

# Assignment 1

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*D: Single Correct*

- 1) Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length  $2\sqrt{7}$  on the y-axis is (are)

(JEEAdv.2013)

- a)  $x^2 + y^2 - 6x + 8y + 9 = 0$
- b)  $x^2 + y^2 - 6x + 7y + 9 = 0$
- c)  $x^2 + y^2 - 6x - 8y + 9 = 0$
- d)  $x^2 + y^2 - 6x - 7y + 9 = 0$

- 2) A circle  $S$  passes through the point  $(0, 1)$  and is orthogonal to the circle  $(x - 1)^2 + y^2 = 16$  and  $x^2 + y^2 = 1$ . Then

(JEEAdv.2014)

- a) Radius of  $S$  is 8
- b) Radius of  $S$  is 7
- c) Centre of  $S$  is  $(-7, 1)$
- d) Centre of  $S$  is  $(-8, 1)$

- 3) Let  $RS$  be the diameter of the Circle  $x^2 + y^2 = 1$ , where  $S$  is the point  $(1, 0)$ . Let  $P$  be a variable point (other than  $R$  and  $S$ ) on the circle and tangents to the circle at  $S$  and  $P$  meet at the point  $Q$ . The normal to the circle at  $P$  intersects a line drawn through  $Q$  parallel to  $RS$  at point  $E$ . Then the locus of  $E$  passes through the point(s)

(JEEAdv.2016)

- a)  $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$
- b)  $\left(\frac{1}{4}, \frac{1}{2}\right)$
- c)  $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$
- d)  $\left(\frac{1}{4}, -\frac{1}{2}\right)$

- 4) Let  $T$  be a line passing through the points  $P(-2, 7)$  and  $Q(2, -5)$ . Let  $F_1$  be the set of all pairs of circles  $(S_1, S_2)$  such that  $T$  is tangent to  $S_1$  at  $P$  and tangent to  $S_2$  at  $Q$ , and also such that  $S_1$  and  $S_2$  touch each other at a point, say  $M$ . Let  $E_1$  be the set representing the locus of  $M$  as the pair  $(S_1, S_2)$  varies in  $F_1$ . Let the set of all straight line segments joining a pair of distinct points of  $E_1$  and passing through the point  $R(1, 1)$  be  $F_2$ . Then which of the

following statements is (are) TRUE?

(JEEAdv.2018)

- a) The point  $(-2, 7)$  lies on  $E_1$
- b) The point  $\left(\frac{4}{5}, \frac{7}{5}\right)$  does **NOT** lie on  $E_1$
- c) The point  $\left(\frac{1}{3}, 1\right)$  lies on  $E_1$
- d) The point  $\left(0, \frac{3}{2}\right)$  does not lie on  $E_1$

*E: Subjective*

- 1) Find the equation of the circle whose radius is 5 and which touches the circle  $x^2 + y^2 - 2x - 4y - 20 = 0$  at the point  $(5, 5)$

(1978)

- 2) Let  $A$  be the centre of circle  $x^2 + y^2 - 2x - 4y - 20 = 0$ . Suppose that the tangents at the points  $B(1, 7)$  and  $D(4, -2)$  on the circle meet at point  $C$ . Find the area of the quadrilateral  $ABCD$ .

(1981 - 4marks)

- 3) Find the equations of the circle passing through  $(-4, 3)$  and touching the lines  $x + y = 2$  and  $x - y = 2$

(1981 - 4marks)

- 4) Through a fixed point  $(h, k)$  secants are drawn to the circle  $x^2 + y^2 = r^2$ . Show that the locus of the mid-points of the secants intercepted is  $x^2 + y^2 = hx + ky$

(1983 - 5marks)

- 5) The abscissa of two points  $A$  and  $B$  are roots of the equation  $x^2 + 2ax - b^2 = 0$  and their ordinates are roots of the equation  $x^2 + 2px - q^2 = 0$ . Find the equation and the radius of the circle with  $AB$  as diameter.

(1984 - 4marks)

- 6) Lines  $5x + 12y - 10 = 0$  and  $5x - 12y - 40 = 0$  touch a Circle  $C_1$  of diameter 6. If the centre of  $C_1$  lies in the first quadrant, find the equation of circle  $C_2$  which is concentric with  $C_1$  and cuts intercepts of length 8 on these lines

(1986 - 5marks)

- 7) Let a given Line  $L_1$  intersects the  $x$  and  $y$  axes at  $P$  and  $Q$  respectively. Let another line  $L_2$ , perpendicular to  $L_1$ , cut the  $x$  and  $y$  axes at  $R$  and  $S$ , respectively. Show that the locus of the

point of intersection of  $PS$  and  $QR$  is a circle passing through origin.

(1987 – 3marks)

- 8) The circle  $x^2 + y^2 - 4x - y + 4 = 0$  is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of the triangle is  $x + y - xy + k(x^2 + y^2)^{1/2}$ . Find  $k$ .

(1987 – 4marks)

- 9) If  $(m_i, \frac{1}{m_i})$ ,  $m_i > 0$ ,  $i = 1, 2, 3, 4$  are four distinct points on a circle, then show that  $m_1 m_2 m_3 m_4 = 1$

(1989 – 2marks)

- 10) A circle touches the line  $y = x$  at a point  $P$  such that  $OP = 4\sqrt{2}$ , where  $O$  is the origin. The circle contains the point  $(-10, 2)$  in its interior and the length of its chord on the line  $x + y = 0$  is  $6\sqrt{2}$ . Determine the equation of circle.

(1990 – 5marks)

- 11) Two circles, each of radius 5 units, touch each other at  $(1, 2)$ . If the equation of common tangent is  $4x + 3y = 10$ , find the equations of circles.

(1991 – 4marks)