11.16.3.15.2

EE24BTECH11019 - Dwarak A

Question:

If A and B are events such that

$$P(A) = 0.25 \tag{0.1}$$
$$P(B) = 0.5 \tag{0.2}$$
$$P(AB) = 0.125 \tag{0.3}$$

Find

$$P(A'B') \tag{0.4}$$

Solution:

Theoretical Solution (Boolean Logic):

For 2 Boolean variables A and B, the axioms of Boolean Algebra are defined as:

$$A + A = A \tag{0.5}$$
$$AA = A \tag{0.6}$$
$$A + A' = 1 \tag{0.7}$$
$$AA' = 0 \tag{0.8}$$
$$AB = BA \tag{0.9}$$
$$A + B = B + A \tag{0.10}$$
$$(A + B) + C = A + (B + C) \tag{0.11}$$
$$(AB)C = A(BC) \tag{0.12}$$
$$A(B + C) = AB + AC \tag{0.13}$$
$$A + BC = (A + B)(A + C) \tag{0.14}$$
$$P(1) = 1 \tag{0.15}$$
$$P(A + B) = P(A) + P(B), \text{ if } P(AB) = 0 \tag{0.16}$$

De Morgan's Theorems:

$$(A + B)' = A'B' \tag{0.17}$$
$$(AB)' = A' + B' \tag{0.18}$$

Using these axioms,

$$A = A(B + B') \tag{0.19}$$
$$= AB + AB' \tag{0.20}$$
$$B = (A + A')B \tag{0.21}$$
$$= AB + A'B \tag{0.22}$$
$$P(A) = P(AB) + P(AB') \tag{0.23}$$
$$P(B) = P(AB) + P(A'B) \tag{0.24}$$

On adding (0.20) and (0.22),

$$A + B = AB + AB + AB' + A'B \tag{0.25}$$
$$A + B = AB + AB' + A'B \tag{0.26}$$
$$P(A + B) = P(AB + AB' + A'B) \tag{0.27}$$
$$P(A + B) = P(AB) + P(AB') + P(A'B) \tag{0.28}$$
$$P(A + B) = P(AB) + P(A) - P(AB) + P(B) - P(AB) \tag{0.29}$$
$$\implies P(A + B) = P(A) + P(B) - P(AB) \tag{0.30}$$

Using (0.26) and (0.17),

$$(A + B)' = A'B' \tag{0.31}$$
$$P((A + B)') = P(A'B') \tag{0.32}$$
$$1 - P(A + B) = P(A'B') \tag{0.33}$$

Using (0.33) and (0.30),

$$P(A'B') = 1 - (P(A) + P(B) - P(AB)) \tag{0.34}$$
$$P(A'B') = 1 + P(AB) - P(A) - P(B) \tag{0.35}$$

Using the given values of $P(A)$, $P(B)$ and $P(AB)$,

$$P(A'B') = 1 + 0.125 - 0.25 - 0.5 \tag{0.36}$$
$$P(A'B') = 0.375 \tag{0.37}$$

Therefore, the value of $P(A'B')$ is 0.375.

Computational Solution:

Let X_1 be an indicator random variable of the event A.
X_1 is defined as:

$$X_1 = \begin{cases} 1, & A \\ 0, & A' \end{cases} \tag{0.38}$$

Let X_2 be the indicator random variable of the event B.

X_2 is defined as:

$$X_2 = \begin{cases} 1, & B \\ 0, & B' \end{cases} \tag{0.39}$$

Let X_3 be the indicator random variable of the event AB.
X_3 is defined as:

$$X_3 = \begin{cases} 1, & AB \\ 0, & (AB)' \end{cases} \tag{0.40}$$

The PMF of the random variable X_1 is:

$$p_{X_1}(n) = \begin{cases} p_1, & n = 1 \\ 1 - p_1, & n = 0 \end{cases} \tag{0.41}$$

The PMF of the random variable X_2 is:

$$p_{X_2}(n) = \begin{cases} p_2, & n = 1 \\ 1 - p_2, & n = 0 \end{cases} \tag{0.42}$$

The PMF of the random variable X_3 is:

$$p_{X_3}(n) = \begin{cases} p_3, & n = 1 \\ 1 - p_3, & n = 0 \end{cases} \tag{0.43}$$

where,

$$p_1 = 0.25 \tag{0.44}$$
$$p_2 = 0.50 \tag{0.45}$$
$$p_3 = 0.125 \tag{0.46}$$

Let Y be the random variable which is defined as follows:

$$Y = 1 - X_1 - X_2 + X_3 \tag{0.47}$$

But we know that Y is another indicator random variable whose PMF is defined as:

$$p_Y(n) = \begin{cases} p, & n = 1 \\ 1 - p, & n = 0 \end{cases} \tag{0.48}$$

$$E(Y) = E(1 - X_1 - X_2 + X_3) \tag{0.49}$$
$$E(Y) = E(1) - E(X_1) - E(X_2) + E(X_3) \tag{0.50}$$
$$1.(p) + 0.(1 - p) = 1.(1) - 1.(p_1) - 1.(p_2) + 1.(p_3) \tag{0.51}$$
$$p = 1 - p_1 - p_2 + p_3 \tag{0.52}$$

Through our definition, we know that,

$$P(A) = p_1 \tag{0.53}$$
$$P(B) = p_2 \tag{0.54}$$
$$P(AB) = p_3 \tag{0.55}$$

Therefore, by comparison of the axiom

$$P(A'B') = 1 - P(A) - P(B) + P(AB) \tag{0.56}$$
$$P(A'B') = 1 - 0.25 - 0.50 + 0.125 \tag{0.57}$$
$$\implies P(A'B') = 0.375 \tag{0.58}$$