6.5.1.3

EE24BTECH11013 - MANIKANTA D

Question:
Find the Maximum and Minimum values of the function if exists

$$y(x) = -(x-1)^2 + 10$$

Reformulating the Problem:
The given function can be rewritten as:

$$y(x) = -(x-1)^2 + 10 \tag{0.1}$$
$$= -x^2 + 2x - 1 + 10 \tag{0.2}$$
$$= -x^2 + 2x + 9 \tag{0.3}$$

To maximize $y(x)$, we can equivalently minimize $-y(x)$:

$$\min_x \quad -(-x^2 + 2x + 9) \tag{0.4}$$

Expanding the negative sign:

$$\min_x \quad x^2 - 2x - 9 \tag{0.5}$$

Quadratic Programming Formulation:
A quadratic programming problem is expressed as:

$$\min_x \quad \frac{1}{2}x^T Q x + c^T x \tag{0.6}$$

where:

- Q is the Hessian matrix representing the quadratic term:
- c is the vector representing the linear term:
- The constant term does not affect the optimization:

For our problem:

$$f(x) = x^2 - 2x - 9 \tag{0.7}$$

We identify the components as:

$$Q = \begin{bmatrix} 2 \end{bmatrix}, \quad c = \begin{bmatrix} -2 \end{bmatrix}, \quad r = -9 \tag{0.8}$$

Ignoring the constant term r, the problem reduces to:

$$\min_{x} \quad \frac{1}{2}x^T Q x + c^T x \tag{0.9}$$

Solution Using CVXPY:
The problem can be solved using the Python library CVXPY as follows:

```
import cvxpy as cp

# Define the optimization variable
x = cp.Variable()

# Define the quadratic programming problem
Q = 2  # Quadratic term
c = -2  # Linear term

# Objective function
objective = cp.Minimize(0.5 * Q * cp.square(x) + c * x)

# Solve the problem
problem = cp.Problem(objective)
problem.solve()

# Display the result
optimal_x = x.value
optimal_y = -(optimal_x - 1)**2 + 10

print(f"The value of x that maximizes the function is: {optimal_x}")
print(f"The maximum value of the function is: {optimal_y}")
```

result:
The value of x that maximizes the function is: 1.0
The maximum value of the function is: 10.0

Analytical Solution:
The derivative of $y(x)$ is:

$$\frac{dy}{dx} = -2x + 2 \tag{0.10}$$

Setting $\frac{dy}{dx} = 0$ gives:

$$-2x + 2 = 0 \implies x = 1 \tag{0.11}$$

Substituting $x = 1$ into $y(x)$:

$$y(1) = -(1 - 1)^2 + 10 = 10 \tag{0.12}$$

Result:

The maximum value of the function occurs at:

$$x = 1, \quad y(x) = 10 \tag{0.13}$$