# Assignment 2

## EE24Btech11024 - G. Abhimanyu Koushik

### A. Multiple Choice

1) Let $x$ be a random variable such that the probability function of a distribution given by $P(X = 0) = \frac{1}{2}$, $P(X = j) = \frac{1}{3^j}$ ($j = 1, 2, 3, \ldots, \infty$). Then the mean of the distribution and $P(X$ is positive and even) respectively are:

(Jul 2021)

   a) $\frac{3}{8}$ and $\frac{1}{8}$
   b) $\frac{3}{4}$ and $\frac{1}{8}$
   c) $\frac{3}{4}$ and $\frac{1}{9}$
   d) $\frac{3}{4}$ and $\frac{1}{16}$

2) If the tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at **B** and **C**, then the circle with $BC$ as diameter pass through the point:

(Jul 2021)

   a) $\left(\sqrt{3}, 0\right)$
   b) $\left(\sqrt{2}, 0\right)$
   c) $(1, 1)$
   d) $(-1, 1)$

3) Let the equation of pair of lines , $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(Jul 2021)

   a) $x^2 - 3xy + y^2 = 0$
   b) $x^2 + 4xy - y^2 = 0$
   c) $x^2 + 3xy - y^2 = 0$
   d) $x^2 - 3xy - y^2 = 0$

4) If $^nP_r = {}^nP_{r+1}$ and $^nC_r = {}^nC_{r+1}$, then the value of $r$ is equal to:

(Jul 2021)

   a) 1
   b) 4
   c) 2
   d) 3

5) Let $y = y(x)$ be the solution of the differential equation $xdy = \left(y + x^3 \cos x\right)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(Jul 2021)

   a) $\frac{\pi^2}{4} + \frac{\pi}{2}$
   b) $\frac{\pi^2}{2} + \frac{\pi}{4}$
   c) $\frac{\pi^2}{2} - \frac{\pi}{4}$
   d) $\frac{\pi^2}{4} - \frac{\pi}{2}$

### B. Numericals

1) Let $n \in \mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to $x$. If the sum of $(n + 1)$ terms $^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3 \ldots$ is equal to $2^{100} \cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to _____.

(Jul 2021)

2) Consider the function $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)} & x \neq 2, \\ 7 & x = 2. \end{cases}$ where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to _____.

(Jul 2021)

3) The equation of a circle is $Re\left(z^2\right) + 2\left(Im(z)\right)^2 + 2Re(z) = 0$, where $z = x + iy$. A line which passes through the centre of the given circle and the vertex of parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to _____.

(Jul 2021)

4) If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _____.

5) If $\left(\vec{a} + 3\vec{b}\right)$ is perpendicular to $\left(7\vec{a} - 5\vec{b}\right)$ and $\left(\vec{a} - 4\vec{b}\right)$ is perpendicular to $\left(7\vec{a} - 2\vec{b}\right)$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _____.

6) Let a curve $y = f(x)$ pass through the point $\left(2, (\log_e 2)^2\right)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real values of $x$. Then the value of $f(e)$ is equal to _____.

7) If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to _____.

8) A fair coin is tossed $n$-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of $n$ is _____.

9) If the co-efficient of $x^7$ and $x^8$ in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to _____.

10) If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar then, the value of $k$ is _____.