NCERT - 12.6.5.28

EE1003

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Question: For all real values of x find the minimum and maximum value of $\frac{1-x+x^2}{1+x+x^2}$.
Theoretical solution:
Given,

$$f(x) = \frac{1-x+x^2}{1+x+x^2} \tag{1}$$

For finding the values, we first find the critical points where $f'(x) = 0$.

$$f'(x) = \frac{(2x-1)\left(1+x+x^2\right) - (2x+1)\left(1-x+x^2\right)}{(1+x+x^2)^2} \tag{2}$$

$$f'(x) = \frac{2\left(x^2-1\right)}{(1+x+x^2)^2} = 0 \tag{3}$$

Solving, we get $x = 1$ and $x = -1$. To find the minimum and maximum, we do the second derivative test.

$$f''(x) = \frac{(4x)\left(1+x+x^2\right)^2 - 4\left(x^2-1\right)\left(1+x+x^2\right)(2x+1)}{(1+x+x^2)^4} \tag{4}$$

$$\tag{5}$$

$f''(1) > 0$, so it is the minimum, while $f''(-1) < 0$ so it is the maximum. Therefore, the minimum value is $f(1) = \frac{1}{3}$, which occurs at $x = 1$ and the maximum value is $f(-1) = 3$.
Gradient Descent method :
We can use the gradient descent method to find the minimum of our curve. The algorithm iterates as follows:

$$x_{n+1} = x_n - \alpha f'(x) \tag{6}$$

Here, α controls the step size and we stop when the difference between x_{n+1} and x_n becomes very small beyond a convergence threshold.

$$x_{n+1} = x_n - \alpha\left(\frac{2\left(x_n^2-1\right)}{(1+x_n+x_n^2)^2}\right) \tag{7}$$

Gradient ascent method: We can find the maximum value of the function using gradient ascent method.

$$x_{n+1} = x_n + \alpha f'(x) \tag{8}$$

$$x_{n+1} = x_n + \alpha \left(\frac{2\left(x_n^2 - 1\right)}{\left(1 + x_n + x_n^2\right)^2} \right) \tag{9}$$

Plotting:
Taking

$$x_0 = 0.5 \tag{10}$$

$$\alpha = 0.01 \tag{11}$$

$$threshold = 10^{-6} \tag{12}$$

$$n = 1000 \tag{13}$$

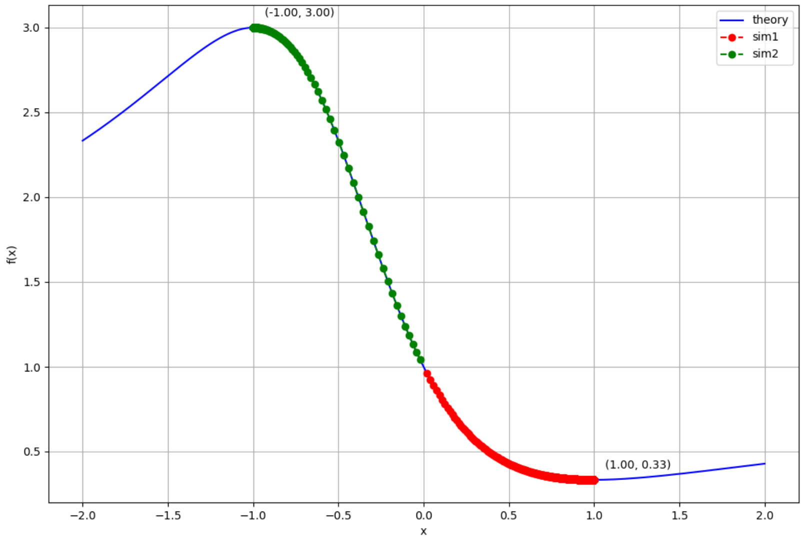

Fig. 0: Plot