

Assignment 1

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D: Single Correct

- Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are) (JEE Adv. 2013)
 - $x^2 + y^2 - 6x + 8y + 9 = 0$
 - $x^2 + y^2 - 6x + 7y + 9 = 0$
 - $x^2 + y^2 - 6x - 8y + 9 = 0$
 - $x^2 + y^2 - 6x - 7y + 9 = 0$
- A circle **S** passes through the point (0, 1) and is orthogonal to the circle $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then (JEE Adv. 2014)
 - Radius of **S** is 8
 - Radius of **S** is 7
 - Centre of **S** is (-7, 1)
 - Centre of **S** is (-8, 1)
- Let **RS** be the diameter of the Circle $x^2 + y^2 = 1$, where **S** is the point (1, 0). Let **P** be a variable point (other than R and S) on the circle and tangents to the circle at **S** and **P** meet at the point **Q**. The normal to the circle at **P** intersects a line drawn through **Q** parallel to **RS** at point **E**. Then the locus of **E** passes through the point (s) (JEE Adv. 2016)
 - $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$
 - $\left(\frac{1}{4}, \frac{1}{2}\right)$
 - $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$
 - $\left(\frac{1}{4}, -\frac{1}{2}\right)$
- Let **T** be a line passing through the points **P**(-2, 7) and **Q**(2, -5). Let **F**₁ be the set of all pairs of circles (**S**₁, **S**₂) such that **T** is tangent to **S**₁ at **P** and tangent to **S**₂ at **Q**, and also such that **S**₁ and **S**₂ touch each other at a point, say **M**. Let **E**₁ be the set representing the locus of **M** as the pair (**S**₁, **S**₂) varies in **F**₁. Let the set of all straight line segments joining a pair of distinct points of **E**₁ and passing through the point R(1, 1) be **F**₂. Then which of the following statement(s) is (are) TRUE? (JEE Adv. 2018)

- The point (-2, 7) lies on **E**₁
- The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie on **E**₂
- The point $\left(\frac{1}{3}, 1\right)$ lies on **E**₂
- The point $\left(0, \frac{3}{2}\right)$ does not lie on **E**₁

E: Subjective

- Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point (5, 5) (1978)
- Let **A** be the centre of circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points **B**(1, 7) and **CD** (4, -2) on the circle meet at point **C**. Find the area of the quadrilateral **ABCD**. (1981 - 4 Marks)
- Find the equations of the circle passing through (-4, 3) and touching the lines $x + y = 2$ and $x - y = 2$ (1981 - 4 Marks)
- Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted is $x^2 + y^2 = hx + ky$ (1983 - 5 Marks)
- The abscissa of two points **A** and **B** are roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with **AB** as diameter. (1984 - 4 Marks)
- Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a Circle **C**₁ of diameter 6. If the centre of **C**₁ lies in the first quadrant, find the equation of circle **C**₂ which is concentric with **C**₁ and cuts intercepts of length 8 on these lines (1986 - 5 Marks)
- Let a given Line **L**₁ intersects the x and y axes at **P** and **Q** respectively. Let another line **L**₂, perpendicular to **L**₁, cut the x and y axes at **R** and **S**, respectively. Show that the locus of the point of intersection of **PS** and **QR** is a circle passing through origin. (1987 - 3 Marks)
- The circle $x^2 + y^2 - 4x - y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of

the triangle is $x + y - xy + k(x^2 + y^2)^{1/2}$. Find **k**.
(1987 - 4 Marks)

- 9) If $\left(m_i, \frac{1}{m_i}\right), m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$
(1989 - 2 Marks)
- 10) A circle touches the line $y = x$ at a point **P** such that $OP = 4\sqrt{2}$, where **O** is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of circle. (1990 - 5 Marks)
- 11) Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.
(1991 - 4 Marks)