10.3.2.4.2

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Is the following pair of linear equations consistent or inconsistent? If consistent, obtain the solution graphically.

$$x - y = 8$$
$$3x - 3y = 16$$

Solution:

We represent the system in matrix form:

$$A = \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ 16 \end{pmatrix}, \quad x = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{1}$$

LU factorization using update equaitons

Given a matrix **A** of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

Step-by-Step Procedure:

1. Initialization: - Start by initializing **L** as the identity matrix **L** = **I** and **U** as a copy of **A**.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix **A** is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.

Using a code we get L,U as

$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}. \tag{2}$$

Solving $Ax = b$

Forward Substitution: Solve $Ly = b$:

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \end{pmatrix}. \tag{3}$$

From the first row:

$$y_1 = 8. \tag{4}$$

From the second row:

$$3y_1 + y_2 = 16 \tag{5}$$

$$3(8) + y_2 = 16 \tag{6}$$

$$y_2 = -8. \tag{7}$$

Thus:

$$y = \begin{pmatrix} 8 \\ -8 \end{pmatrix}. \tag{8}$$

Back Substitution: Solve $Ux = y$:

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \end{pmatrix}. \tag{9}$$

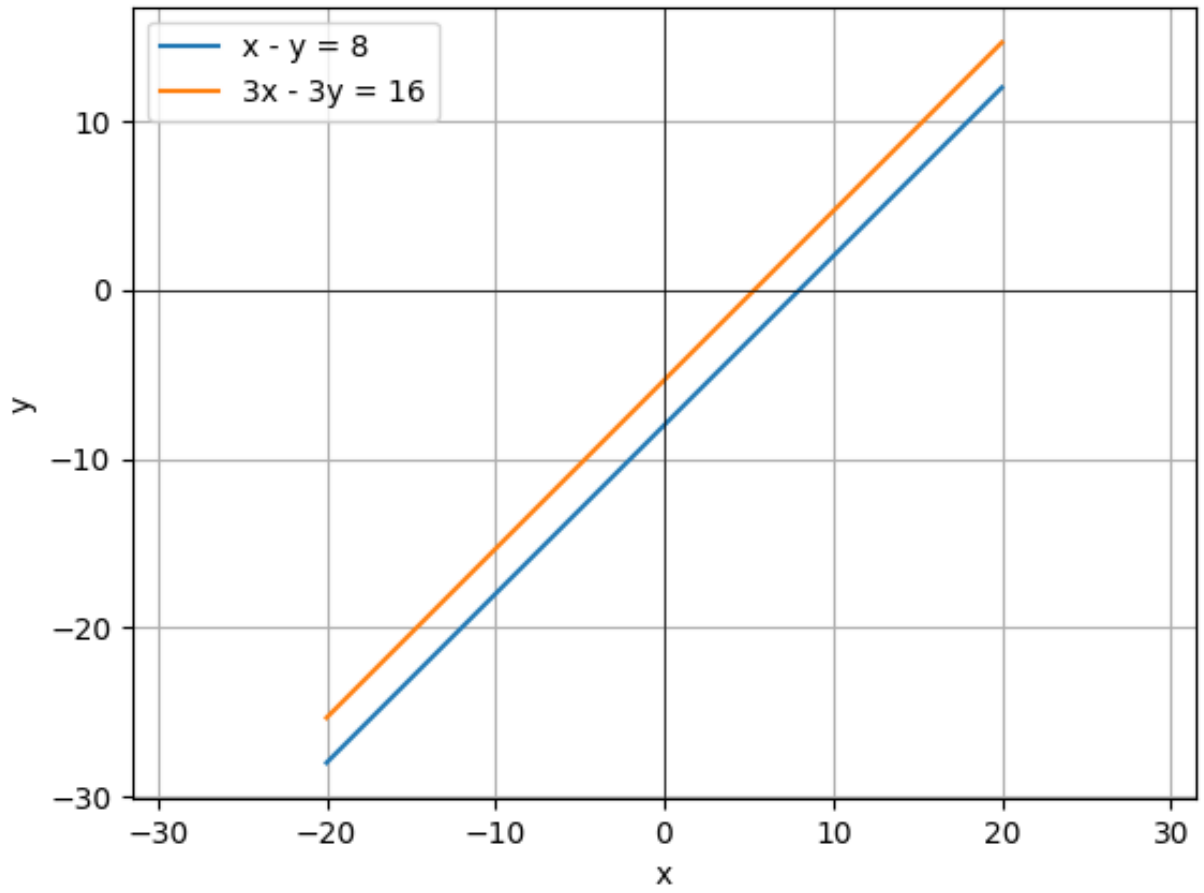
From the first row:

$$x - y = 8. \tag{10}$$

From the second row:

$$0 = -8 \quad \text{(contradiction)}. \tag{11}$$

The system of equations is inconsistent and has no solution. The matrix A is singular (non-invertible), as indicated by the zero u_{22} in the U-matrix.


x - y = 8
3x - 3y = 16
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