9.4.22

EE24BTECH11051 - Prajwal

Question:
In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present

variable	description
N	Number of bacteria at any time t
t	time in hours
C	primary arbitrary constant
C_1	secondary arbitrary constant
k	proportionality constant
N_0	initial Number of bacteria

TABLE 0
Variables Used

Let N be the number of bacteria at any time t. According to the given problem, Rate of change in number of bacteria can be given as

$$\frac{dN}{dt} \propto N \tag{0.1}$$

$$\frac{dN}{dt} = k \times N \tag{0.2}$$

Seperating the variables in the equation (0.2), We get

$$\frac{dN}{N} = k \times dt \tag{0.3}$$

On integrating both sides

$$\int \frac{dN}{N} = k \int dt \tag{0.4}$$

$$logN = k \times t + C \tag{0.5}$$

$$N = e^{kt+C} \tag{0.6}$$

$$N = e^{kt}.e^{C} \tag{0.7}$$

$$N = e^{kt}.C_1 \tag{0.8}$$

Given, at time t=0,N_0=1,00,000 then, from (0.8)

$$N_0 = C_1 = 1,00,000 \tag{0.9}$$

number of bacteria can be given as

$$N = N_0 e^{kt} \tag{0.10}$$

At time t=2,number of bacteria is increased by 10% of 1,00,000 from (0.8)

$$1,10,000 = 1,00,000 \times e^{kt} \tag{0.11}$$

$$1,10,000 = 1,00,000 \times e^{k \times 2} \tag{0.12}$$

$$1.1 = e^{k \times 2} \tag{0.13}$$

$$k = \frac{log(1.1)}{2} \tag{0.14}$$

$$\tag{0.15}$$

number of bacteria can be given as

$$N = N_0 e^{kt} \tag{0.16}$$

rearranging the variables,

$$\frac{1}{k} log \frac{N}{N_0} = t \tag{0.17}$$

for N=2,00,000 time is,

$$t = 14.55 \textbf{ hours} \tag{0.18}$$

Logic used for programming:-
Method of finite differences: This method is used to find the approximate solution of the given differential equation by using the values of the function at discrete points.
From the defination of derivative of a function

$$\frac{dy}{dx} \approx \frac{yx + h - yx}{h} \tag{0.19}$$

by rearranging the terms, we get the function

$$yx + h = yx + h \times \frac{dy}{dx} \tag{0.20}$$

$$Nt + h = Nt + h \times N \times k \tag{0.21}$$

Let t$_0$, $N_0 bepointsonthecurve, Ongeneralisingtheaboveequations, Wherehisaverysmalldivision(ex$
$1,00,000$ and $t = 0$, till $t_n = 15$. Then we get the number of bacteria after 15 hours. If we plot all the points (t, N), we get the function N varying with t, i.e N vs T graph.

Finding the solution of this equation using the Z-Transform: By using the z-transform method we can convert the differential equation into a linear equation in Z-domain,after finding the solution in z-domin,inverse of it is the solution of the given differential

equation.
The differential equation for this question is,

$$\frac{dN}{dt} = N \times k \tag{0.22}$$

from (**??**),

$$N_{n+1} = N_n + h \times N \times k \tag{0.23}$$
$$N_{n+1} = N_n(1 + h \times k) \tag{0.24}$$

Applying Z-tranform on both sides,We get,

$$ZN_{n+1} = ZN_n(1 + h \times k) \tag{0.25}$$
$$Z(N_n + 1) = (1 + h \times k)Z(N_n) \tag{0.26}$$

Let,

$$ZN_n = Nz \tag{0.27}$$

Then,

$$ZN_{n+1} = zN(z) - zN_0 \tag{0.28}$$

Now,

$$zN(z) - zN_0 = N(z)(1 + h \times k) \tag{0.29}$$
$$N(z)z - 1 + h \times k = zN_0 \tag{0.30}$$
$$\tag{0.31}$$

$$N(z) = N_0 \frac{z}{z - 1 + h \times k} \tag{0.32}$$

By inversing, we get

$$N_n = N_0 \times 1 + h \times k^n \tag{0.33}$$

We know that,

$$1 + h \times k \approx e^{h \times k} \tag{0.34}$$

then,

$$N_n = N_0 e^{h \times k^n} \tag{0.35}$$
$$N_n = N_o e^{n \times k \times h} \tag{0.36}$$

As h is the small division of time and n are the total no.of divisions, nh turns to be t at that point,Then

$$Nt = N_0 e^{kt} \tag{0.37}$$

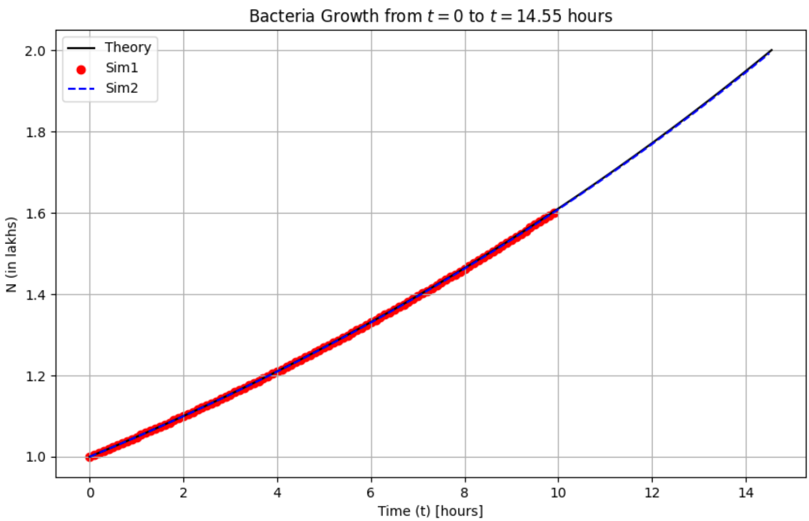
Bacteria Growth from $t = 0$ to $t = 14.55$ hours
Theory
Sim1
Sim2
N (in lakhs)
Time (t) [hours]
2.0
1.8
1.6
1.4
1.2
1.0
0
2
4
6
8
10
12
14

Fig. 0.1. Plot