Bandpass Filter using Sallen-Key Second-Order Filters

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1 Introduction

Bandpass filters are essential components in signal processing systems, allowing frequencies within a specific range to pass while attenuating frequencies outside this range. This experiment focuses on implementing a bandpass filter using the Sallen-Key topology, which is widely used for its simplicity and stability. The bandpass filter is constructed by cascading a high-pass filter (HPF) and a low-pass filter (LPF), with carefully selected cutoff frequencies to define the desired passband.

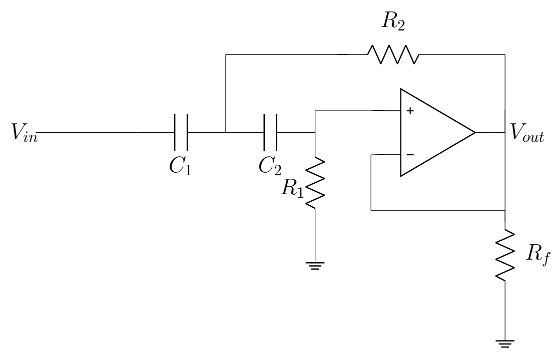
2 Theory

2.1 Bandpass Filter Fundamentals

A bandpass filter (BPF) allows signals within a specific frequency range to pass while attenuating signals outside this range. The filter is characterized by:

- Lower cutoff frequency (f_l): Frequencies below this are attenuated
- Upper cutoff frequency (f_h): Frequencies above this are attenuated
- Bandwidth = $f_h - f_l$
- Center frequency (f_0) = $\sqrt{f_h \times f_l}$

2.2 High Pass Filter Topology



Given the circuit diagram, we write the equations as follows:

$$\frac{V_{\text{in}} - V_1}{X_{C_1}} + \frac{V_2 - V_1}{X_{C_2}} + \frac{V_{\text{out}} - V_1}{R_2} = 0$$

Simplifying:

$$\frac{V_{\text{in}}}{X_{C_1}} + \frac{V_2}{X_{C_2}} + \frac{V_{\text{out}}}{R_2} = V_1 \left(\frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{R_2} \right)$$

Next, consider the relationship between V_1 and V_2:

$$\frac{V_1 - V_2}{X_{C_2}} - \frac{V_2}{R_1} = 0$$

Rewriting:

$$V_1 = V_2 \left(1 + \frac{X_{C_2}}{R_1} \right)$$

Using the virtual short property ($V_2 = V_{\text{out}}$):

$$V_1 = V_{\text{out}} \left(1 + \frac{X_{C_2}}{R_1} \right)$$

Substituting X_{C_1} and X_{C_2} as:

$$X_{C_1} = \frac{1}{j\omega C_1}, \quad X_{C_2} = \frac{1}{j\omega C_2}$$

We get:

$$V_1 = V_{\text{out}} \left(1 + \frac{1}{j\omega C_1 R_2}\right)$$

Substituting back into the first equation:

$$\frac{V_{\text{in}}}{X_{C_1}} + \frac{V_{\text{out}}}{R_2} + j\omega C_2 V_{\text{out}} = V_{\text{out}} \left(1 + \frac{1}{j\omega C_1 R_2}\right) \left(\frac{1}{R_2} + j\omega C_1 + j\omega C_2\right)$$

Expanding further:

$$\frac{V_{\text{in}}}{X_{C_1}} + V_{\text{out}} \left(\frac{1}{R_2} + j\omega C_2\right) = V_{\text{out}} \left(1 + \frac{1}{j\omega C_1 R_2}\right) \left(\frac{1}{R_2} + j\omega C_1 + j\omega C_2\right)$$

Rewriting the transfer function:

$$s^2 V_{\text{in}} C_1 C_2 R_1 R_2 = V_{\text{out}} \left(1 + sR_2(C_1 + C_2) + s^2 C_1 C_2 R_1 R_2\right)$$

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{s^2 C_1 C_2 R_1 R_2}{s^2 C_1 C_2 R_1 R_2 + sR_2(C_1 + C_2) + 1}$$

Assume $C_1 = C_2 = C$ and $R_1 = R_2 = R$,

$$\text{Gain} = |H(s)| = \frac{(\omega RC)^2}{(\omega RC)^2 + 1}$$

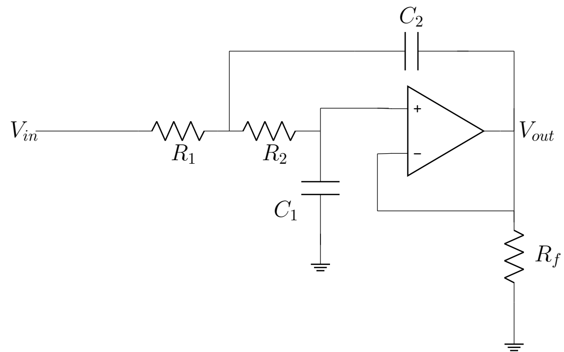
Cutoff angular frequency (w_c) is given by,

$$w_c = \frac{1}{RC}$$

Cutoff frequency (f_c) is given by,

$$f_c = \frac{1}{2\pi RC}$$

2.3 Low-Pass Filter Topology



Given the circuit diagram, we write the equations as follows:

$$\frac{V_{\text{in}} - V_1}{R_1} + \frac{V_2 - V_1}{R_2} + \frac{V_{\text{out}} - V_1}{X_{C_2}} = 0 \tag{1}$$

Simplifying:

$$\frac{V_{\text{in}}}{R_1} + \frac{V_2}{R_2} + \frac{V_{\text{out}}}{X_{C_2}} = V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{X_{C_2}} \right) \tag{2}$$

Next, consider the relationship between V_1 and V_2:

$$\frac{V_1 - V_2}{R_2} - \frac{V_2}{X_{C_1}} = 0 \tag{3}$$

Rewriting:

$$V_1 = V_2 \left(1 + \frac{R_2}{X_{C_1}} \right) \tag{4}$$

Using the virtual short property ($V_2 = V_{\text{out}}$):

$$V_1 = V_{\text{out}} \left(1 + \frac{R_2}{X_{C_1}} \right) \tag{5}$$

Substituting X_{C_1} and X_{C_2} as:

$$X_{C_1} = \frac{1}{j\omega C_1}, \quad X_{C_2} = \frac{1}{j\omega C_2} \tag{6}$$

