## Assignment 3

## EE24Btech11024 - G. Abhimanyu Koushik

<i>A</i> .	Mu	ltipl	e (	CI	hoice
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1) Let X be a random variable such that the probability function of a distribution given by  $P(X=0)=\frac{1}{2}$ ,  $P(X=j)=\frac{1}{3^j}$   $(j=1,2,3,\ldots,\infty)$ . Then the mean of the distribution and P(X is positive and even)respectively are:

(Jul 2021)

a)  $\frac{3}{8}$  and  $\frac{1}{8}$ 

b)  $\frac{3}{4}$  and  $\frac{1}{8}$ 

c)  $\frac{3}{4}$  and  $\frac{1}{9}$ 

d)  $\frac{3}{4}$  and  $\frac{1}{16}$ 

2) If the tangent to the ellipse  $x^2 + 4y^2 = 4$  meets the tangents at the extremities of its major axis at **B** and C, then the circle with BC as diameter pass through the point:

(Jul 2021)

a)  $(\sqrt{3}, 0)$ 

b)  $(\sqrt{2},0)$ 

c) (1,1)

d) (-1,1)

3) Let the equation of pair of lines , y = px and y = qx, can be written as (y - px)(y - qx) = 0. Then the equation of the pair of angle bisectors of the lines  $x^2 - 4xy - 5y^2 = 0$  is:

(Jul 2021)

a)  $x^2 - 3xy + y^2 = 0$  b)  $x^2 + 4xy - y^2 = 0$  c)  $x^2 + 3xy - y^2 = 0$  d)  $x^2 - 3xy - y^2 = 0$ 

4) If  ${}^{n}P_{r} = {}^{n}P_{r+1}$  and  ${}^{n}C_{r} = {}^{n}C_{r+1}$ , then the value of r is equal to:

(Jul 2021)

a) 1

b) 4

c) 2

d) 3

5) Let y = y(x) be the solution of the differential equation  $xdy = (y + x^3 \cos x) dx$  with  $y(\pi) = 0$ , then  $y\left(\frac{\pi}{2}\right)$  is equal to:

(Jul 2021)

a)  $\frac{\pi^2}{4} + \frac{\pi}{2}$  b)  $\frac{\pi^2}{2} + \frac{\pi}{4}$  c)  $\frac{\pi^2}{2} - \frac{\pi}{4}$  d)  $\frac{\pi^2}{4} - \frac{\pi}{2}$ 

## B. Numericals

1) Let  $n \in \mathbb{N}$  and [x] denote the greatest integer less than or equal to x. If the sum of (n+1) terms  ${}^{n}C_{0}$ ,  $3 \cdot {}^{n}C_{1}$ ,  $5 \cdot {}^{n}C_{2}$ ,  $7 \cdot {}^{n}C_{3}$ ... is equal to  $2^{100} \cdot 101$ , then  $2\left[\frac{n-1}{2}\right]$  is equal to \_\_\_\_\_.

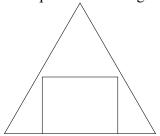
(Jul 2021)

2) Consider the function  $f(x) = \begin{cases} \frac{P(x)}{\sin(x-2)} & x \neq 2, \\ 7 & x = 2. \end{cases}$  where P(x) is a polynomial such that P''(x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to \_\_\_\_\_.

3) The equation of a circle is  $Re(z^2) + 2(Im(z))^2 + 2Re(z) = 0$ , where z = x + iy. A line which passes through the centre of the given circle and the vertex of parabola,  $x^2 - 6x - y + 13 = 0$ , has y-intercept equal to \_\_\_\_\_.

(Jul 2021)

4) If a rectangle is inscribed in an equilateral triangle of side length  $2\sqrt{2}$  as shown in the figure, then the square of the largest area of such a rectangle is \_



(Jul 2021)

5) If  $(\mathbf{a} + 3\mathbf{b})$  is perpendicular to  $(7\mathbf{a} - 5\mathbf{b})$  and  $(\mathbf{a} - 4\mathbf{b})$  is perpendicular to  $(7\mathbf{a} - 2\mathbf{b})$ , then the angle between **a** and **b** (in degrees) is \_\_\_\_\_

(Jul 2021)

6) Let a curve y = f(x) pass through the point  $\left(2, (\log_e 2)^2\right)$  and have slope  $\frac{2y}{x \log_e x}$  for all positive real values of x. Then the value of f(e) is equal to \_\_\_\_\_.

(Jul 2021)

- 7) If a+b+c=1, ab+bc+ca=2 and abc=3, then the value of  $a^4+b^4+c^4$  is equal to \_ (Jul 2021)
- 8) A fair coin is tossed *n*-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is \_\_\_\_\_.

(Jul 2021)

9) If the co-efficient of  $x^7$  and  $x^8$  in the expansion of  $\left(2+\frac{x}{3}\right)^n$  are equal, then the value of n is equal to

(Jul 2021)

10) If the lines  $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$  and  $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$  are co-planar then, the value of k is \_\_\_\_\_ (Jul 2021)