

# Assignment 3

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## A. Multiple Choice

- 1) Let  $X$  be a random variable such that the probability function of a distribution given by  $P(X = 0) = \frac{1}{2}$ ,  $P(X = j) = \frac{1}{3^j}$  ( $j = 1, 2, 3, \dots, \infty$ ). Then the mean of the distribution and  $P(X \text{ is positive and even})$  respectively are:

(Jul 2021)

- a)  $\frac{3}{8}$  and  $\frac{1}{8}$                       b)  $\frac{3}{4}$  and  $\frac{1}{8}$                       c)  $\frac{3}{4}$  and  $\frac{1}{9}$                       d)  $\frac{3}{4}$  and  $\frac{1}{16}$

- 2) If the tangent to the ellipse  $x^2 + 4y^2 = 4$  meets the tangents at the extremities of its major axis at **B** and **C**, then the circle with  $BC$  as diameter pass through the point:

(Jul 2021)

- a)  $(\sqrt{3}, 0)$                       b)  $(\sqrt{2}, 0)$                       c)  $(1, 1)$                       d)  $(-1, 1)$

- 3) Let the equation of pair of lines,  $y = px$  and  $y = qx$ , can be written as  $(y - px)(y - qx) = 0$ . Then the equation of the pair of angle bisectors of the lines  $x^2 - 4xy - 5y^2 = 0$  is:

(Jul 2021)

- a)  $x^2 - 3xy + y^2 = 0$       b)  $x^2 + 4xy - y^2 = 0$       c)  $x^2 + 3xy - y^2 = 0$       d)  $x^2 - 3xy - y^2 = 0$

- 4) If  ${}^nP_r = {}^nP_{r+1}$  and  ${}^nC_r = {}^nC_{r+1}$ , then the value of  $r$  is equal to:

(Jul 2021)

- a) 1                      b) 4                      c) 2                      d) 3

- 5) Let  $y = y(x)$  be the solution of the differential equation  $xdy = (y + x^3 \cos x)dx$  with  $y(\pi) = 0$ , then  $y\left(\frac{\pi}{2}\right)$  is equal to:

(Jul 2021)

- a)  $\frac{\pi^2}{4} + \frac{\pi}{2}$                       b)  $\frac{\pi^2}{2} + \frac{\pi}{4}$                       c)  $\frac{\pi^2}{2} - \frac{\pi}{4}$                       d)  $\frac{\pi^2}{4} - \frac{\pi}{2}$

## B. Numericals

- 1) Let  $n \in \mathbb{N}$  and  $[x]$  denote the greatest integer less than or equal to  $x$ . If the sum of  $(n + 1)$  terms  ${}^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3 \dots$  is equal to  $2^{100} \cdot 101$ , then  $2\left[\frac{n-1}{2}\right]$  is equal to \_\_\_\_.

(Jul 2021)

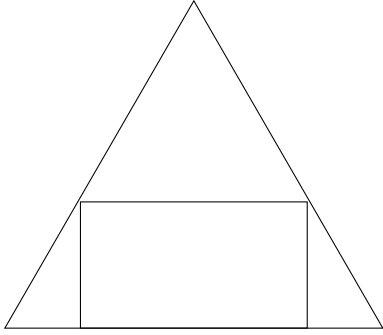
- 2) Consider the function  $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)} & x \neq 2, \\ 7 & x = 2. \end{cases}$  where  $P(x)$  is a polynomial such that  $P''(x)$  is always a constant and  $P(3) = 9$ . If  $f(x)$  is continuous at  $x = 2$ , then  $P(5)$  is equal to \_\_\_\_.

(Jul 2021)

- 3) The equation of a circle is  $Re(z^2) + 2(Im(z))^2 + 2Re(z) = 0$ , where  $z = x + iy$ . A line which passes through the centre of the given circle and the vertex of parabola,  $x^2 - 6x - y + 13 = 0$ , has y-intercept equal to \_\_\_\_.

(Jul 2021)

- 4) If a rectangle is inscribed in an equilateral triangle of side length  $2\sqrt{2}$  as shown in the figure, then the square of the largest area of such a rectangle is \_\_\_\_\_.



(Jul 2021)

- 5) If  $(\mathbf{a} + 3\mathbf{b})$  is perpendicular to  $(7\mathbf{a} - 5\mathbf{b})$  and  $(\mathbf{a} - 4\mathbf{b})$  is perpendicular to  $(7\mathbf{a} - 2\mathbf{b})$ , then the angle between  $\mathbf{a}$  and  $\mathbf{b}$  (in degrees) is \_\_\_\_\_.

(Jul 2021)

- 6) Let a curve  $y = f(x)$  pass through the point  $(2, (\log_e 2)^2)$  and have slope  $\frac{2y}{x \log_e x}$  for all positive real values of  $x$ . Then the value of  $f(e)$  is equal to \_\_\_\_\_.

(Jul 2021)

- 7) If  $a + b + c = 1$ ,  $ab + bc + ca = 2$  and  $abc = 3$ , then the value of  $a^4 + b^4 + c^4$  is equal to \_\_\_\_\_.

(Jul 2021)

- 8) A fair coin is tossed  $n$ -times such that the probability of getting at least one head is at least 0.9. Then the minimum value of  $n$  is \_\_\_\_\_.

(Jul 2021)

- 9) If the co-efficient of  $x^7$  and  $x^8$  in the expansion of  $(2 + \frac{x}{3})^n$  are equal, then the value of  $n$  is equal to \_\_\_\_\_.

(Jul 2021)

- 10) If the lines  $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$  and  $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$  are co-planar then, the value of  $k$  is \_\_\_\_\_.

(Jul 2021)