

1.1.8.25

EE24BTECH11024 - G.Abhimanyu Koushik

Question:

Find a point on the Y axis which is equidistant from the points $\mathbf{A}\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $\mathbf{B}\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Solution:

Symbol	Value	Description
A	$\begin{pmatrix} 6 \\ 5 \end{pmatrix}$	First point
B	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	Second point
Y	$\begin{pmatrix} 0 \\ y \end{pmatrix}$	Point on Y-Axis equidistant from A and B

TABLE 0: Variables Used

$$\|\mathbf{A} - \mathbf{Y}\| = \|\mathbf{B} - \mathbf{Y}\| \quad (0.1)$$

$$\|\mathbf{A} - \mathbf{Y}\|^2 = \|\mathbf{B} - \mathbf{Y}\|^2 \quad (0.2)$$

$$\|\mathbf{A}\|^2 + \|\mathbf{Y}\|^2 - 2(\mathbf{A}^\top)(\mathbf{Y}) = \|\mathbf{B}\|^2 + \|\mathbf{Y}\|^2 - 2(\mathbf{B}^\top)(\mathbf{Y}) \quad (0.3)$$

$$\|\mathbf{A}\|^2 - 2(\mathbf{A}^\top)(\mathbf{Y}) = \|\mathbf{B}\|^2 - 2(\mathbf{B}^\top)(\mathbf{Y}) \quad (0.4)$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2(\mathbf{A}^\top)(\mathbf{Y}) - 2(\mathbf{B}^\top)(\mathbf{Y}) \quad (0.5)$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2(\mathbf{A}^\top - \mathbf{B}^\top)(\mathbf{Y}) \quad (0.6)$$

$$\mathbf{A}^\top \mathbf{A} - \mathbf{B}^\top \mathbf{B} = 2(\mathbf{A}^\top - \mathbf{B}^\top)(\mathbf{Y}) \quad (0.7)$$

$$\begin{pmatrix} 6 & 5 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} -4 & 3 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 2 \left(\begin{pmatrix} 6 & 5 \end{pmatrix} - \begin{pmatrix} -4 & 3 \end{pmatrix} \right) \begin{pmatrix} 0 \\ y \end{pmatrix} \quad (0.8)$$

$$36 + 25 - (16 + 9) = 2 \begin{pmatrix} 10 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} \quad (0.9)$$

$$36 = 4y \quad (0.10)$$

$$y = 9 \quad (0.11)$$

$$(0.12)$$

The point on the Y axis which is equidistant to \mathbf{A} and \mathbf{B} is $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$

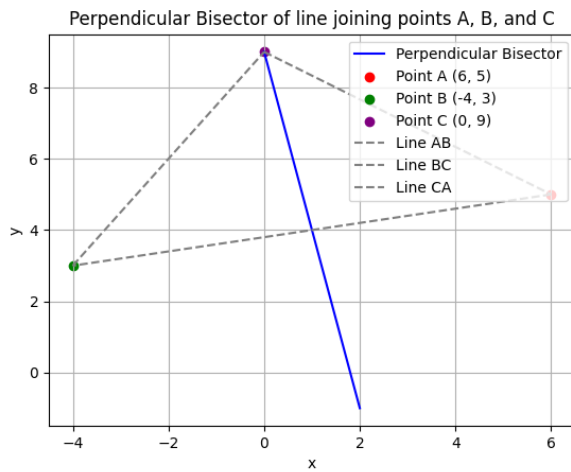


Fig. 0.1: Plot of the triangle