# 10.4.3.1.1

EE24BTECH11001 - Aditya Tripathy

**Question**:
Find the roots of the equation $2x^2 - 3x + 5 = 0$
**Solution:**
Below are two methods to find the solutions of this quadratic equation,
Fixed Point Iterations: Take an initial guess $x_0$. The update difference equation will use the following function:

$$x = g(x) \tag{0.1}$$

For our problem,

$$g(x) = \frac{2}{3}x^2 + \frac{5}{3} \tag{0.2}$$

Now the update equation will be,

$$x_{n+1} = g(x_n) \tag{0.3}$$

When we try to run the iterations however, we realize that whatever be the initial guess, the subsequent updated values grow without bound. This is becaue of the following theorem

**Theorem:** (0.4)

Let $x = s$ be a solution of $x = g(x)$ and suppose that $g$ has a continuous derivative in some interval $J$ containing $s$. Then if $|g'| \leq K < 1$ in $J$, the iteration process defined above converges for any $x_0$ in $J$. The limit of the sequence $[x_n]$ is s

Since there is no solution (evident by quadratic formula) there exists no interval J for which the process converges to a point.

The same behaviour is shown by the Newton-Raphson Method,
Start with an initial guess $x_0$, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.5}$$

where ,

$$f(x) = 2x^2 - 3x + 5 \tag{0.6}$$
$$f'(x) = 4x - 3 \tag{0.7}$$

The behaviour shown here is that regardless of which guess we take, it reaches a point of

extrema(derivative $\approx 0$) and then the process halts, or the updated point grow with bound. To get the complex solutions, however , we can Just take the initial guess point to be a random complex number.
The ouput of a program written to find roots is shown below:

```
Running Newton iterations:
x got too big
Trying fixed point iterations:
x got too big
Trying complex Newton's iterations:
Solution = 0.750000 + -1.391941 i
```

And on a second run,

```
Running Newton iterations:
Failure
Trying fixed point iterations:
x got too big
Trying complex Newton's iterations:
Solution = 0.750000 + 1.391941 i
```