9.3.12

EE24BTECH11028 - Jadhav Rajesh

Question: Which of the following differential equations has $y = x$ as one of its particular solution?

$$(C)\,\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 \tag{0.1}$$

Solution: By first principle of derivatives,

$$y'(t) = \lim_{h\to 0}\frac{y(t+h) - y(t)}{h} \tag{0.2}$$

$$y(t+h) = y(t) + hy'(t) \tag{0.3}$$

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 \tag{0.4}$$

Rewriting the given equation , we get:

$$\frac{d^2y}{dx^2} = x^2\frac{dy}{dx} - xy \tag{0.5}$$

To solve this equation numeriacally,we apply Euler's method. we start by introducing the following substitution:
Let:

$$y'_{n+1} = y'_n + h\left(y''_n\right) \tag{0.6}$$

Then when we substition eq (0.5) in eq (0.6)

$$y'_{n+1} = y'_n + h\left(x^2y'_n - xy_n\right) \tag{0.7}$$

then

$$y_{n+1} = y_n + h\left(y'_n\right) \tag{0.8}$$

$$y_{n+1} = y_n + h\left(y'_{n-1}\right) + h\left(x^2y'_{n-1} - xy_{n-1}\right) \tag{0.9}$$

We need to assume two initial conditions as it is a second order differential equation.
So here we assume the initial conditions as

$$x_0 = 0 \tag{0.10}$$
$$y_0 = 0 \tag{0.11}$$
$$y'_0 = 1 \tag{0.12}$$
$$h = 0.1 \tag{0.13}$$

substitute eq (0.10), eq (0.11) and eq (0.10) in eq (0.1)
we get

$$y''(0) = 0 \tag{0.14}$$

Substitute eq (0.10) in eq (0.8)

$$y_1 = y_0 + y_0'(0.1) \tag{0.15}$$
$$y_1 = 0.1 \tag{0.16}$$

For the rest of the points use eq (0.8) we get the other points.