NCERT-9.4.10

EE24BTECH11039 - MANDALA RANJITH

Question: Solve the differential equation:

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0. \tag{1}$$

Solution: Rewriting the equation:

$$\frac{e^x \tan y}{1 - e^x} \, dx + \sec^2 y \, dy = 0. \tag{2}$$

Rearranging terms:

$$\frac{e^x}{1 - e^x} \, dx = -\frac{\sec^2 y}{\tan y} \, dy. \tag{3}$$

where C is the constant of integration.Let us assume it as 1.

$$(1 - e^x)(\tan y) = 1 \tag{4}$$

Final solution:

$$y = \tan^{-1}\left(\frac{1}{1 - e^x}\right) \tag{5}$$

Numerical Approach:
1. I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$. where h is the step size, representing the rate of change.
2. Assigned the values of y for different x-values using a for loop.

Using the Method of Finite Differences
The Method of Finite Differences is a numerical technique used to approximate solutions to differential equations.

We know that:

$$\lim_{h \to 0} \frac{x(y + h) - x(y)}{h} = \frac{dx}{dy} \tag{6}$$

For the given differential equation,

$$\frac{dx}{dy} = \frac{(e^x - 1) \sec^2 y}{(e^x) tany} \tag{7}$$

we approximate:

$$\frac{x_{n+1} - x_n}{h} \approx \frac{(e^{x_n} - 1) \sec^2 y_n}{e^{x_n} \tan y_n} \tag{8}$$

this implies:

$$x_{n+1} = x_n + \frac{(e^{x_n} - 1)\sec^2 y_n}{e^{x_n}\tan y_n}h \tag{9}$$

Here, h is the step size, y_n is the approximation of $y(x)$ at the n-th step, and x_n is the corresponding x-value at the n-th step.
The iterative formula for updating x-values is:

$$x_n = x_{n-1} + \left(\frac{dx}{dy}\right)h, \tag{10}$$

The iterative formula for updating y-values is:

$$y_n = y_{n-1} + h \tag{11}$$

Initial Conditions:

- $x = 0.693$
- $y = 1.107$
- $h = 0.0001$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match.

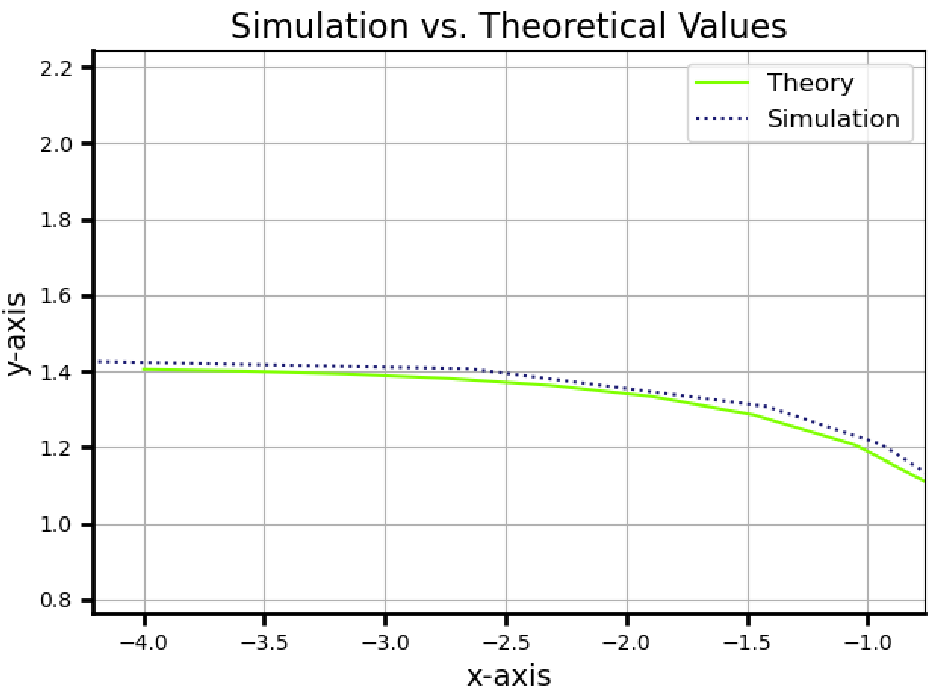

Fig. 0.1: verifying through graph of sim and theory values