

12.6.6.14

EE24BTECH11029 - J SHRETHAN REDDY

Question:

Find the absolute maximum and minimum values of the function f given by

$$f(x) = \cos^2 x + \sin x, x \in [0, \pi]$$

Answer

Theoretical Solution

$$f(x) = \cos^2 x + \sin x \quad (0.1)$$

$$f'(x) = -\sin 2x + \cos x \quad (0.2)$$

$$f'(x) = \cos x(1 - 2 \sin x) \quad (0.3)$$

solve $f'(x) = 0$

$$\cos x = 0 \quad \text{or} \quad 1 - 2 \sin x = 0 \quad (0.4)$$

Case 1: $\cos x = 0$

$$x = \frac{\pi}{2} \quad (0.5)$$

Case 2: $1 - 2 \sin x = 0$

$$x = \frac{\pi}{6} \quad \text{or} \quad x = \frac{5\pi}{6} \quad (0.6)$$

critical points are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

endpoints are $x = 0, \pi$ Evaluation of $f(x)$ at critical points and endpoints.

At $x = 0$

$$f(0) = \cos^2 0 + \sin 0 = 1 \quad (0.7)$$

At $x = \pi$

$$f(\pi) = \cos^2 \pi + \sin \pi = 1 \quad (0.8)$$

At $x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \cos^2 \frac{\pi}{2} + \sin \frac{\pi}{2} \quad (0.9)$$

$$f\left(\frac{\pi}{2}\right) = 1 \quad (0.10)$$

At $x = \frac{\pi}{6}$

$$f\left(\frac{\pi}{6}\right) = \cos^2 \frac{\pi}{6} + \sin \frac{\pi}{6} \quad (0.11)$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} \quad (0.12)$$

$$f\left(\frac{\pi}{6}\right) = \frac{5}{4} \quad (0.13)$$

At $x = \frac{5\pi}{6}$

$$f\left(\frac{5\pi}{6}\right) = \cos^2 \frac{5\pi}{6} + \sin \frac{5\pi}{6} \quad (0.14)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} \quad (0.15)$$

$$f\left(\frac{5\pi}{6}\right) = \frac{5}{4} \quad (0.16)$$

absolute maxima is $\frac{5}{4}$ at $x = \frac{\pi}{6}, \frac{5\pi}{6}$
 absolute minima is 1 at $x = 0, \frac{\pi}{2}, \pi$

Numerical method

Finding maximum and minimum value of function can be done using **Gradient Ascent and Descent method**

maximum value

$$x_{n+1} = x_n + \alpha f'(x) \quad (0.17)$$

$$x_{n+1} = x_n + \alpha (\cos x (1 - 2 \sin x)) \quad (0.18)$$

minimum value

$$x_{n+1} = x_n - \alpha f'(x) \quad (0.19)$$

$$x_{n+1} = x_n - \alpha (\cos x (1 - 2 \sin x)) \quad (0.20)$$

Where α is learning rate. Taking

$$h = 0.001 \quad (0.21)$$

$$\alpha = 0.001 \quad (0.22)$$

we have

$$x_{max} = 0.5230988543174608, y_{max} = 1.2499998125049454 \quad (0.23)$$

$$x_{min} = 1.5702963169279682, y_{min} = 1.0000001250049153 \quad (0.24)$$

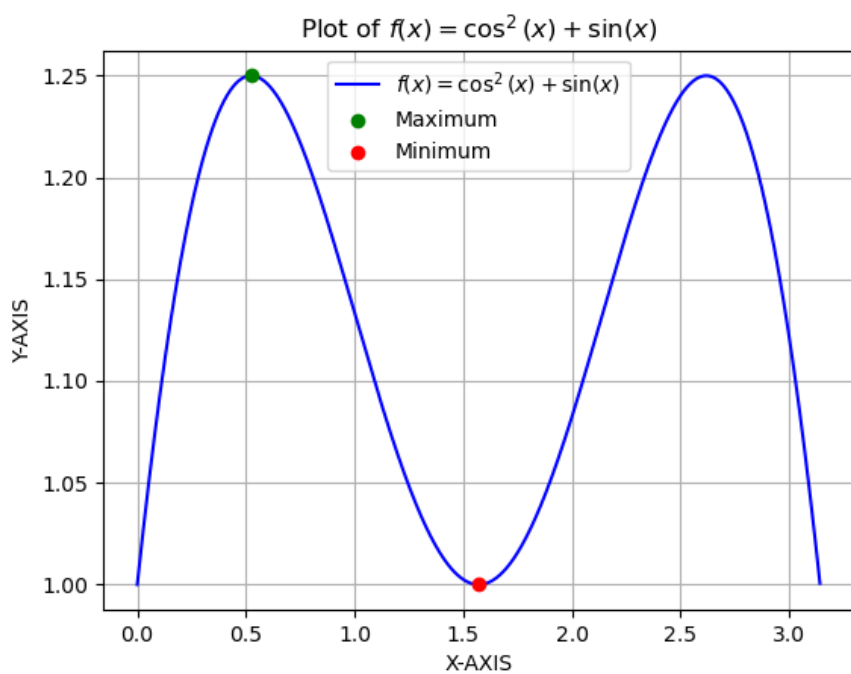


Fig. 0.1: plot