12.8.ex.9

EE24BTECH11028 - Jadhav Rajesh

QUESTION: Using integration find the area of region bounded by the triangle whose verticle are $(1, 0), (2, 2)$ and (3.1).

SOLUTION:
THEORETICAL SOLUTION Let $A(1, 0), B(2, 2)$ and $C(3.1)$ be the vertices of a triangle $ABC\,(fig8.18)$.

Area of $\triangle ABC$ = Area of $\triangle ABD$ + Area of trapezium $BDEC-$ Area of $\triangle AEC$
Now equation of the sides AB, BC and CA given by

$$y = 2\left(x-1\right), y = 4 - x, y = \frac{1}{2}\left(x-1\right), respectively. \tag{0.1}$$

Hence, area of $\triangle ABC$

$$= \int_1^2 2\left(x-1\right)dx + \int_2^3 \left(4-x\right)dx - \int_1^3 \frac{x-1}{2}dx \tag{0.2}$$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^3 \tag{0.3}$$

$$= 2\left[\left(\frac{2^2}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right] + \left[\left(4*3 - \frac{3^2}{2}\right) - \left(4*2 - \frac{2^2}{2}\right)\right] - \frac{1}{2}\left[\left(\frac{3^2}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right] \tag{0.4}$$

$$= \frac{3}{2} \tag{0.5}$$

Computational Solution:
Using the trapezoidal rule to get the area
The trapezoidal rule is as follows.

$$A = \int_a^b f(x)\, dx \approx h\left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b)\right) \tag{0.6}$$

$$h = \frac{b-a}{n} \tag{0.7}$$

$$A = j_n, \quad where, \quad j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{0.8}$$

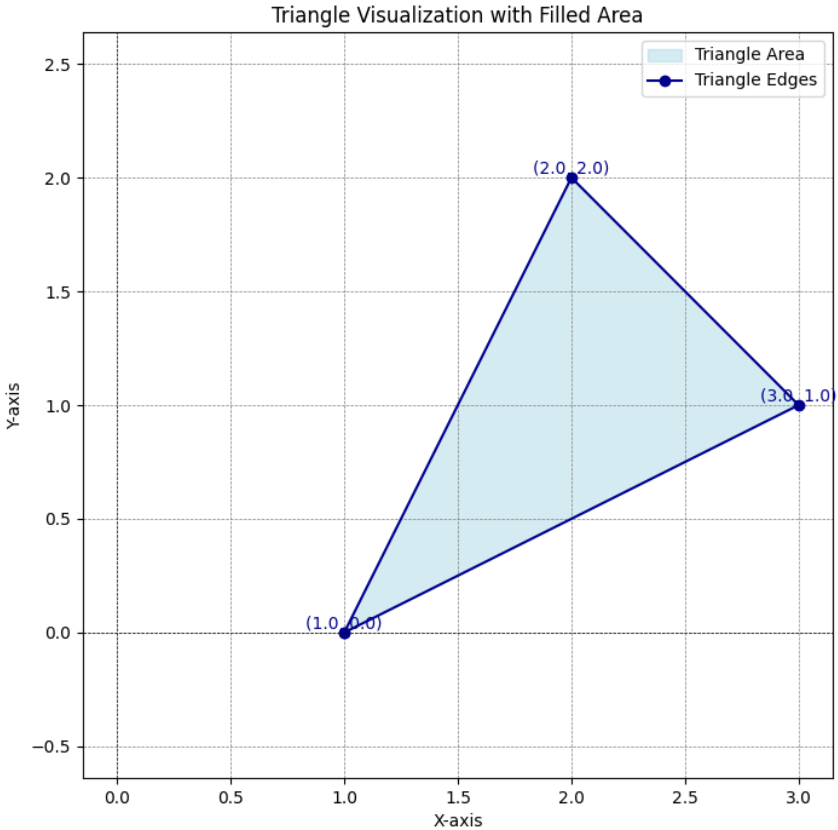
$$j_{i+1} = j_i + h\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{0.9}$$

$$x_{i+1} = x_i + h \tag{0.10}$$

$$h = \frac{1}{30000} \tag{0.11}$$

$$n = 30000 \tag{0.12}$$

Using the code answer obtained is $A = 1.5000 sq$.units


Triangle Visualization with Filled Area
Triangle Area
Triangle Edges
(2.0, 2.0)
(3.0, 1.0)
(1.0, 0.0)
Y-axis
X-axis
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0