# Assignment 1

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*D: Single Correct*

1) Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are)
(*JEEAdv*.2013)
a) $x^2 + y^2 - 6x + 8y + 9 = 0$
b) $x^2 + y^2 - 6x + 7y + 9 = 0$
c) $x^2 + y^2 - 6x - 8y + 9 = 0$
d) $x^2 + y^2 - 6x - 7y + 9 = 0$

2) A circle $S$ passes through the point $(0, 1)$ and is orthogonal to the circle $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then
(*JEEAdv*.2014)
a) Radius of $S$ is 8
b) Radius of $S$ is 7
c) Centre of $S$ is $(-7, 1)$
d) Centre of $S$ is $(-8, 1)$

3) Let $RS$ be the diameter of the Circle $x^2+y^2 = 1$, where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to $RS$ at point E. Then the locus of E passes through the point(s)
(*JEEAdv*.2016)
a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$
b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$
d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

4) Let $T$ be a line passing through the points P$(-2, 7)$ and Q$(2, -5)$. Let $F_1$ be the set of all pairs of circles ($S_1$,$S_2$ such that $T$ is tangent to $S_1$ at P and tangent to $S_2$ at Q, and also such that $S_1$ and $S_2$ touch each other at a point, say M. Let $E_1$ be the set representing the locus of M as the pair ($S_1$,$S_2$ varies in $F_1$. Let the set of all straight line segments joining a pair of distinct points of $E_1$ and passing through the point R$(1, 1)$ be $F_2$. Then which of the following statements is (are) TRUE?
(*JEEAdv*.2018)
a) The point $(-2, 7)$ lies on $E_1$
b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie on $E_1$
c) The point $\left(\frac{1}{3}, 1\right)$ lies on $E_1$
d) The point $\left(0, \frac{3}{2}\right)$ does not lie on $E_1$

*E: Subjective*

1) Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point $(5, 5)$
(1978)

2) Let A be the centre of circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B$(1, 7)$ and D $(4, -2)$ on the circle meet at point C. Find the area of the quadrilateral $ABCD$.
(1981 – 4*marks*)

3) Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$
(1981 – 4*marks*)

4) Through a fixed point $(h, k)$ secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted is $x^2 + y^2 = hx + ky$
(1983 – 5*marks*)

5) The abscissa of two points A and B are roots of the equation $x^2+2ax-b^2 = 0$ and their ordinates are roots of the equation $x^2+2px-q^2 = 0$. Find the equation and the radius of the circle with $AB$ as diameter.
(1984 – 4*marks*)

6) Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a Circle $C_1$ of diameter 6. If the centre of $C_1$ lies in the first quadrant, find the equation of circle $C_2$ which is concentric with $C_1$ and cuts intecepts of length 8 on these lines
(1986 – 5*marks*)

7) Let a given Line $L_1$ intersects the $x$ and $y$ axes at P and Q respectively. Let another line $L_2$, perpendicular to $L_1$, cut the $x$ and $y$ axes at R

and S, respectively. Show that the locus of the point of intersection of $PS$ and $QR$ is a circle passing through origin.

$(1987 - 3marks)$

8) The circle $x^2+y^2-4x-y+4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of the triangle is $x+y-xy+k(x^2+y^2)^{1/2}$. Find $k$.

$(1987 - 4marks)$

9) If $\left(m_i, \frac{1}{m_i}\right), m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1m_2m_3m_4 = 1$

$(1989 - 2marks)$

10) A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$ , where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x+y = 0$ is $6\sqrt{2}$. Determine the equation of circle.

$(1990 - 5marks)$

11) Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.

$(1991 - 4marks)$