9.6.15

EE24BTECH11003 - Akshara Sarma Chennubhatla

Question: Solve the differential equation $\frac{dy}{dx} - \sin 2x = 3y \cot x$, with the point $\left(\frac{\pi}{2}, 2\right)$ lying on the graph

Solution:
Theoretical Solution:

$$\frac{dy}{dx} - 3y \cot x = \sin 2x \tag{1}$$

$$\tag{2}$$

This is a linear differential equation. So the Integrating factor is,

$$I.F = e^{\int -3 \cot x} \tag{3}$$

$$I.F = e^{3 \log \operatorname{cosec} x} \tag{4}$$

$$I.F = \operatorname{cosec}^3 x \tag{5}$$

$$\tag{6}$$

Multiplying both sides of the equation by the integrating factor and integrating,

$$\int \operatorname{cosec}^3 x \left(\frac{dy}{dx} - 3y \cot x\right) dx = \int \operatorname{cosec}^3 x \sin 2x dx \tag{7}$$

$$y \operatorname{cosec}^3 x = \int \operatorname{cosec}^3 x \sin 2x dx \tag{8}$$

$$y \operatorname{cosec}^3 x = \int \frac{2 \sin x \cos x}{\sin^3 x} dx \tag{9}$$

$$y \operatorname{cosec}^3 x = \int 2 \frac{\cos x}{\sin^2 x} dx \tag{10}$$

$$y \operatorname{cosec}^3 x = -\frac{2}{\sin x} + C \tag{11}$$

$$\tag{12}$$

Since $\left(\frac{\pi}{2}, 2\right)$ satisfies the function,

$$2\left(1^3\right) = -\frac{2}{1} + C \tag{13}$$

$$\implies C = 4 \tag{14}$$

$$\tag{15}$$

So the function $y(x)$ is,

$$y\operatorname{cosec}^3 x = -\frac{2}{\sin x} + 4 \tag{16}$$

$$\implies y = 4\sin^3 x - 2\sin^2 x \tag{17}$$

$$\tag{18}$$

Simulated Solution:

By first principle of derivatives,

$$y'(x) = \lim_{h\to 0} \frac{y(x+h) - y(x)}{h} \tag{19}$$

$$y(x+h) = y(x) + hy'(x) \tag{20}$$

Given differential equation can be written as,

$$y' = \sin 2x + 3y\cot x \tag{21}$$

So, by using the method of finite diffferences,

$$y_1 = y_0 + h(\sin 2x_0 + 3y_0\cot x_0) \tag{22}$$

Similarly, by iterating for $y_2, y_3 ...$, The general difference equation is:

$$y_{n+1} = y_n + h(\sin 2x_n + 3y_n\cot x_n) \tag{23}$$

Below is the simulated plot and the theoretical plot for given curve based on initial conditions, obtained by iterating through the values of x with step size of h

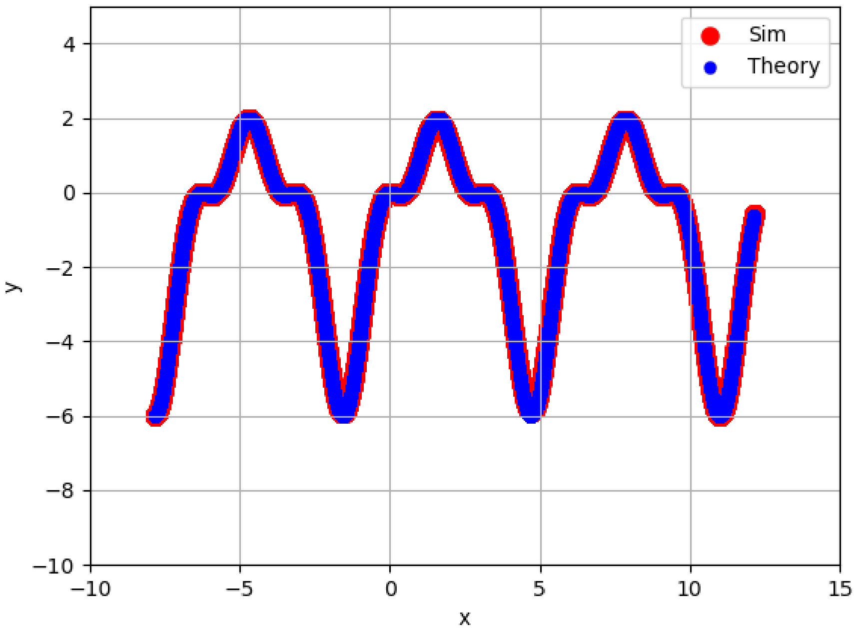

Fig. 1: Plot of the solution of $\frac{dy}{dx} - \sin 2x = 3y \cot x$