

8.1.1

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant.

Solution:

Theoretical Solution:

Finding Area

$$A = \int_a^b y_2(x) - y_1(x) dx \quad (1)$$

$$y_2(x) = \sqrt{x} \quad (2)$$

$$y_1(x) = 0 \quad (3)$$

$$b = 4 \quad (4)$$

$$a = 1 \quad (5)$$

$$A = \int_1^4 \sqrt{x} dx \quad (6)$$

$$A = \frac{2}{3} \left[x^{\frac{3}{2}} \right]_1^4 \quad (7)$$

$$A = \frac{2}{3} [8 - 1] \quad (8)$$

$$\therefore A = 4.666666666666667 \quad (9)$$

Computational Solution:

Using the trapezoidal rule to get the area

The trapezoidal rule is as follows.

$$A = \int_a^b f(x) dx \approx h \left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (10)$$

$$h = \frac{b - a}{n} \quad (11)$$

$$A = j_n, \text{ where, } j_{i+1} = j_i + h \frac{f(x_{i+1}) + f(x_i)}{2} \quad (12)$$

$$\rightarrow j_{i+1} = j_i + h \left(\sqrt{x_{i+1}} + \sqrt{x_i} \right) \quad (13)$$

$$x_{i+1} = x_i + h \quad (14)$$

$$h = 0.00001 \quad (15)$$

$$n = 300000 \quad (16)$$

Using the code answer obtained is $A = 4.666666666667341$ sq. units

