

Assignment 3

EE24Btech11024 - G. Abhimanyu Koushik

A. Multiple Choice

- 1) Let X be a random variable such that the probability function of a distribution given by $P(X = 0) = \frac{1}{2}$, $P(X = j) = \frac{1}{3^j}$ ($j = 1, 2, 3, \dots, \infty$). Then the mean of the distribution and $P(X \text{ is positive and even})$ respectively are:

(Jul 2021)

- a) $\frac{3}{8}$ and $\frac{1}{8}$ b) $\frac{3}{4}$ and $\frac{1}{8}$ c) $\frac{3}{4}$ and $\frac{1}{9}$ d) $\frac{3}{4}$ and $\frac{1}{16}$

- 2) If the tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at **B** and **C**, then the circle with BC as diameter pass through the point:

(Jul 2021)

- a) $(\sqrt{3}, 0)$ b) $(\sqrt{2}, 0)$ c) $(1, 1)$ d) $(-1, 1)$

- 3) Let the equation of pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(Jul 2021)

- a) $x^2 - 3xy + y^2 = 0$ b) $x^2 + 4xy - y^2 = 0$ c) $x^2 + 3xy - y^2 = 0$ d) $x^2 - 3xy - y^2 = 0$

- 4) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r+1}$, then the value of r is equal to:

(Jul 2021)

- a) 1 b) 4 c) 2 d) 3

- 5) Let $y = y(x)$ be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(Jul 2021)

- a) $\frac{\pi^2}{4} + \frac{\pi}{2}$ b) $\frac{\pi^2}{2} + \frac{\pi}{4}$ c) $\frac{\pi^2}{2} - \frac{\pi}{4}$ d) $\frac{\pi^2}{4} - \frac{\pi}{2}$

B. Numericals

- 1) Let $n \in \mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to x . If the sum of $(n + 1)$ terms ${}^nC_0, 3 \cdot {}^nC_1, 5 \cdot {}^nC_2, 7 \cdot {}^nC_3 \dots$ is equal to $2^{100} \cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to ____.

(Jul 2021)

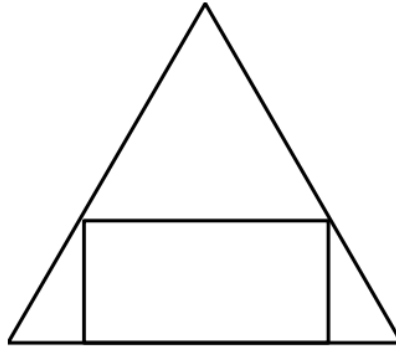
- 2) Consider the function $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)} & x \neq 2, \\ 7 & x = 2. \end{cases}$ where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to ____.

(Jul 2021)

- 3) The equation of a circle is $Re(z^2) + 2(Im(z))^2 + 2Re(z) = 0$, where $z = x + iy$. A line which passes through the centre of the given circle and the vertex of parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ____.

(Jul 2021)

- 4) If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ____.



(Jul 2021)

- 5) If $(\mathbf{a} + 3\mathbf{b})$ is perpendicular to $(7\mathbf{a} - 5\mathbf{b})$ and $(\mathbf{a} - 4\mathbf{b})$ is perpendicular to $(7\mathbf{a} - 2\mathbf{b})$, then the angle between \mathbf{a} and \mathbf{b} (in degrees) is _____.

(Jul 2021)

- 6) Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real values of x . Then the value of $f(e)$ is equal to _____.

(Jul 2021)

- 7) If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to _____.

(Jul 2021)

- 8) A fair coin is tossed n -times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _____.

(Jul 2021)

- 9) If the co-efficient of x^7 and x^8 in the expansion of $(2 + \frac{x}{3})^n$ are equal, then the value of n is equal to _____.

(Jul 2021)

- 10) If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar then, the value of k is _____.

(Jul 2021)