11.16.4.7.3

EE24BTECH11028 - Jadhav Rajesh

QUESTION: A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ $P(A \cap B) = 0.35$. Find $(iii)\, P(A \cap B')$
Theoretical Solution:

$$(A \cap B) = (A \cdot B) \tag{0.1}$$

For 2 Boolean variables A and B, the axioms of Boolean Algebra are defined as:

$$A = AB + AB' \tag{0.2}$$

$$A \cdot A = A \tag{0.3}$$

$$B \cdot B' = 0 \tag{0.4}$$

Now let's take
$X.Y = AB.AB' = 0$,because it is a disjoint
using these axioms, we will try to prove that

$$Pr(A) = Pr(AB) + Pr(AB') \tag{0.5}$$

$$Pr(A) - Pr(AB) = Pr(AB') \tag{0.6}$$

Using the given values of $Pr(A)$, $Pr(B)$ and $Pr(A \cdot B)$

$$Pr(AB') = Pr(A) - Pr(AB) \tag{0.7}$$

$$Pr(AB') = 0.54 - 0.35 \tag{0.8}$$

$$Pr(AB') = 0.19 \tag{0.9}$$

Simulated Solution:
Let X_1 be an indicator random variable of the event A.
X_1 is defined as:

$$X_1 = \begin{cases} 1, & A \\ 0, & A' \end{cases} \tag{0.10}$$

Let X_2 be the indicator random variable of the event B.
X_2 is defined as:

$$X_2 = \begin{cases} 1, & B \\ 0, & B' \end{cases} \tag{0.11}$$

Let X_3 be the indicator random variable of the event AB.
X_3 is defined as:

$$X_3 = \begin{cases} 1, & AB \\ 0, & (AB)' \end{cases} \tag{0.12}$$

The PMF of the random variable X_1 is:

$$p_{X_1}(n) = \begin{cases} p_1, & n = 1 \\ 1 - p_1, & n = 0 \end{cases} \tag{0.13}$$

The PMF of the random variable X_2 is:

$$p_{X_2}(n) = \begin{cases} p_2, & n = 1 \\ 1 - p_2, & n = 0 \end{cases} \tag{0.14}$$

The PMF of the random variable X_3 is:

$$p_{X_3}(n) = \begin{cases} p_3, & n = 1 \\ 1 - p_3, & n = 0 \end{cases} \tag{0.15}$$

where,

$$p_1 = 0.54 \tag{0.16}$$
$$p_2 = 0.69 \tag{0.17}$$
$$p_3 = 0.35 \tag{0.18}$$
$$\tag{0.19}$$

Let Y be the random variable which is defined as follows:

$$Y = X_1 - X_3 \tag{0.20}$$

$$p_Y(n) = \begin{cases} p, & n = 1 \\ 1 - p, & n = 0 \end{cases} \tag{0.21}$$

Where

$$p = P(A.B') \tag{0.22}$$

Using Expectation to Find p:From linearity of expectation

$$E(Y) = E(X_1) - E(X_3) \tag{0.23}$$

Since

$$E(X_1) = p_1 = 0.54, E(X_3) = p_3 = 0.35 \tag{0.24}$$

We have

$$p = E(Y) = p_1 - p_3 \tag{0.25}$$

Substitute the

$$p = 0.54 - 0.35 \tag{0.26}$$

$$p = (AB') = 0.19 \tag{0.27}$$

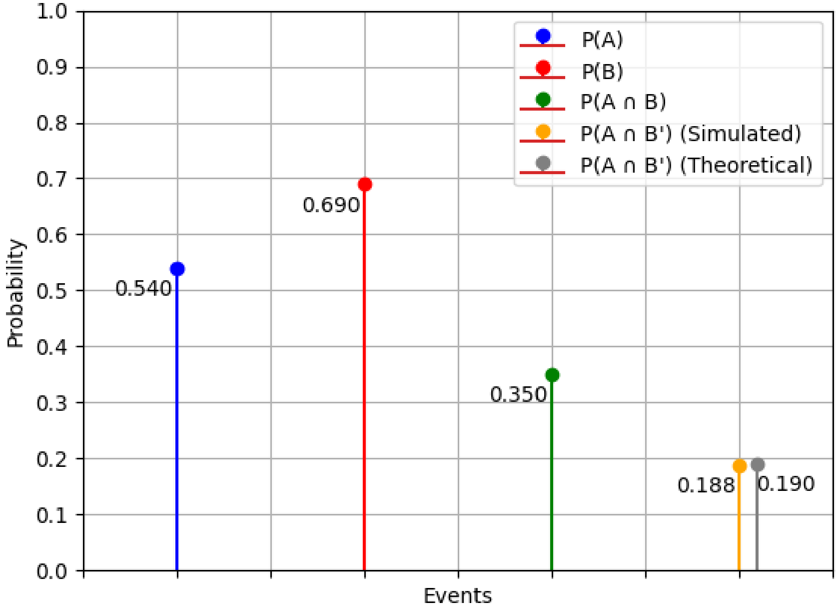

Fig. 0.1: Solving the system of equations