12-9-6-2

EE24BTECH11060

Question:
Find the general equation of

$$\frac{dy}{dx} + 3y = e^{-2x} \tag{0.1}$$

Theoretical solution by laplace transform:
Laplace transform of the derivative $\frac{dy}{dx}$ is given by

$$\mathcal{L}\left\{\frac{dy}{dx}\right\} = sY(s) - y(0) \tag{0.2}$$

where Y(s) is the laplace transform of y(x),and y(0) is the initial condition. taking laplace transform on both sides of the equation:

$$\mathcal{L}\left\{\frac{dy}{dx} + 3y\right\} = \mathcal{L}\left\{e^{-2x}\right\} \tag{0.3}$$

$$\implies \mathcal{L}\left\{\frac{dy}{dx}\right\} + \mathcal{L}\{y\} = \mathcal{L}\left\{e^{-2x}\right\} \tag{0.4}$$

$$\mathcal{L}\left\{e^{-2x}\right\} = \frac{1}{s+2} \tag{0.5}$$

$$\implies sY(s) - y(0) + 3Y(s) = \frac{1}{s+2} \tag{0.6}$$

rearranging the equation

$$Y(s) = \frac{1}{(s+2)(s+3)} + \frac{y(0)}{s+3} \tag{0.7}$$

$$Y(s) = \frac{1}{(s+2)} - \frac{1}{(s+3)} + \frac{y(0)}{s+3} \tag{0.8}$$

Taking the inverse laplace transform of each term:

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} \tag{0.9}$$

thus,the solution by laplace transform is

$$y(x) = e^{-2x} - e^{-3x} + y(0)e^{-3x} \tag{0.10}$$

$$\implies y(x) = e^{-2x} + (y(0) - 1)e^{-3x} \tag{0.11}$$

Method of finite differences The derivative of f(x) can be written as

$$\frac{df}{dx} = \frac{f(x+h) - f(x)}{h} \tag{0.12}$$

$$\implies f(x+h) = f(x) + h \cdot \frac{df}{dx} \tag{0.13}$$

from the above question

$$\frac{dy}{dx} + 3y = e^{-2x} \tag{0.14}$$

$$\implies \frac{dy}{dx} = e^{-2x} - 3x \tag{0.15}$$

$$\implies y(x+h) = y(x) + h\left(e^{-2x} - 3x\right) \tag{0.16}$$

for $x \in [x_0, x_n]$ divide into equal parts by difference h
Let us assume that $x_0 = 0$,$y_0 = 1$
Let x_1=$x_0 + h$ then

$$y_1 = y_0 + h\left(e^{-2x_0} - 3x_0\right) \tag{0.17}$$

To obtain the graph repeat the process until sufficient points to pllot the graph and the general equation will be

$$x_{n+1} = x_n + h \tag{0.18}$$

$$y_{n+1} = y_n + h\left(e^{-2x_n} - 3x_n\right) \tag{0.19}$$

The curve generalised using the method of finite differences for the given question taking $x_0 = 0$,$y_0 = 1$, $h = 0.01$ and running iterations for 100 times