10.3.2.5

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.
Solution:
Let length and width of the garden be x and y respectively

$$x + y = 36 \tag{1}$$

$$x - y = 4 \tag{2}$$

We represent the system in matrix form:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 36 \\ 4 \end{pmatrix}, \quad x = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3}$$

LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:
Step-by-Step Procedure:
1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}. \tag{4}$$

Solving $Ax = b$

Forward Substitution: Solve $Ly = b$:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 36 \\ 4 \end{pmatrix}. \tag{5}$$

From the first row:

$$y_1 = 36. \tag{6}$$

From the second row:

$$y_1 + y_2 = 4 \tag{7}$$

$$36 + y_2 = 4 \tag{8}$$

$$y_2 = -32. \tag{9}$$

Thus:

$$y = \begin{pmatrix} 36 \\ -32 \end{pmatrix}. \tag{10}$$

Back Substitution: Solve $Ux = y$:

$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 36 \\ -32 \end{pmatrix}. \tag{11}$$

From the first row:

$$x + y = 36. \tag{12}$$

From the second row:

$$-2y = -32 \tag{13}$$

$$y = 16. \tag{14}$$

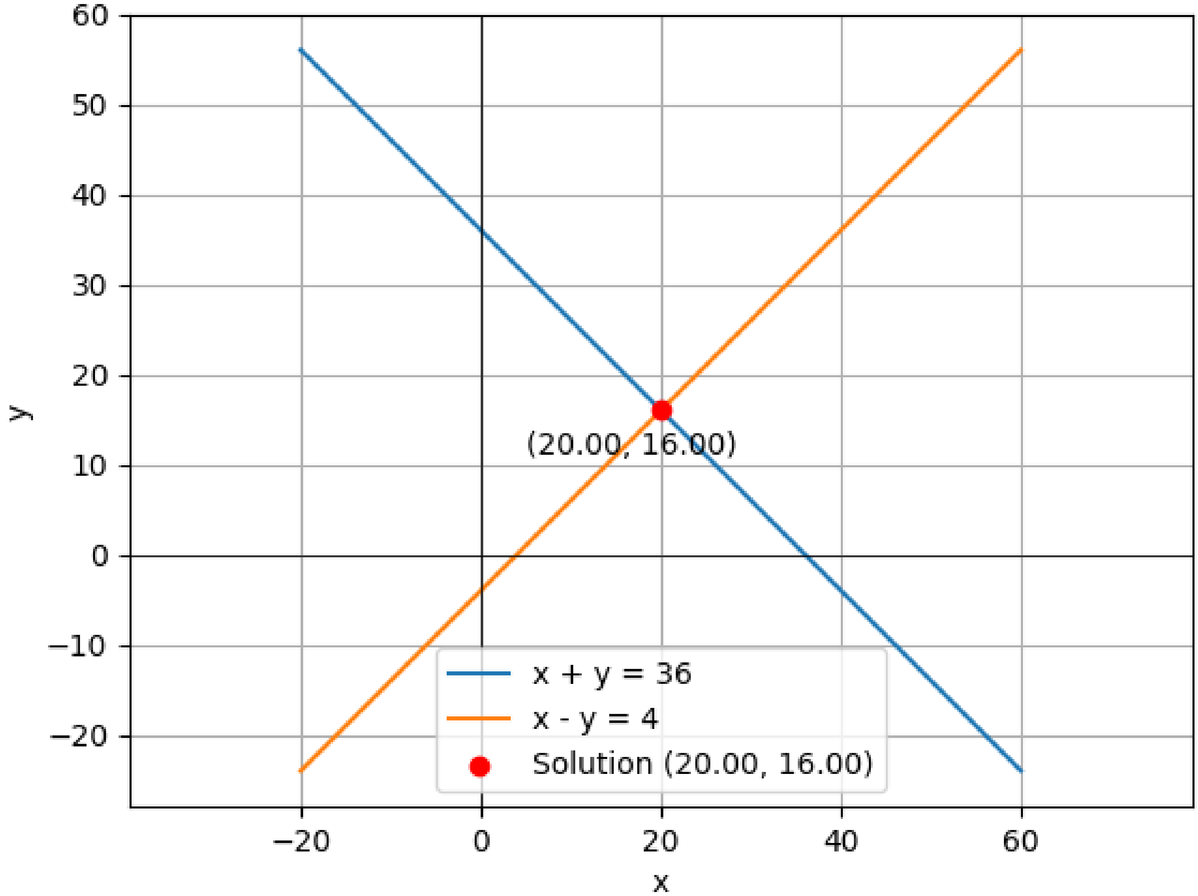
Substitute $y = 16$ into the first equation:

$$x + 16 = 36 \tag{15}$$

$$x = 20. \tag{16}$$

Thus:

$$x = 20, \quad y = 16. \tag{17}$$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x
(20.00, 16.00)
x + y = 36
x - y = 4
Solution (20.00, 16.00)