NCERT 12.8.ex.12

EE24BTECH11051 - Prajwal

Question: Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

Therotical logic:

1) Set up the integral:
   The area under the curve can be calculated as:

$$\text{Area} = \int_a^b f(x)dx \tag{1}$$

   Here:

$$f(x) = 3x + 2, \quad x_1 = -1, \quad x_2 = 1 \tag{2}$$

   Check whether the line touches the x-axis in the interval $x \in -1, 1$

$$y = 0 = 3x + 2 \tag{3}$$

$$x = \frac{-2}{3} \tag{4}$$

$$\tag{5}$$

   As $x = \frac{-2}{3} \in (-1, 1)$ Thus, the integral becomes:

$$\text{Area} = -\int_{-1}^{-2/3} (3x + 2)\, dx + \int_{-2/3}^{1} (3x + 2)\, dx \tag{6}$$

2) Compute the integral:
   The integral of $3x + 2$ is:

$$\int 3x + 2\, dx = \frac{3x^2}{2} + 2x \tag{7}$$

3) Evaluate the definite integral:
   Substitute the limits of integration:

$$\text{Area} = -\frac{3x^2}{2} + 2x \Big|_{-1}^{-2/3} + \frac{3x^2}{2} + 2x \Big|_{-2/3}^{1} \tag{8}$$

$$\text{Area} = \frac{13}{3} \tag{9}$$

   Computational Logic: Using the trapezoidal rule to get the area. The trapezoidal rule is as follows.

$$\int_a^b fxdx \approx \sum_{k=1}^{N} \frac{fx_{k+1} + fx_k}{2} h \tag{10}$$

where

$$h = \frac{b-a}{N} \tag{11}$$

∴The difference equation obtained is

$$A = \int_a^b fx\,dx \approx h\frac{1}{2}fa + fx_1 + fx_2 \cdots + fx_{n-1} + \frac{1}{2}fb \tag{12}$$

$$h = \frac{b-a}{n} \tag{13}$$

$$A = j_n, \text{ where, } j_{i+1} = j_i + h\frac{fx_{i+1} + fx_i}{2} \tag{14}$$

$$\rightarrow j_{i+1} = j_i + hx_{i+1}^2 + x_i^2 \tag{15}$$

$$x_{i+1} = x_i + h \tag{16}$$

$$h = 0.00001 \tag{17}$$

$$n = 300000 \tag{18}$$

Using the code answer obtained is 4.333333333346957