3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

Question:
A Triangle ABC can be constructed with side $BC = 7cm$, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC
Solution:

Symbol	Description
a	length of side BC of first triangle
b	length of side CA of first triangle
c	length of side AB of first triangle
a_0	length of side BC of second triangle
b_0	length of side CA of second triangle
c_0	length of side AB of second triangle
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C

TABLE 0: Variables Used

$\angle C$ can be found as

$$A + B + C = 180° \tag{1}$$
$$105° + 45° + C = 180° \tag{2}$$
$$C = 30° \tag{3}$$

From properties of triangles we get the following equations

$$a = a \tag{4}$$
$$a = b\cos(C) + c\cos(B) \tag{5}$$
$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{6}$$

Rewriting the equations will give

$$a + (0)\,b + (0)\,c = a \tag{7}$$
$$(0)\,a + b\cos(C) + c\cos(B) = a \tag{8}$$
$$(0)\,a + b\,(\sin(C)) + c\,(-\sin(B)) = 0 \tag{9}$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{10}$$

We can find all the side lengths by solving the above matrix equation where $x = \frac{a}{a}$, $y = \frac{b}{a}$, and $z = \frac{c}{a}$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{11}$$

$$\tag{12}$$

The augmented matrix for this will be

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & 1 \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{2}{\sqrt{3}} R_2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \tag{13}$$

$$\xleftrightarrow{R_3 \leftarrow 2R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 1 & -\sqrt{2} & 0 \end{pmatrix} \tag{14}$$

$$\xleftrightarrow{R_3 \leftarrow R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 0 & \frac{\sqrt{2}+\sqrt{6}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \tag{15}$$

$$\xleftrightarrow{R_3 \leftarrow \left(\frac{\sqrt{3}}{\sqrt{2}+\sqrt{6}}\right) R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\ 0 & 0 & 1 & \frac{\sqrt{2}}{1+\sqrt{3}} \end{pmatrix} \tag{16}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - \left(\frac{\sqrt{2}}{\sqrt{3}}\right) R_3} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{2}{1+\sqrt{3}} \\ 0 & 0 & 1 & \frac{\sqrt{2}}{1+\sqrt{3}} \end{pmatrix} \tag{17}$$

The values of x,y,z are

$$x = 1 \tag{18}$$

$$y = \frac{2}{1 + \sqrt{3}} \tag{19}$$

$$z = \frac{\sqrt{2}}{1 + \sqrt{3}} \tag{20}$$

The values of $\frac{a}{a}, \frac{b}{a}$ and $\frac{c}{a}$ are

$$\frac{a}{a} = 1 \tag{21}$$

$$\frac{b}{a} = \frac{2}{1+\sqrt{3}} \tag{22}$$

$$\frac{c}{a} = \frac{\sqrt{2}}{1+\sqrt{3}} \tag{23}$$

The lengths of sides of first triangle are

$$a = 7 \tag{24}$$

$$b = \frac{14}{1+\sqrt{3}} \tag{25}$$

$$c = \frac{7\sqrt{2}}{1+\sqrt{3}} \tag{26}$$

The lengths of sides of second triangle are

$$a_0 = 21/4 \tag{27}$$

$$b_0 = \frac{21}{2\left(1+\sqrt{3}\right)} \tag{28}$$

$$c_0 = \frac{21\sqrt{2}}{4\left(1+\sqrt{3}\right)} \tag{29}$$

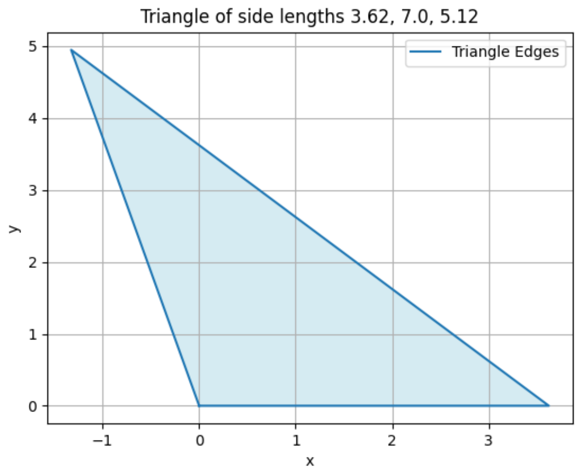

Fig. 0: Triangle with $\angle B = 45°$, $\angle A = 105°$ and $BC = 7cm$

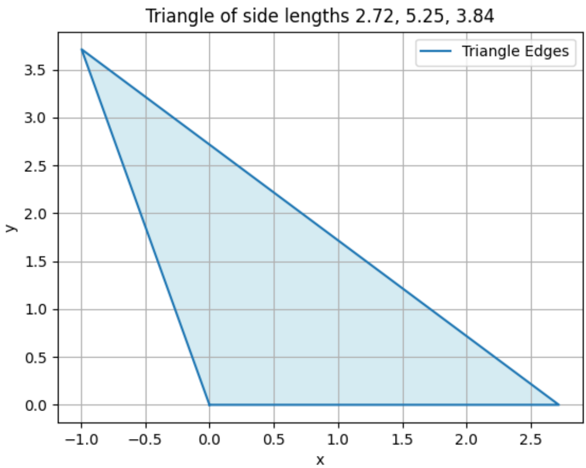

Fig. 0: Triangle with $\angle B = 45°$, $\angle A = 105°$ and $BC = 7 \times \left(\frac{3}{4}\right) cm$