6.5.3.5

EE24BTECH11021 - Eshan Ray

Question:
Find the local maximum and minimum value of the function $f(x) = x^3 - 6x^2 + 9x + 15$
Solution: For the function,

$$y(x) = x^3 - 6x^2 + 9x + 15 \tag{1}$$
$$y\prime(x) = 3x^2 - 12x + 9 \tag{2}$$
$$y\prime\prime(x) = 6x - 12 \tag{3}$$

For critical points,

$$y\prime(x) = 0 \tag{4}$$
$$\implies 3x^2 - 12x + 9 = 0 \tag{5}$$
$$\implies x = 1, 3 \tag{6}$$

For, critical points to be local minimum or local maximum, it should follow the following:-

$$\text{Local Minimum:} y\prime\prime(x)>0 \tag{7}$$
$$\text{Local Maximum:} y\prime\prime(x)<0 \tag{8}$$

For,$x = 1$ we get,

$$y\prime\prime(1) = 6(1) - 12 \tag{9}$$
$$y\prime\prime(1) = -6 <0 \tag{10}$$

So, $x = 1$ is a point of local maxima.

For, $x = 3$ we get,

$$y\prime\prime(3) = 6(3) - 12 \tag{11}$$
$$y\prime\prime(3) = 6 >0 \tag{12}$$

So, $x = 3$ is a point of local minima.
Computational Solution:
Finding the Local Maxima using Gradient Ascent we get,

$$x_{n+1} = x_n + \alpha f\prime(x_n) \tag{13}$$
$$x_{n+1} = x_n + \alpha\left(3x^2 - 12x + 9\right) \tag{14}$$

Finding the Local Minima using Gradient Decent we get,

$$x_{n+1} = x_n - \alpha f\prime(x_n) \tag{15}$$

$$x_{n+1} = x_n - \alpha\left(3x^2 - 12x + 9\right) \tag{16}$$

Taking the following conditions, we have

$$x_0 = 0 \tag{17}$$

$$y_0 = 15 \tag{18}$$

$$h = 0.01 \tag{19}$$

$$\alpha = 0.01 \tag{20}$$

After computing we get,

$$\text{Local maxima: } x = 1.0000153989160618,\ f(x) = 18.999999999288626 \tag{21}$$

$$\text{Local Minima: } x = 2.9999846010839377,\ f(x) = 15.00000000071137 \tag{22}$$

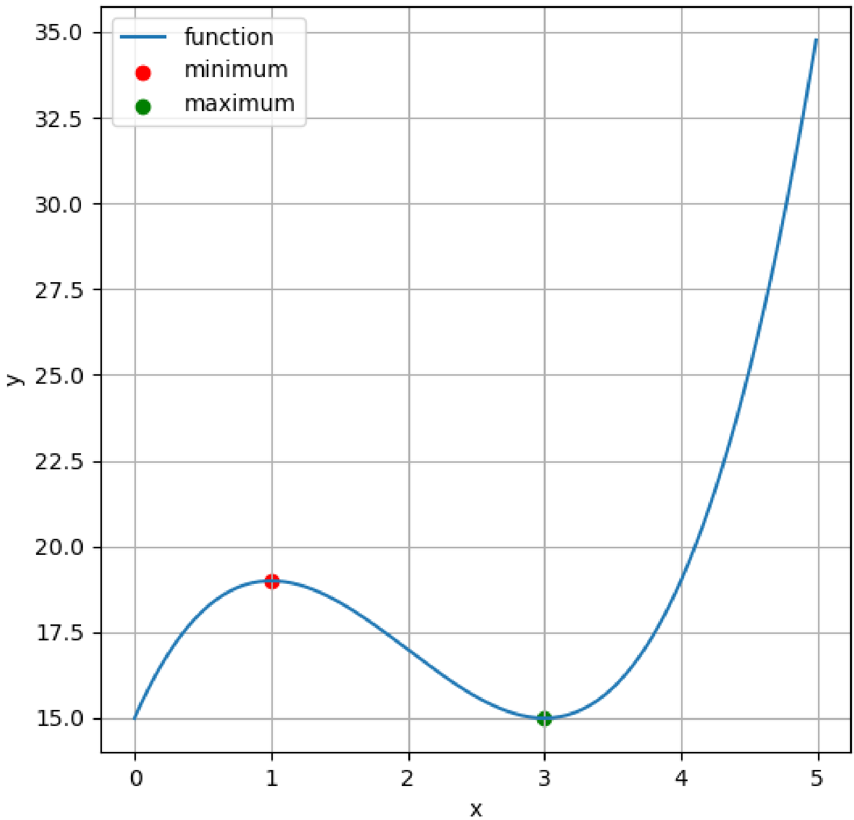

Fig. 0: Maxima and Minima points of the given function