

## 3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

### Question:

A Triangle  $ABC$  can be constructed with side  $BC = 7\text{cm}$ ,  $\angle B = 45^\circ$ ,  $\angle A = 105^\circ$ . Then construct another triangle whose sides are  $\frac{3}{4}$  times the corresponding sides of  $\triangle ABC$

### Solution:

Symbol	Description
$a$	length of side BC of first triangle
$b$	length of side CA of first triangle
$c$	length of side AB of first triangle
$a_0$	length of side BC of second triangle
$b_0$	length of side CA of second triangle
$c_0$	length of side AB of second triangle
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C

TABLE 0: Variables Used

$\angle C$  can be found as

$$A + B + C = 180^\circ \quad (1)$$

$$105^\circ + 45^\circ + C = 180^\circ \quad (2)$$

$$C = 30^\circ \quad (3)$$

From properties of triangles we get the following equations

$$a = a \quad (4)$$

$$a = b \cos(C) + c \cos(B) \quad (5)$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad (6)$$

Rewriting the equations will give

$$a + (0)b + (0)c = a \quad (7)$$

$$(0)a + b \cos(C) + c \cos(B) = a \quad (8)$$

$$(0)a + b(\sin(C)) + c(-\sin(B)) = 0 \quad (9)$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad (10)$$

We can find all the side lengths by solving the above matrix equation where  $x = \frac{a}{a}$ ,  $y = \frac{b}{a}$ , and  $z = \frac{c}{a}$ .

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad (11)$$

$$(12)$$

The augmented matrix for this will be

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & 1 \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{2}{\sqrt{3}} R_2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \quad (13)$$

$$\xrightarrow{R_3 \leftarrow 2R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 1 & -\sqrt{2} & 0 \end{pmatrix} \quad (14)$$

$$\xrightarrow{R_3 \leftarrow R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 0 & \frac{\sqrt{2} + \sqrt{6}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \quad (15)$$

$$\xrightarrow{R_3 \leftarrow \left( \frac{\sqrt{3}}{\sqrt{2} + \sqrt{6}} \right) R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 0 & 1 & \frac{\sqrt{2}}{1 + \sqrt{3}} \end{pmatrix} \quad (16)$$

$$\xrightarrow{R_2 \leftarrow R_2 - \left( \frac{\sqrt{2}}{\sqrt{3}} \right) R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{2}{1 + \sqrt{3}} \\ 0 & 0 & 1 & \frac{\sqrt{2}}{1 + \sqrt{3}} \end{pmatrix} \quad (17)$$

The values of  $x, y, z$  are

$$x = 1 \quad (18)$$

$$y = \frac{2}{1 + \sqrt{3}} \quad (19)$$

$$z = \frac{\sqrt{2}}{1 + \sqrt{3}} \quad (20)$$

The values of  $\frac{a}{a}, \frac{b}{a}$  and  $\frac{c}{a}$  are

$$\frac{a}{a} = 1 \quad (21)$$

$$\frac{b}{a} = \frac{2}{1 + \sqrt{3}} \quad (22)$$

$$\frac{c}{a} = \frac{\sqrt{2}}{1 + \sqrt{3}} \quad (23)$$

The lengths of sides of first triangle are

$$a = 7 \quad (24)$$

$$b = \frac{14}{1 + \sqrt{3}} \quad (25)$$

$$c = \frac{7\sqrt{2}}{1 + \sqrt{3}} \quad (26)$$

The lengths of sides of second triangle are

$$a_0 = 21/4 \quad (27)$$

$$b_0 = \frac{21}{2(1 + \sqrt{3})} \quad (28)$$

$$c_0 = \frac{21\sqrt{2}}{4(1 + \sqrt{3})} \quad (29)$$

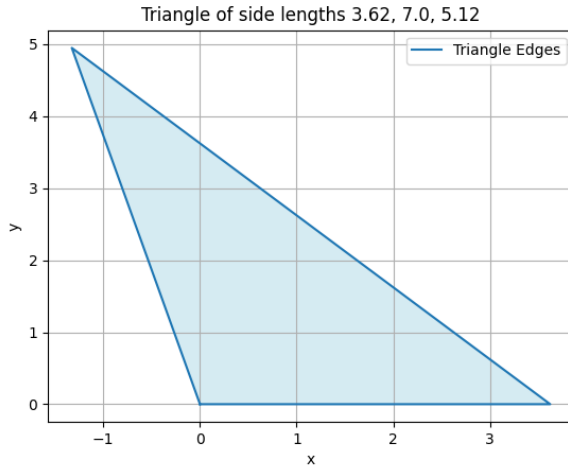


Fig. 0: Triangle with  $\angle B = 45^\circ$ ,  $\angle A = 105^\circ$  and  $BC = 7cm$

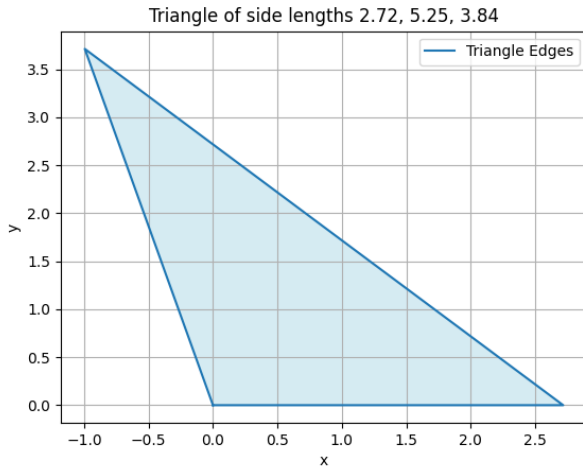


Fig. 0: Triangle with  $\angle B = 45^\circ$ ,  $\angle A = 105^\circ$  and  $BC = 7 \times \left(\frac{3}{4}\right) cm$