10.3.6.1.2

EE24BTECH11043 - Murra Rajesh Kumar Reddy

Question

Solve the following pair of linear equations using LU decomposition:
Solution:

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \tag{1}$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \tag{2}$$

First, we rewrite the question as a system of linear equations.

$$x_1 \implies \frac{1}{\sqrt{x}} \tag{3}$$

$$x_2 \implies \frac{1}{\sqrt{y}} \tag{4}$$

Converting into matrix form, we get:

$$\begin{pmatrix} 2 & 3 \\ 4 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{5}$$

$$\mathbf{A}x = \mathbf{b} \tag{6}$$

To solve the above equation, we apply LU decomposition to matrix **A**.

Step 2: LU Factorization Using Update Equations

Given a matrix **A** of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:
1. **Initialization:** - Start by initializing **L** as the identity matrix $\mathbf{L} = \mathbf{I}$ and **U** as a copy of **A**.

2. **Iterative Update:** - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of **U** using the first update equation. - Compute the entries of **L** using the second update equation.

3. **Result:** - After completing the iterations, the matrix **A** is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of **U***)*

For each column $j \geq k$, the entries of **U** in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of $\mathbf{L}$)

For each row $i > k$, the entries of $\mathbf{L}$ in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

LU Factorizing $\mathbf{A}$, we get:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 0 & -15 \end{pmatrix}, \tag{7}$$

$$\mathbf{L} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \tag{8}$$

$$\mathbf{U} = \begin{pmatrix} 2 & 3 \\ 0 & -15 \end{pmatrix} \tag{9}$$

The solution can now be obtained as:

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{10}$$

Solving for y, we get:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{11}$$

Now, solving for x via back substitution:

$$\begin{pmatrix} 2 & 3 \\ 0 & -15 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{12}$$

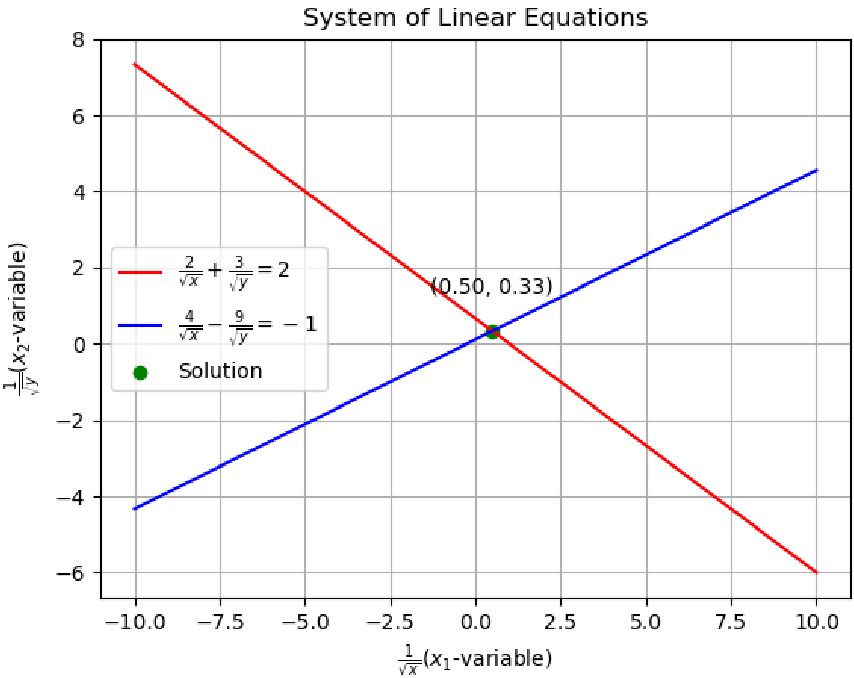
$$x_2 = \frac{1}{3}, \tag{13}$$

$$2x_1 + 3x_2 = 2 \implies x_1 = \frac{1}{2} \tag{14}$$

Thus, the solution is:

$$\frac{1}{\sqrt{x}} = \frac{1}{2}, \ \frac{1}{\sqrt{y}} = \frac{1}{3} \tag{15}$$

$$x = 4, \ y = 9 \tag{16}$$



System of Linear Equations
$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$
$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$
Solution
(0.50, 0.33)
$\frac{1}{\sqrt{x}}$(x_1-variable)
$\frac{1}{\sqrt{y}}$(x_2-variable)
8
6
4
2
0
−2
−4
−6
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0