10.3.2.7

EE24BTECH11051 - Prajwal

1) Draw the graph of the equation $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by the lines and x-axis
Solution:-
Given,

$$x - y + 1 = 0 \tag{1.1}$$
$$3x + 2y - 12 = 0 \tag{1.2}$$
$$y = 0 \tag{1.3}$$

lines $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ touches ‘x-axis at,

$$x = -1 \tag{1.4}$$
$$x = 4 \tag{1.5}$$

Both given line touches at,

$$x = 2 \tag{1.6}$$
$$y = 3 \tag{1.7}$$

CODING LOGIC

Let us assume the given system of equations are consistent and we will try solving using LU decomposition
Given the system of linear equations:

$$x - y + 1 = 0 \tag{1.8}$$
$$3x + 2y - 12 = 0 \tag{1.9}$$
$$y = 0 \tag{1.10}$$

We rewrite the equations as:

$$x_1 = x, \tag{1.11}$$
$$x_2 = y, \tag{1.12}$$

giving the system:

$$x_1 - x_2 = -1 \tag{1.13}$$
$$3x_1 + 2x_2 = 12 \tag{1.14}$$
$$x_2 = 0 \tag{1.15}$$

Step 1: Convert to Matrix Form

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{1.16}$$

where:

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \tag{1.17}$$

$$A_2 = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}, \tag{1.18}$$

$$A_3 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}, \tag{1.19}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{1.20}$$

$$\mathbf{b_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \tag{1.21}$$

$$\mathbf{b_2} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}, \tag{1.22}$$

$$\mathbf{b_3} = \begin{bmatrix} -1 \\ 12 \end{bmatrix}. \tag{1.23}$$

Step 2: LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

$$L_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, U_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \tag{1.24}$$

$$L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, U_2 = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \tag{1.25}$$

$$L_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, U_3 = \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \tag{1.26}$$

Step 3: Solve $L\mathbf{y} = \mathbf{b}$ (Forward Substitution)

We solve:

$$L_1\mathbf{y_1} = \mathbf{b_1} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{1.27}$$

$$L_2\mathbf{y_2} = \mathbf{b_2} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{1.28}$$

$$L_3\mathbf{y_3} = \mathbf{b_3} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 15 \end{bmatrix}. \tag{1.29}$$

Thus:

$$\mathbf{y_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{1.30}$$

$$\mathbf{y_2} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{1.31}$$

$$\mathbf{y_3} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{1.32}$$

$$\tag{1.33}$$

Step 4: Solve $U\mathbf{x} = \mathbf{y}$ (Backward Substitution)

We solve:

$$U_1\mathbf{x_1} = \mathbf{y_1} \quad \text{or} \quad \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{1.34}$$

$$U_2\mathbf{x_2} = \mathbf{y_2} \quad \text{or} \quad \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}. \tag{1.35}$$

$$U_3\mathbf{x_3} = \mathbf{y_3} \quad \text{or} \quad \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \end{bmatrix}. \tag{1.36}$$

$$\mathbf{x_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}. \tag{1.37}$$

$$\mathbf{x_2} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}. \tag{1.38}$$

$$\mathbf{x_3} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}. \tag{1.39}$$

Hence ,there exist infinity many values of x_1 and x_2. So, both lines are same.