9.1.12

EE24BTECH11011 - Pranay Kumar

Question:
Solve the following differential equation:

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \tag{0.1}$$

Solution:
By first principle of derivatives,

$$y'(t) = \lim_{h\to 0}\frac{y(t+h) - y(t)}{h} \tag{0.2}$$
$$y(t+h) = y(t) + hy'(t) \tag{0.3}$$

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \tag{0.4}$$

Rewriting the given equation, we get:

$$\frac{d^2y}{dx^2} = \frac{3\frac{dy}{dx} - y}{2x^2} \tag{0.5}$$

To solve this equation numerically, we apply Euler's method. We start by introducing the following substitutions:
Let:

$$y_1 = y, \quad y_2 = \frac{dy}{dx} \tag{0.6}$$

Thus, the system becomes:

$$y_1' = y_2 \tag{0.7}$$

$$y_2' = \frac{3y_2 - y_1}{2x^2} \tag{0.8}$$

The system of equations in matrix form:

$$\mathbf{y}' = \begin{bmatrix} y_2 \\ \frac{3y_2 - y_1}{2x^2} \end{bmatrix} \tag{0.9}$$

Using Euler's method, the update formulas become:

$$y_1(x+h) = y_1(x) + h \cdot y_2(x) \tag{0.10}$$

$$y_2(x+h) = y_2(x) + h \cdot \left(\frac{3y_2 - y_1}{2x^2} \right) \tag{0.11}$$

This can be expressed in matrix form as:

$$\mathbf{y}_{n+1} = \mathbf{y}_n + h \begin{bmatrix} 0 & 1 \\ -\frac{1}{2x^2} & \frac{3}{2x^2} \end{bmatrix} \cdot \mathbf{y}_n \tag{0.12}$$

We will use the initial conditions:

$$y_1(1) = 1, \quad y_2(1) = 0 \tag{0.13}$$

Now, applying Euler's method and iterating, we obtain the numerical solution. The following plot represents the solution based on these initial conditions.

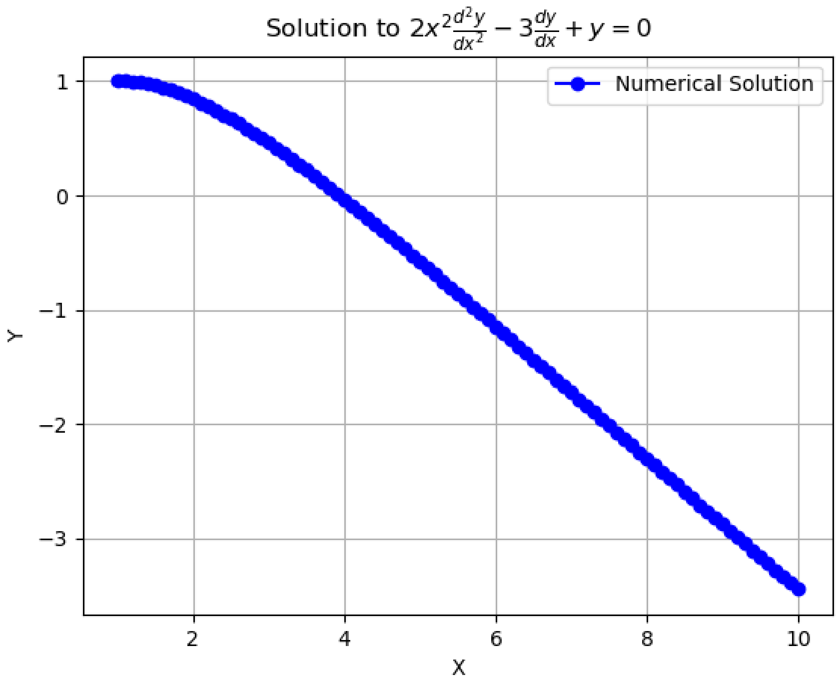

Fig. 0.1: Numerical Solution