

# Assignment 1

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- 1) For which of the following ordered pairs  $(\mu, \delta)$  the system of linear equations

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + 4y + 5z &= \mu \\ 4x + 4y + 4z &= \delta \end{aligned}$$

is inconsistent?

(Jan 2020)

- a) (4, 6)                      b) (3, 4)                      c) (1, 0)                      d) (4, 3)

- 2) Let  $y = f(x)$  be a solution to the differential equation

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, \quad |x| < 1$$

If  $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ , then  $y\left(-\frac{1}{\sqrt{2}}\right)$  is equal to

(Jan 2020)

- a)  $-\frac{1}{\sqrt{2}}$                       b)  $-\frac{\sqrt{3}}{2}$                       c)  $\frac{1}{\sqrt{2}}$                       d)  $\frac{\sqrt{3}}{2}$

- 3) If  $a$ ,  $b$  and  $c$  are the greatest values of  ${}^{19}C_p$ ,  ${}^{20}C_q$ ,  ${}^{21}C_r$  respectively, then:

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- a)  $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{42}\right)$     b)  $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{42}\right)$     c)  $\left(\frac{a}{11}\right) = \left(\frac{b}{22}\right) = \left(\frac{c}{21}\right)$     d)  $\left(\frac{a}{10}\right) = \left(\frac{b}{11}\right) = \left(\frac{c}{21}\right)$

- 4) Which of the following is a tautology?

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- a)  $(P \wedge (P \rightarrow Q)) \rightarrow Q$     b)  $P \wedge (P \vee Q)$                       c)  $(Q \rightarrow (\wedge (P \rightarrow Q)))$     d)  $P \vee (P \wedge Q)$

- 5) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be such that for all  $x \in \mathbb{R}$ ,  $(2^{1+x} + 2^{1-x})$ ,  $f(x)$  and  $(3^x + 3^{-x})$  are in A.P, then the minimum value of  $f(x)$  is:

(Jan 2020)

- a) 0                      b) 4                      c) 3                      d) 2

- 6) The locus of a point which divides the line segment joining the point  $(0, -1)$  and a point on parabola,  $x^2 = 4y$ , internally in the ratio 1 : 2 is:

(Jan 2020)

- a)  $9x^2 - 12y = 8$                       b)  $4x^2 - 3y = 2$                       c)  $x^2 - 3y = 2$                       d)  $9x^2 - 3y = 2$

- 7) For  $a > 0$ , let the curves  $C_1: y^2 = ax$  and  $C_2: x^2 = ay$  intersect at origin **O** and a point **P**. Let the line  $x = b$  ( $0 < b < a$ ) intersect the chord  $OP$  and the x-axis at points **Q** and **R**, respectively. If the line  $x = b$  bisects the area bounded by the curves,  $C_1$  and  $C_2$ , and the area of  $\Delta OQR = \frac{1}{2}$ , then  $a$  satisfies the equation

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a)  $x^6 - 12x^3 + 4 = 0$     b)  $x^6 - 12x^3 - 4 = 0$     c)  $x^6 + 6x^3 - 4 = 0$     d)  $x^6 - 6x^3 + 4 = 0$

8) The inverse of the function  $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$  is

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a)  $\frac{1}{4}(\log_8 e) \log_e \left(\frac{1+x}{1-x}\right)$     b)  $\frac{1}{4}(\log_8 e) \log_e \left(\frac{1-x}{1+x}\right)$     c)  $\frac{1}{4} \log_e \left(\frac{1+x}{1-x}\right)$     d)  $\frac{1}{4} \log_e \left(\frac{1-x}{1+x}\right)$

9)  $\lim_{x \rightarrow 0} \left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$  is equal to

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a)  $e$     b)  $\frac{1}{e^2}$     c)  $\frac{1}{e}$     d)  $e^2$

10) Let  $f(x) = \left(\sin(\tan^{-1} x) + \sin(\cot^{-1} x)\right)^2 - 1$  where  $|x| > 1$ . If

$$\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx} \left(\sin^{-1} f(x)\right)$$

and  $y(\sqrt{3}) = \frac{\pi}{6}$ , then  $y(-\sqrt{3})$  is equal to:

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a)  $\frac{\pi}{3}$     b)  $\frac{2\pi}{3}$     c)  $-\frac{\pi}{6}$     d)  $\frac{5\pi}{6}$

11) If the equation,  $x^2 + bx + 45 = 0$  ( $b \in \mathbb{R}$ ) has conjugate complex roots and they satisfy  $|z + 1| = 2\sqrt{10}$ , then:

(Jan 2020)

a)  $b^2 + b = 12$     b)  $b^2 - b = 42$     c)  $b^2 - b = 30$     d)  $b^2 + b = 72$

12) The mean and standard deviation of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by  $p$  and then reduced by  $q$ , where  $p \neq 0$  and  $q \neq 0$ . If the new mean and standard deviation become half of their original values, then  $q$  is equal to:

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a)  $-20$     b)  $-5$     c)  $10$     d)  $-10$

13) If  $\int \frac{\cos x}{\sin^3 x (1 + \sin^6 x)^{\frac{2}{3}}} dx = f(x) (1 + \sin^6 x)^{\frac{1}{3}} + c$ , where  $c$  is a constant of integration, then  $\lambda f\left(\frac{\pi}{3}\right)$  is equal to:

(Jan 2020)

a)  $-\frac{9}{8}$     b)  $\frac{9}{8}$     c)  $2$     d)  $-2$

14) Let  $A$  and  $B$  be two independent events such that  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{6}$ . Then which of the following is **TRUE**?

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a)  $P\left(\frac{A}{A \cup B}\right) = \frac{1}{4}$     b)  $P\left(\frac{A}{B}\right) = \frac{1}{3}$     c)  $P\left(\frac{A}{B}\right) = \frac{2}{3}$     d)  $P\left(\frac{A'}{B'}\right) = \frac{1}{3}$

15) If volume of a parallelepiped whose coterminous edges are given by

$$\mathbf{u} = \mathbf{i} + \mathbf{j} + \lambda \mathbf{k},$$

$$\mathbf{v} = \mathbf{i} + \mathbf{j} + 3\mathbf{k} \text{ and}$$

$$\mathbf{w} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$$

be 1cu.unit. If  $\theta$  is the angle between the edges  $\mathbf{u}$  and  $\mathbf{w}$  then,  $\cos \theta$  will be:

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a)  $\frac{7}{6\sqrt{6}}$

b)  $\frac{5}{7}$

c)  $\frac{7}{6\sqrt{3}}$

d)  $\frac{5}{3\sqrt{3}}$