9.9.2.28

EE24BTECH11024 - Abhimanyu Koushik

Question:
Find the area of region bounded by the line $x = 2$ and the parabola $y^2 = 8x$
Solution:

Variable	Description
$\mathbf{x_1}$	First intersection point
$\mathbf{x_2}$	Second intersection point
$\mathbf{h}$	Point on the given line
$\mathbf{m}$	Direction vector of given line
A	Area of the region

TABLE I: Variables Used

The equation of a parabola in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

The equation of a line in vector form is

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{2}$$

For the given parabola $y^2 = 8x$, The values of $\mathbf{V}$,$\mathbf{u}$,f are

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{4}$$

$$f = 0 \tag{5}$$

For the given line $x = 2$, The values of $\mathbf{h}$, $\mathbf{m}$ are

$$\mathbf{h} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7}$$

Substituing the line equation in parabola equation gies the values of κ

$$(\mathbf{h}+\kappa\mathbf{m})^\top \mathbf{V}(\mathbf{h}+\kappa\mathbf{m})+2\mathbf{u}^\top(\mathbf{h}+\kappa\mathbf{m})+f=0 \tag{8}$$

$$\left(\begin{pmatrix}2\\0\end{pmatrix}+\kappa\begin{pmatrix}0\\1\end{pmatrix}\right)^\top\begin{pmatrix}0&0\\0&1\end{pmatrix}\left(\begin{pmatrix}2\\0\end{pmatrix}+\kappa\begin{pmatrix}0\\1\end{pmatrix}\right)+2\begin{pmatrix}-4\\0\end{pmatrix}^\top\left(\begin{pmatrix}2\\0\end{pmatrix}+\kappa\begin{pmatrix}0\\1\end{pmatrix}\right)+0=0 \tag{9}$$

$$\begin{pmatrix}2&\kappa\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}2\\\kappa\end{pmatrix}+2\begin{pmatrix}-4&0\end{pmatrix}\begin{pmatrix}2\\\kappa\end{pmatrix}=0 \tag{10}$$

$$\begin{pmatrix}2&\kappa\end{pmatrix}\begin{pmatrix}0\\\kappa\end{pmatrix}+2(-8)=0 \tag{11}$$

$$\kappa^2-16=0 \tag{12}$$

$$\kappa_1=4 \tag{13}$$

$$\kappa_2=-4 \tag{14}$$

$$\tag{15}$$

The intersection points are

$$\mathbf{x_1}=\mathbf{h}+\kappa_1\mathbf{m} \tag{16}$$

$$\mathbf{x_1}=\begin{pmatrix}2\\4\end{pmatrix} \tag{17}$$

$$\mathbf{x_2}=\mathbf{h}+\kappa_2\mathbf{m} \tag{18}$$

$$\mathbf{x_2}=\begin{pmatrix}2\\-4\end{pmatrix} \tag{19}$$

The Area under the curve is given by

$$A=\left(\int_{-4}^{4}\frac{y^2}{8}\right)dy \tag{20}$$

$$A=\left(\frac{1}{8}\right)\left(\frac{4^3-\left(-4^3\right)}{3}\right) \tag{21}$$

$$A=\left(\frac{1}{24}\right)(128) \tag{22}$$

$$A=\left(\frac{1}{24}\right)(128) \tag{23}$$

$$A=\frac{16}{3} \tag{24}$$

The area of region bounded by the line $x=2$ and the parabola $y^2=8x$ is $\frac{16}{3}$

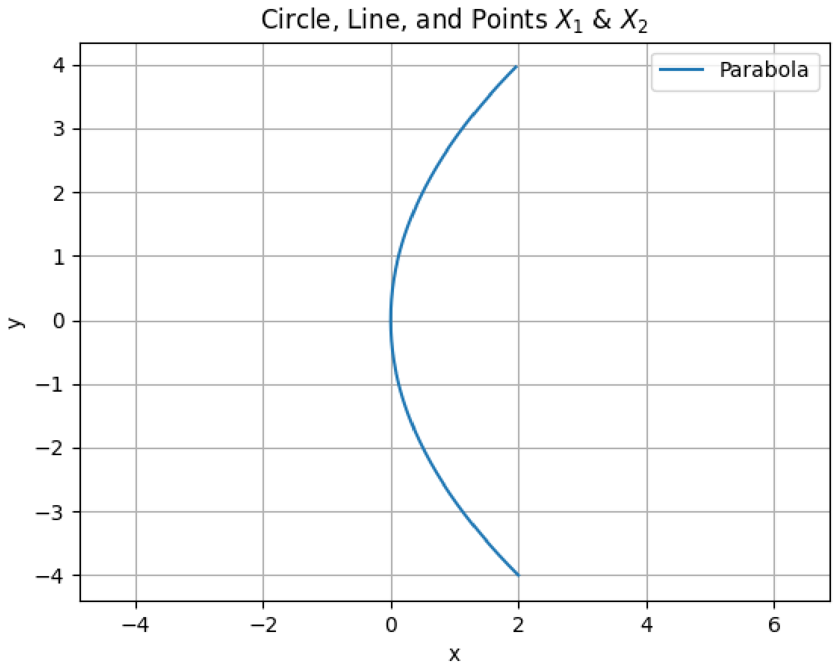

Fig. 1: The circle which satisfies the conditions