Assignment 3

EE24Btech11024 - G. Abhimanyu Koushik

A. Multiple Choice

1) Let X be a random variable such that the probability function of a distribution given by $P(X=0)=\frac{1}{2}$, $P(X=j)=\frac{1}{3^j}$ $(j=1,2,3,\ldots,\infty)$. Then the mean of the distribution and $P(X$ is positive and even$)$ respectively are:

(Jul 2021)

a) $\frac{3}{8}$ and $\frac{1}{8}$ b) $\frac{3}{4}$ and $\frac{1}{8}$ c) $\frac{3}{4}$ and $\frac{1}{9}$ d) $\frac{3}{4}$ and $\frac{1}{16}$

2) If the tangent to the ellipse $x^2+4y^2=4$ meets the tangents at the extremities of its major axis at **B** and **C**, then the circle with BC as diameter pass through the point:

(Jul 2021)

a) $\left(\sqrt{3},0\right)$ b) $\left(\sqrt{2},0\right)$ c) $(1,1)$ d) $(-1,1)$

3) Let the equation of pair of lines , $y=px$ and $y=qx$, can be written as $(y-px)(y-qx)=0$. Then the equation of the pair of angle bisectors of the lines $x^2-4xy-5y^2=0$ is:

(Jul 2021)

a) $x^2-3xy+y^2=0$ b) $x^2+4xy-y^2=0$ c) $x^2+3xy-y^2=0$ d) $x^2-3xy-y^2=0$

4) If ${}^nP_r={}^nP_{r+1}$ and ${}^nC_r={}^nC_{r+1}$, then the value of r is equal to: (Jul 2021)

a) 1 b) 4 c) 2 d) 3

5) Let $y=y(x)$ be the solution of the differential equation $xdy=\left(y+x^3\cos x\right)dx$ with $y(\pi)=0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(Jul 2021)

a) $\frac{\pi^2}{4}+\frac{\pi}{2}$ b) $\frac{\pi^2}{2}+\frac{\pi}{4}$ c) $\frac{\pi^2}{2}-\frac{\pi}{4}$ d) $\frac{\pi^2}{4}-\frac{\pi}{2}$

B. Numericals

1) Let $n\in\mathbb{N}$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of $(n+1)$ terms ${}^nC_0$, $3\cdot{}^nC_1$, $5\cdot{}^nC_2$, $7\cdot{}^nC_3\ldots$ is equal to $2^{100}\cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to _____.

(Jul 2021)

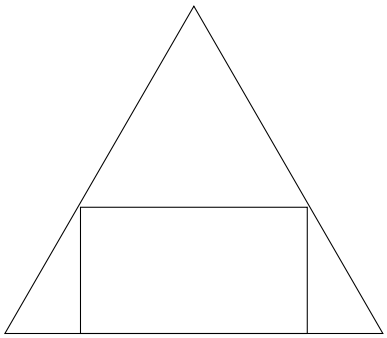
2) Consider the function $f(x)=\begin{cases}\frac{P(x)}{\sin(x-2)} & x\neq 2,\\ 7 & x=2.\end{cases}$ where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3)=9$. If $f(x)$ is continuous at $x=2$, then $P(5)$ is equal to _____.

(Jul 2021)

3) The equation of a circle is $Re\left(z^2\right)+2\left(Im(z)\right)^2+2Re(z)=0$, where $z=x+iy$. A line which passes through the centre of the given circle and the vertex of parabola, $x^2-6x-y+13=0$, has y-intercept equal to _____.

(Jul 2021)

4) If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _____.



(Jul 2021)

5) If $(\mathbf{a} + 3\mathbf{b})$ is perpendicular to $(7\mathbf{a} - 5\mathbf{b})$ and $(\mathbf{a} - 4\mathbf{b})$ is perpendicular to $(7\mathbf{a} - 2\mathbf{b})$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ (in degrees) is _____.

(Jul 2021)

6) Let a curve $y = f(x)$ pass through the point $\left(2, (\log_e 2)^2\right)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real values of x. Then the value of $f(e)$ is equal to _____.

(Jul 2021)

7) If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to _____.

(Jul 2021)

8) A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _____.

(Jul 2021)

9) If the co-efficient of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to _____.

(Jul 2021)

10) If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar then, the value of k is _____.

(Jul 2021)