We get:

$$V_1 = V_{\text{out}} \left(1 + j\omega C_1 R_2\right) \tag{7}$$

Substituting back into the first equation:

$$\frac{V_{\text{in}}}{R_1} + \frac{V_{\text{out}}}{R_2} + j\omega C_2 V_{\text{out}} = V_{\text{out}}(1 + j\omega C_1 R_2)\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_2\right) \tag{8}$$

Expanding further:

$$\frac{V_{\text{in}}}{R_1} + V_{\text{out}}\left(\frac{1}{R_2} + j\omega C_2\right) = V_{\text{out}}\left(1 + j\omega C_1 R_2\right)\left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_2\right) \tag{9}$$

Rewriting the transfer function:

$$V_{\text{in}} = V_{\text{out}}\left(1 + sC_1(R_1 + R_2) + s^2 C_1 C_2 R_1 R_2\right) \tag{10}$$

$$H(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{s^2 C_1 C_2 R_1 R_2 + sC_1(R_1 + R_2) + 1} \tag{11}$$

Assume $C_1 = C_2 = C$ and $R_1 = R_2 = R$,

$$\text{Gain} = |H(s)| = \frac{1}{(\omega RC)^2 + 1}$$

Cutoff angular frequency (w_c) is given by,

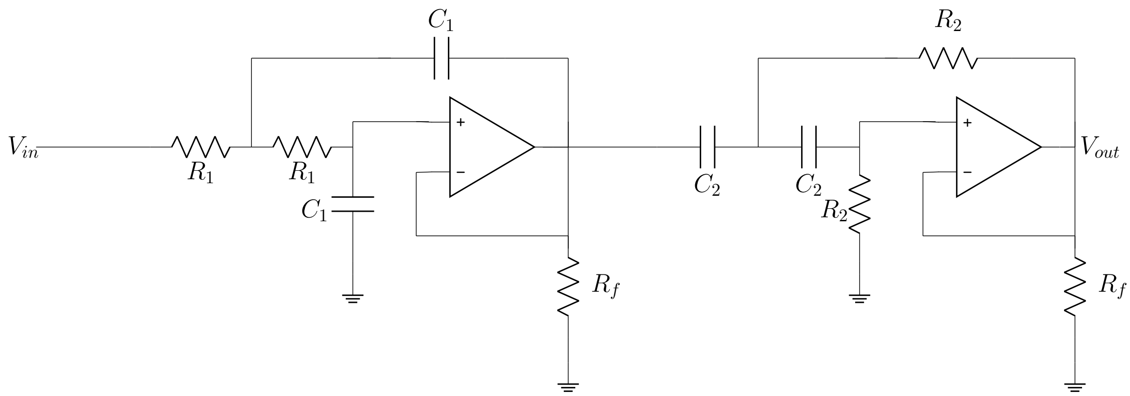
$$w_c = \frac{1}{RC}$$

Cutoff frequency (f_c) is given by,

$$f_c = \frac{1}{2\pi RC}$$

2.4 Band-Pass Filter Topology

Band-Pass Filter is just low-pass and high-pass filters cascaded, i.e. output of low-pass filter is passed to high-pass filter and the final out is V_{out} of high-pass filter.



Cutoff frequency (low-pass): $f_{low} = \frac{1}{R_1 C_1}$.

Cutoff frequency (high-pass): $f_{high} = \frac{1}{R_2 C_2}$.

Bandwidth = $f_{low} - f_{high} = \frac{1}{R_1 C_1} - \frac{1}{R_2 C_2}$.

3 Apparatus

- Operational Amplifiers
- Resistors: R_1, R_2, R_3, R_4 (in kΩ)
- Capacitors: C_1, C_2, C_3, C_4 (in nF)
- Function Generator
- Oscilloscope
- DC Power Supply
- Breadboard and connecting wires

4 Procedure

4.0.1 High-Pass Filter Implementation

1. Assemble the Sallen-Key HPF circuit on the breadboard.

2. Connect the function generator to the input and set it to generate a sine wave.
3. Connect the oscilloscope to measure the output voltage.
4. Vary the input frequency from 100 Hz to 100 kHz in appropriate steps.
5. For each frequency, record the input and output voltages to calculate the gain.
6. Plot the frequency response (gain vs. frequency) on a logarithmic scale.

4.0.2 Low-Pass Filter Implementation

1. Assemble the Sallen-Key LPF circuit on the breadboard.
2. Repeat the same procedure as in Step 1.

4.0.3 Bandpass Filter Implementation

1. Connect the output of the LPF to the input of the HPF to form the complete bandpass filter.
2. Repeat the frequency response measurements as in the previous steps.

5 Results and Analysis

5.1 Low-Pass Filter

$$R = 1k\Omega, C = 85nF$$

$\log(\omega)$	$20\log(\|H(s)\|)$
1.7982	-0.7242
2.4971	0.0
2.7982	0.0
3.0992	0.0
3.4971	-0.3546
3.7013	-1.1103
3.7982	-1.9382
3.8774	-2.2905
3.9743	-3.6593
4.0992	-5.8112
4.2753	-10.5683
4.4002	-14.3340
4.4971	-16.8328
4.5763	-19.0156
4.6732	-21.9382
4.7982	-25.0362

