Assignment 1

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D: Single Correct

1) Circle(s) touching the x-axis at a distance (3) from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are)

(*JEEAdv*.2013)

a) $x^2+y^2-6x+8y+9=0$
b) $x^2+y^2-6x+7y+9=0$
c) $x^2+y^2-6x-8y+9=0$
d) $x^2+y^2-6x-7y+9=0$

2) A circle S passes through the point $(0,1)$ and is orthogonal to the circle $(x-1)^2+y^2=16$ and $x^2+y^2=1$. Then

(*JEEAdv*.2014)

a) Radius of S is 8
b) Radius of S is 7
c) Centre of S is $(-7,1)$
d) Centre of S is $(-8,1)$

3) Let RS be the diameter of the Circle $x^2+y^2=1$, where **S** is the point $(1,0)$. Let **P** be a variable point (other than R and S) on the circle and tangents to the circle at **S** and **P** meet at the point **Q**. The normal to the circle at **P** intersects a line drawn through **Q** parallel to RS at point **E**. Then the locus of **E** passes through the point(s)

(*JEEAdv*.2016)

a) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$
b) $\left(\frac{1}{4},\frac{1}{2}\right)$
c) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$
d) $\left(\frac{1}{4},-\frac{1}{2}\right)$

4) Let T be a line passing through the points $\mathbf{P}(-2,7)$ and $\mathbf{Q}(2,-5)$. Let F_1 be the set of all pairs of circles (S_1,S_2) such that T is tangent to S_1 at **P** and tangent to S_2 at **Q**, and also such that S_1 and S_2 touch each other at a point, say **M**. Let E_1 be the set representing the locus of **M** as the pair (S_1,S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $\mathbf{R}(1,1)$ be F_2. Then which of the following statements is (are) TRUE?

(*JEEAdv*.2018)

a) The point $(-2,7)$ lies on E_1
b) The point $\left(\frac{4}{5},\frac{7}{5}\right)$ does **NOT** lie on E_1
c) The point $\left(\frac{1}{3},1\right)$ lies on E_1
d) The point $\left(0,\frac{3}{2}\right)$ does not lie on E_1

E: Subjective

1) Find the equation of the circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at the point $(5,5)$

(1978)

2) Let **A** be the centre of circle $x^2+y^2-2x-4y-20=0$. Suppose that the tangents at the points $\mathbf{B}(1,7)$ and $\mathbf{D}(4,-2)$ on the circle meet at point **C**. Find the area of the quadrilateral $ABCD$.

(1981 – 4*marks*)

3) Find the equations of the circle passing through $(-4,3)$ and touching the lines $x+y=2$ and $x-y=2$

(1981 – 4*marks*)

4) Through a fixed point (h,k) secants are drawn to the circle $x^2+y^2=r^2$. Show that the locus of the mid-points of the secants intercepted is $x^2+y^2=hx+ky$

(1983 – 5*marks*)

5) The abscissa of two points **A** and **B** are roots of the equation $x^2+2ax-b^2=0$ and their ordinates are roots of the equation $x^2+2px-q^2=0$. Find the equation and the radius of the circle with AB as diameter.

(1984 – 4*marks*)

6) Lines $5x+12y-10=0$ and $5x-12y-40=0$ touch a Circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of circle C_2 which is concentric with C_1 and cuts intecepts of length 8 on these lines

(1986 – 5*marks*)

7) Let a given Line L_1 intersects the x and y axes at **P** and **Q** respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at **R** and **S**, respectively. Show that the locus of the point of intersection of PS and QR is a circle passing through origin.

$(1987 - 3marks)$

8) The circle $x^2+y^2-4x-y+4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2}$. Find k.

$(1987 - 4marks)$

9) If $\left(m_i, \frac{1}{m_i}\right), m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1m_2m_3m_4 = 1$

$(1989 - 2marks)$

10) A circle touches the line $y = x$ at a point **P** such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x+y = 0$ is $6\sqrt{2}$. Determine the equation of circle.

$(1990 - 5marks)$

11) Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.

$(1991 - 4marks)$