

3.3.2.25

EE24BTECH11024 - G. Abhimanyu Koushik

Question:

A Triangle ABC can be constructed in which $\angle B = 60^\circ$, $\angle C = 45^\circ$ and $AB + BC + CA = 12cm$

Solution:

Symbol	Description
a	length of side BC
b	length of side CA
c	length of side AB
$\angle A$	angle at vertex A
$\angle B$	angle at vertex B
$\angle C$	angle at vertex C
K	Perimeter of triangle

TABLE 0: Variables Used

From properties of triangles we get the following equations

$$a + b + c = K \quad (1)$$

$$a = b \cos(C) + c \cos(B) \quad (2)$$

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad (3)$$

Rewriting the equations will give

$$a + b + c = K \quad (4)$$

$$-a + b \cos(C) + c \cos(B) = 0 \quad (5)$$

$$b \sin(C) - c \sin(B) = 0 \quad (6)$$

$$(7)$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (8)$$

We can find all the side lengths by solving the above matrix equation where $x = \frac{a}{K}$, $y = \frac{b}{K}$,

and $z = \frac{c}{K}$.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \quad (10)$$

$$\xleftrightarrow{R_3 \leftarrow R_2 - (\sqrt{2}+1)R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & (\sqrt{3} + \sqrt{2} + 1)\frac{\sqrt{3}}{2} & 1 \end{pmatrix} \quad (11)$$

$$\xleftrightarrow{R_3 \leftarrow \left(\frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)}\right)R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \end{pmatrix} \quad (12)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - \left(\frac{3}{2}\right)R_3} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} + 1 & 0 & \frac{\sqrt{2}+1}{\sqrt{3} + \sqrt{2} + 1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \end{pmatrix} \quad (13)$$

$$\xleftrightarrow{R_2 \leftarrow \left(\frac{\sqrt{2}}{\sqrt{2}+1}\right)R_2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \end{pmatrix} \quad (14)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 1 & \frac{\sqrt{3}+1}{\sqrt{3} + \sqrt{2} + 1} \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \end{pmatrix} \quad (15)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{1+\sqrt{3}}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \\ 0 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} \\ 0 & 0 & 1 & \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \end{pmatrix} \quad (16)$$

$$(17)$$

The values of x, y, z are

$$x = \frac{1 + \sqrt{3}}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (18)$$

$$y = \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2} + 1} \quad (19)$$

$$z = \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (20)$$

The values of $\frac{a}{K}$, $\frac{b}{K}$ and $\frac{c}{K}$ are

$$\frac{a}{K} = \frac{1 + \sqrt{3}}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (21)$$

$$\frac{b}{K} = \frac{\sqrt{2}}{(\sqrt{3} + \sqrt{2} + 1)} \quad (22)$$

$$\frac{c}{K} = \frac{2}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (23)$$

The length of sides of triangle are

$$a = \frac{12 + 12\sqrt{3}}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (24)$$

$$b = \frac{12\sqrt{2}}{(\sqrt{3} + \sqrt{2} + 1)} \quad (25)$$

$$c = \frac{24}{\sqrt{3}(\sqrt{3} + \sqrt{2} + 1)} \quad (26)$$

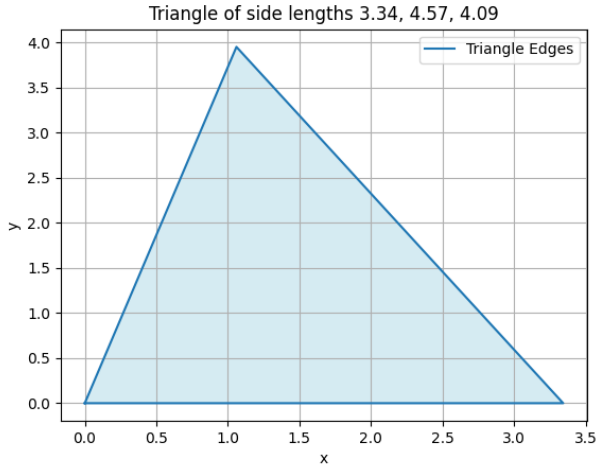


Fig. 0: Triangle with $\angle B = 60^\circ$, $\angle C = 45^\circ$ and Perimeter = 12cm