Table 1: Logarithmic frequency response data for low-pass filter

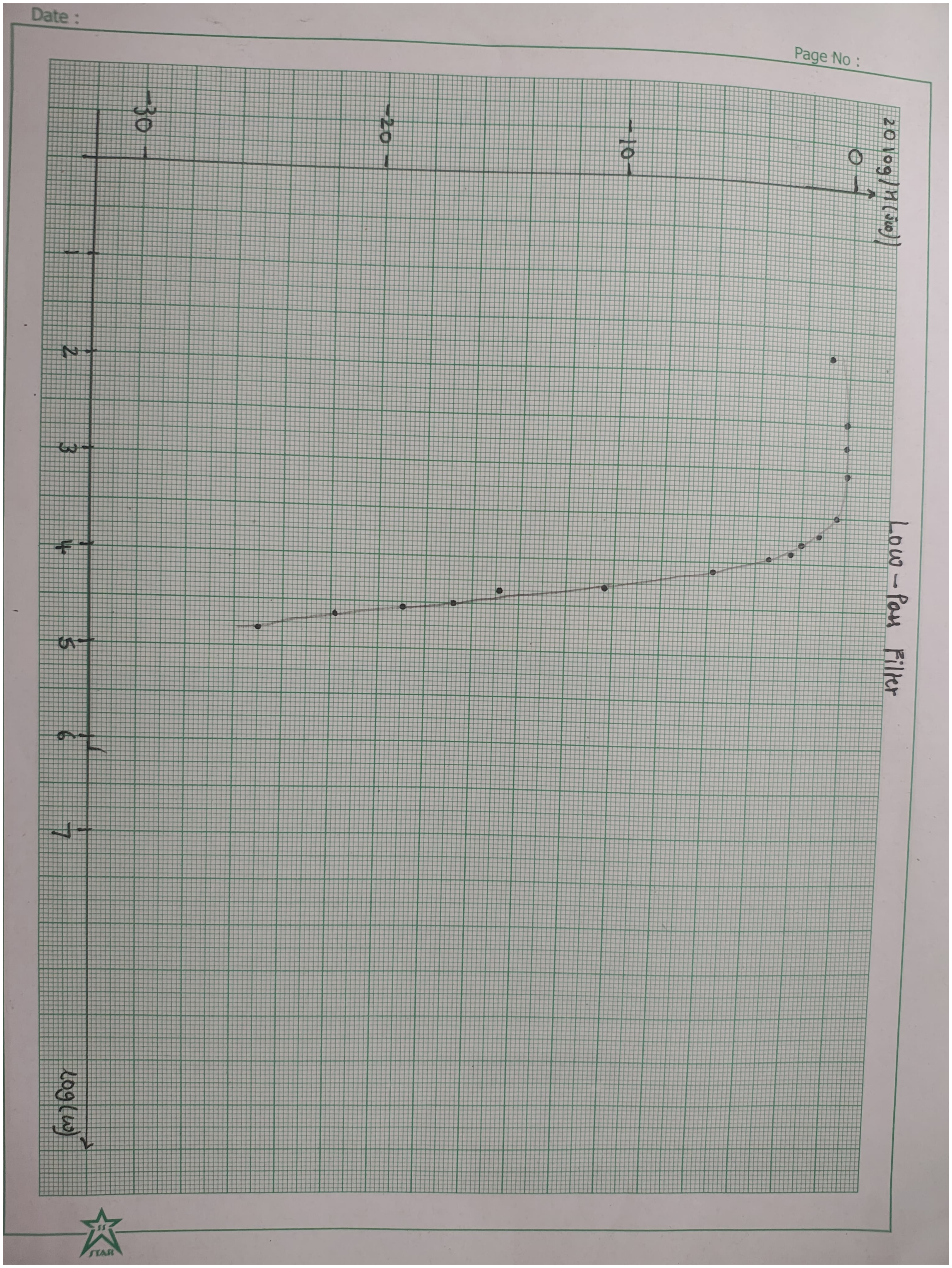


Figure 1: Bode Plot for Low-Pass Filter

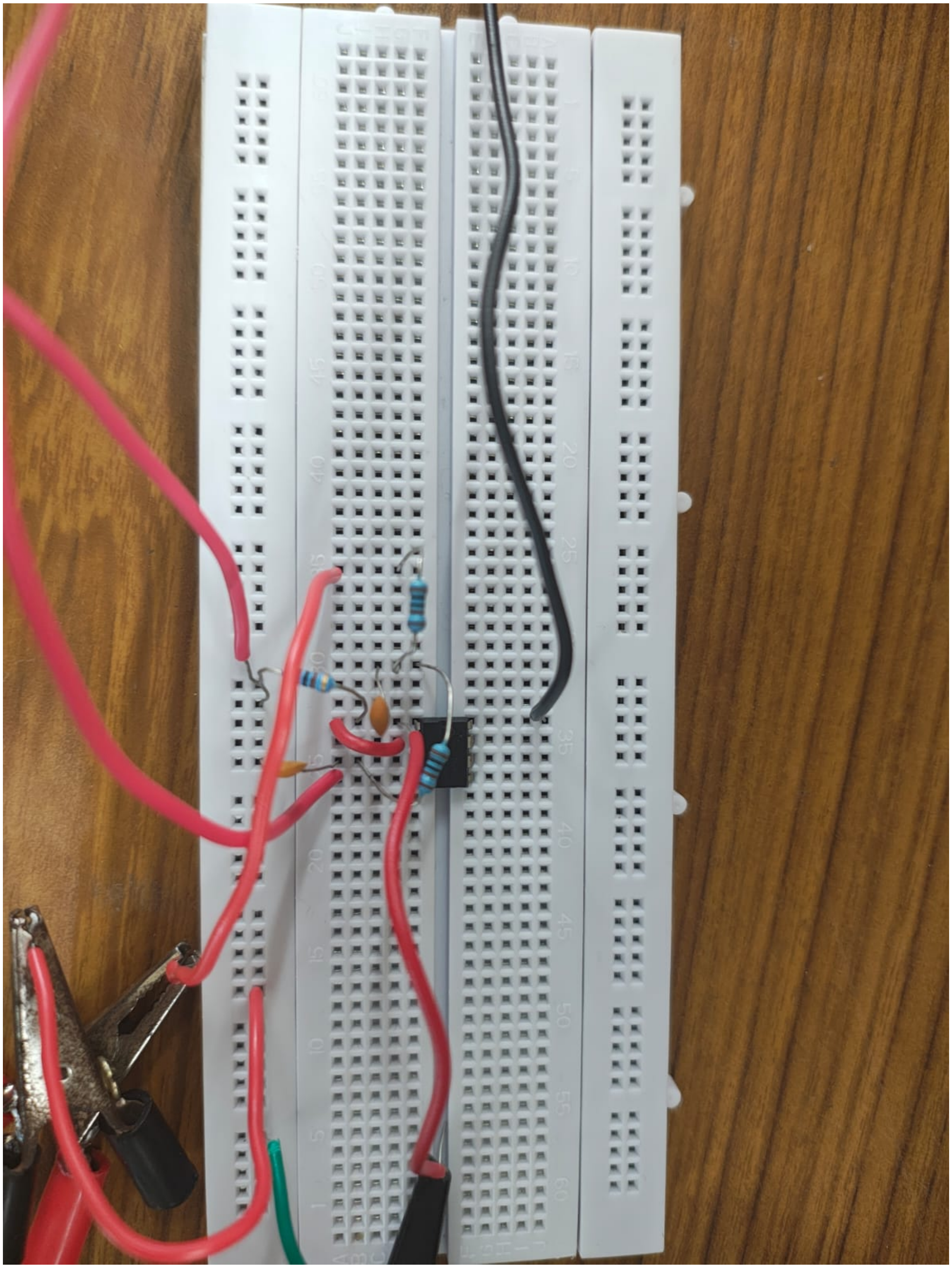
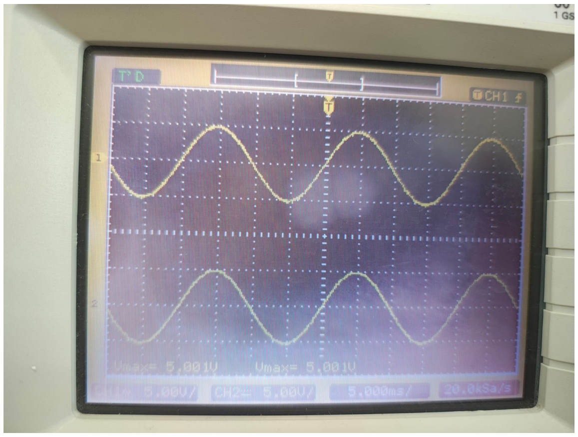
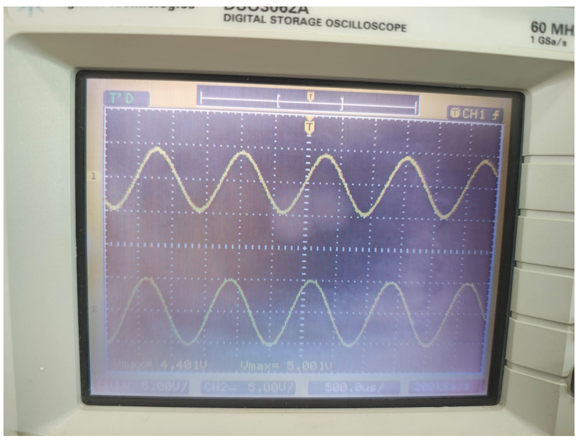
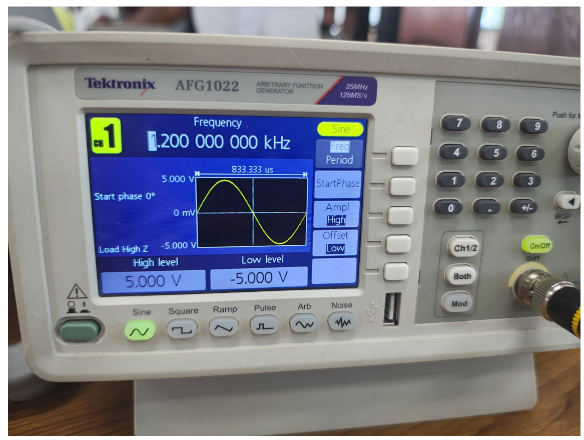


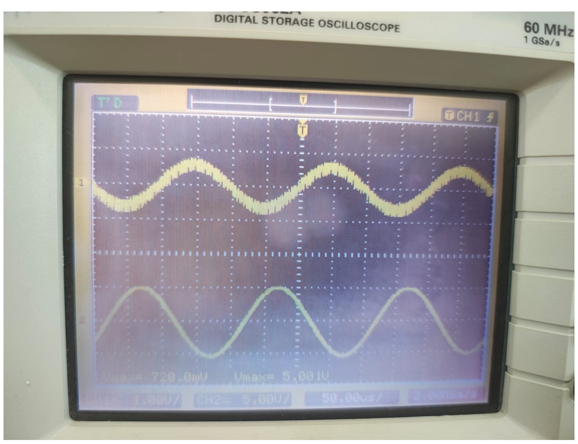
Figure 2: Circuit for Low-Pass Filter

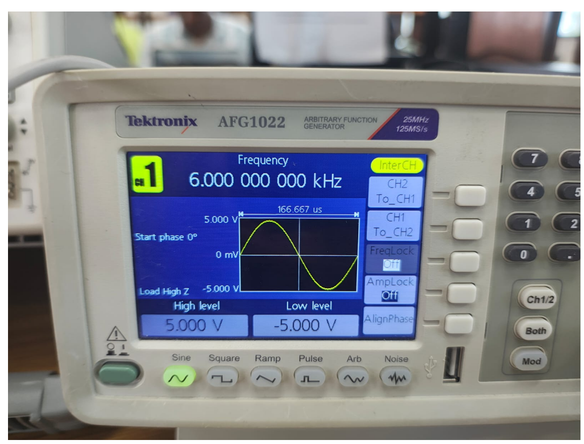

CH1
Vmax= 5.001V
Vmax= 5.001V
CH2= 5.00V/
5.000ms/


Tektronix AFG1022
Frequency
50.000 000 Hz
Sine
Freq
Period
StartPhase
Ampl
High
Offset
Low
20.0000 ms
5.000 V
Start phase 0°
0 mV
Load High Z
-5.000 V
High level
Low level
5.000 V
-5.000 V
Sine Square Ramp Pulse Arb Noise
Ch1/2
Both
Mod


DIGITAL STORAGE OSCILLOSCOPE
1 GSa/s
CH1
Vmax= 4.401V
Vmax= 5.001V
CH2= 5.00V/
500.0us/


Tektronix AFG1022
Frequency
1.200 000 000 kHz
Sine
Freq
Period
StartPhase
Ampl
High
Offset
Low
833.333 us
5.000 V
Start phase 0°
0 mV
Load High Z
-5.000 V
High level
Low level
5.000 V
-5.000 V
Sine Square Ramp Pulse Arb Noise
Ch1/2
Both
Mod


