11.16.2.2.5

EE24BTECH11052 - Rongali Charan

Question: A Die is thrown.Describe the following events:
E: an even number greater than 4

Solution:

1) **Total Number of Possible Outcomes**
Let X be the random variable representing the outcome of a single die roll. The sample space is $S = \{1,2,3,4,5,6\}$. We are interested in the event E where the outcome is an even number greater than 4. The only outcome satisfying this condition is 6.
2) **Probability of Success**
The probability of event E is the number of favorable outcomes divided by the total number of possible outcomes:
$$P(E) = \frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in S}} = \frac{1}{6}$$
3) **Defining the Random Variable**
The PMF of the random variable X is given by:
$$P_X(x) = \begin{cases} \frac{1}{6}, & x \in \{1,2,3,4,5,6\} \\ 0, & \text{otherwise} \end{cases}$$
Desired probability i.e. probability that an even number greater than 4 is when die comes up as 6
$$P(X = 6) = \frac{1}{6} \tag{3.1}$$
For the event E (rolling a 6), we can define a new random variable Y such that:
Let Y be the random variable that represents the die turn up to be a 6:
$$Y = 1, \text{ If the number is 6, } \left(\text{With probability } p = \frac{1}{6}\right) \tag{3.2}$$
$$Y = 0, \text{ if number gets in } \{1,2,3,4,5\}, \left(\text{With probability } 1 - p = \frac{5}{6}\right) \tag{3.3}$$
4) **Probability Mass Function (PMF):**
The PMF of a Bernoulli random variable Y is given by:
$$P(Y = y) = p^y (1-p)^{1-y}, y \in \{0,1\} \tag{4.1}$$

substituting $p = \frac{1}{6}$,

$$P(Y=1) = 0.166666, P(Y=0) = 0.833333 \tag{4.2}$$

$$P(Y=y) = \begin{cases} 0.166666, & y=1 \\ 0.833333, & y=0 \\ 0, & \text{otherwise} \end{cases} \tag{4.3}$$

5) **Cumulative Distribution function (CDF):**
The CDF of a discrete uniform random variable X is:

$$F_X(k) = P(X \leq k) = \frac{k}{6}, \quad k \in \{1,2,3,4,5,6\} \tag{5.1}$$

+

$$F_X(k) = \begin{cases} 0, & k < 1 \\ \frac{k}{6}, & 1 \leq k \leq 6 \\ 1, & k > 6 \end{cases} \tag{5.2}$$

The CDF of a Bernoulli random variable Y is defined as:

$$F_Y(y) = P(Y \leq y) \tag{5.3}$$

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ \frac{5}{6}, & 0 \leq y < 1 \\ 1, & y \geq 1 \end{cases} \tag{5.4}$$

6) **Numerical Solution (Monte Carlo)**
We can estimate the probability using the Monte Carlo method. We simulate a large number of die rolls and count how many times we get a 6.

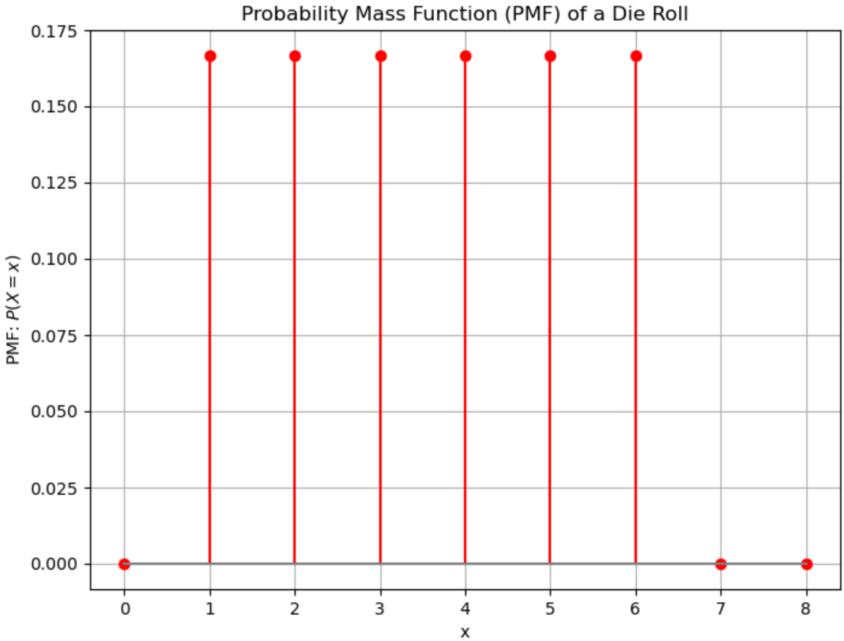

Fig. 6.1: PMF of the Random Variable

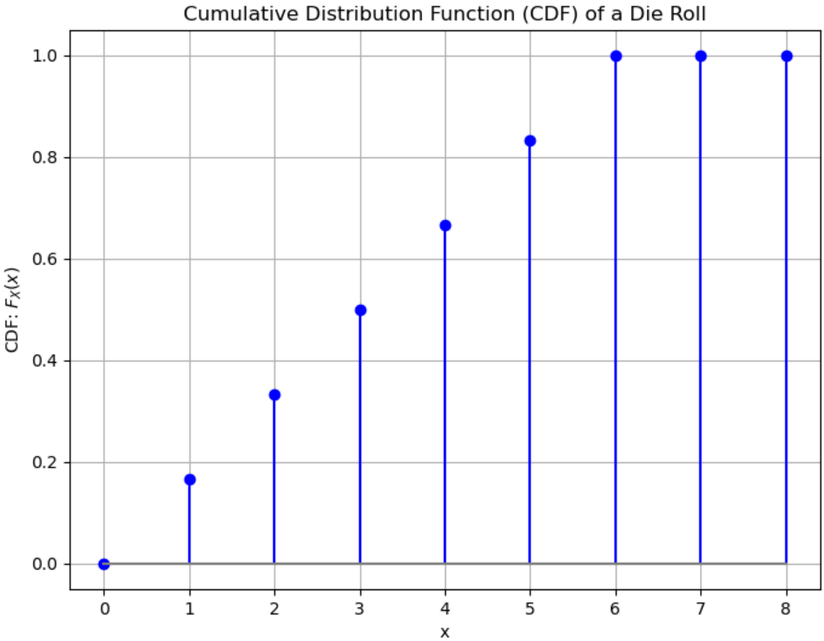

Fig. 6.2: CDF of the Random Variable