# NCERT 12.6.5.15

EE24BTECH11051 - Prajwal

**Question**:Find two positive numbers $x$ and $y$ such that their sum is 35 and the product $x^2 \times y^5$ is a maximum.

**Solution:**

**Therotical logic:**

1) Given

$$x + y = 35, max(x^2 \times y^5) = ? \tag{1}$$

2) Value of $x$ in terms of $y$ from (1)

$$x = 35 - y \tag{2}$$

3) let $F(x, y) = x^2 y^5$

4) Substituting value of $x$ in equation $F(x, y)$ to convert it into single variable

$$F(x, y) = x^2 y^5 \tag{3}$$

$$F(y) = (35 - y)^2 y^5 \tag{4}$$

$$F(y) = (y^2 - 70y + 1225)y^5 \tag{5}$$

$$F(y) = y^7 - 70y^6 + 1225y^5 \tag{6}$$

Differentiate equation (6) with respect to $y$ set it to '0' for critical points

$$\frac{dF}{dy} = 7y^6 - 420y^5 + 6125y^4 \tag{7}$$

$$0 = 7y^4(y^2 - 60y + 875) \tag{8}$$

$$0 = 7y^4(y - 25)(y - 35) \tag{9}$$

5) From equation (9) critical points are $y = 0, y = 25$ and $y = 35$

6) Differentiate equation (7) with respect to $y$

$$\frac{d^2F}{dy^2} = 7(6y^5 - 300y^4 + 3500y^3) \tag{10}$$

7) If $\frac{d^2M}{dy^2} < 0$ at a critical point,that is point of maximum

$$\frac{d^2F}{dy^2}\bigg|_{y=0} = 0 \tag{11}$$

$$\frac{d^2F}{dy^2}\bigg|_{y=25} = -27,343,750 \tag{12}$$

$$\frac{d^2F}{dy^2}\bigg|_{y=35} = 105,043,750 \tag{13}$$

8) From equation (12) $y = 25$ is the point of maximum

9) Value of $x$ and $y$ at which maximum value of the $x^2y^5$ is

$$y = 25 \tag{14}$$

$$x = 35 - y = 10 \tag{15}$$

**Computational Logic:** Using the method of gradient accent

$$y_{n+1} = y_n + h * F'(y_n) \tag{16}$$

$$\tag{17}$$

from equation (7),

$$y_{n+1} = y_n + h(7y_n^6 - 420y_n^5 + 6125y_n^4) \tag{18}$$

where,

$$h = 10^{-10} \tag{19}$$



Graph of f(y) from y=0 to y=35 with Maximum Point

f(y) = y^7 - 70y^6 + 1225y^5
Maximum Point: (y = 25.000, f(y) = 976562498.187)