Assignment 7

EE24Btech11024 - G. Abhimanyu Koushik

A. Multiple Choice

1) Let $\mathbf{A}(-1, 1)$ and $\mathbf{B}(2, 3)$ be two points and $\mathbf{P}$ be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of $\mathbf{P}$ is $ax + by = 15$, then $5a + 2b$ is:

(Apr 2024)

a) 6 b) 4 c) $-\frac{12}{5}$ d) $-\frac{6}{5}$

2) Let $\alpha\beta \neq 0$ and $A = \begin{pmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\beta \end{pmatrix}$. If $B = \begin{pmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{pmatrix}$ is the matrix of cofactor elements of A, then $\det(AB)$ is equal to:

(Apr 2024)

a) 216
b) 343
c) 64
d) 125

3) The value of m, n for which the system of linear equations
$x + y + z = 4$,
$2x + 5y + 5z = 17$,
$x + 2y + mz = n$
has infinitely many solutions satisfy the equation:

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a) $m^2 + n^2 - m - n = 46$
b) $m^2 + n^2 + mn = 68$
c) $m^2 + n^2 + m + n = 64$
d) $m^2 + n^2 - mn = 39$

4) Let $ABCD$ and $AEFG$ be squares of side 4 and 2 units respectively. The point $\mathbf{E}$ is on the line segment AB and the point $\mathbf{F}$ is on the diagonal AC. Then the radius r of the circle passing through the point $\mathbf{F}$ and touching the line segments BC and CD satisfies:

a) $r = 1$ b) $r^2 - 8r + 8 = 0$ c) $2r^2 - 8r + 7 = 0$ d) $2r^2 - 4r + 1 = 0$

5) Let $\mathbf{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\mathbf{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\mathbf{c}$ be three vectors such that $\left(\mathbf{c} + \hat{i}\right) \times \left(\mathbf{a} + \mathbf{b} + \hat{i}\right) = \mathbf{a} \times \left(\mathbf{c} + \hat{i}\right)$. If $\mathbf{a} \cdot \mathbf{c} = -29$, then $\mathbf{c} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right)$ is equal to:

(Apr 2024)

a) 15 b) 12 c) 5 d) 10

B. Numericals

1) Let the maximum and minimum values of $\left(\sqrt{8x - x^2 - 12} - 4\right)^2 + (x - 7)^2$, $x \in \mathbb{R}$ be M and m, respectively. Then $M^2 - m^2$ is equal to _____.

(Apr 2024)

2) Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha = \beta)^2$ is equal to ______.

(Apr 2024)

3) The number of real solutions of the equation $x\,|x+5| + 2\,|x+7| - 2 = 0$ is ______.

(Apr 2024)

4) Let $y = y(x)$ be the solution to the differential equation $\frac{dy}{dx} + \frac{2x}{\left(1+x^2\right)^2} y = xe^{\frac{1}{1+x^2}}; y(0) = 0$. Then the area enclosed by the curve $f(x) = y(x)\,e^{-\frac{1}{1+x^2}}$ and the line $y - x = 4$ is ______.

(Apr 2024)

5) Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x-9)$ at the point **P**. The distance of the point **P** from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is ______.

(Apr 2024)

6) The number of solutions of $\sin^2 x + \left(2 + 2x - x^2\right)\sin x - 3(x-1)^2 = 0$, where $-\pi \le x \le \pi$, is ______.

(Apr 2024)

7) Let the mean and the standard deviation of a probability distribution

X	α	1	0	-3
$P(X)$	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. Then $\sigma + \mu$ is equal to ______.

(Apr 2024)

8) If $1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \ldots$ upto $\infty = 2 + \left(\sqrt{\frac{b}{a}} + 1\right)\log_e \frac{a}{b}$, where a and b are integers with $\gcd(a,b) = 1$, then $11a + 18b$ is equal to ______.

(Apr 2024)

9) If $f(t) = \int_0^{\pi} \frac{2x\,dx}{1-\cos^2 t \sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)}$ equals ______.

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10) Let $a > 0$ be a root of the equation $2x^2 + x - 2 = 0$. If $\lim_{x \to \frac{1}{a}} \frac{16\left(1-\cos\left(2+x-2x^2\right)\right)}{(1-ax)^2} = \alpha + \beta\sqrt{17}$, where $\alpha, \beta \in \mathbb{Z}$, then $\alpha + \beta$ is equal to ______.

(Apr 2024)