Assignment 2

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1) The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in \mathbb{R}$ is:

(Mar 2021)

a) $\sqrt{7}$ b) $\sqrt{5}$ c) 5 d) $\frac{3}{4}$

2) Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then the probability of event A is equal to:

(Mar 2021)

a) $\frac{4}{9}$ b) $\frac{9}{56}$ c) $\frac{3}{7}$ d) $\frac{11}{27}$

3) Let $\alpha \in \mathbb{R}$ be such that the function $f(x) = \begin{cases} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3} & x \neq 0, \\ \alpha & x = 0. \end{cases}$ is continuous at $x = 0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x. Then:

(Mar 2021)

a) $\alpha = \frac{\pi}{4}$ b) No such α exists c) $\alpha = 0$ d) $\alpha = \frac{\pi}{\sqrt{2}}$

4) If (x, y, z) be an arbitrary point lying on the plane P which passes through the points $(42, 0, 0)$, $(0, 42, 0)$, and $(0, 0, 42)$, then the value of expression $3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$ is equal to

(Mar 2021)

a) 3 b) 0 c) 39 d) -45

5) Consider the integral $I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}}dx$, where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal to :

(Mar 2021)

a) $45(e-1)$ b) $45(e+1)$ c) $9(e-1)$ d) $9(e+1)$

6) Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $\mathbf{P}(2, 1)$ is:

(Mar 2021)

a) $2x + y = 5$ b) $x + 2y = 4$ c) $x + 3y = 5$ d) $x - y = 1$

7) If $y = y(x)$ is the solution of the differential equation $\left(\frac{dy}{dx}\right) + (\tan x)\, y = \sin x$, $0 \leq x \leq \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to:

(Mar 2021)

a) $\log_e 2$ b) $\frac{1}{2}\log_e 2$ c) $\frac{1}{2\sqrt{2}}\log_e 2$ d) $\frac{1}{4}\log_e 2$

8) Let $A = \{2, 3, 4, 5, \ldots, 30\}$ and '=' be an equivalence relation on $A \times A$, defined by $(a, b) = (c, d)$. if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to:

(Mar 2021)

a) 5 b) 6 c) 8 d) 7

9) Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to:

(Mar 2021)

a) $\sqrt{10}$ b) $\sqrt{6}$ c) $\sqrt{11}$ d) $\sqrt{7}$

10) The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7} + 9i\right|$, $i = \sqrt{-1}$, is equal to

(Mar 2021)

a) 8 b) 3 c) $\sqrt{5}$ d) 2

11) Consider a rectangle $ABCD$ having 5, 7, 6, 9 points in the interior of the line segments AB, BC, CD, DA respectively. Let α be the number of triangles having these points from the different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

(Mar 2021)

a) 1890 b) 795 c) 717 d) 1173

12) If the points of intersection of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(Mar 2021)

a) 12 b) 5 c) 6 d) 10

13) Given that the inverse trigonometric functions take principal values only. Then, the number of real value of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$ is

(Mar 2021)

a) 1 b) 2 c) 3 d) 0

14) Let $\mathbf{A}(-1, 1)$, $\mathbf{B}(3, 4)$ and $\mathbf{C}(2, 0)$ be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point $\mathbf{P}$ and $\mathbf{Q}$ respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to:

(Mar 2021)

a) $\frac{4}{15}$ b) 1 c) 2 d) 3

15) Let f be a real-valued function, defined on $\mathbb{R} - \{-1, 1\}$ and given by $f(x) = 3\log_e\left(\frac{|(x-1)|}{|(x+1)|}\right) - \frac{2}{x-1}$. Then in which of the following intervals, function $f(x)$ is increasing?

(Mar 2021)

a) $(-\infty, -1) \cup \left(\left[\frac{1}{2} , \infty \right) - \{1\} \right)$

b) $(-1 , \frac{1}{2}]$

c) $(-\infty, \infty) - \{-1, 1\}$

d) $(-\infty , \frac{1}{2}] - \{-1\}$