9.4.9

EE24BTECH11015 - Dhawal

Question:
Find the solution of the differential equation $\frac{dy}{dx} = \sin^{-1} x$.
Solution: Solving the given D.E. , we get,

$$\frac{dy}{dx} = \sin^{-1} x \tag{1}$$

Integrate both sides with respect to x:

$$y = \int \sin^{-1} x \, dx \tag{2}$$

Using integration by parts:

$$\int u \, dv = uv - \int v \, du \tag{3}$$

Let:

$$u = \sin^{-1} x, \; dv = dx \tag{4}$$

Then:

$$du = \frac{1}{\sqrt{1 - x^2}} \, dx, \; v = x \tag{5}$$

Substituting into the integration by parts formula:

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int x \frac{1}{\sqrt{1 - x^2}} \, dx \tag{6}$$

For the remaining integral, let $u = 1 - x^2$, so $du = -2x \, dx$:

$$\int x \frac{1}{\sqrt{1 - x^2}} \, dx = -\frac{1}{2} \int \frac{1}{\sqrt{u}} \, du \tag{7}$$

$$= -\sqrt{u} + C \tag{8}$$

$$= -\sqrt{1 - x^2} + C \tag{9}$$

Thus, the solution to the differential equation is:

$$y = x \sin^{-1} x + \sqrt{1 - x^2} + C \tag{10}$$

Computational Solution:
Using a classical definition of derivative, we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{11}$$

$$\implies f(x+h) = f(x) + hf'(x) \tag{12}$$

By increasing x n each iteration by h and let $C = 0$, we are getting y by,

$$x_0 = 0 \tag{13}$$

$$y_0 = 1 \tag{14}$$

$$h = 0.01 \tag{15}$$

$$n = 100 \tag{16}$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h\frac{dy}{dx}\bigg|_{(x_n, y_n)} \tag{17}$$

$$y_{n+1} = y_n + h\sin^{-1} x_n \tag{18}$$

Using bilinear transfrom:;

$$\frac{dy}{dx} = \sin^{-1} x \tag{19}$$

Taking Laplase of $\frac{dy}{dx}$ and putting $\sin^{-1} x$ as $F(s)$

$$sY(s) = F(s) \tag{20}$$

$$\frac{Y(s)}{F(s)} = \frac{1}{s} = H(s) \tag{21}$$

Now, convert it into Z-transform

$$Y(z) = \frac{h}{2}\frac{1+z^{-1}}{1-z^{-1}}F(z) \tag{22}$$

$$Y(z)\left(1 - z^{-1}\right) = \frac{h}{2}\left(1 + z^{-1}\right)F(z) \tag{23}$$

Applying inverse Z-transform, we get

$$y_n - y_{n-1} = \frac{h}{2}\left(\sin^{-1} x_n + \sin^{-1} x_{n-1}\right) \tag{24}$$

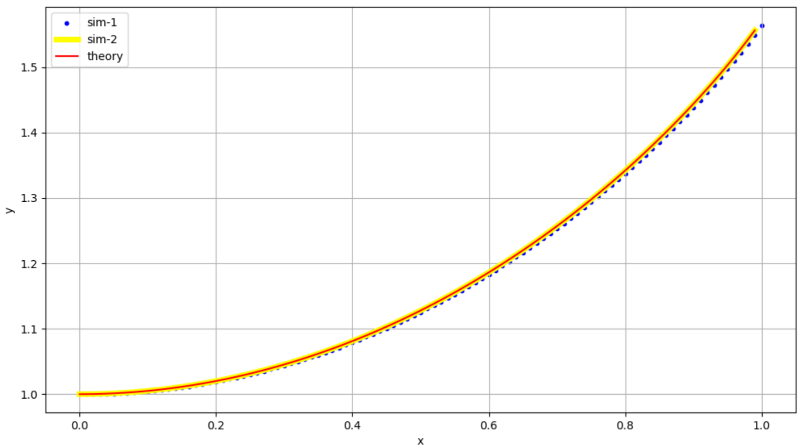

Fig. 0: Plot of the differential equation