11.16.3.11

EE24BTECH11006- Arnav Mahishi

Question:- In a lottery, a person chooses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers aldready fixed by the lotttery committee, he wins a prize. What is the probability of winning the prize in the game?
Solution:
The sample space is

$$\Omega = [\text{All possible collections of 6 numbers from 1 to 20}] \tag{0.1}$$

$$\implies |\Omega| = \binom{20}{6} \tag{0.2}$$

Assuming equally likely outcomes,

$$\Pr(\omega \in \Omega) = \frac{1}{|\Omega|} = \frac{1}{\binom{20}{6}} \tag{0.3}$$

Let X represent the event:
X=1, The person wins the prize (All the numbers match)
X=0: The person does not win (numbers don't match).
The Probability Mass Function (PMF) for the given random variable is

$$P(X = k) = \begin{cases} 1 - \frac{1}{\binom{20}{6}}, & k = 0 \\ \frac{1}{\binom{20}{6}}, & k = 1 \end{cases} \tag{0.4}$$

The Cumulative Distribution Function (CDF) for the given random variable is

$$F_X(k) = P(X \le k) = \begin{cases} 0, & k < 0 \\ 1 - \frac{1}{\binom{20}{6}}, & 0 \le k < 1 \\ 1, & k \le 1 \end{cases} \tag{0.5}$$

The probability of winnig the lottery is

$$\Pr(X = 1) = \frac{1}{\binom{20}{6}} \tag{0.6}$$

Simulation

To simulate the lottery, the process follows these steps:

1) **Random Number Generation:** Generate random numbers uniformly from 1 to 20 for both the player and the committee using a uniform random number generator.
2) **Simulate Multiple Trials:** Run the simulation for a large number of trials (e.g., 1,000,000), where in each trial the player's six numbers are compared to the committee's six numbers.

3) **Match Calculation:** If all six numbers match, the player wins. The number of wins is tracked.

4) **Probability Calculation:** Calculate the probability of winning as the ratio of winning trials to total trials. The true probability is $\frac{1}{\binom{20}{6}}$.

5) **Relative Frequency:** Track and plot the relative frequency of winning over time to observe convergence to the true probability.

6) **Final Results:** After all trials, generate:
   - Probability Mass Function (PMF)
   - Cumulative Distribution Function (CDF)
   - Convergence of relative frequency to the true probability

The following shows how the relative frequency reaches true probability with increasing number of trials of the event.
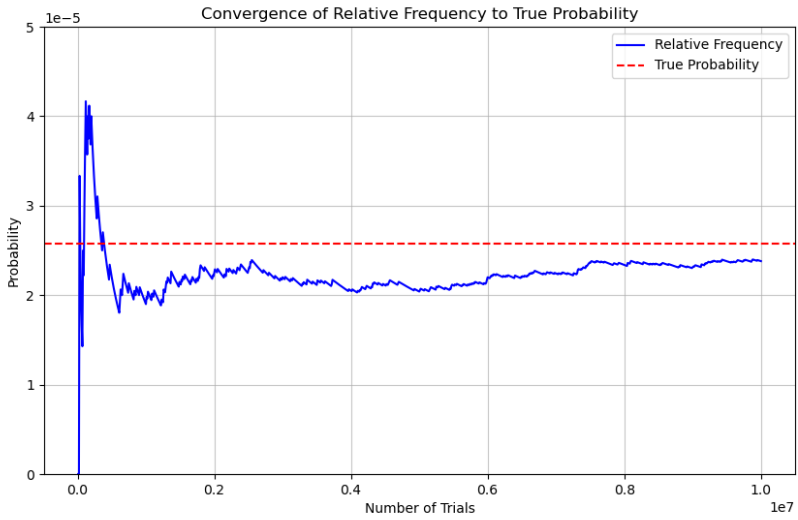
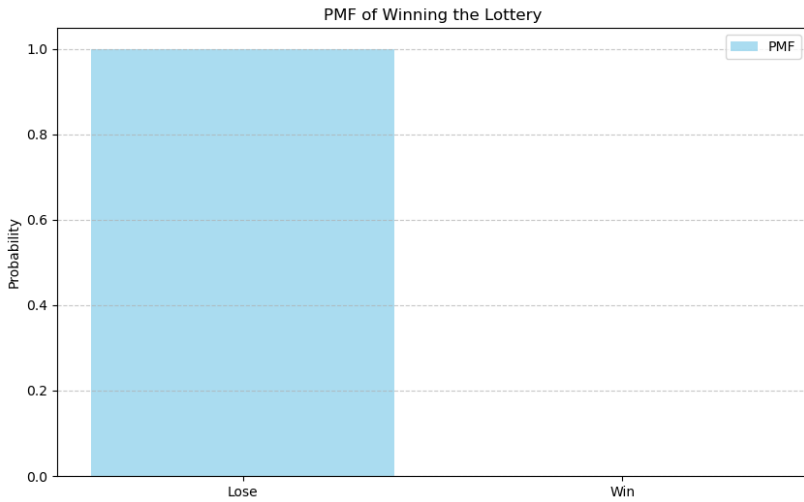Fig. 6.1: Relative Frequency tends to True Probability
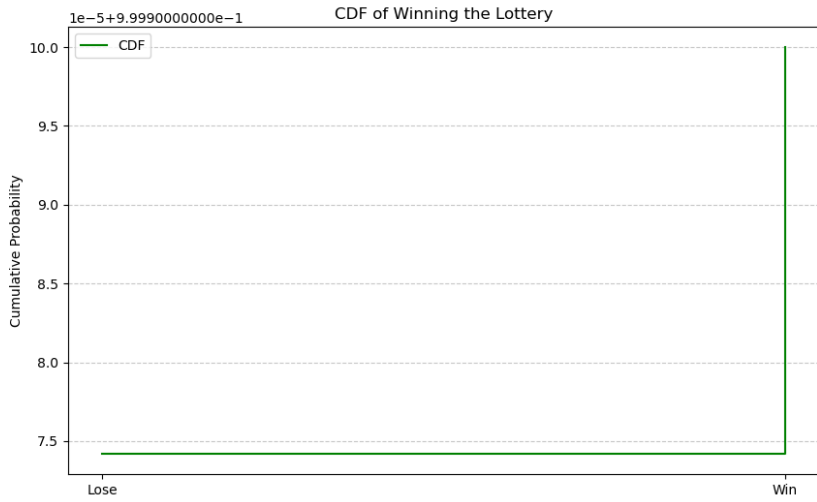

Fig. 6.2: Probability Mass Function of given Random variable

Fig. 6.3: Cumulative Distribution Function of given Random variable