3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

Question:

A Triangle ABC can be constructed in which $\angle B = 60°$, $\angle C = 45°$ and $AB + BC + CA = 12cm$ **Solution:**

Symbol	Description
a	length of side BC
b	length of side CA
c	length of side AB
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C
K	Perimeter of triangle

TABLE 0: Variables Used

$$a + b + c = K \tag{1}$$

$$b\cos(C) + c\cos(B) - a = 0 \tag{2}$$

$$b\sin(C) - c\sin(B) = 0 \tag{3}$$

$$\tag{4}$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

We can find all the side lengths by solving the above matrix equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 12\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \tag{7}$$

$$\xleftarrow{R_3 \leftarrow R_2 - (\sqrt{2}+1)R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & (\sqrt{3} + \sqrt{2} + 1)\frac{\sqrt{3}}{2} & 1 \end{pmatrix} \tag{8}$$

By using Back-Substitution we get

$$\frac{a}{K} = \frac{1 + \sqrt{3}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{9}$$

$$\frac{b}{K} = \frac{\sqrt{2}}{\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{10}$$

$$\frac{c}{K} = \frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{11}$$

The length of sides of triangle are

$$a = \frac{12 + 12\sqrt{3}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{12}$$

$$b = \frac{12\sqrt{2}}{\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{13}$$

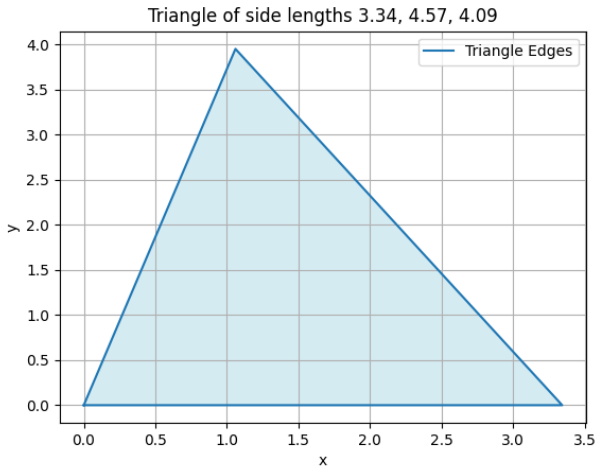$$c = \frac{24}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{14}$$

Fig. 0: Triangle with $\angle B = 60°$, $\angle C = 45°$ and Perimeter $= 12cm$