

# NCERT-9.3.11

EE24BTECH11023 - RASAGNA

**Question:** Find the solution of the following differential equation:

$$(x + y) \frac{dy}{dx} = 1$$

**Solution:** From the question,

$$\frac{dy}{dx} = \frac{1}{x + y} \quad (0.1)$$

let,

$$x + y = v \quad (0.2)$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1 \quad (0.3)$$

Substituting into the equation (0.1), we get:

$$\frac{dv}{dx} = \frac{1}{v} + 1 \quad (0.4)$$

$$\int (1 - \frac{1}{1 + v}) dv = \int dx \quad (0.5)$$

$$v - \ln |1 + v| = x + c \quad (0.6)$$

putting  $v = x + y$ ,

$$y - \ln |1 + x + y| = c \quad (0.7)$$

let us assume the initial condition to be  $y = 0$  and  $x = 1$   
substituting in equation (0.7), we get the value of  $c$ ,

$$\therefore c = -\ln 2 \quad (0.8)$$

the final required equation is,

$$x = 2e^y - (1 + y) \quad (0.9)$$

**Logic for writing the code(method of finite difference)**

The slope of a tangent at a point  $(x_0, y_0)$  on the curve is:

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \quad (0.10)$$

By moving infinitesimally small distance( $h$ ) along the tangent , we get another point  $(x_1, y_1)$  .the value of  $(x_1, y_1)$

$$x_1 = x_0 + h \quad (0.11)$$

$$y_1 = y_0 + \frac{dy}{dx}h \quad (0.12)$$

Here,

$$\frac{dy}{dx} = \frac{1}{x_0 + y_0} \quad (0.13)$$

On substituting the value of  $\frac{dy}{dx}$  in equation (0.15) we get,

$$y_1 = y_0 + \frac{1}{x_0 + y_0}h \quad (0.14)$$

Similarly we can obtain n number of points where

$$x_n = x_{n-1} + h \quad (0.15)$$

$$y_n = y_{n-1} + \frac{1}{x_{n-1} + y_{n-1}}h \quad (0.16)$$

Together these points form the curve representing one of the general solutions of the given Differential Equation. The plot is generated by choosing a known point  $(x_0, y_0)$  which satisfies the equation. The value of  $h$  is taken to be very small. We generate a large number of points and then plot them.