DIGITAL STORAGE OSCILLOSCOPE
60 MHz
1 GSa/s
CH1
Vmax= 720.0mV
Vmax= 5.001V
CH2= 5.00V/
50.00us/


Tektronix AFG1022
Frequency
6.000 000 000 kHz
InterCH
CH2 To_CH1
CH1 To_CH2
FreqLock Off
AmpLock Off
AlignPhase
166.667 us
5.000 V
Start phase 0°
0 mV
Load High Z
-5.000 V
High level
Low level
5.000 V
-5.000 V
Sine Square Ramp Pulse Arb Noise
Ch1/2
Both
Mod

5.2 High-Pass Filter

$$R = 1k\Omega, C = 220nF$$

$\log(\omega)$	$20\log(\lvert H(s)\rvert)$
3.0992	-20.3546
3.2753	-15.4938
3.4971	-9.4680
3.5763	-7.2863
3.6433	-6.0869
3.7982	-3.6593
4.0992	-1.9360
4.4971	-1.1084
4.7982	-1.1084

Table 2: Logarithmic frequency response data for high-pass filter

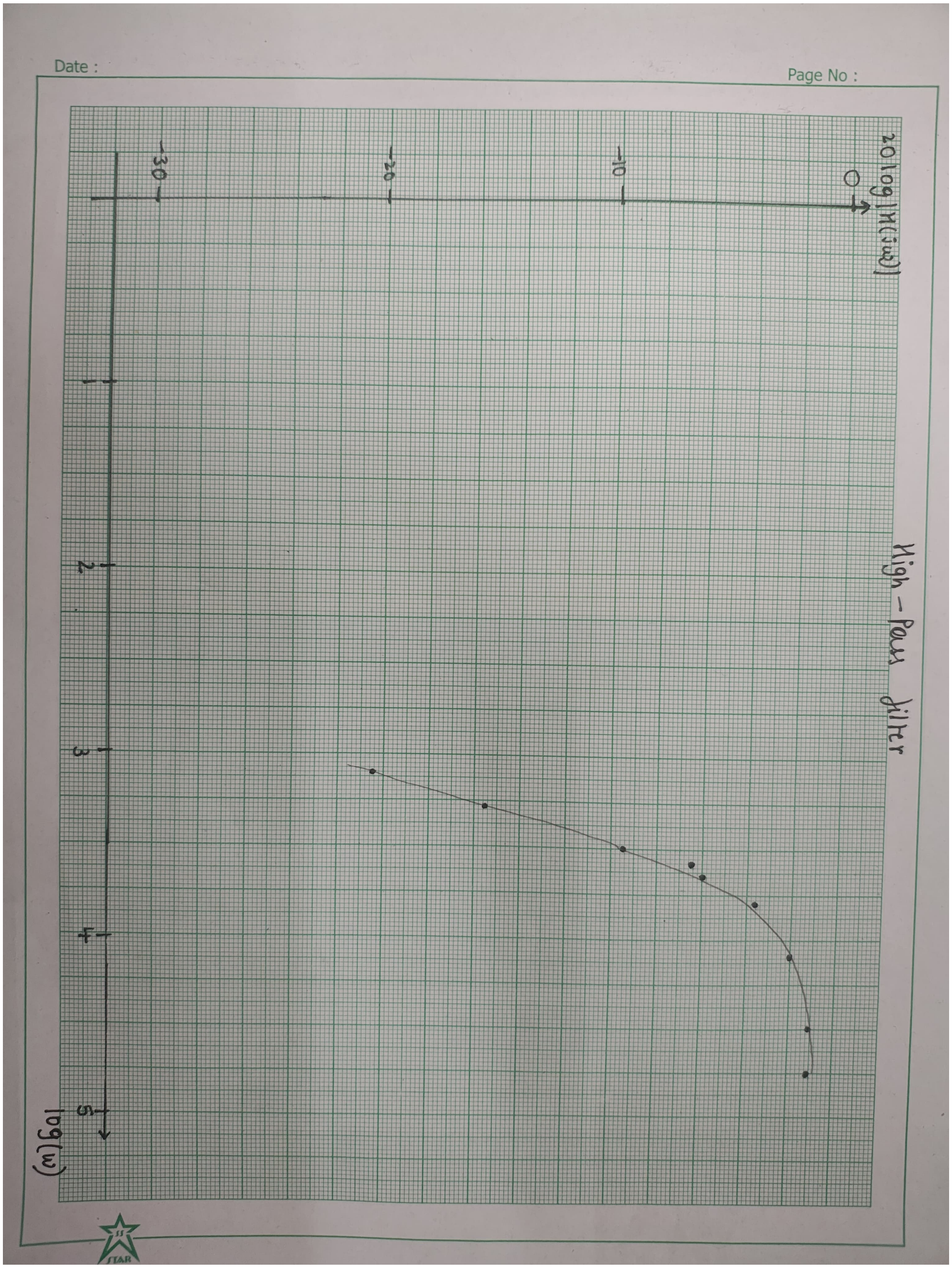


Figure 3: Bode Plot for High-Pass Filter

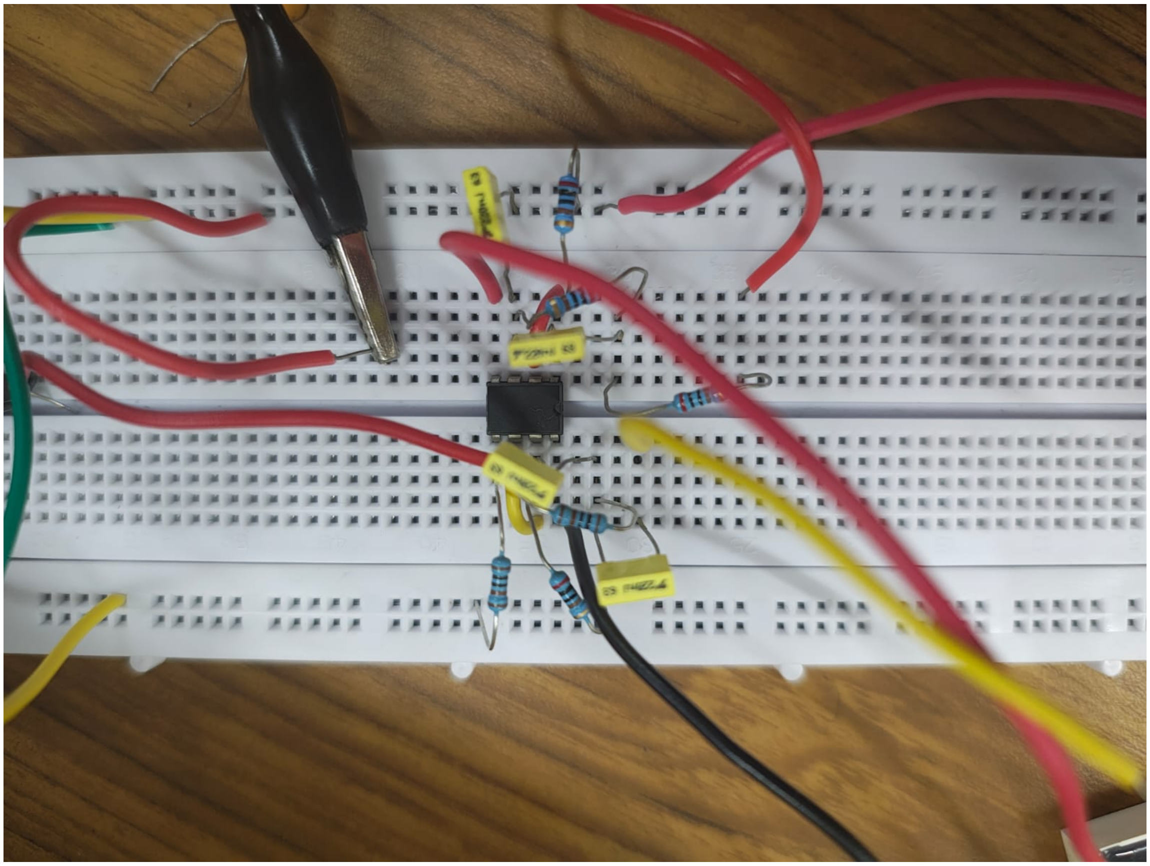
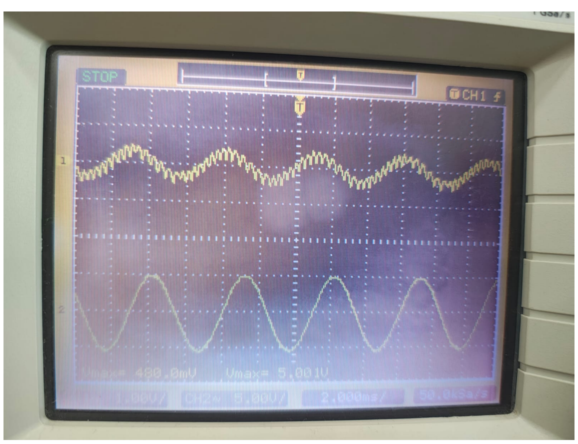
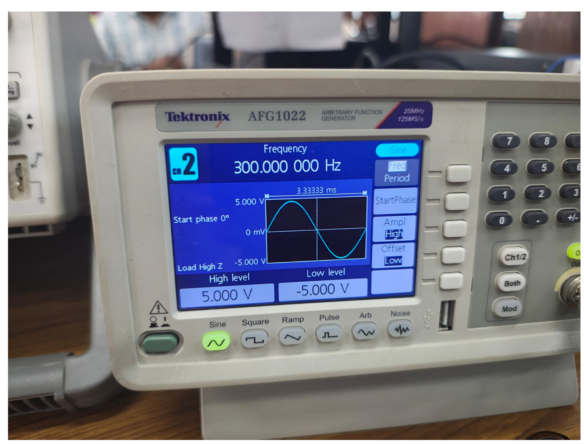
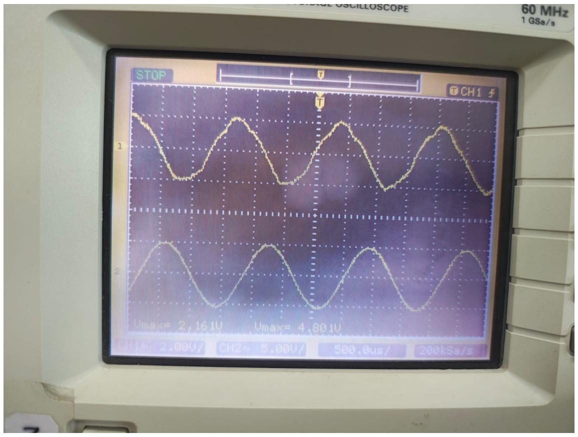
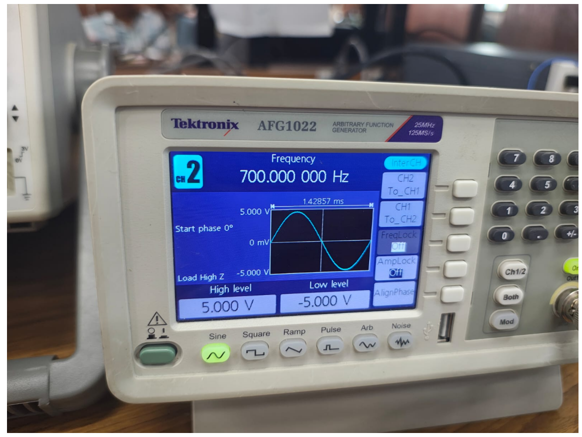


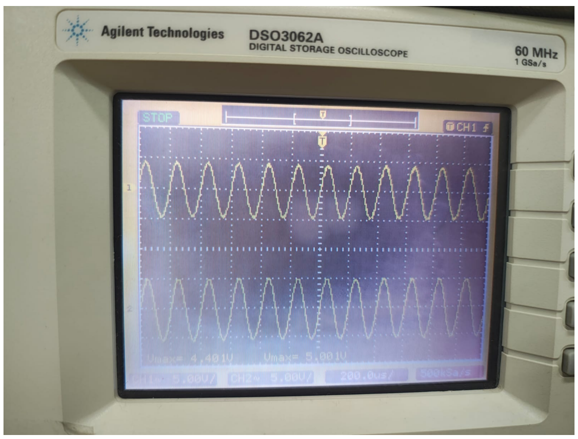
Figure 4: Circuit for High-Pass Filter











5.3 Band-Pass Filter

Let R_1, C_1 be for Low-Pass filter, and R_2, C_2 be for High-pass Filter.

