6.6.2

EE24BTECH11012 - Bhavanisankar G S

QUESTION :
Show that the function given by $f(x) = \frac{log(x)}{x}$ has maximum at $x = e$.
SOLUTION :

Theoritical solution :
Given function,

$$y(x) = \frac{\log(x)}{x} \tag{0.1}$$
$$\implies y'(x) = \frac{1 - \log(x)}{x^2} \tag{0.2}$$
$$\implies y''(x) = \frac{-2}{x^3} - \frac{1}{x^3} + \frac{2\log(x)}{x^3} \tag{0.3}$$
$$y''(x) = \frac{2\log(x) - 3}{x^3} \tag{0.4}$$

To find the critical points, we do

$$y'(x) = 0 \tag{0.5}$$
$$\frac{1 - \log(x)}{x^2} = 0 \tag{0.6}$$
$$\log(x) = 1 \tag{0.7}$$
$$x = e \tag{0.8}$$

For

$$Localmin \implies y''(x) > 0 \tag{0.9}$$
$$Localmax \implies y''(x) < 0 \tag{0.10}$$
$$Inflectionpoint \implies y''(x) = 0 \tag{0.11}$$

Substituting (0.8) in (0.4), we have

$$y'' = \frac{-1}{e^3} \tag{0.12}$$
$$\implies y'' < 0 \tag{0.13}$$

Hence, (0.8) is a point of maximum.

$$Maxvalue = \frac{1}{e} \tag{0.14}$$

Computational solution :

Finding maximum value of a function can be done using **Gradient Ascent method**

$$x_{n+1} = x_n + \alpha f'(x_n) \tag{0.15}$$

$$x_{n+1} = x_n + \alpha\left(\frac{1 - \log(x_n)}{x_n^2}\right) \tag{0.16}$$

where α is the learning rate. Taking

$$h = 0.001 \tag{0.17}$$

$$\alpha = 0.001 \tag{0.18}$$

we have

$$x_{max} = 2.717779896744937 \tag{0.19}$$

$$y_{max} = 0.36787943489794817 \tag{0.20}$$

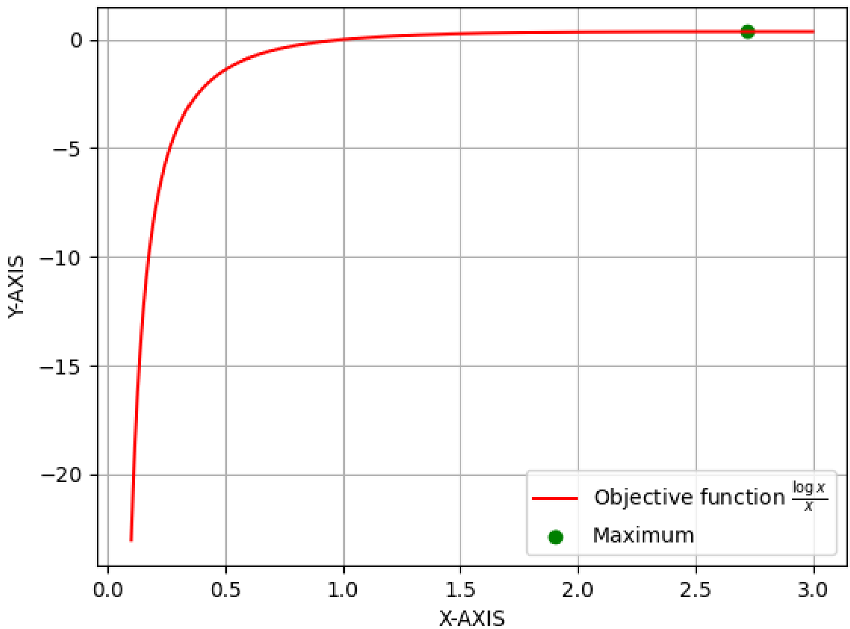

Fig. 0.1: Plot of the given question.