Assignment 1

EE24Btech11024 - G. Abhimanyu Koushik

D: Single Correct

1) Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are)

(*JEEAdv.*2013)

(a) $x^2 + y^2 - 6x + 8y + 9 = 0$
(b) $x^2 + y^2 - 6x + 7y + 9 = 0$
(c) $x^2 + y^2 - 6x - 8y + 9 = 0$
(d) $x^2 + y^2 - 6x - 7y + 9 = 0$

2) A circle S passes through the point $(0,1)$ and is orthogonal to the circle $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(*JEEAdv.*2014)

(a) Radius of S is 8
(b) Radius of S is 7
(c) Centre of S is $(-7, 1)$
(d) Centre of S is $(-8, 1)$

3) Let RS be the diameter of the Circle $x^2 + y^2 = 1$, where S is the point $(1,0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point (s)

(*JEEAdv.*2016)

(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$
(b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$
(d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

4) Let T be a line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be F_2. Then which of the

following statements is (are) TRUE?

(*JEEAdv.*2018)

a) The point $(-2, 7)$ lies on E_1
b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie on E_1
c) The point $\left(\frac{1}{3}, 1\right)$ lies on E_1
d) The point $\left(0, \frac{3}{2}\right)$ does not lie on E_1

E: Subjective

1) Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point $(5, 5)$

(1978)

2) Let A be the centre of circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points $B(1, 7)$ and $CD(4, -2)$ on the circle meet at point C. Find the area of the quadrilateral $ABCD$.

(1981 − 4*marks*)

3) Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$

(1981 − 4*marks*)

4) Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted is $x^2 + y^2 = hx + ky$

(1983 − 5*marks*)

5) The abscissa of two points A and B are roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter.

(1984 − 4*marks*)

6) Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a Circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of circle C_2 which is concentric with C_1 and cuts intecepts of length 8 on these lines

(1986 − 5*marks*)

7) Let a given Line L_1 intersects the x and y axes at P and Q respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R

and S, respectively. Show that the locus of the point of intersection of PS and QR is a circle passing through origin.

<div align="right">(1987 − 3<i>marks</i>)</div>

8) The circle $x^2 + y^2 - 4x - y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2}$. Find k.

<div align="right">(1987 − 4<i>marks</i>)</div>

9) If $\left(m_i, \frac{1}{m_i}\right), m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$

<div align="right">(1989 − 2<i>marks</i>)</div>

10) A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of circle.

<div align="right">(1990 − 5<i>marks</i>)</div>

11) Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.

<div align="right">(1991 − 4<i>marks</i>)</div>