6.5.7

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

Solution:

Theoretical Solution:

$$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25 \tag{1}$$

Differentiating with respect to x on both sides of (1)

$$f'(x) = 12x^3 - 24x^2 + 24x - 48 \tag{2}$$

Using the property that $f'(x) = 0$ at extrema

$$f'(x) = 0 \tag{3}$$

$$12x^3 - 24x^2 + 24x - 48 = 0 \tag{4}$$

$$(x-2)^3 = 0 \tag{5}$$

Thus extrema exists on $x = 2$.Now differentiating (2) on both sides to get $f''(x)$

$$f''(x) = 36x^2 - 48x + 24 \tag{6}$$

putting x=2 in (6)

$$f''(2) = 72 \tag{7}$$

$$\implies f''(2) > 0 \tag{8}$$

Hence minima at x=2 of value $f(2) = -39$. Now checking for global maxima on boundaries

$$f(0) = 25 \tag{9}$$

$$f(3) = 16 \tag{10}$$

Hence for $f(x)$ in interval $[0, 3]$ minimum value is -39 and maximum value is 25

Computational Solution:

Using Gradient Descent to find extrema

$$h = 0.001 \tag{11}$$

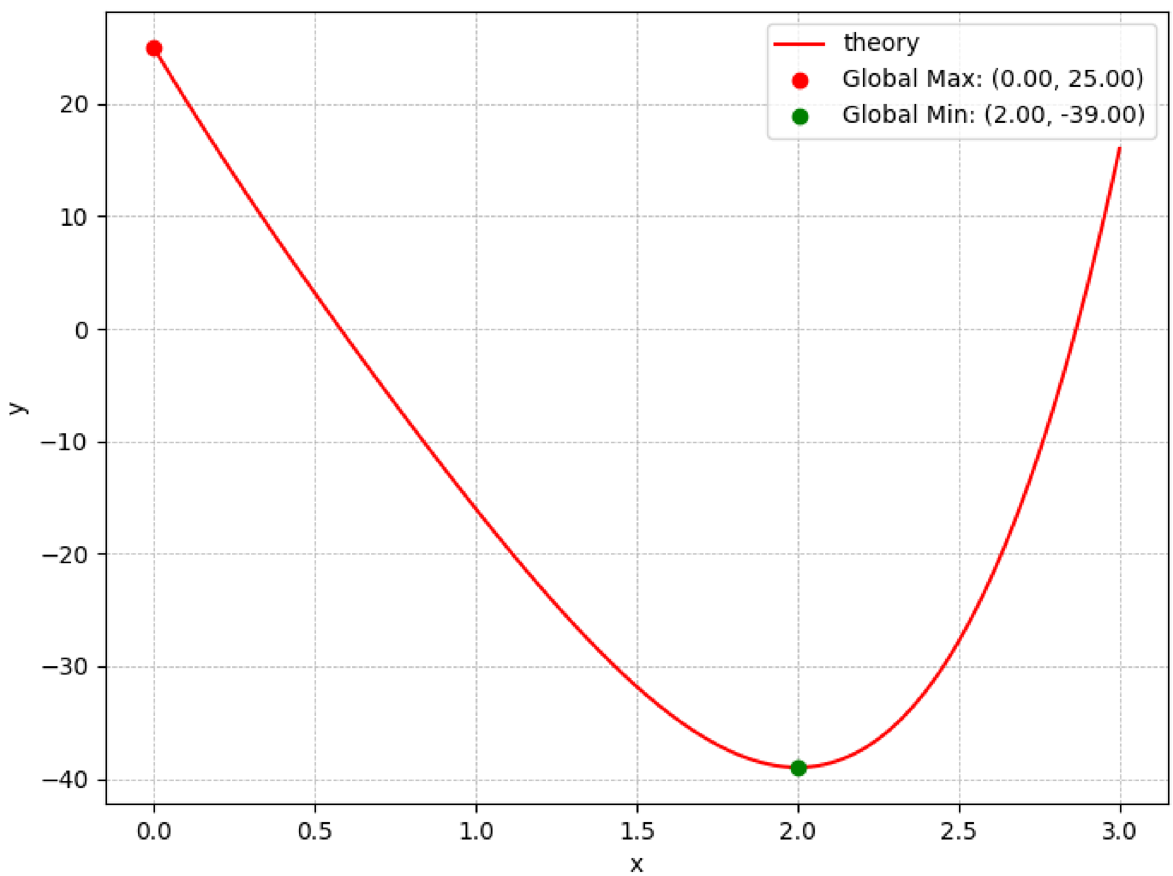
$$x_{n+1} = x_n - h(f'(x_n)) \tag{12}$$

$$y_{n+1} = y_n - (x_{n+1} - x_n)(f'(x_n)) \tag{13}$$

$$y_{n+1} = y_n - (x_{n+1} - x_n)\left(12x_n^3 - 24x_n^2 + 24x_n - 48\right) \tag{14}$$

$$\tag{15}$$

We get global maxima $y(0) = 25$ and global minima $y(2) = -39$ from the code



theory
Global Max: (0.00, 25.00)
Global Min: (2.00, -39.00)
20
10
0
−10
−20
−30
−40
y
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x