NCERT-11.16.3.13.1

EE24BTECH11023 - RASAGNA

Question: Given:

$$Pr(A) = \frac{1}{3} \tag{0.1}$$

$$Pr(B) = \frac{1}{5} \tag{0.2}$$

$$Pr(AB) = \frac{1}{15} \tag{0.3}$$

Then find the value of $P(A + B)$.

Theoritical Solution:

Let A and B be two sets ;

$$A = AB^{'} + AB \tag{0.4}$$

$$B = A^{'}B + AB \tag{0.5}$$

Adding the equations (0.4) and (0.5) we get;

$$A + B = A^{'}B + AB^{'} + AB \tag{0.6}$$

Here $A^{'}B, AB^{'}, AB$ are disjoint.

$$\therefore Pr(A + B) = Pr(A) + Pr(B) - Pr(AB) \tag{0.7}$$

Substituting the values of $Pr(A), Pr(B)$ and $Pr(A \cap B)$ in the equation (0.7) we get the value of $Pr(A + B)$ as,

$$Pr(A + B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} \tag{0.8}$$

$$\therefore Pr(A + B) = \frac{7}{15} \tag{0.9}$$

Computational Solution

Let X, Y, Z be an indicator random variables of the event A, B, AB.

Where X, Y, Z are defined as:

$$X = \begin{cases} 1 & ; A \\ 0 & ; A^{'} \end{cases} \tag{0.10}$$

$$Y = \begin{cases} 1 & ; B \\ 0 & ; B^{'} \end{cases} \tag{0.11}$$

$$Z = \begin{cases} 1 & ; AB \\ 0 & ; (AB)' \end{cases} \tag{0.12}$$

The PMF of the random variables X, Y, Z are:

$$p_X(n) = \begin{cases} p_1 & ; n = 1 \\ 1 - p_1 & ; n = 0 \end{cases} \tag{0.13}$$

$$p_Y(n) = \begin{cases} p_2 & ; n = 1 \\ 1 - p_2 & ; n = 0 \end{cases} \tag{0.14}$$

$$p_Z(n) = \begin{cases} p_3 & ; n = 1 \\ 1 - p_3 & ; n = 0 \end{cases} \tag{0.15}$$

Here,

$$p_1 = \frac{1}{3} \tag{0.16}$$

$$p_2 = \frac{1}{5} \tag{0.17}$$

$$p_3 = \frac{1}{15} \tag{0.18}$$

Now let us define another random variable K.
Where,

$$K = X + Y - Z \tag{0.19}$$

Z can never be 0 whenever X and Y are 1 (Because (1).(1) is never equal to 0).
So, K is also an Indicator Random variable.
Its PMF is defined as:

$$p_K(n) = \begin{cases} p & ; n = 1 \\ 1 - p & ; n = 0 \end{cases} \tag{0.20}$$

From (0.20),

$$E(K) = E(X + Y - Z) \tag{0.21}$$

$$E(K) = E(X) + E(Y) - E(Z) \tag{0.22}$$

$$1.(p) + 0.(1 - p) = 1.(p_1) + 0.(1 - p_1) + 1.(p_2) + 0.(1 - p_2) - 1.(p_3) - 0.(1 - p_3) \tag{0.23}$$

$$p = p_1 + p_2 - p_3 \tag{0.24}$$

Here

$$\Pr(A) = p_1 \tag{0.25}$$

$$\Pr(B) = p_2 \tag{0.26}$$

$$\Pr(AB) = p_3 \tag{0.27}$$

Therefore, from the axiom and equation (0.25)

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{0.28}$$

$$p = \Pr(A+B) \tag{0.29}$$

$$\Pr(A+B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{15} \tag{0.30}$$

$$\therefore \Pr(A+B) = \frac{7}{15} \tag{0.31}$$

Here is the graph (Stem plot) of probabilities $Pr(A), Pr(B), Pr(AB)$ and $Pr(A + B)$
Blue colour represents theoritical values of the probabilities.
Brown colour represents Computed values of the probabilities.