

10.3.2.2.1

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Question:

Find out whether the lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ intersect at a point, parallel or coincident

Variable	Description
A	Matrix consisting of coefficients in the linear equation
L	Lower triangular matrix
U	Upper triangular matrix
\mathbf{x}	Solution to the linear equation

TABLE 0: Variables Used

Theoretical Solution:

Let a_1, b_1 , and c_1 and a_2, b_2 , and c_2 be the coefficients of x, y , and 1 in lines 1 and 2 respectively.

We get:

$$\frac{a_1}{a_2} = \frac{5}{7} \quad (0.1)$$

$$\frac{b_1}{b_2} = \frac{-2}{3} \quad (0.2)$$

$$\frac{c_1}{c_2} = \frac{-8}{9} \quad (0.3)$$

$$m_1 = \frac{-a_1}{b_1} = \frac{5}{4} \quad (0.4)$$

$$m_2 = \frac{-a_2}{b_2} = \frac{7}{6} \quad (0.5)$$

As all the ratios aren't equal to each other neither are m_1 and m_2 equal

\therefore The lines intersect at a point

Computational Solution:

The set of linear equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ can be represented by the following equation

$$\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \quad (0.6)$$

Any non-singular matrix can be represented as a product of a lower triangular matrix L and an upper triangular matrix U :

$$A = L \cdot U \quad (0.7)$$

1. Initialization: - Start by initializing \mathbf{L} as the identity matrix $\mathbf{L} = \mathbf{I}$ and \mathbf{U} as a copy of \mathbf{A} .

2. Iterative Update: - For each pivot $k = 1, 2, \dots, n$: - Compute the entries of \mathbf{U} using the first update equation. - Compute the entries of \mathbf{L} using the second update equation.

3. Result: - After completing the iterations, the matrix \mathbf{A} is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where \mathbf{L} is a lower triangular matrix with ones on the diagonal, and \mathbf{U} is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of \mathbf{U})

For each column $j \geq k$, the entries of \mathbf{U} in the k -th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix \mathbf{U} by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of \mathbf{L})

For each row $i > k$, the entries of \mathbf{L} in the k -th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix \mathbf{L} , where each entry in the column is determined by the values in the rows above it.

Using a code we get \mathbf{L}, \mathbf{U} as

Step-by-Step Process:

1. Initial Matrix:

$$\mathbf{A} = \begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix} \quad (0.8)$$

2. Compute \mathbf{U} (Upper Triangular Matrix):

Using the update equation for \mathbf{U} :

$$U_{11} = A_{11} = 5, \quad U_{12} = A_{12} = -4 \quad (0.9)$$

For U_{22} :

$$U_{22} = A_{22} - L_{21} \cdot U_{12} = 6 - \frac{7}{5} \cdot (-4) = \frac{58}{5} \quad (0.10)$$

3. Compute \mathbf{L} (Lower Triangular Matrix):

Using the update equation for \mathbf{L} :

$$L_{21} = \frac{A_{21}}{U_{11}} = \frac{7}{5} \quad (0.11)$$

The final L matrix is:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \quad (0.12)$$

4. Solving the System:

Using the equations $Ly = \mathbf{b}$ and $Ux = \mathbf{y}$:

• Forward Substitution:

$$\begin{pmatrix} 1 & 0 \\ \frac{7}{5} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -8 \\ 9 \end{pmatrix} \quad (0.13)$$

Solving gives:

$$y_1 = -8, \quad y_2 = \frac{101}{5} \quad (0.14)$$

• Backward Substitution:

$$\begin{pmatrix} 5 & -4 \\ 0 & \frac{58}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -8 \\ \frac{101}{5} \end{pmatrix} \quad (0.15)$$

Solving gives:

$$x_2 = \frac{101}{58}, \quad x_1 = \frac{-6}{29} \quad (0.16)$$

Thus, the solution is:

$$\mathbf{x} = \begin{pmatrix} \frac{-6}{29} \\ \frac{101}{58} \end{pmatrix} \quad (0.17)$$

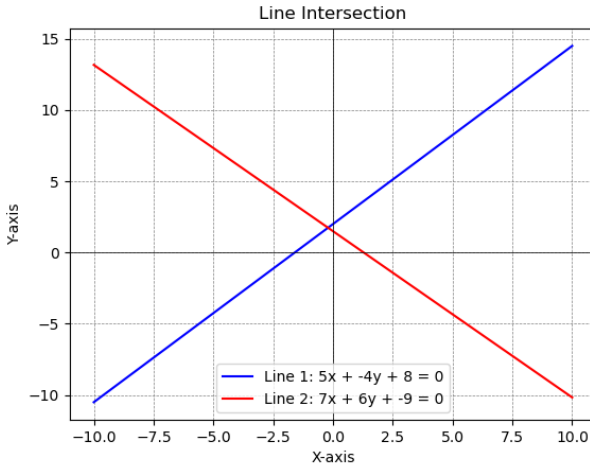


Fig. 0.1: Solution to set of linear equations