12.6.5.3.4

EE24BTECH11010 - Balaji B

Question:
Find the local maximum and local minimum values of the function $f(x) = \sin x - \cos x$ for x the interval $[0, 2\pi]$.

Theoretical Solution:
To find critical points, we equalize $\frac{df(x)}{dx} = 0$. Let $y = f(x)$

$$\frac{dy}{dx} = \cos x + \sin x \tag{0.1}$$

$$\cos x + \sin x = 0 \tag{0.2}$$

$$\tan x = -1 \tag{0.3}$$

For $x \in [0, 2\pi]$, $\tan x = -1$ for $x = \frac{3\pi}{4}$ and $\frac{7\pi}{4}$.

To find maxima and minima, we should double the derivative of the function $f(x)$. On double derivating we get,

$$\frac{d^2y}{dx^2} = \cos x - \sin x \tag{0.4}$$

On applying the critical point $x = \frac{3\pi}{4}$, we get

$$\frac{d^2y}{dx^2} = -\sqrt{2} \tag{0.5}$$

As the double derivative is negative at $x = \frac{3\pi}{4}$, so we have maxima at that point.

Similarly, when we apply the critical point $x = \frac{7\pi}{4}$, we get

$$\frac{d^2y}{dx^2} = \sqrt{2} \tag{0.6}$$

As the double derivative is positive at $x = \frac{7\pi}{4}$, so we have minima at that point.
$\therefore$ Local maxima at $x = \frac{3\pi}{4}$
Local minima at $x = \frac{7\pi}{4}$

Computational Solution:
We use the gradient descent method to find the local maximum and local minimum of the given function.

$$f'(x_n) = \cos(x_n) + \sin(x_n) \tag{0.7}$$

Gradient decent to find local minimum

$$x_{n+1} = x_n - \eta f'(x_n) \tag{0.8}$$
$$x_{n+1} = x_n - \eta(\cos(x_n) + \sin(x_n)) \tag{0.9}$$

Gradient ascent to find local maximum,

$$x_{n+1} = x_n + \eta f'(x_n) \tag{0.10}$$
$$x_{n+1} = x_n + \eta(\cos(x_n) + \sin(x_n)) \tag{0.11}$$

Where η is the learning rate.

Assuming,

$$\eta = 0.1 \tag{0.12}$$
$$\text{tolerance} = 1e - 6 \tag{0.13}$$
$$x_0 = 0.0 \tag{0.14}$$

We get,

$$x_{min} = 2.356194, \quad y_{min} = 1.414214 \tag{0.15}$$
$$x_{max} = 5.497788, \quad y_{max} = -1.414214 \tag{0.16}$$

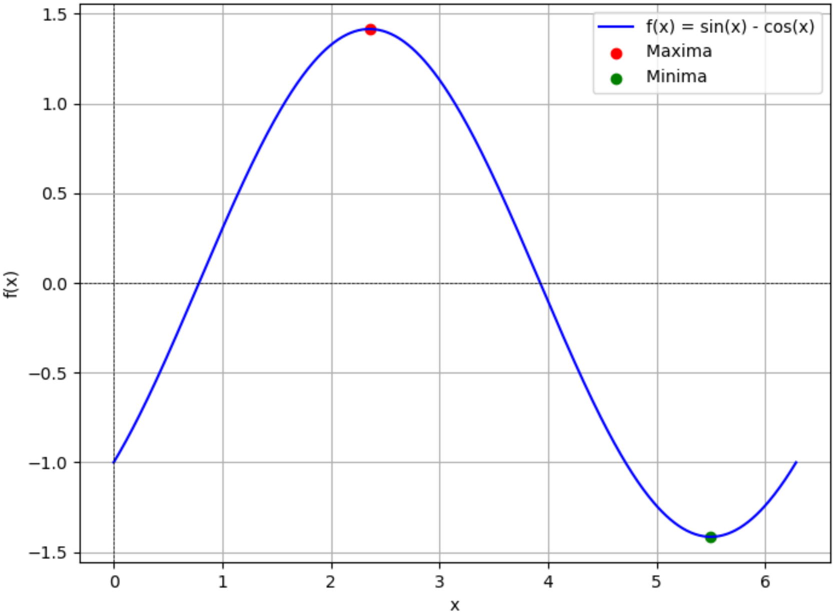

Fig. 0.1: Plot of local maximum and minimum