NCERT-9.7.2.3

EE24BTECH11042 - SRUJANA

QUESTION:

Verify that the given function is a solution of the corresponding differential equation.

$\frac{d^2y}{dx^2} + 9y - 6cos3x = 0 : y = xsin3x$

Theoretical Solution:

$$\frac{d^2y}{dx^2} = 6\cos 3x - 9y \tag{0.1}$$

Solution have two parts homogeneous solution and particular solution
Homogeneous part :

$$\frac{d^2y}{dx^2} = -9y \tag{0.2}$$

assume solution is of the form ce^{rx}

$$r^2 = -9 \tag{0.3}$$

$$r = \pm 3i \tag{0.4}$$

As the roots are exponential , then the solution will be of the form $C_1cos3x + C_2sin3x$
Particular solution:

Since the non-homogeneous term is $6\cos 3x$, which is a trigonometric function, we assume the particular solution has the same form:

$$y_p(x) = Ax\cos 3x + Bx\sin 3x \tag{0.5}$$

First derivative:

$$y_p'(x) = A\cos 3x - 3Ax\sin 3x + B\sin 3x + 3Bx\cos 3x \tag{0.6}$$

Second derivative:

$$y_p''(x) = -6A\sin 3x - 6Ax\cos 3x + 6B\cos 3x - 6Bx\sin 3x \tag{0.7}$$

Now substitute $y_p(x)$ and $y_p''(x)$ into the original differential equation $\frac{d^2y}{dx^2} = 6\cos 3x$ - 9y.

Simplifying and comparing coefficients gives the values of A and B.

we get A = 0 and B =1

Assume Initial Conditions as 0,0

Overall solution $y_h + y_p = y$

$$y(0) = C_1 = 0 \tag{0.8}$$
$$y''(x) = 6\cos 3x + C_2 \cos 3x \tag{0.9}$$
$$y''(0) = 6 + C_2 \tag{0.10}$$

But ,

$$y''(0) = 6 \tag{0.11}$$
$$C_2 = 0 \tag{0.12}$$

Final solution is $y = x \sin 3x$

Programming appproach

Step 1 :

Using Taylor's expansion around a point x, we approximate $y(x + h)$ as:

$$y(x + h) \approx y(x) + hy'(x) + \frac{h^2}{2} y''(x) \tag{0.13}$$

we can neglect $y'(x)$ as its contribution to the solution is minimal.

$$y(x + h) \approx y(x) + \frac{h^2}{2} y''(x), \tag{0.14}$$

By substituting x as 0 , $y''(x)$ as 6 and h as 0.01 we get y(h)
Step II :

Finite differences method

The finite difference method is used to calculate the solutions of differential equations.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} \tag{0.15}$$

Similarly ,

$$y''(x) \approx \frac{y(x + h) + y(x - h) - 2y(x)}{h^2}. \tag{0.16}$$

$$y_{n+1} = \frac{d^2y}{dx^2} \cdot h^2 - y_{n-1} + 2y_n \quad (0.17)$$

$$y_{n+1} = (6\cos 3x_n - 9y_n n) \cdot h^2 - y_{n-1} + 2y_n \quad (0.18)$$

$$x_{n+1} = x_n + h \quad (0.19)$$

By substituting the results obtained above, we calculate $y(-h)$,

Now, use $x + h$ and $f(x + h)$ as the new initial conditions.

Repeat this process until you have sufficient points to plot the graph.

Connect all points to obtain an approximate plot of the differential equation.

This is a graphical representation of both the simulation and theoretical approaches of the differential equation.