12.9.2.2

EE24BTECH11013 - MANIKANTA D

Question:
Consider the differential equation

$$y' - 2x - 2 = 0 \tag{0.1}$$

Verify that

$$y = x^2 + 2x + C \tag{0.2}$$

is a solution for it.
Theoretical Solution:
The given differential equation is:

$$y' - 2x - 2 = 0 \tag{0.3}$$

Rearrange the terms to group all x and $y$related terms:

$$y' = 2x + 2 \tag{0.4}$$

Now integrate both sides with respect to x:

$$\int y' dx = \int 2x + 2dx \tag{0.5}$$

The left-hand side simplifies to y, and the right-hand side is integrated term by term:

$$y = \int 2xdx + \int 2dx \tag{0.6}$$

$$y = x^2 + 2x + C \tag{0.7}$$

This matches the assumed solution:

$$y = x^2 + 2x + C \tag{0.8}$$

Integrating Factor Approach:

$$y' - 2x = 2 \tag{0.9}$$

Rearrange to match the standard form:

$$y' = 2x + 2 \tag{0.10}$$

Integrate both sides:

$$y = \int (2x + 2)dx \tag{0.11}$$

$$y = x^2 + 2x + C \tag{0.12}$$

Thus, we recover the same solution:

$$y = x^2 + 2x + C \tag{0.13}$$

Difference equation method

The difference equation is:

$$y_{n+1} = y_n + h \cdot y'(x), \tag{0.14}$$

where:

- y_n is the value of the function at step n,
- h is the step size,
- $y'x$ is the derivative of the function.

Step 1: Substitute y_n and $y'(x)$

Assume $y_n = x_n^2 + 2x_n + C$. Substituting $y'x = 2x_n + 2$ into the difference equation gives:

$$y_{n+1} = y_n + h \cdot (2x_n + 2). \tag{0.15}$$

Substituting $y_n = x_n^2 + 2x_n + C$, we get:

$$y_{n+1} = (x_n^2 + 2x_n + C) + h \cdot (2x_n + 2). \tag{0.16}$$

Step 2: Expand y_{n+1}

Expanding the terms:

$$y_{n+1} = x_n^2 + 2x_n + C + 2hx_n + 2h, \tag{0.17}$$

$$y_{n+1} = x_n^2 + (2x_n + 2hx_n) + (C + 2h). \tag{0.18}$$

Step 3: Difference Equation Solution

Starting with the expanded difference equation:

$$y_{n+1} = x_n^2 + (2x_n + 2hx_n) + (C + 2h). \tag{0.19}$$

We can further simplify by grouping terms:

$$y_{n+1} = x_n^2 + 2x_n(1 + h) + (C + 2h). \tag{0.20}$$

Thus, the solution for y_{n+1} becomes:

$$y_{n+1} = x_n^2 + 2x_n(1 + h) + (C + 2h). \tag{0.21}$$

This matches the original function $y = x^2 + 2x + C$ when $h \to 0$, verifying the consistency of the difference equation method with the exact solution.

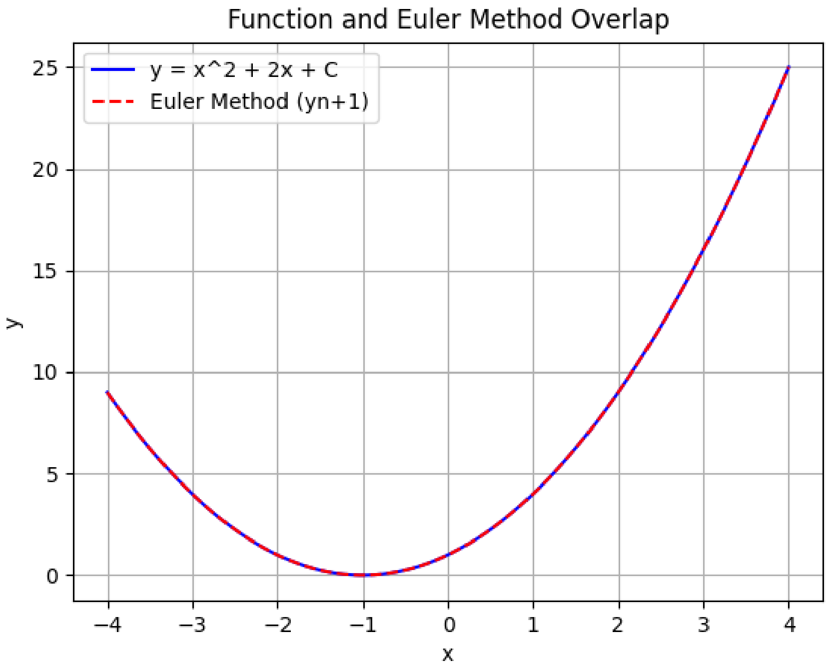

Fig. 0.1: Plot of the differential equation