NCERT-11.16.3.21.3

EE24BTECH11065 - Spoorthi yellamanchali

Question:
In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that the student has opted for NSS but not NCC.
Solution: Let us define A and B as shown in the table 0

Event	Denotation
A	probability of choosing NSS
A'	probability of not choosing NSS
B	probability of choosing NCC
B'	probability of not choosing NCC
AB	probability of choosing both NSS and NCC

TABLE 0: Denotations of events

Then, according to the given question,

$$Pr(A) = \frac{32}{60} = \frac{8}{15} \tag{0.1}$$

Let B be the event that a student opts for NCC , then

$$Pr(B) = \frac{30}{60} = \frac{1}{2} \tag{0.2}$$

then, $A.B$ will denote the event that a student opts for both NSS and NCC,and,

$$Pr(AB) = \frac{24}{60} = \frac{2}{5} \tag{0.3}$$

On using the following postulates given in table 0 and the additivity axiom, which is,
Additivity axiom of probability
If A_1,A_2 and A_3,....are mutually exclusive events(disjoint), then,

$$Pr\left(A_iA_j\right) = 0, \forall 1 \le i, j \le n \tag{0.4}$$

then ,

$$Pr(A_1 + A_2 + ...A_n) = Pr(A_1) + Pr(A_2) + ...PR(A_n) \tag{0.5}$$

Then , by applying the additivity axiom on the postulate (5) in the table below, we can write,

$$Pr(A) + Pr(A') = 1 \tag{0.6}$$

	(a)	(b)
Postulate 5	$A + A' = 1$	$A \cdot A' = 0$
Theorem 1	$A + A = A$	$A \cdot A = A$
Theorem 2	$A + 1 = 1$	$A \cdot 0 = 0$
Theorem 3, involution	$(A')' = A$	-
Postulate 3, commutative	$A + B = B + A$	$AB = BA$
Theorem 4, associative	$A + (B + C) = (A + B) + C$	$A(BC) = (AB)C$
Postulate 4, distributive	$A(B + C) = AB + AC$	$A + BC = (A + B)(A + C)$

TABLE 0: Caption

For any two events R and S, we can write,

$$\because S + S' = 1 \tag{0.7}$$

$$RS + RS' = R \tag{0.8}$$

$$\implies Pr(RS) + \Pr(RS') = Pr(R) \tag{0.9}$$

$$\therefore Pr(RS') = Pr(R) - Pr(RS) \tag{0.10}$$

$\therefore$ for the given events A and B, we can write,

$$Pr(AB') = Pr(A) - Pr(AB) \tag{0.11}$$

$$Pr(AB') = \frac{8}{15} - \frac{6}{15} \tag{0.12}$$

$$\therefore Pr(AB') = \frac{2}{15} \approx 0.13333 \tag{0.13}$$

$\therefore$ The probability that a student opts for NSS but not NCC is $\frac{2}{15}$.

Finding probability computationally

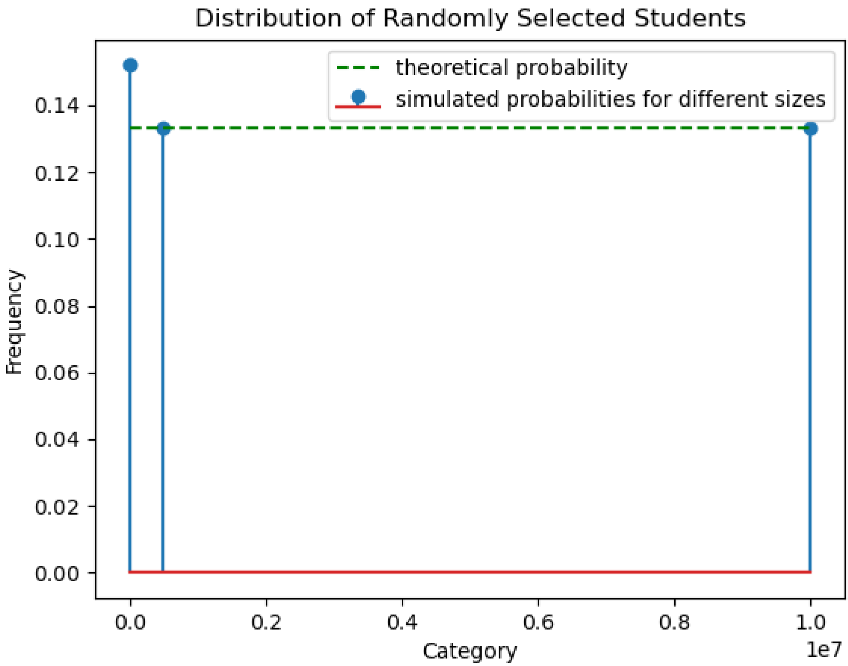
On finding the prbability computationally we get

Simulated Probability of only NSS size(1000): 0.14100

Simulated Probability of only NSS size(500000): 0.13420

Simulated Probability of only NSS size(1000000): 0.13344

As size or number of times selection is done increases, the accuracy increases.


Distribution of Randomly Selected Students
theoretical probability
simulated probabilities for different sizes
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Frequency
0.0
0.2
0.4
0.6
0.8
1.0
1e7
Category