10.4.1.2

EE24BTECH11001 - Aditya Tripathy

Question:
Find the roots of the equation $x^2 - 4x + 6 = 0$
Solution:
Below are two methods to find the solutions of this quadratic equation,

Fixed Point Iterations: Take an initial guess x_0. The update difference equation will use the following function:

$$x = g(x) \tag{0.1}$$

For our problem,

$$g(x) = \frac{1}{4}x^2 + \frac{3}{2} \tag{0.2}$$

Now the update equation will be,

$$x_{n+1} = g(x_n) \tag{0.3}$$

When we try to run the iterations however, we realize that whatever be the initial guess, the subsequent updated values grow without bound. This is becaue of the following theorem

Theorem: (0.4)

Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative in some interval J containing s. Then if $|g'| \leq K < 1$ in J, the iteration process defined above converges for any x_0 in J. The limit of the sequence $[x_n]$ is s

Since there is no solution (evident by quadratic formula) there exists no interval J for which the process converges to a point.

The same behaviour is shown by the Newton-Raphson Method,
Start with an initial guess x_0, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.5}$$

where ,

$$f(x) = x^2 - 4x + 6 \tag{0.6}$$
$$f'(x) = 2x - 4 \tag{0.7}$$

The behaviour shown here is that regardless of which guess we take, it reaches a point of

extrema(derivative ≈ 0) and then the process halts, or the updated point grow with bound. To get the complex solutions, however , we can Just take the initial guess point to be a random complex number.
The ouput of a program written to find roots is shown below:

```
Running Newton iterations:
x got too big
Trying fixed point iterations:
x got too big
Trying complex Newton's iterations:
Solution = 2.000000 + 1.414214 i
```

And on a second run,

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Running Newton iterations:
Failure
Trying fixed point iterations:
x got too big
Trying complex Newton's iterations:
Solution = 2.000000 + -1.414214 i
```