NCERT - 12.9.4.11

EE1003

Homa Harshitha Vuddanti
(EE24BTECH11062)

Question: Find the particular solution of the differential equation $\left(x^3+x^2+x+1\right)\frac{dy}{dx} = 2x^2+x; y=1$ when $x=0$

Theoretical Solution:

Given,

$$\frac{dy}{dx} = \frac{2x^2+x}{x^3+x^2+x+1} \tag{1}$$

$$\frac{dy}{dx} = \frac{2x^2+x}{(x+1)(x^2+1)} \tag{2}$$

$$\tag{3}$$

Splitting into partial fractions,

$$\frac{dy}{dx} = \frac{2x^2+x}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{3x-1}{2(x^2+1)} \tag{4}$$

$$\int dy = \int \frac{1}{2(x+1)}dx + \int \frac{3x-1}{2(x^2+1)}dx \tag{5}$$

$$y = \frac{1}{2}\ln|x+1| + \frac{3}{4}\ln\left|x^2+1\right| - \frac{1}{2}\tan^{-1}x + c \tag{6}$$

Substituting $(0,1)$ in the equation, we get the value of $c=1$

$$y = \frac{1}{2}\ln|x+1| + \frac{3}{4}\ln\left|x^2+1\right| - \frac{1}{2}\tan^{-1}x + 1 \tag{7}$$

Solving by Bilinear transformation : Applying Laplace transformation

$$\mathcal{L}(y') = \mathcal{L}\left(\frac{2x^2+x}{x^3+x^2+x+1}\right) \tag{8}$$

$$\tag{9}$$

Let

$$\mathcal{L}\left(\frac{2x^2+x}{x^3+x^2+x+1}\right) = X(s) \tag{10}$$

$$sY(s) - y(0) = X(s) \tag{11}$$

$$sY(s) = X(s) + 1 \tag{12}$$

From bilinear transformation,

$$s = \frac{2}{h}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) \tag{13}$$

Substituting in (**??**),

$$Y(s) - z^{-1}Y(s) = \frac{h}{2}\left(X(s) + z^{-1}X(s) + 1 + z^{-1}\right) \tag{14}$$

Applying inverse $\mathcal{Z}$-transformation,

$$y_{n+1} - y_n = \frac{h}{2}\left(\frac{2x_{n+1}^2 + x_{n+1}}{x_{n+1}^3 + x_{n+1}^2 + x_{n+1} + 1} + \frac{2x_n^2 + x_n}{x_n^3 + x_n^2 + x_n + 1} + \delta[n+1] + \delta[n]\right) \tag{15}$$

$$x_{n+1} = x_n + h \tag{16}$$

Trapezoidal Rule:
Area is given by

$$\int_a^b f(x)\,dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{17}$$

$$h = \frac{b-a}{n} \tag{18}$$

$$A_{n+1} = A_n + \frac{h}{2}\left(f\left(x_{n+1} + f\left(x_n\right)\right)\right) \tag{19}$$

Substituting, we finally get

$$y_{n+1} = y_n + \frac{h}{2}\left(\frac{2x_{n+1}^2 + x_{n+1}}{x_{n+1}^3 + x_{n+1}^2 + x_{n+1} + 1} + \frac{2x_n^2 + x_n}{x_n^3 + x_n^2 + x_n + 1}\right) \tag{20}$$

$$x_{n+1} = x_n + h \tag{21}$$

Plotting: Taking the initial point (x_0, y_0) as $(0, 1)$ and the value of $h = 0.01$

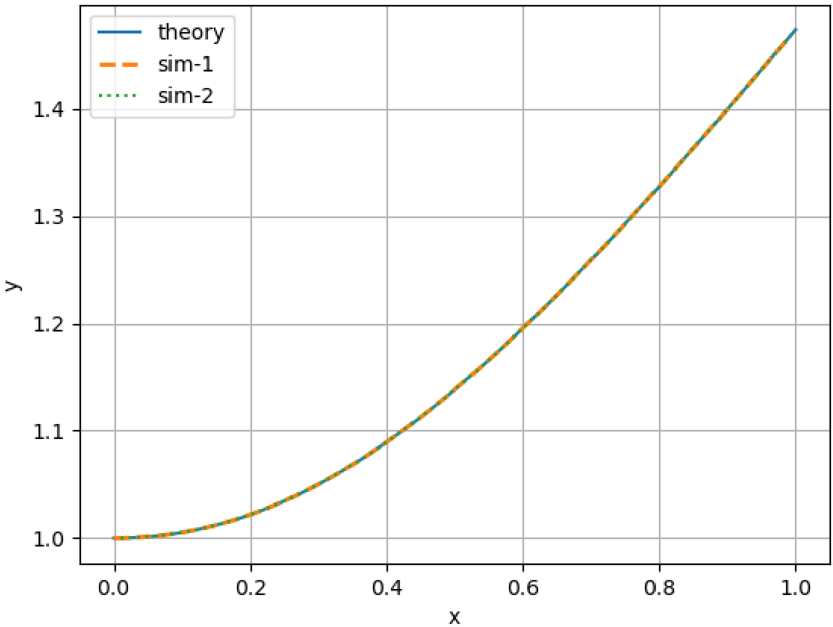

Fig. 0: Plot