

11.16.3.3.5

EE24BTECH11059 - Y Siddhanth

Question:

A die is thrown, find the probability of a number less than 6 to appear is?

Solution:

The sample space (Ω) for rolling a die is:

$$\Omega = \{1, 2, 3, 4, 5, 6\} \quad (0.1)$$

Each outcome is equally likely. Let the random variable X represent the number rolled on the die.

Probability Mass Function (PMF):

The PMF for a fair die is:

$$P(X = x) = \begin{cases} \frac{1}{n}, & k \in \{1, 2, \dots, n\} \\ 0, & \text{otherwise} \end{cases} \quad (0.2)$$

Cumulative Distribution Function (CDF):

The CDF for the die roll is:

$$F_X(k) = P(X \leq k) = \begin{cases} 0, & k < 1 \\ \frac{k}{n}, & 1 \leq k < n \\ 1, & k \geq n \end{cases} \quad (0.3)$$

Where n is 6.

\therefore The probability of getting a number less than 6 is as below, from (0.3)

$$P(X < 6) = F_X(5) \quad (0.4)$$

$$= \frac{5}{6} \quad (0.5)$$

Probability simulation:

We will use the hardware random number generator of an ESP32 to simulate true randomness. I have referred the esp32-technical-reference-manual, for the proper usage of it's random number generator. These true random numbers are generated based on the thermal noise in the system and the asynchronous clock mismatch.

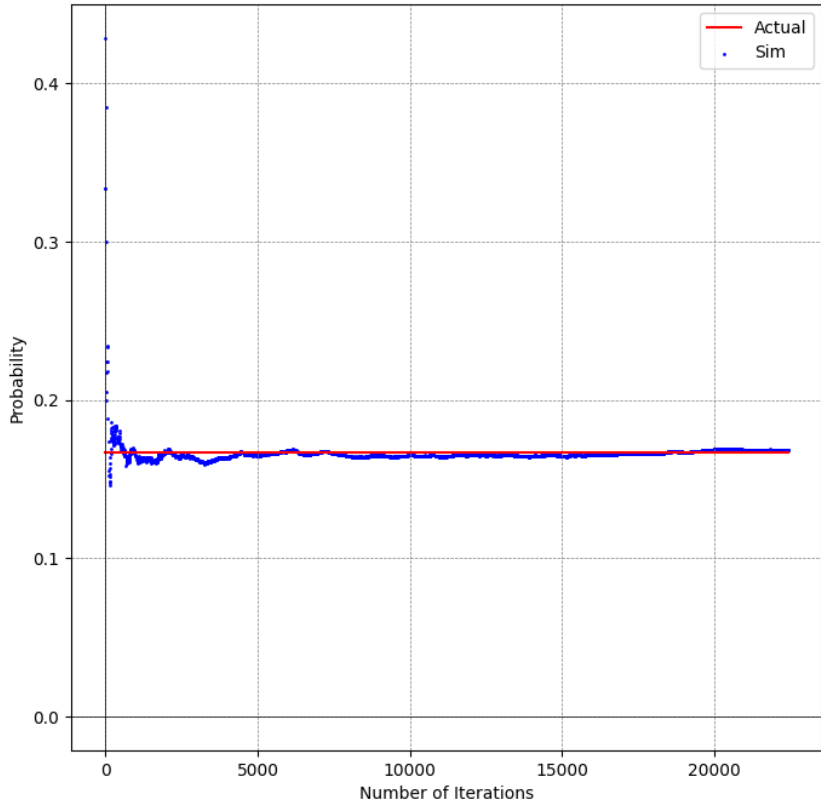


Fig. 0.1: Simulation using ESP32

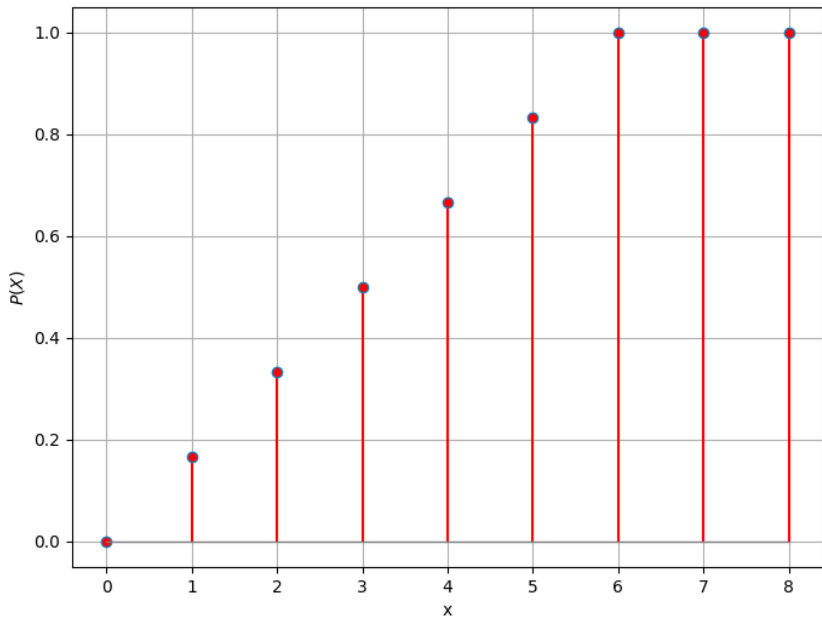


Fig. 0.2: Cumulative Distribution Function