ c) $b^2 - b = 30$ d) $b^2 + b = 72$

14) For $f(x) = \ln\left(\frac{x^2+\alpha}{7x}\right)$ Rolle's theorem is applicable on $[3, 4]$, the value of $f''(c)$ is equal to

(Jan 2020)

a) $\frac{1}{12}$ b) $-\frac{1}{12}$ c) $\frac{1}{6}$ d) $-\frac{1}{6}$

15) Let $f(x) = x\cos^{-1}(\sin(-|x|))$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Then

(Jan 2020)

a) $f'(0) = -\frac{\pi}{2}$
b) $f'(x)$ is not defined at $x = 0$
c) $f'(x)$ is increasing in $\left(\frac{-\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$
d) $f'(x)$ is decreasing in $\left(\frac{-\pi}{2}, 0\right)$ and $f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$