$$R_1 = 1k\Omega, C_1 = 200nF$$
$$R_2 = 4.7k\Omega, C_2 = 200nF$$

$\log(\omega)$	$20\log(\lvert H(s)\rvert)$
2.4971	-16.8328
2.6433	-13.6309
2.7982	-9.6772
2.9743	-5.8112
3.0992	-4.0935
3.1961	-3.0461
3.4002	-2.6601
3.4971	-3.2457
3.5763	-4.0963
3.7982	-7.9545
3.9743	-11.8281
4.0992	-15.9176
4.2753	-19.6593
4.4971	-23.2482

Table 3: Logarithmic frequency response data for band-pass filter

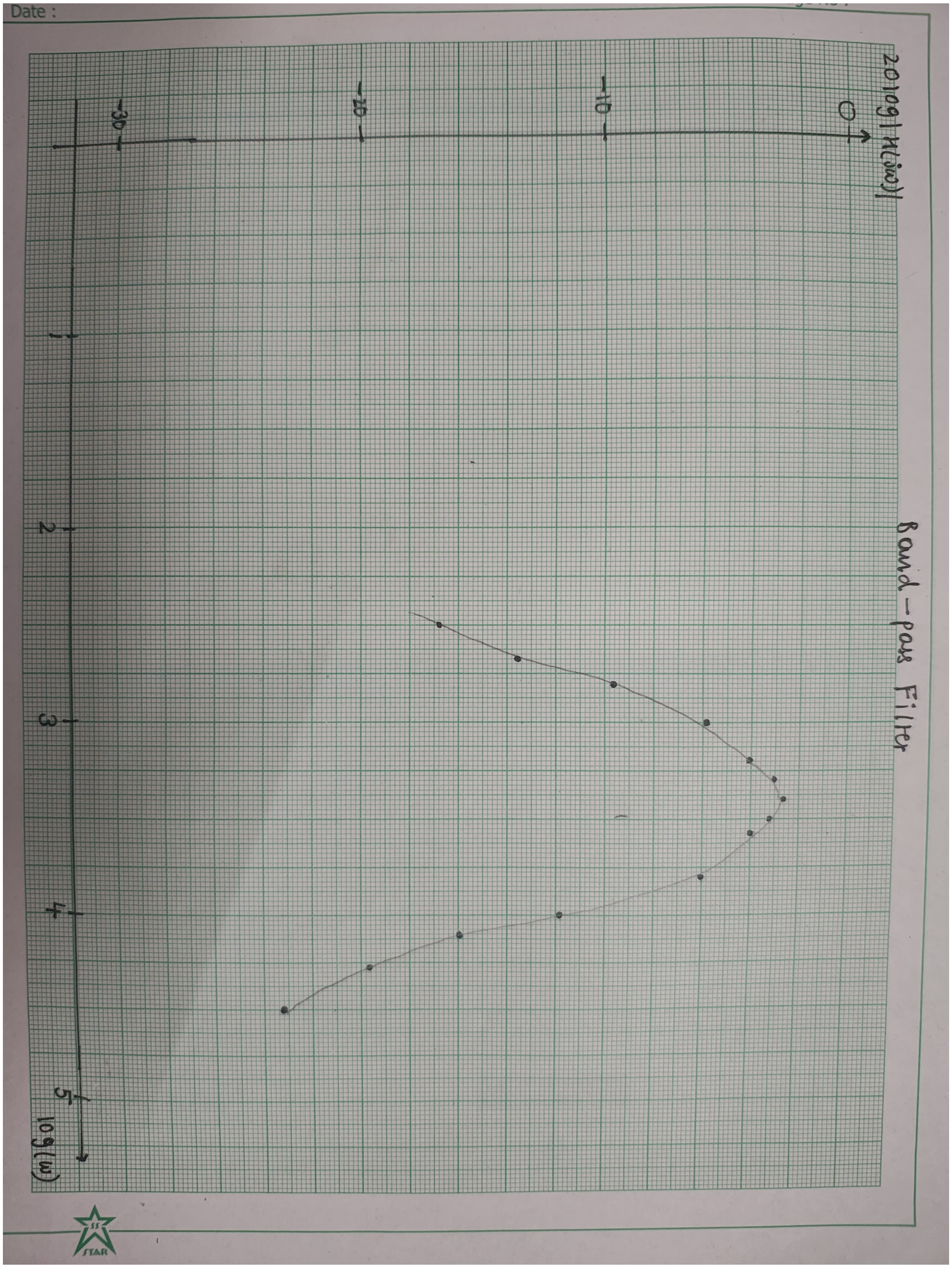


Figure 5: Bode Plot for Band-Pass Filter

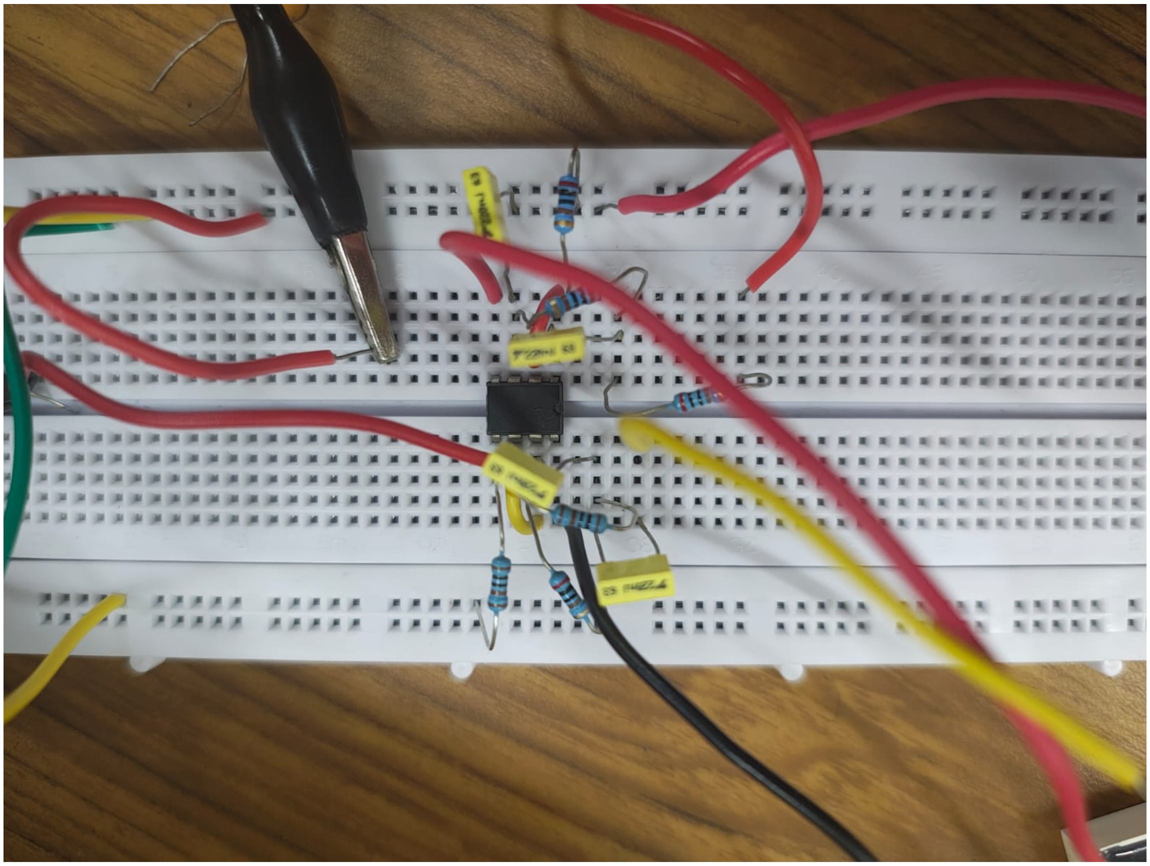
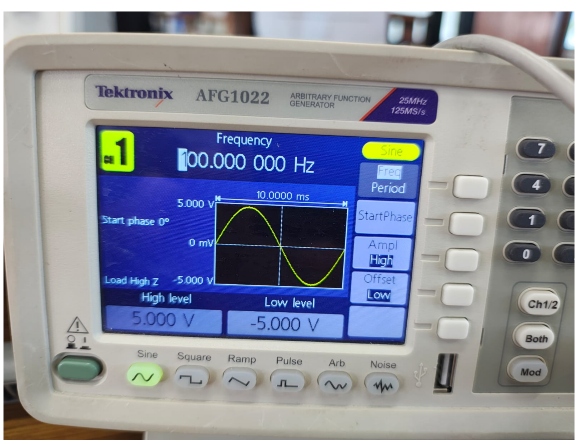
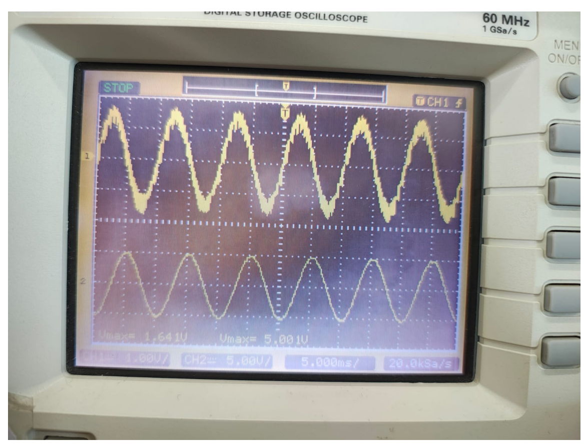
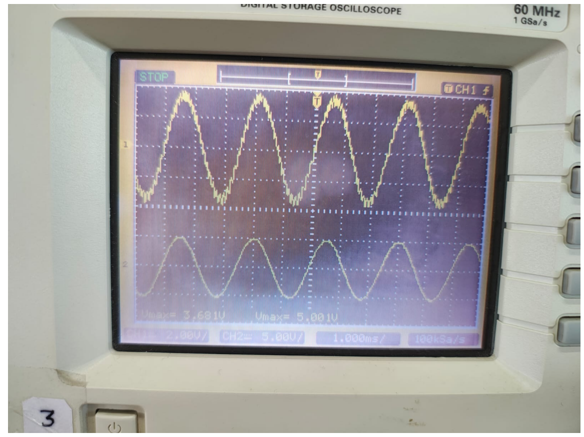
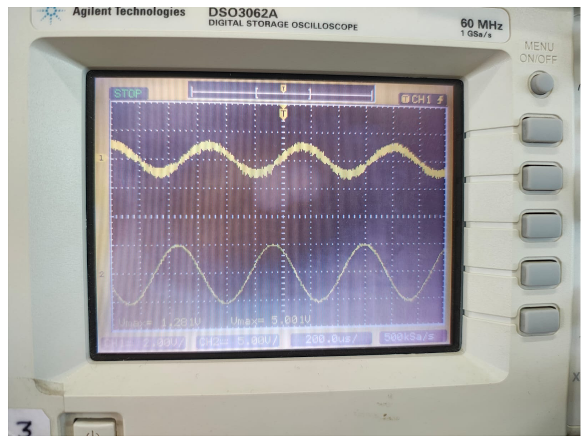


Figure 6: Circuit for Band-Pass Filter









6 Verification Codes and Pictures

All the pictures taken are provided below, `https://github.com/AbhimanyuKoushik/EE1200/tree/main/Lab6/Pictures`

Python verification codes are provided below, `https://github.com/AbhimanyuKoushik/EE1200/tree/main/Lab6`

7 Conclusion

This experiment successfully demonstrated the design and implementation of a band-pass filter using cascaded Sallen-Key second-order filters.

The Sallen-Key topology proved to be an effective and practical solution for implementing active filters with predictable characteristics. The cascading method for creating a bandpass filter from high-pass and low-pass sections was verified to be a viable approach.