NCERT-12.6.5.3.8

EE24BTECH11023 - RASAGNA

Question: Find the local maxima of the function $f(x) = x\sqrt{1-x}, 0 < x < 1$.Also find the local maximum and the local minimum values,as the case may be.
Theoritical Solution: The first derivative $g'(x)$ gives the critical points:

$$g'(x) = \frac{2-3x}{2\sqrt{1-x}} = 0 \tag{0.1}$$

$$\implies x = \frac{2}{3}. \tag{0.2}$$

Critical point is $x = \frac{2}{3}$. The second derivative $g''(x)$ helps to determine the nature of the critical points:

$$g''(x) = \frac{-1}{4(1-x)^{3/2}}(4-3x) \tag{0.3}$$

$$g''\left(\frac{2}{3}\right) = \frac{-3\sqrt{3}}{2} < 0. \tag{0.4}$$

At $x = \frac{2}{3}$, $g(x)$ has a maximum. This indicates a local maximum at $x = \frac{2}{3}$.
Calculating Local Maximum, At $x = \frac{2}{3}$:

$$g\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1-\frac{2}{3}} \tag{0.5}$$

$$= \frac{2}{3\sqrt{3}}. \tag{0.6}$$

Thus, the local maximum value is ,$\frac{2}{3\sqrt{3}}$. **Computational Solution**
We adapt the gradient descent approach to find the numerical solution of the function.

$$\lambda_{n+1} = \lambda_n - \mu f'(\lambda_n) \tag{0.7}$$

$$f(x) = x\sqrt{1-x}$$

$$f'(x) = \frac{2-3x_n}{2\sqrt{1-x}}$$

. Therefore, equation (0.7) becomes:

$$\lambda_{n+1} = \lambda_n - \frac{2-3\lambda_n}{2\sqrt{1-\lambda_n}}\mu \tag{0.8}$$

The equation (0.8) is nonlinear.
To linearize f'(x) around x=c,

Now, $f'(x) \approx 2a(x-c)+b$, where: $a = \frac{f''(c)}{2}$ $b = f'(c)$ Using Taylor expansion around $x = c$:

$$f'(x) \approx f'(c) + f''(c)(n-c) \tag{0.9}$$

We can linearize $f'(n)$ around $x = 0.5$, a convenient point for linearization. At $c = 0.5$, we calculate:

$$f'(0.5) = \frac{1}{2\sqrt{2}}, \quad f''(0.5) = -\frac{5}{2\sqrt{2}} \tag{0.10}$$

So, $a = -\frac{5}{4\sqrt{2}}$ and $b = \frac{1}{2\sqrt{2}}$. Substituting these into equation (0.7):

$$\lambda_{n+1} = \lambda_n - \mu\,(2a\lambda_n + b) \tag{0.11}$$

$$\lambda_{n+1} = \lambda_n\,(1-2a\mu) - \mu b \tag{0.12}$$

Applying the Z-transform:

$$z(\lambda(z)) - 2\lambda_0 = (1-2a\mu)\lambda(z) - \frac{\mu bz}{z-1} \tag{0.13}$$

Finally:

$$\lambda(z) = \sum_{n=0}^{\infty}\left[\left(\lambda_0 - \frac{\mu b}{(1-(1-2a\mu)}\right)(1-2a\mu)^{\eta} + \frac{\mu b}{1-(1-2a\mu)}\right] \tag{0.14}$$

For $n \to \infty$ $(1-2a\mu$ vanishes if $|1-2a\mu| < 1$,ensuring convergence. $\therefore$ ROC is

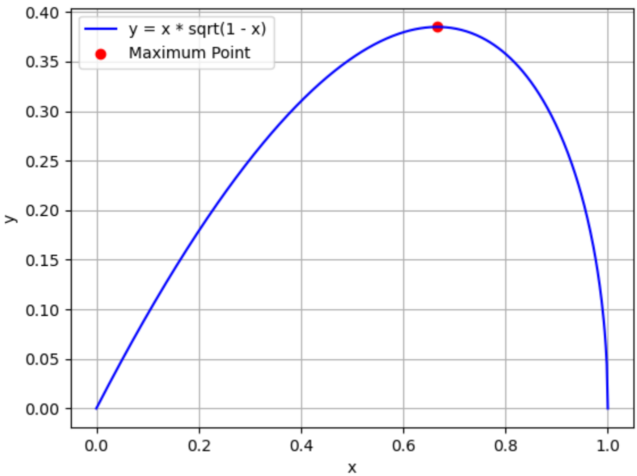
$$\mu\epsilon R - \{0\}$$

Assuming the condition for convergence holds, the result is:

$$\lim_{n\to\infty} \lambda_n = \frac{\mu b}{(1-(1-2a\mu))} \tag{0.15}$$

$$= \frac{b}{2a} \tag{0.16}$$

Taking initial value =0.5
$h = 0.001$ (step size)
Tolerance= 10^{-5} (minimum value of gradient).
The computed local maxima:0.6666662992697517
The computed local maximum:0.38490017945957516


y = x * sqrt(1 - x)
Maximum Point
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
y
0.0
0.2
0.4
0.6
0.8
1.0
x