10.3.2.5

EE24BTECH11006 - Arnav Mahishi

Question:
Half the perimeter of a rectangular garden, whose length is $4m$ more than its width, is 36m. Find the dimensions of the garden

Variable	Description
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
$\mathbf{x}$	Solution to the linear equation

TABLE 0: Variables Used

Theoretical Solution:
Taking x to be length and y to be width We get:

$$x - y = 4 \tag{0.1}$$

$$x + y = 72 \tag{0.2}$$

$$\implies 2x = 76 \tag{0.3}$$

$$\implies x = 38 \tag{0.4}$$

$$\implies y = 34 \tag{0.5}$$

Computational Solution:
The set of linear equations $x - y - 4 = 0$ and $x + y - 72 = 0$ can be represented by the following equation

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 72 \end{pmatrix} \tag{0.6}$$

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U:

$$A = L \cdot U \tag{0.7}$$

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

Step-by-Step Process:

1. Initial Matrix:

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{0.8}$$

2. Compute U (Upper Triangular Matrix):
Using the update equation for U:

$$U_{11} = A_{11} = 1, \quad U_{12} = A_{12} = -1 \tag{0.9}$$

For U_{22}:

$$U_{22} = A_{22} - L_{21} \cdot U_{12} = 1 - 1 \cdot (-1) = 2 \tag{0.10}$$

3. Compute L (Lower Triangular Matrix):
Using the update equation for L:

$$L_{21} = \frac{A_{21}}{U_{11}} = 1 \tag{0.11}$$

The final L matrix is:

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \tag{0.12}$$

4. Solving the System:
Using the equations $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$:

- **Forward Substitution:**

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 72 \end{pmatrix} \tag{0.13}$$

Solving gives:

$$y_1 = 4, \quad y_2 = 68 \tag{0.14}$$

- **Backward Substitution:**

$$\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 68 \end{pmatrix} \tag{0.15}$$

Solving gives:

$$x_2 = 34, \quad x_1 = 38 \tag{0.16}$$

Thus, the solution is:

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 38 \\ 34 \end{pmatrix} \tag{0.17}$$

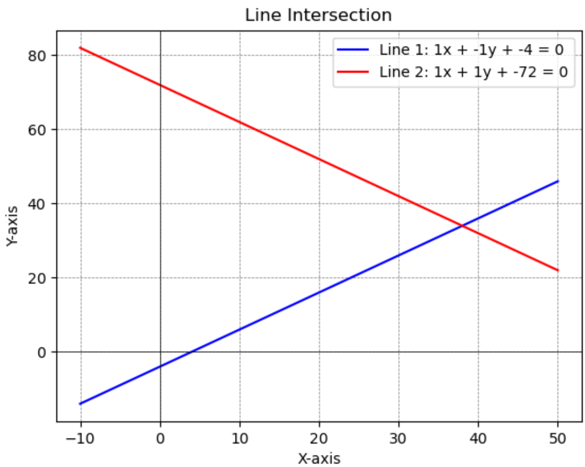

Fig. 0.1: Solution to set of linear equations