6.5.21

EE24BTECH11005 - Arjun Pavanje

Question: Of all the closed right circular cylindrical cans of given volume $100cm^3$, find the dimensions of the can which has minimum surface area
Solution:
Surface Area of cylinder is given by,

$$2\pi rh + 2\pi r^2 \tag{1}$$

where r is the radius of the cylinder, h is the height of the cylinder.
Volume of a cylinder is given by,

$$\pi r^2 h \tag{2}$$

where r is the radius of the cylinder, h is the height of the cylinder.
Given that volume is $100cm^3$,

$$\pi r^2 h = 100 \tag{3}$$

$$h = \frac{100}{\pi r^2} \tag{4}$$

Equation (1) becomes,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{5}$$

Theoretical Solution
To minimize surface area, differentiate equation (1) and set it to zero,

$$\frac{4\pi r^3 - 200}{r^2} = 0 \tag{6}$$

value of r at which satisfies is, $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$. We can verify that this is a minima by differentiating equation (6),

$$\frac{400}{r^3} + 4\pi \tag{7}$$

Thus we see that at the above value of r, it is a minima.
Can of given volume will have maximum surface area when radius is $\left(\frac{50}{\pi}\right)^{\frac{1}{3}} cm$, height is $2\left(\frac{\pi}{50}\right)^{\frac{1}{3}}$
Computational Solution:
We need to minimize,

$$\left(\frac{200}{r} + 2\pi r^2\right) \tag{8}$$

Applying gradient descent theorem,

$$r_{n+1} = r_n - \mu f'(r_n) \tag{9}$$
$$\tag{10}$$

where μ is the step size,

$$f'(r_n) = -\frac{200}{r_n^2} + 4\pi r_n \tag{11}$$

Final Difference Equation comes out to be,

$$r_{n+1} = r_n(1 - 4\pi) + \frac{200}{r_n^2} \tag{12}$$

Taking inital guess as 2, step size 0.01, tolerence as 0.0001.
We get minimum value of r to be $2.515397787094116cm \approx \left(\frac{50}{\pi}\right)^{\frac{1}{3}} cm$

Alternate Computational Solution:

We can also solve it using *cvxpy* module in python. On running the code we get, Minimum value of r is, $2.515299390016942cm$, Minimum surface area is, $119.26542080485049cm^2$

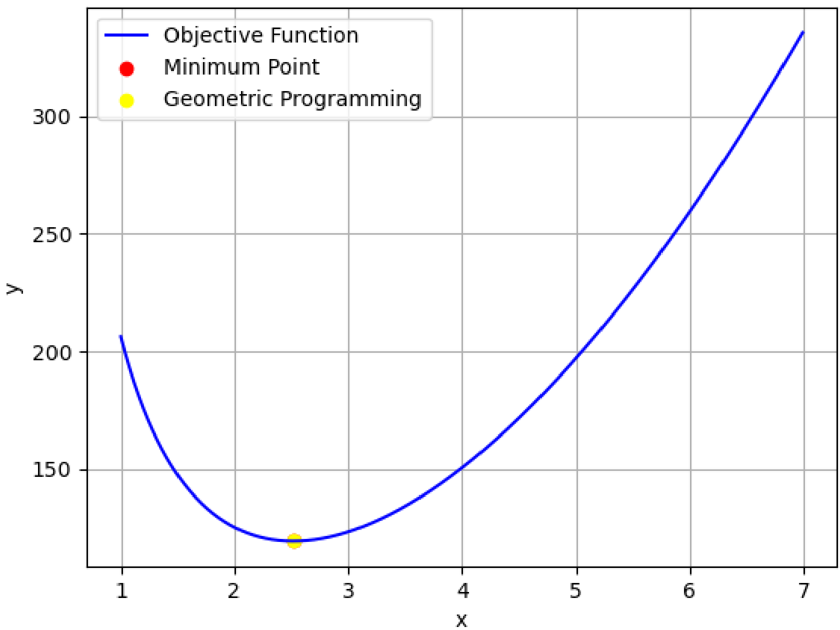

Fig. 1: Minimizing $\left(\frac{200}{r} + 2\pi r^2\right)$. Surface Area function with point of minima