

# Assignment 6

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## A. Multiple Choice

- 1) Let  $\mathbf{A}(-1, 1)$  and  $\mathbf{B}(2, 3)$  be two points and  $\mathbf{P}$  be a variable point above the line  $AB$  such that the area of  $\Delta PAB$  is 10. If the locus of  $\mathbf{P}$  is  $ax + by = 15$ , then  $5a + 2b$  is:

(Jan 2023)

- a) 6                      b) 4                      c)  $-\frac{12}{5}$                       d)  $-\frac{6}{5}$

- 2) Let  $\alpha\beta \neq 0$  and  $A = \begin{pmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\beta \end{pmatrix}$ . If  $B = \begin{pmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{pmatrix}$  is the matrix of cofactor elements of  $A$ , then  $\det(AB)$  is equal to:

(Jan 2023)

- a) 216  
b) 343  
c) 64  
d) 125

- 3) The value of  $m, n$  for which the system of linear equations

$$x + y + z = 4,$$

$$2x + 5y + 5z = 17,$$

$$x + 2y + mz = n$$

has infinitely many solutions satisfy the equation:

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- a)  $m^2 + n^2 - m - n = 46$   
b)  $m^2 + n^2 + mn = 68$   
c)  $m^2 + n^2 + m + n = 64$   
d)  $m^2 + n^2 - mn = 39$

- 4) Let  $ABCD$  and  $AEFG$  be squares of side 4 and 2 units respectively. The point  $\mathbf{E}$  is on the line segment  $AB$  and the point  $\mathbf{F}$  is on the diagonal  $AC$ . Then the radius  $r$  of the circle passing through the point  $\mathbf{F}$  and touching the line segments  $BC$  and  $CD$  satisfies:

- a)  $r = 1$                       b)  $r^2 - 8r + 8 = 0$                       c)  $2r^2 - 8r + 7 = 0$                       d)  $2r^2 - 4r + 1 = 0$

- 5) Let  $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ ,  $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$  and  $\mathbf{c}$  be three vectors such that  $(\mathbf{c} + \mathbf{i}) \times (\mathbf{a} + \mathbf{b} + \mathbf{i}) = \mathbf{a} \times (\mathbf{c} + \mathbf{i})$ . If  $\mathbf{a} \cdot \mathbf{c} = -29$ , then  $\mathbf{c} \cdot (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$  is equal to:

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- a) 15                      b) 12                      c) 5                      d) 10

## B. Numericals

- 1) Let the maximum and minimum values of  $\left(\sqrt{8x - x^2 - 12} - 4\right)^2 + (x - 7)^2$ ,  $x \in \mathbb{R}$  be  $M$  and  $m$ , respectively. Then  $M^2 - m^2$  is equal to \_\_\_\_\_.

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- 2) Let the point  $(-1, \alpha, \beta)$  lie on the line of the shortest distance between the lines  $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$  and  $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$ . Then  $(\alpha = \beta)^2$  is equal to \_\_\_\_\_.  
(Jan 2023)
- 3) The number of real solutions of the equation  $x|x+5| + 2|x+7| - 2 = 0$  is \_\_\_\_\_.  
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- 4) Let  $y = y(x)$  be the solution to the differential equation  $\frac{dy}{dx} + \frac{2x}{(1+x^2)^2}y = xe^{\frac{1}{1+x^2}}$ ;  $y(0) = 0$ . Then the area enclosed by the curve  $f(x) = y(x)e^{-\frac{1}{1+x^2}}$  and the line  $y - x = 4$  is \_\_\_\_\_.  
(Jan 2023)
- 5) Let a line perpendicular to the line  $2x - y = 10$  touch the parabola  $y^2 = 4(x - 9)$  at the point **P**. The distance of the point **P** from the centre of the circle  $x^2 + y^2 - 14x - 8y + 56 = 0$  is \_\_\_\_\_.  
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- 6) The number of solutions of  $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x - 1)^2 = 0$ , where  $-\pi \leq x \leq \pi$ , is \_\_\_\_\_.  
(Jan 2023)
- 7) Let the mean and the standard deviation of a probability distribution

$X$	$\alpha$	1	0	-3
$P(X)$	$\frac{1}{3}$	$K$	$\frac{1}{6}$	$\frac{1}{4}$

be  $\mu$  and  $\sigma$ , respectively. Then  $\sigma + \mu$  is equal to \_\_\_\_\_.

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- 8) If  $1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \dots$  upto  $\infty = 2 + \left(\sqrt{\frac{b}{a}} + 1\right)\log_e \frac{a}{b}$ , where  $a$  and  $b$  are integers with  $\gcd(a, b) = 1$ , then  $11a + 18b$  is equal to \_\_\_\_\_.  
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- 9) If  $f(t) = \int_0^\pi \frac{2x dx}{1 - \cos^2 t \sin^2 x}$ ,  $0 < t < \pi$ , then the value of  $\int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)}$  equals \_\_\_\_\_.  
(Jan 2023)
- 10) Let  $a > 0$  be a root of the equation  $2x^2 + x - 2 = 0$ . If  $\lim_{x \rightarrow \frac{1}{a}} \frac{16(1 - \cos(2+x-2x^2))}{(1-ax)^2} = \alpha + \beta\sqrt{17}$ , where  $\alpha, \beta \in \mathbb{Z}$ , then  $\alpha + \beta$  is equal to \_\_\_\_\_.  
(Jan 2023)