NCERT-10.3.3.3.3

EE24BTECH11023 - RASAGNA

Question

Form the pair of linear equations for the following problems and find their solution by substitution method.The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball

Theoritical Solution

Let the cost of each bat be x (in rupees) and the cost of each ball be y (in rupees).
From the problem, we can form the following equations:

$$7x + 6y = 3800 \tag{0.1}$$

$$3x + 5y = 1750 \tag{0.2}$$

Solve one equation for one variable
From equation (0.2)

$$3x + 5y = 1750 \tag{0.3}$$

$$3x = 1750 - 5y \tag{0.4}$$

$$x = \frac{1750 - 5y}{3} \tag{0.5}$$

Substitute x into equation (0.1)

$$7x + 6y = 3800 \tag{0.6}$$

$$7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800 \tag{0.7}$$

$$\frac{7(1750 - 5y)}{3} + 6y = 3800 \tag{0.8}$$

$$\frac{12250 - 35y}{3} + 6y = 3800 \tag{0.9}$$

Simplifying further,

$$12250 - 35y + 18y = 11400 \tag{0.10}$$

$$-17y = -850 \tag{0.11}$$

$$\therefore y = 50 \tag{0.12}$$

Now,Substitute $y = 50$ into equation (0.5);

$$x = \frac{1750 - 5(50)}{3} \tag{0.13}$$

$$x = \frac{1500}{3} \tag{0.14}$$

$$\therefore x = 500 \tag{0.15}$$

Computational Solution

LU Factorization Solution

Given a matrix A of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows: We Start by initializing L as the identity matrix $L = I$ and U as a copy of A. For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \forall\, j \geq k \tag{0.16}$$

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \forall\, i > k \tag{0.17}$$

We are solving the following system of linear equations:

$$7x + 6y = 3800 \tag{0.18}$$

$$3x + 5y = 1750 \tag{0.19}$$

In matrix form:

$$\begin{bmatrix} 7 & 6 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 7 & 6 \\ 3 & 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix}$$

The LU decomposition expresses A as:

$$A = L \cdot U \tag{0.20}$$

Where,

$$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} \tag{0.21}$$

Compute L and U

$$u_{11} = 7, \quad u_{12} = 6 \tag{0.22}$$

$$l_{21} = \frac{A[2,1]}{u_{11}} = \frac{3}{7} \tag{0.23}$$

$$u_{22} = A[2,2] - l_{21} \cdot u_{12} = 5 - \frac{3}{7} \cdot 6 = \frac{17}{7} \tag{0.24}$$

Thus:

$$L = \begin{bmatrix} 1 & 0 \\ \frac{3}{7} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 7 & 6 \\ 0 & \frac{17}{7} \end{bmatrix}$$

Solve $L \cdot \mathbf{y} = \mathbf{b}$

$$\begin{bmatrix} 1 & 0 \\ \frac{3}{7} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix} \tag{0.25}$$

Solving,

$$\implies y_1 = 3800 \tag{0.26}$$

$$\implies \frac{3}{7} \cdot 3800 + y_2 = 1750 \tag{0.27}$$

$$y_2 = 1750 - \frac{3}{7} \cdot 3800 \tag{0.28}$$

$$y_2 = 1750 - \frac{11400}{7} = \frac{850}{7} \tag{0.29}$$

Thus,

$$\mathbf{y} = \begin{bmatrix} 3800 \\ \frac{850}{7} \end{bmatrix} \tag{0.30}$$

Solve $U \cdot \mathbf{x} = \mathbf{y}$

$$\begin{bmatrix} 7 & 6 \\ 0 & \frac{17}{7} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3800 \\ \frac{850}{7} \end{bmatrix} \tag{0.31}$$

Solving,

$$\implies \frac{17}{7} \cdot y = \frac{850}{7} \tag{0.32}$$

$$y = \frac{850}{7} \cdot \frac{7}{17} = 50 \tag{0.33}$$

$$\implies 7x + 6y = 3800 \tag{0.34}$$
$$7x + 6 \cdot 50 = 3800 \tag{0.35}$$
$$7x = 3800 - 300 = 3500 \tag{0.36}$$
$$x = \frac{3500}{7} = 500 \tag{0.37}$$

Final computed Solution
Cost of one bat (x): 500.00
Cost of one ball (y): 50.00