3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
A Triangle ABC can be constructed in which $\angle B = 60°$, $\angle C = 45°$ and $AB + BC + CA = 12cm$ **Solution:**

Symbol	Description
a	length of side BC
b	length of side CA
c	length of side AB
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C
K	Perimeter of triangle
x	$\frac{a}{K}$
y	$\frac{b}{K}$
z	$\frac{c}{K}$

TABLE 0: Variables Used

From properties of triangles we get the following equations

$$a + b + c = K \tag{1}$$

$$a = b\cos(C) + c\cos(B) \tag{2}$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{3}$$

Rewriting the equations will give

$$a + b + c = K \tag{4}$$

$$b\cos(C) + c\cos(B) - a = 0 \tag{5}$$

$$b\sin(C) - c\sin(B) = 0 \tag{6}$$

$$\tag{7}$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

We can find all the side lengths by solving the above matrix equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \tag{10}$$

$$\xleftrightarrow{R_3 \leftarrow R_2 - \left(\sqrt{2}+1\right) R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & \left(\sqrt{3} + \sqrt{2} + 1\right) \frac{\sqrt{3}}{2} & 1 \end{pmatrix} \tag{11}$$

By using Back-Substitution we get The values of x,y,z

$$z = \frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{12}$$

Value of y

$$\left(\frac{1}{\sqrt{2}} + 1\right) y + \left(\frac{3}{2}\right) z = 1 \tag{13}$$

$$\left(\frac{1}{\sqrt{2}} + 1\right) y + \left(\frac{3}{2}\right) \left(\frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)}\right) = 1 \tag{14}$$

$$\left(\frac{1}{\sqrt{2}} + 1\right) y = 1 - \left(\frac{3}{2}\right) \left(\frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)}\right) \tag{15}$$

$$\left(\frac{1 + \sqrt{2}}{\sqrt{2}}\right) y = 1 - \left(\frac{\sqrt{3}}{\sqrt{3} + \sqrt{2} + 1}\right) \tag{16}$$

$$\left(\frac{1 + \sqrt{2}}{\sqrt{2}}\right) y = \frac{\sqrt{2} + 1}{\sqrt{3} + \sqrt{2} + 1} \tag{17}$$

$$\left(\frac{1}{-} \sqrt{2}\right) y = \frac{1}{\sqrt{3} + \sqrt{2} + 1} \tag{18}$$

$$y = \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} \tag{19}$$

Value of x

$$x + y + z = 1 \tag{20}$$

$$x + \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} + \frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} = 1 \tag{21}$$

$$x + \frac{\sqrt{6}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} + \frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} = 1 \tag{22}$$

$$1 - \frac{2 + \sqrt{6}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} = x \tag{23}$$

$$\frac{3 + \sqrt{6} + \sqrt{3} - 2 - \sqrt{6}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} = x \tag{24}$$

$$\frac{1 + \sqrt{3}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} = x \tag{25}$$

From these we get the value of $\frac{a}{K}$, $\frac{b}{K}$ and $\frac{c}{K}$

$$\frac{a}{K} = \frac{1 + \sqrt{3}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{26}$$

$$\frac{b}{K} = \frac{\sqrt{2}}{\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{27}$$

$$\frac{c}{K} = \frac{2}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{28}$$

The length of sides of triangle are

$$a = \frac{12 + 12\sqrt{3}}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{29}$$

$$b = \frac{12\sqrt{2}}{\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{30}$$

$$c = \frac{24}{\sqrt{3}\left(\sqrt{3} + \sqrt{2} + 1\right)} \tag{31}$$

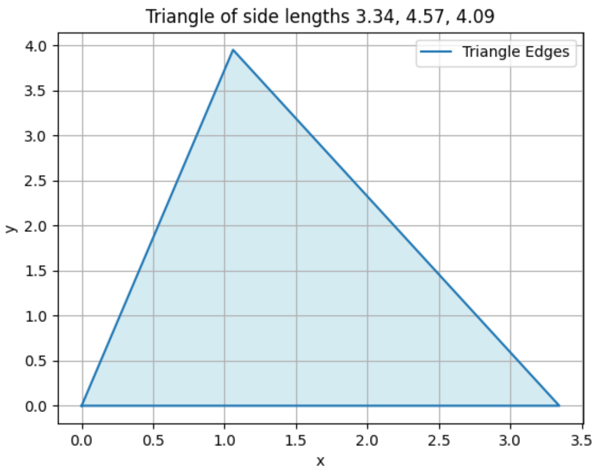

Fig. 0: Triangle with $\angle B = 60°$, $\angle C = 45°$ and Perimeter = $12cm$