NCERT-8.1.ex3

EE24BTECH11042 - SRUJANA

QUESTION:

Find the area of the region bounded by the curve $y = x^2$ and the line y=4
Theoretical Solution:

Intersection points are

$$4 = x^2 \implies x = \pm 2 \tag{0.1}$$

Area

$$\int_{-2}^{2} f(x)\,dx \tag{0.2}$$

$$f(x) = 4 - x^2 \tag{0.3}$$

$$\int_{-2}^{2} (4 - x^2)\,dx \tag{0.4}$$

$$\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) \tag{0.5}$$

$$\text{Area} = \frac{32}{3} = 10.666 \tag{0.6}$$

Trapezoidal method
The trapezoidal rule approximates the integral using the formula:

$$A \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \tag{0.7}$$

where:

1) $h = \frac{b-a}{n}$ is the width of each subinterval.
2) $f(x) = 4 - x^2$.
3) $a = -2, b = 2$.
4) n is the number of subintervals.

Taking trapezoid shaped strips of small area and adding them all up.. Say we have to find the area of $y(x)$ from $x = x_0$ to $x = x_n$,discretize points on the x axis $x_0, x_1, x_2, ..., x_n$ such that they are equally spaced with step-size h.Sum of all trapezoidal areas is given by,

$$A = \frac{1}{2}h\left[y(x_1) + y(x_0)\right] + \frac{1}{2}h\left[y(x_2) + y(x_1)\right] + \frac{1}{2}h\left[y(x_3) + y(x_2)\right] + \cdots + \frac{1}{2}h\left[y(x_n) + y(x_{n-1})\right] \tag{4.1}$$

$$= h\left[\frac{1}{2}\left(y(x_0) + y(x_n)\right) + y(x_1) + y(x_2) + \cdots + y(x_{n-1})\right] \tag{4.2}$$

Let $A(x_n)$ be the area enclosed by the curve $y(x)$ from $x = x_0$ to $x = x_n$:

$$A(x_n + h) = A(x_n) + \frac{1}{2}h\left[y(x_n + h) + y(x_n)\right] \tag{4.3}$$

We can repeat this till we get the required area:

$$A_{n+1} = A_n + \frac{1}{2}h\left[y_{n+1} + y_n\right] \tag{4.4}$$

We can write y_{n+1} in terms of y_n as:

$$y_{n+1} = y_n + h \cdot y'_n \tag{4.5}$$

Substituting this into the equation, we get:

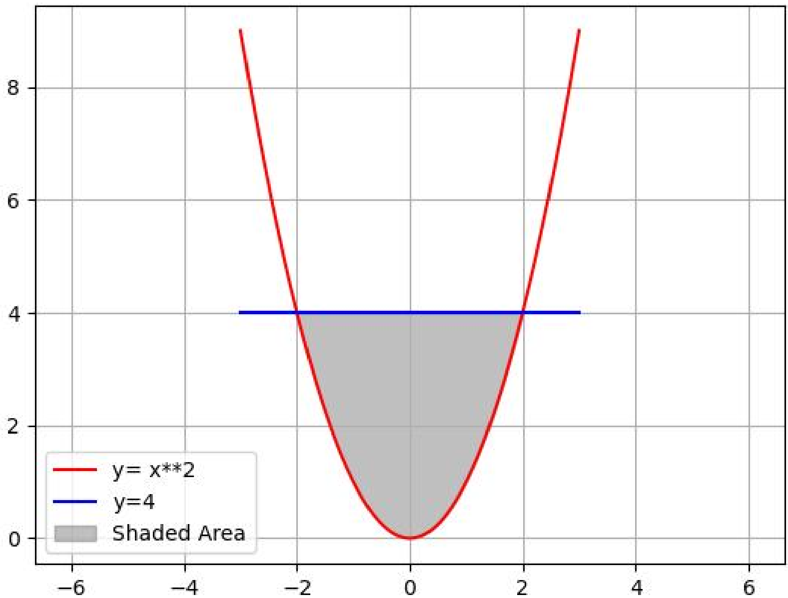
$$A_{n+1} = A_n + \frac{1}{2}h\left[(y_n + h \cdot y'_n) + y_n\right] \tag{4.6}$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2 y'_n \tag{4.7}$$

$$A_{n+1} = A_n + h(4 - x_n^2) + \frac{1}{2}h^2(-2x_n) \tag{4.8}$$

$$x_{n+1} = x_n + h \tag{4.9}$$

By assuming some value for n area is obtained


y= x**2
y=4
Shaded Area
0
2
4
6
8
−6
−4
−2
0
2
4
6