1.1.8.22

EE24BTECH11024 - G.Abhimanyu Koushik

Question:

If point $\mathbf{A}\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ is equidistant from $\mathbf{P}\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ and $\mathbf{Q}\begin{pmatrix} -10 \\ y \end{pmatrix}$, find the values of y. Also find distance PQ.

Solution:

Symbol	Value	Description
A	$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	First point
P	$\begin{pmatrix} 3 \\ 8 \end{pmatrix}$	Second point
Q	$\begin{pmatrix} -10 \\ y \end{pmatrix}$	Point such that **A** is equidistant from **P** and **Q**

TABLE 0: Variables Used

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{A} - \mathbf{Q}\|^2 \tag{0.1}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (\mathbf{A} - \mathbf{Q})^\top (\mathbf{A} - \mathbf{Q}) \tag{0.2}$$

$$\left(\mathbf{A}^\top\right)(\mathbf{A}) + \left(\mathbf{P}^\top\right)(\mathbf{P}) - 2\left(\mathbf{A}^\top\right)(\mathbf{P}) = \left(\mathbf{A}^\top\right)(\mathbf{A}) + \left(\mathbf{Q}^\top\right)(\mathbf{Q}) - 2\left(\mathbf{A}^\top\right)(\mathbf{Q}) \tag{0.3}$$

$$\left(\mathbf{P}^\top\right)(\mathbf{P}) - 2\left(\mathbf{A}^\top\right)(\mathbf{P}) = \left(\mathbf{Q}^\top\right)(\mathbf{Q}) - 2\left(\mathbf{A}^\top\right)(\mathbf{Q}) \tag{0.4}$$

$$\begin{pmatrix} 3 & 8 \end{pmatrix}\begin{pmatrix} 3 \\ 8 \end{pmatrix} - 2\begin{pmatrix} 2 & -4 \end{pmatrix}\begin{pmatrix} 3 \\ 8 \end{pmatrix} = \begin{pmatrix} -10 & y \end{pmatrix}\begin{pmatrix} -10 \\ y \end{pmatrix} - 2\begin{pmatrix} 2 & -4 \end{pmatrix}\begin{pmatrix} -10 \\ y \end{pmatrix} \tag{0.5}$$

$$73 - 2(-26) = 100 + y^2 - 2(-4y - 20) \tag{0.6}$$

$$y^2 + 8y + 15 = 0 \tag{0.7}$$

$$y^2 + 8y + 16 = 0 \tag{0.8}$$

$$y = -3, -5 \tag{0.9}$$

The value of y is -3 or -5.

The Distance d between P and Q is

$$d = \|P - Q\| \tag{0.10}$$

$$d = \sqrt{(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q})} \tag{0.11}$$

$$\tag{0.12}$$

$$d = \sqrt{\begin{pmatrix} 13 & 11 \end{pmatrix}\begin{pmatrix} 13 \\ 11 \end{pmatrix}} \text{ or } d = \sqrt{\begin{pmatrix} 13 & 13 \end{pmatrix}\begin{pmatrix} 13 \\ 13 \end{pmatrix}} \tag{0.13}$$
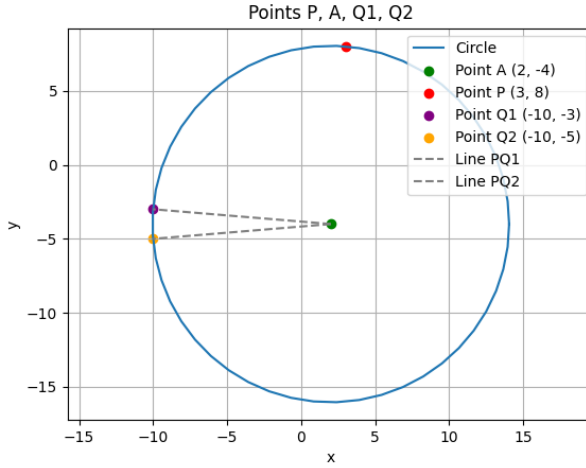
$$d = \sqrt{290} \text{ or } d = 13\sqrt{2} \tag{0.14}$$

Fig. 0.1: Plot of the given points and the bisector