Assignment 2

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1) The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$, $i=\sqrt{-1}$, is equal to

(Mar 2021)

a) 8 b) 3 c) $\sqrt{5}$ d) 2

2) Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g : S \to \mathbb{R}$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to

(Mar 2021)

a) $\frac{197}{144}$ b) $\frac{187}{144}$ c) $\frac{205}{144}$ d) 1

3) If $y = y(x)$ is the solution of the differential equation $\left(\frac{dy}{dx}\right) + (\tan x)\, y = \sin x$, $0 \leq x \leq \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to: (Mar 2021)

a) $\log_e 2$ b) $\frac{1}{2}\log_e 2$ c) $\frac{1}{2\sqrt{2}}\log_e 2$ d) $\frac{1}{4}\log_e 2$

4) If the foot of perpendicular from the point $(4, 2, 8)$ on the $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is $(3, 5, 7)$, then find the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to

(Mar 2021)

a) $\frac{\sqrt{2}}{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{6}}$

5) If (x, y, z) be an arbitrary point lying on the plane P which passes through the points $(42, 0, 0)$, $(0, 42, 0)$, and $(0, 0, 42)$, then the value of expression $3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to

(Mar 2021)

a) 3 b) 0 c) 39 d) -45

6) Consider the integral $I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx$, where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal to :

(Mar 2021)

a) $45(e-1)$ b) $45(e+1)$ c) $9(e-1)$ d) $9(e+1)$

7) Let $\mathbf{A}(-1, 1)$, $\mathbf{B}(3, 4)$ and $\mathbf{C}(2, 0)$ be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point $\mathbf{P}$ and $\mathbf{Q}$ respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to:

(Mar 2021)

a) $\frac{4}{15}$ b) 1 c) 2 d) 3

8) Let f be a real-valued function, defined on $\mathbb{R} - \{-1, 1\}$ and given by $f(x) = 3\log_e\left(\frac{|(x-1)|}{|(x+1)|}\right) - \frac{2}{x-1}$. Then in which of the following intervals, function $f(x)$ is increasing?

(Mar 2021)

a) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$

b) $\left(-1, \frac{1}{2}\right]$

c) $(-\infty, \infty) - \{-1, 1\}$

d) $\left(-\infty, \frac{1}{2}\right] - \{-1\}$

9) Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to:

(Mar 2021)

a) $\sqrt{10}$ b) $\sqrt{6}$ c) $\sqrt{11}$ d) $\sqrt{7}$

10) Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then the probability of event A is equal to:

(Mar 2021)

a) $\frac{4}{9}$ b) $\frac{9}{56}$ c) $\frac{3}{7}$ d) $\frac{11}{27}$

11) Let $\alpha \in \mathbb{R}$ be such that the function $f(n) = \begin{cases} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3} & x \neq 0, \\ \alpha & x = 0. \end{cases}$ is continuous at $x = 0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x. Then:

(Mar 2021)

a) $\alpha = \frac{\pi}{4}$ b) No such α exists c) $\alpha = 0$ d) $\alpha = \frac{\pi}{\sqrt{2}}$

12) The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & b\cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in \mathbb{R}$ is:

(Mar 2021)

a) $\sqrt{7}$ b) $\sqrt{5}$ c) 5 d) $\frac{3}{4}$

13) Consider a rectangle $ABCD$ having 5, 7, 6, 9 points in the interior of the line segments AB, BC, CD, DA respectively. Let α be the number of triangles having these points from the different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

(Mar 2021)

a) 1890 b) 795 c) 717 d) 1173

14) Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $\mathbf{P}(2, 1)$ is:

(Mar 2021)

a) $2x + y = 5$ b) $x + 2y = 4$ c) $x + 3y = 5$ d) $x - y = 1$

15) Given that the inverse trigonometric functions take principal values only. Then, the number of real value of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$ is

(Mar 2021)

a) 1 b) 2 c) 3 d) 0