Assignment 5

EE24Btech11024 - G. Abhimanyu Koushik

A. Multiple Choice

1) The relation $R = \{(a,b) : \gcd(a,b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is:_____ (Jan 2023)

a) Transitive but not reflexive
b) Symmetric but not transitive
c) Reflexive but not symmetric
d) Neither symmetric nor transitive

2) The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \implies ((\sim P) \wedge (\sim Q))$ is equivalent to (Jan 2023)

a) $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$
b) $(\sim Q) \vee P$
c) $((\sim P) \vee Q) \wedge (\sim Q)$
d) $(\sim P) \vee Q$

3) Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0, \\ 0 & x = 0. \end{cases}$; Then at $x = 0$ (Jan 2023)

a) f is continuous but not differentiable
b) f is continuous but f' is not continuous
c) f and f' both are continuous
d) f' is continuous but not differentiable

4) The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes greatest integer function, has: (Jan 2023)

a) Exactly two solutions in $(-\infty, \infty)$
b) No solution
c) A unique solution in $(-\infty, 1)$
d) A unique solution in $(-\infty, \infty)$

5) Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements:
(S1): If $P(A) = 0$, then $A = \phi$
(S2): If $P(A) = 1$, then $A = \Omega$
Then (Jan 2023)

a) Only (S1) is true
b) Only (S2) is true
c) Both (S1) and (S2) are true
d) Both (S1) and (S2) are false

B. Numericals

1) Let C be the largest circle centred at $(2, 0)$ and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\alpha^2$ is equal to _____. (Jan 2023)

2) Suppose $\sum_{r=0}^{2023} r^2 \times {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _____. (Jan 2023)

3) The value of $12 \int_0^3 \left|x^2 - 3x + 2\right| dx$ is _____. (Jan 2023)

4) The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places is _____.

(Jan 2023)

5) Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in set $S = \{x + \lambda : x \text{ is an integer solution of } E\}$ is _____.

(Jan 2023)

6) A boy needs to select 5 courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most 2 language courses, then the number of ways he can choose five courses is _____.

(Jan 2023)

7) Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect coordinate axes at points **A** and **B**. Then, the minimum length of the line segment AB is _____.

(Jan 2023)

8) The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is _____.

(Jan 2023)

9) The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to _____.

(Jan 2023)

10) The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in \mathbb{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 > S_6 + \frac{1}{2}$, then the number of possible values of m is _____.

(Jan 2023)