9.7.8

EE24BTECH11050 - Pothuri Rahul

QUESTION: Find the equation of the curve passing through the points $\left(0, \frac{\pi}{4}\right)$ whose differential equation is $\sin x \cos y dx + \cos x \sin y dy = 0$

Solution: Given differential equation is

$$\sin x \cos y dx + \cos x \sin y dy = 0 \tag{0.1}$$

By rearranging the terms in (0.1)

$$\sin x \ cosydx = -\cos x \sin y dy \tag{0.2}$$

$$\tan x dx = -\tan y dy \tag{0.3}$$

Integrating both sides of (0.3),

$$\int \tan x dx = \int -\tan y dy \tag{0.4}$$

$$-\ln \cos x = -(-\ln \cos y) + C \tag{0.5}$$

$$-\ln \cos x = (\ln \cos y) + C \tag{0.6}$$

Where C is the integration constant, To find C,Lets use the given condition that the curve passes through $\left(0, \frac{\pi}{4}\right)$

By substituting given point in (0.5),

$$-\ln 1 = \ln \frac{1}{\sqrt{2}} + C \tag{0.7}$$

$$0 = \ln \frac{1}{\sqrt{2}} + C \tag{0.8}$$

$$C = -\ln \frac{1}{\sqrt{2}} \tag{0.9}$$

$$C = \ln \sqrt{2} \tag{0.10}$$

By substituting (0.10) in (0.6) ,

$$-\ln \cos x = \ln \cos y + \ln \sqrt{2} \tag{0.11}$$

$$\ln \cos y = -\ln \cos x - \ln \sqrt{2} \tag{0.12}$$

$$\ln \cos y = -\left(\ln\left(\cos x \sqrt{2}\right)\right) \tag{0.13}$$

$$\cos y = \frac{1}{\cos x \sqrt{2}} \tag{0.14}$$

$$y = \cos^{-1}\left(\frac{1}{\cos x\sqrt{2}}\right) \tag{0.15}$$

Logic used for programming:-

Method of finite differences: This method is used to find the approximate solution of the given differential equation by using the values of the function at discrete points.
From the defination of derivative of a function

$$\frac{dy}{dx} \approx \frac{y(x+h) - y(x)}{h} \tag{0.16}$$

by rearranging the terms, we get the function

$$y(x+h) = y(x) + h \times \frac{dy}{dx} \tag{0.17}$$

For this question,

$$\frac{dy}{dx} = -\frac{\tan x}{\tan y} \tag{0.18}$$

By substituting in (0.17),

$$y(x+h) = y(x) - h \times \frac{\tan x}{\tan y} \tag{0.19}$$

Let (t_0, P_0) be points on the curve,

$$x_1 = x_0 + h \tag{0.20}$$

$$y_1 = y_0 - h \times \frac{\tan x_n}{\tan y_n} \tag{0.21}$$

On generalising the above equations,

$$x_{n+1} = x_n + h \tag{0.22}$$

$$y_{n+1} = y_n - h \times \frac{\tan x_n}{\tan y - n} \tag{0.23}$$

Where h is a very small division (example 0.1),We need iterate this algorithm by taking $y_0 = \frac{\pi}{4}$ and $x = 0$. If we plot all the points (x, y), we get the function y varying with x, i.e y vs x graph.

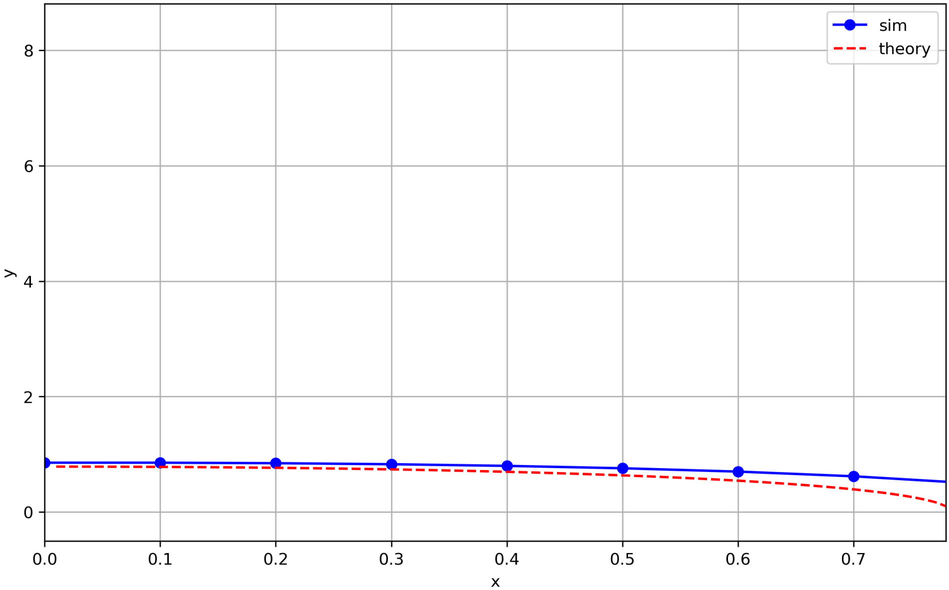

Fig. 0.1: Plot