9.7.13

EE24BTECH11003 - Akshara Sarma Chennubhatla

Question: Solve the differential equation $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$, with the point $\left(\frac{\pi}{2}, 0\right)$ lying on the graph

Solution:
Theoretical Solution:

$$\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x \tag{1}$$
$$\tag{2}$$

This is a linear differential equation. So the Integrating factor is,

$$I.F = e^{\int \cot x} \tag{3}$$
$$I.F = e^{\log \sin x} \tag{4}$$
$$I.F = \sin x \tag{5}$$
$$\tag{6}$$

Multiplying both sides of the equation by the integrating factor and integrating,

$$\int \sin x\left(\frac{dy}{dx} + y\cot x\right)dx = \int \sin x 4x\,\mathrm{cosec}\,x dx \tag{7}$$
$$y\sin x = \int 4x dx \tag{8}$$
$$y\sin x = 2x^2 + C \tag{9}$$
$$\tag{10}$$

Since $\left(\frac{\pi}{2}, 0\right)$ satisfies the function,

$$0\,(1) = 2\left(\frac{\pi}{2}\right)^2 + C \tag{11}$$
$$\implies C = -\frac{\pi^2}{2} \tag{12}$$
$$\tag{13}$$

So the function $y(x)$ is,

$$y \sin x = 2x^2 - \frac{\pi^2}{2} \tag{14}$$

$$\implies y = \frac{2x^2}{\sin x} - \frac{\pi^2}{2\sin x} \tag{15}$$

$$\tag{16}$$

Simulated Solution:

By first principle of derivatives,

$$y'(x) = \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} \tag{17}$$

$$y(x+h) = y(x) + hy'(x) \tag{18}$$

Given differential equation can be written as,

$$y' = 4x \operatorname{cosec} x - y \cot x \tag{19}$$

So, by using the method of finite diffferences,

$$y_1 = y_0 + h(4x_0 \operatorname{cosec} x_0 - y_0 \cot x_0) \tag{20}$$

Similarly, by iterating for $y_2, y_3 ...$, The general difference equation is:

$$y_{n+1} = y_n + h(4x_n \operatorname{cosec} x_n - y_n \cot x_n) \tag{21}$$

Below is the simulated plot and the theoretical plot for given curve based on initial conditions, obtained by iterating through the values of x with step size of h

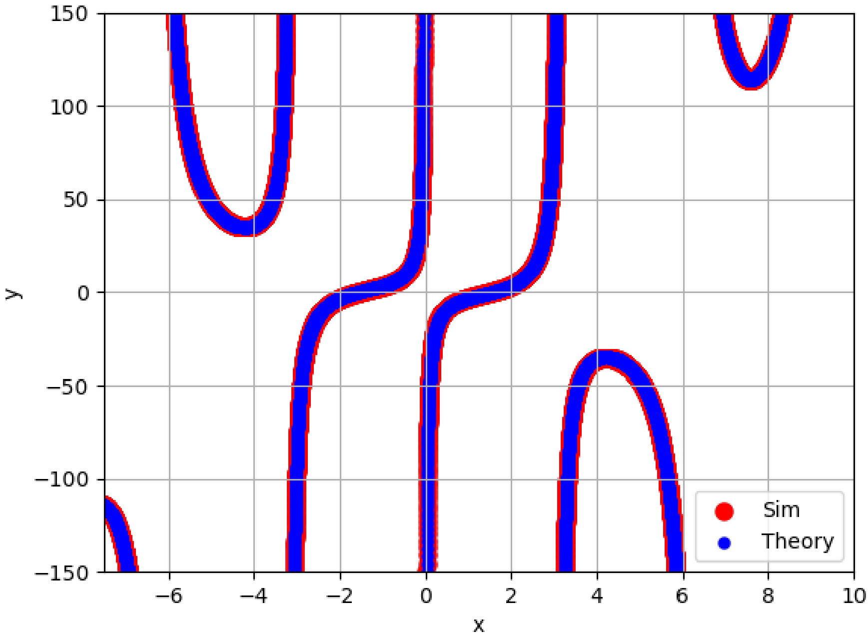

Fig. 1: Plot of the solution of $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$