10.3.3.1.4

EE24BTECH110021 - Eshan Ray

Question: Solve the following system of equations,

$$0.2x + 0.3y = 1.3 \tag{1}$$
$$0.4x + 0.5y = 2.3 \tag{2}$$

Solution:
The following system of equations can be written in matrix form:-

$$Ax = b \tag{3}$$

where,

$$A = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} \tag{4}$$
$$b = \begin{pmatrix} 1.3 \\ 2.3 \end{pmatrix} \tag{5}$$
$$x = \begin{pmatrix} x \\ y \end{pmatrix} \tag{6}$$
$$\begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.3 \\ 2.3 \end{pmatrix} \tag{7}$$

Using row reduction on augmented matrix $[A|b]$ we get,

$$\begin{pmatrix} 0.2 & 0.3 & |1.3 \\ 0.4 & 0.5 & |2.3 \end{pmatrix} \tag{8}$$

$R_2 \to R_2 - 2R_1$

$$\begin{pmatrix} 0.2 & 0.3 & |1.3 \\ 0 & -0.1 & |-0.3 \end{pmatrix} \tag{9}$$

$R_1 \to R_1 + 3R_2$

$$\begin{pmatrix} 0.2 & 0 & |0.4 \\ 0 & -0.1 & |-0.3 \end{pmatrix} \tag{10}$$

$R_1 \to \frac{R_1}{0.2}, R_2 \to \frac{R_2}{-0.1}$

$$\begin{pmatrix} 1 & 0 & |2 \\ 0 & 1 & |3 \end{pmatrix} \tag{11}$$

So, the point of intersection is $(2, 3)$
Computational Solution: **LU Decomposition**
Solving the system of equations matrix form $Ax = b$ by LU Decomposition where A can

be represented as a product of a lower triangular matrix L and an upper triangular matrix U

$$A\mathbf{x} = LU\mathbf{x} = \mathbf{b} \tag{12}$$

Applying row reduction on A, we get,

$$\begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \tag{13}$$

Let

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \tag{14}$$

l_{21} is the multiplier used to zero a_{21}, so $l_{21} = 2$.
Now,

$$A = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \tag{15}$$

We have obtained LU Decomposition of the matrix A.
The LU Decomposition of matrix A can also be obtained by Doolittle's Algorithm. This gives us update equations to construct the L and U matrix.
Elements of U matrix:
For each column j,

$$U_{ij} = \begin{cases} A_{ij} & i = 0 \\ A_{ij} - \sum_{k=0}^{i-1} L_{ik} U_{kj} & i > 0 \end{cases} \tag{16}$$

Elements of L matrix:
For each row i,

$$L_{ij} = \begin{cases} \frac{A_{ij}}{U_{ij}} & j = 0 \\ \frac{A_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{U_{ij}} & j > 0 \end{cases} \tag{17}$$

The above process decomposes any non-singular matrix A into an upper-triangular matrix U and a lower-triangular matrix L. Now, let

$$U\mathbf{x} = \mathbf{y} \tag{18}$$

$$L\mathbf{y} = \mathbf{b} \tag{19}$$

Substituting L in equation (19),

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1.3 \\ 2.3 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} \tag{21}$$

Back substituting into equation (18),

$$\begin{pmatrix} 0.2 & 0.3 \\ 0 & -0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} \tag{22}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{23}$$

Thus, the point of intersection is $(2, 3)$

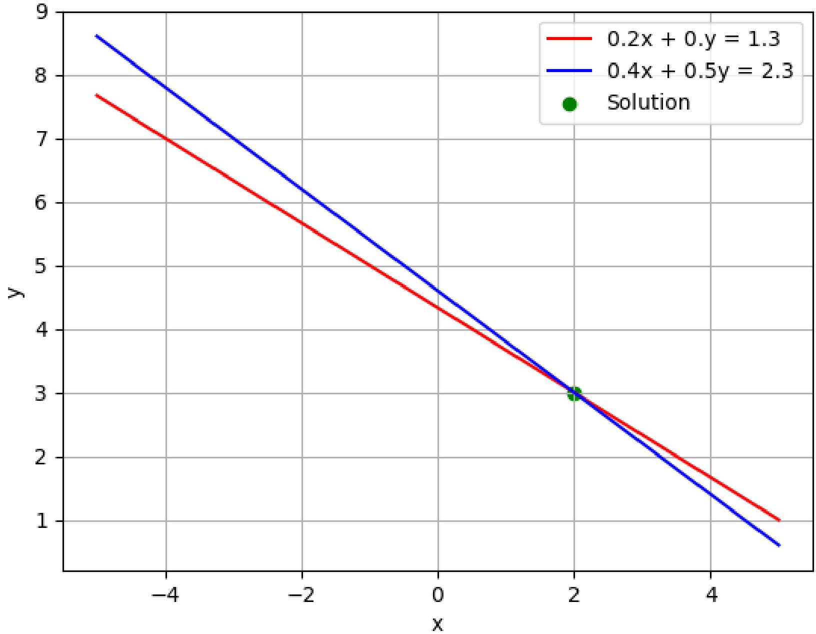

Fig. 1: Solving the system of equations, $0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3$