1.1.2.2

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
Find the values of x, y, z so that the vectors $x\hat{i} + 2\hat{j} + z\hat{k}$ and $2\hat{i} + y\hat{j} + \hat{k}$ are equal
Solution:

Variable	Description
$\overrightarrow{v}_1$	$x\hat{i} + 2\hat{j} + z\hat{k}$
$\overrightarrow{v}_2$	$2\hat{i} + y\hat{j} + \hat{k}$
x	Component along X-direction of 1^{st} vector
y	Component along Y-direction of 2^{nd} vector
z	Component along Z-direction of 1^{st} vector

TABLE 0: Variables Used

If the vectors are equal then the vector component along the X,Y and Z axes should also be equal

$$\overrightarrow{v}_1 \cdot \hat{i} = \overrightarrow{v}_2 \cdot \hat{i} \tag{0.1}$$

$$\left(x\hat{i} + 2\hat{j} + z\hat{k}\right) \cdot \hat{i} = \left(2\hat{i} + y\hat{j} + \hat{k}\right) \cdot \hat{i} \tag{0.2}$$

$$x \times 1 + 2 \times 0 + z \times 0 = 2 \times 1 + y \times 0 + 1 \times 0 \tag{0.3}$$

$$x + 0 + 0 = 2 + 0 + 0 \tag{0.4}$$

$$x = 2 \tag{0.5}$$

Similarly, taking dot product with $\hat{j}$ and $\hat{k}$ will give the values of y and z.
Dot product with $\hat{j}$

$$\overrightarrow{v}_1 \cdot \hat{j} = \overrightarrow{v}_2 \cdot \hat{j} \tag{0.6}$$

$$\left(x\hat{i} + 2\hat{j} + z\hat{k}\right) \cdot \hat{j} = \left(2\hat{i} + y\hat{j} + \hat{k}\right) \cdot \hat{j} \tag{0.7}$$

$$x \times 0 + 2 \times 1 + z \times 0 = 2 \times 0 + y \times 1 + 1 \times 0 \tag{0.8}$$

$$0 + 2 + 0 = 0 + y + 0 \tag{0.9}$$

$$y = 2 \tag{0.10}$$

Dot product with $\hat{k}$

$$\overrightarrow{v}_1 \cdot \hat{k} = \overrightarrow{v}_2 \cdot \hat{k} \tag{0.11}$$

$$\left(x\hat{i} + 2\hat{j} + z\hat{k}\right) \cdot \hat{k} = \left(2\hat{i} + y\hat{j} + \hat{k}\right) \cdot \hat{k} \tag{0.12}$$

$$x \times 0 + 2 \times 0 + z \times 1 = 2 \times 0 + y \times 0 + 1 \times 1 \tag{0.13}$$

$$0 + 0 + z = 0 + 0 + 1 \tag{0.14}$$

$$z = 1 \tag{0.15}$$

The values of x, y, z are $2, 2, 1$ respectively.

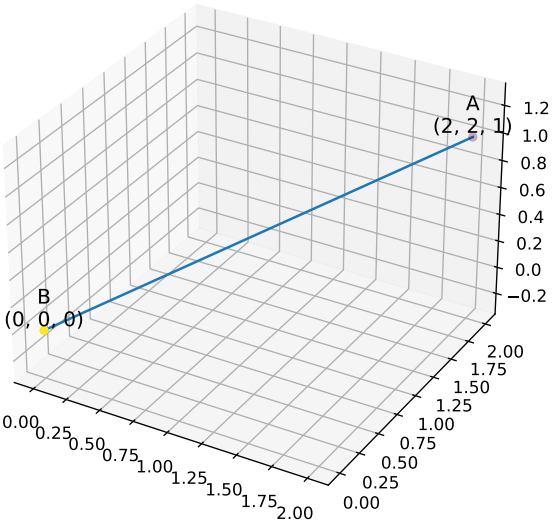

Fig. 0.1: Line segment represent the vector