

Solving differential equation

NCERT-12.9.6.3

EE24BTECH11056 - S.Kavya Anvitha

Question:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad (1)$$

Theoretical Solution:

$$\frac{dy}{dx} + \frac{y}{x} = x^2 \quad (2)$$

$$\text{Integrating Factor: } e^{\int \frac{1}{x} dx} = e^{\ln x} = x \quad (3)$$

$$y \cdot x = \int x^2 \cdot x dx \quad (4)$$

$$y \cdot x = \frac{x^4}{4} + c \quad (5)$$

$$\text{Therefore:} \quad (6)$$

$$y = \frac{x^3}{4} + \frac{c}{x}. \quad (7)$$

Taking C as 1 we get a solution i.e.,

$$y = \frac{x^3}{4} + \frac{1}{x}. \quad (8)$$

Method of finite differences:

$$\frac{dy}{dx} = x^2 - \frac{y}{x} \quad (9)$$

$$\text{where } \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = x^2 - \frac{y}{x} \quad (10)$$

$$(11)$$

Approximaing for small h:

$$y_{n+1} - y_n = h \cdot \left(x_n^2 - \frac{y_n}{x_n} \right) \quad (12)$$

$$(13)$$

Therefore:

$$y_{n+1} = y_n + h \cdot \left(x_n^2 - \frac{y_n}{x_n} \right) \quad (14)$$

$$x_{n+1} = x_n + h \quad (15)$$

By initializing the values of x and y and iterating the process several times and plotting them gives the curve for solution of the differential equation.

