

NCERT - 11.16.1.3

EE1003

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Question: Find the PMF of the binomial random variable for the experiment - A coin is tossed four times. **Solution:**

Let X be the number of heads in four independent tosses of the fair coin.

$$X = X_1 + X_2 + X_3 + X_4 \quad (1)$$

Let X_i be the Bernoulli random variable.

X is the Binomial random variable.

$$X_i = \begin{cases} 1, & \text{outcome is Heads} \\ 0, & \text{outcome is Tails} \end{cases} \quad (2)$$

Compute the moment generating function (MGF) using the \mathcal{Z} -transform:

The \mathcal{Z} -transform of the PMF is given by

$$M_{X_i}(z) = \sum_{n=-\infty}^{\infty} p_{X_i}(n)z^{-n} \quad (3)$$

Since X_i takes only two values (0 or 1):

$$M_{X_i}(z) = (1 - p) + pz^{-1} \quad (4)$$

since X_1, X_2, X_3, X_4 are independent, their total MGF is:

$$M_X(z) = M_{X_1}(z)M_{X_2}(z)M_{X_3}(z)M_{X_4}(z) \quad (5)$$

$$M_X(z) = ((1 - p) + pz^{-1})^4 \quad (6)$$

$$M_X(z) = \sum_{n=-\infty}^{\infty} {}^4C_n(1 - p)^{4-n}p^n z^{-n} \quad (7)$$

$$p_X(n) = {}^4C_n p^n (1 - p)^{4-n} \quad (8)$$

Substituting $p = \frac{1}{2}$

$$p_X(n) = \frac{{}^4C_n}{16} \quad (9)$$

The Probability Mass Function (PMF) for the given random variable is

$$p_X(n) = \begin{cases} \frac{1}{16}, & n = 0 \\ \frac{1}{4}, & n = 1 \\ \frac{3}{8}, & n = 2 \\ \frac{1}{4}, & n = 3 \\ \frac{1}{16}, & n = 4 \end{cases} \quad (10)$$

Plotting:

We generate random numbers between 0 and 1, and classify them as heads if it is less than 0.5 and tails if it is greater than 0.5. We repeat this trial 4 times to get number of heads in an experiment. This is repeated large number of times and number of heads at $k=0,1,2,3,4$ is divided with number of trials to get probability.

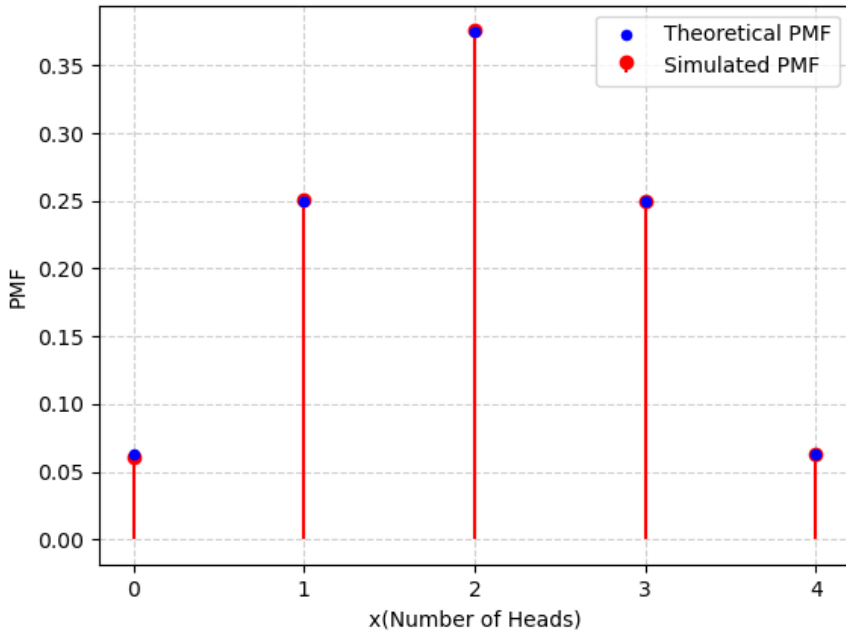


Fig. 0: Plot