9.4.23

EE24BTECH11052 - Rongali Charan

Question: Solve the differential equation $\frac{dy}{dx} = e^{x+y}$.

1) **Theoretical Solution:**

$$\frac{dy}{dx} = e^{x+y} \tag{1.1}$$
$$\frac{dy}{dx} = e^{x} \cdot e^{y} \tag{1.2}$$
$$\frac{1}{e^y} dy = e^x dx \tag{1.3}$$
$$e^{-y} dy = e^x dx \tag{1.4}$$
$$\int e^{-y} dy = \int e^x dx. \tag{1.5}$$
$$-e^{-y} = e^x + C \tag{1.6}$$
$$e^{-y} = -e^x + C \tag{1.7}$$
$$\implies e^x + e^{-y} = C \tag{1.8}$$
$$x_0 = -2; y_0 = -\ln(2 - e^{-2}) \implies C = 2; \tag{1.9}$$
$$\therefore y = -\ln(-e^x + 2). \tag{1.10}$$

2) **Using method of finite differences:**
The Method of finite Differences is a numerical technique used to approximate solutions to differential equations.
We know that;

$$y = f(x) \tag{2.1}$$
$$\lim_{x \to 0} \frac{f(x+h) - f(x)}{h} = \frac{dy}{dx} \tag{2.2}$$
$$\approx \frac{y_{n+1} - y_n}{h} = e^{x_n + y_n} \tag{2.3}$$
$$\implies y_{n+1} = y_n + h\left(e^{x_n + y_n}\right) \tag{2.4}$$

Now the following steps were used:
a) Initialized $x_0 = -2$ and $y_0 = -\ln(2 - e^{-2})$.
b) h was taken to be 0.01, and number of iterations was taken to be 1000 to ensure accuracy.

c) Now the subsequent points of the curve were generated through iterations by using the below equations;

$$x_{n+1} = x_n + h \tag{2.5}$$

$$y_{n+1} = y_n + h\left(e^{x_n+y_n}\right) \tag{2.6}$$

The below graph shows the comparison between the curve that is obtained theoretically and the simulation curve(numerically generated points through iterations).