Finding maximum value NCERT-12.6.5.24

EE24BTECH11056 - S.Kavya Anvitha

Question:

Our aim is to find the maximum value of the function:

$$f(x) = [x(x-1)+1]^{1/3} \tag{1}$$

in the interval $0 \leq x \leq 1$ using the gradient ascent method
Simplify the expression

$$f(x) = \left[x^2 - x + 1\right]^{1/3} \tag{2}$$

Letting , the derivative of using the chain rule is:

$$f'(x) = \frac{d}{dx}\left(g(x)^{1/3}\right) = \frac{1}{3}g(x)^{-2/3} \cdot g'(x), \text{where:} g'(x) = 2x - 1 \tag{3}$$

The gradient ascent update rule is:

$$x_{n+1} = x_n + \eta \cdot f'(x_n) \tag{4}$$

where:

1) x_n is the current estimate.
2) η is the learning rate.
3) $f'(x)$ is the derivative calculated above.

Implementing gradient ascent:

1) We need to Choose a small learning rate η (say, 0.01).
2) Choose a starting point x_0 (say 0.0)

Compute the gradient $f'(x_n)$
Update the current point using the formula

$$x_{n+1} = x_n + \eta \cdot f'(x_n) \tag{5}$$

$$x_{n+1} = x_n + \eta \cdot \frac{1}{3}([x_n^2 - x_n + 1]^{\frac{-2}{3}}) \cdot (2x_n - 1) \tag{6}$$

For finding minimum we use gradient descent method.Update the value of x using the formula:

gradient descent update rule is:

$$x_{n+1} = x_n - \eta \cdot f'(x_n) \tag{7}$$

$$x_{n+1} = x_n - \eta \cdot \frac{1}{3}([x_n^2 - x_n + 1]^{\frac{-2}{3}}) \cdot (2x_n - 1) \tag{8}$$

Check if the stopping condition is met.
The value of x after sufficient iterations will be the approximate point where the function attains its minimum.

Behavior in each region:

for $x > 0.5$: $g'(x) = 2x - 1 > 0$
for $x < 0.5$: $g'(x) = 2x - 1 < 0$
Stop when the gradient is close to zero(e.g.,$|f'(x_n)| < 10^{-6}$)
Stop if the next step takes x out of the interval [0,1].
Since the interval is restricted to $0 \le x \le 1$:

1) Compute the function value at the boundaries $f(0)$ and $f(1)$.
2) Compare with the value obtained using gradient ascent.

Boundary values:

$f(0) = [0(0-1)+1]^{\frac{1}{3}} = 1$
$f(1) = [1(1-1)+1]^{\frac{1}{3}} = 1$