10.3.1.1

EE24BTECH11001 - Aditya Tripathy

Question:
Aftab tells his daughter, ”Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be.”. Represent this situation algebraically and graphically.

Solution:
Let Aftab's present age be denoted by x and his daughter's present age be denoted as y. Now the problem can be represented algebraically as follows :

$$(x-7) = 7(y-7) \tag{0.1}$$

$$(x+3) = 3(y+3) \tag{0.2}$$

Simplifying and using matrix notation,

$$x - 7y = -42 \tag{0.3}$$

$$x - 3y = 6 \tag{0.4}$$

$$\begin{pmatrix} 1 & -7 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -42 \\ 6 \end{pmatrix} \tag{0.5}$$

Any non-sigular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{0.6}$$

The upper triangular matrix U is found by row reducing A,

$$\begin{pmatrix} 1 & -7 \\ 1 & -3 \end{pmatrix} \xrightarrow{R_2 -> R_2 - R_1} \begin{pmatrix} 1 & -7 \\ 0 & 4 \end{pmatrix} \tag{0.7}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{0.8}$$

l_{21} is the multiplier used to zero a_{21}, so $l_{21} = 1$.

Now,

$$A = \begin{pmatrix} 1 & -7 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -7 \\ 0 & 4 \end{pmatrix} \tag{0.9}$$

Now we can get the solution to our problem by the two step process,

$$L\mathbf{y} = \mathbf{b} \tag{0.10}$$

$$U\mathbf{x} = \mathbf{y} \tag{0.11}$$

Using forward substitution to solve the first equation,

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -42 \\ 6 \end{pmatrix} \tag{0.12}$$

$$\rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -42 \\ 48 \end{pmatrix} \tag{0.13}$$

Now using back-substitution for the second equation,

$$\begin{pmatrix} 1 & -7 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -42 \\ 48 \end{pmatrix} \tag{0.14}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 42 \\ 12 \end{pmatrix} \tag{0.15}$$

Therefore Aftab's age is 42 years and 12 years is the age of his daughter.

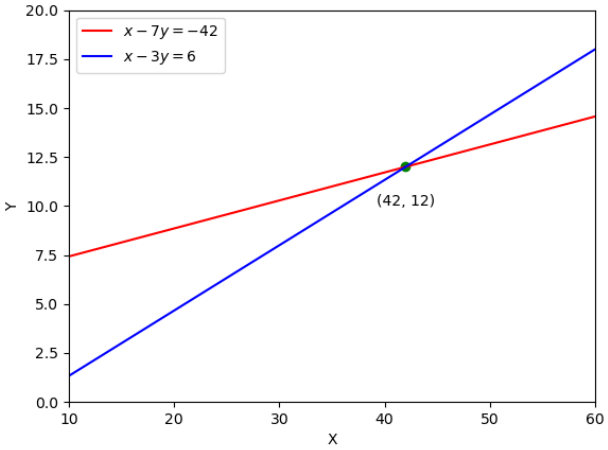

Fig. 0.1: Solution to set of linear equations