NCERT-9.5.13

EE24BTECH11065 - Spoorthi yellamanchali

Question:
The difference between two numbers is 26 and one number is three times the other. Find them.
Theoretical Solution:
Let the two numbers be x and y respectively, Then, by the question, we get the equation,

$$x - y = 26 \tag{0.1}$$
$$x = 3y \tag{0.2}$$

On substituting equation (0.2) in equation (0.1), we get,

$$3y - y = 26 \tag{0.3}$$
$$y = 13 \tag{0.4}$$

Then,

$$x = 3(13) = 39. \tag{0.5}$$

$\therefore$ we get, $x = 13$ and $y = 39$
Solution by LU decomposition.
Given a matrix A of size $n \times n$,LU decomposition is performed row by row ,column by column. We start ny initializing L as identity matrix of same order $L = I_n$,and U as A. The update equations are as follows,
For each column $j \geq k$, the entries of U in the kth row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m}.U_{m,j}, \forall j \geq k \tag{0.6}$$

For each row $i > k$, the entries of L in the kth column are updated as:

$$L_{j,k} = \frac{1}{U_{k,k}} \left(A_{j,k} - \sum_{m=1}^{k-1} .U_{m,k} \right), \forall i > k \tag{0.7}$$

given equations:

$$x - y = 26 \tag{0.8}$$
$$x - 3y = 0 \tag{0.9}$$

we can represent these set of equations as

$$A\bar{x} = b \tag{0.10}$$

where,

$$A = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix} \tag{0.11}$$

$$\bar{x} = \begin{bmatrix} x \\ y \end{bmatrix} \tag{0.12}$$

$$b = \begin{bmatrix} 26 \\ 0 \end{bmatrix} \tag{0.13}$$

Using guassian elimination algorithm,we can decompose matix A into product of lower traingular matrix (L) and upper triangular matrix (U).

$$A = LU. \tag{0.14}$$

let us first initialize

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{0.15}$$

and

$$U = A \tag{0.16}$$

Then on applying guassian elimination algorithm,
On elminating the element (making it zero) of position (2,1) by row operations, we get,

$$R_2 = R_2 - 1.R_1. \tag{0.17}$$

updated U becomes,

$$\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} \tag{0.18}$$

here , '1' is the multiplier we used. So , on updating the position (2,1) in the matrix L with the multiplier, we get the required matrices L and U respectively.

$$\therefore L = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{0.19}$$

Now,equation (0.14) can be written as,

$$LU\bar{x} = b \tag{0.20}$$

Let,

$$U\bar{x} = y \tag{0.21}$$

Then,

$$Ly = b \tag{0.22}$$

On solving using forward substitution, we get ,

$$y = \begin{bmatrix} 26 \\ -26 \end{bmatrix} \tag{0.23}$$

Now, from equation (0.21), on solving for $\bar{x}$ using backward substitution, we get,

$$\bar{x} = \begin{bmatrix} 39 \\ 13 \end{bmatrix} \tag{0.24}$$

$\therefore$ we get,

$$x = 39 \tag{0.25}$$

$$y = 13 \tag{0.26}$$