3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
A Triangle ABC can be constructed in which $\angle B = 60°$, $\angle C = 45°$ and $AB + BC + CA = 12cm$
Solution:

Symbol	Description
a	length of side BC
b	length of side CA
c	length of side AB
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C
K	Perimeter of triangle

TABLE 0: Variables Used

From properties of triangles we get the following equations

$$a + b + c = K \tag{1}$$

$$a = b\cos(C) + c\cos(B) \tag{2}$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{3}$$

Rewriting the equations will give

$$a + b + c = K \tag{4}$$

$$-a + b\cos(C) + c\cos(B) = 0 \tag{5}$$

$$b\sin(C) - c\sin(B) = 0 \tag{6}$$

$$\tag{7}$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

We can find all the side lengths by solving the above matrix equation where $x = \frac{a}{K}$, $y = \frac{b}{K}$, and $z = \frac{c}{K}$.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

$$\tag{10}$$

The augmented matrix for this will be

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \tag{11}$$

$$\xleftrightarrow{R_3 \leftarrow R_2 - \left(\sqrt{2}+1\right) R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & \left(\sqrt{3} + \sqrt{2} + 1\right) \frac{\sqrt{3}}{2} & 1 \end{pmatrix} \tag{12}$$

$$\xleftrightarrow{R_3 \leftarrow \left(\frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)}\right) R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \end{pmatrix} \tag{13}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - \left(\frac{3}{2}\right) R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & 0 & \frac{\sqrt{2}+1}{\sqrt{3}+\sqrt{2}+1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \end{pmatrix} \tag{14}$$

$$\xleftrightarrow{R_2 \leftarrow \left(\frac{\sqrt{2}}{\sqrt{2}+1}\right) R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}+1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \end{pmatrix} \tag{15}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 1 & \frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}+1} \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}+1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \end{pmatrix} \tag{16}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}+1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \end{pmatrix} \tag{17}$$

$$\tag{18}$$

The values of x,y,z are

$$x = \frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{19}$$

$$y = \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}+1} \tag{20}$$

$$z = \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{21}$$

The values of $\frac{a}{K}$,$\frac{b}{K}$ and $\frac{c}{K}$ are

$$\frac{a}{K} = \frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{22}$$

$$\frac{b}{K} = \frac{\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{23}$$

$$\frac{c}{K} = \frac{2}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{24}$$

The length of sides of triangle are

$$a = \frac{12+12\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{25}$$

$$b = \frac{12\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{26}$$

$$c = \frac{24}{\sqrt{3}\left(\sqrt{3}+\sqrt{2}+1\right)} \tag{27}$$

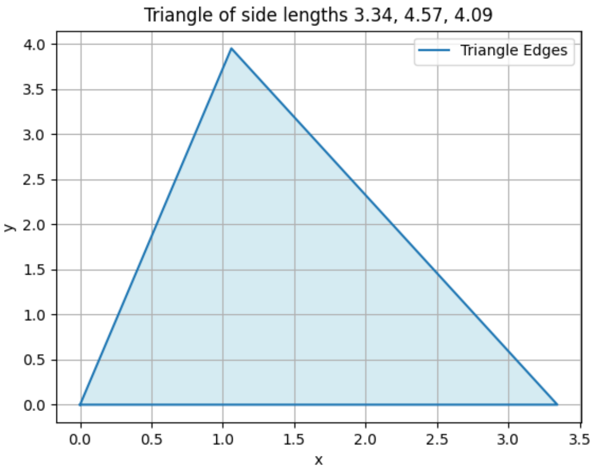

Fig. 0: Triangle with $\angle B = 60°$, $\angle C = 45°$ and Perimeter $= 12cm$