# NCERT - 11.16.2.2.8

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**Question:**
A die is thrown. Consider the following events:
$A$: A number less than 7 is obtained
$B$: A number greater than 7 is obtained
Find $\Pr(AB)$.

**Theoretical Solution:**
The probability mass function (PMF) for throwing a fair six-sided die is:

$$P_X(x) = \begin{cases} \frac{1}{6} & \text{for } x = 1, 2, 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases} \tag{0.1}$$

The cumulative distribution function (CDF) gives the probability of rolling a number less than or equal to some integer $x$.

$$F_X(x) = \Pr(X \le x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{x}{6} & \text{for } x = 1, 2, 3, 4, 5, 6 \\ 1 & \text{for } x > 6 \end{cases} \tag{0.2}$$

Thus, the probability of event $A$ can be calculated as follows:

$$\Pr(A) = F_X(7) - P_X(7) = 1 - 0 = 1 \tag{0.3}$$

Similarly, the probability of event $B$ can also be calculated (using the axiom of Boolean Algebra $E + E' = 1$ for some event $E$ in the sample space):

$$\Pr(B) = \Pr(X > 7) = 1 - \Pr(X \le 7) = 1 - F_X(7) \tag{0.4}$$

$$\implies \Pr B = 1 - 1 = 0 \tag{0.5}$$

$A$ and $B$ can be observed to be mutually exclusive events, as no number $x$ can be lesser than and greater than 7 at the same time. Hence, we can say that:

$$\Pr(AB) = 0 \tag{0.6}$$

**Simulated Solution:**
Let $X_1$ be an indicator random variable of the event $A$. $X_1$ is defined as:

$$X_1 = \begin{cases} 1, & A \\ 0, & A' \end{cases} \tag{0.7}$$

Let $X_2$ be the indicator random variable of the event $B$. $X_2$ is defined as:

$$X_2 = \begin{cases} 1, & B \\ 0, & B' \end{cases} \tag{0.8}$$

Let $X_3$ be the indicator random variable of the event $AB$. $X_3$ is defined as:

$$X_3 = \begin{cases} 1, & AB \\ 0, & (AB)' \end{cases} \tag{0.9}$$

The PMF of the random variable $X_1$ is:

$$P_{X_1}(n) = \begin{cases} p_1, & n = 1 \\ 1 - p_1, & n = 0 \end{cases} \tag{0.10}$$

The PMF of the random variable $X_2$ is:

$$P_{X_2}(n) = \begin{cases} p_2, & n = 1 \\ 1 - p_2, & n = 0 \end{cases} \tag{0.11}$$

The PMF of the random variable $X_3$ is:

$$P_{X_3}(n) = \begin{cases} p_3, & n = 1 \\ 1 - p_3, & n = 0 \end{cases} \tag{0.12}$$

where,

$$p_1 = 1 \tag{0.13}$$
$$p_2 = 0 \tag{0.14}$$
$$p_3 = 0 \tag{0.15}$$
$$\tag{0.16}$$

Through the definitions made earlier:

$$\Pr(A) = p_1 = 1 \tag{0.17}$$
$$\Pr(B) = p_2 = 0 \tag{0.18}$$
$$\Pr(AB) = p_3 = 0 \tag{0.19}$$

**Conclusion:**
The probaility of the event $AB$ is:

$$\Pr(AB) \tag{0.20}$$

**Plots:**

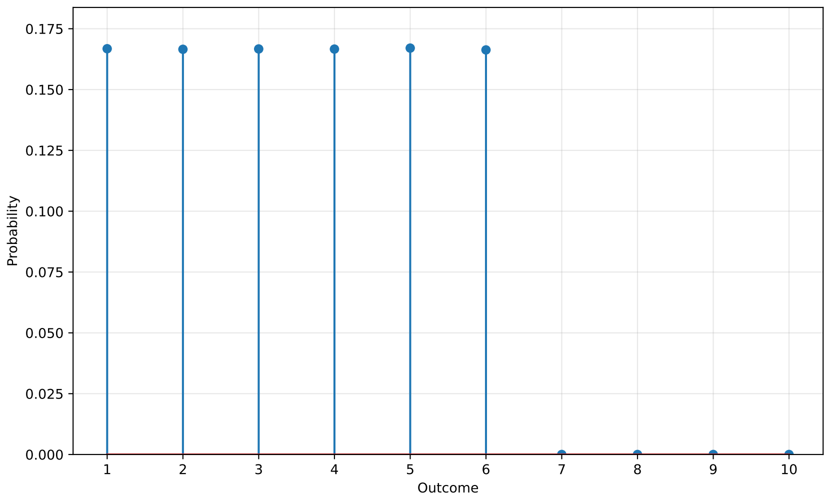


Fig. 0.1: Plot of PMF of die roll

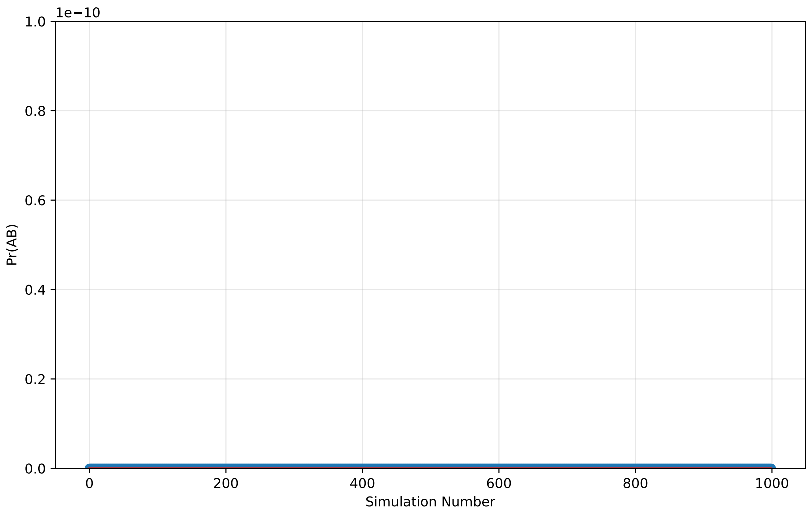


Fig. 0.2: Plot of PMF of $\Pr(AB)$