Reinforcement Learning

CS 760@UW-Madison

Goals for the lecture

you should understand the following concepts

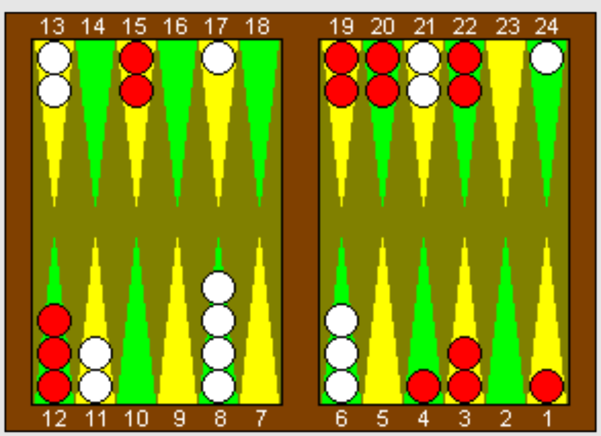
- the reinforcement learning task
- Markov decision process
- value functions
- value iteration
- Q functions
- Q learning
- exploration vs. exploitation tradeoff
- compact representations of Q functions
- reinforcement learning example

Reinforcement learning (RL)

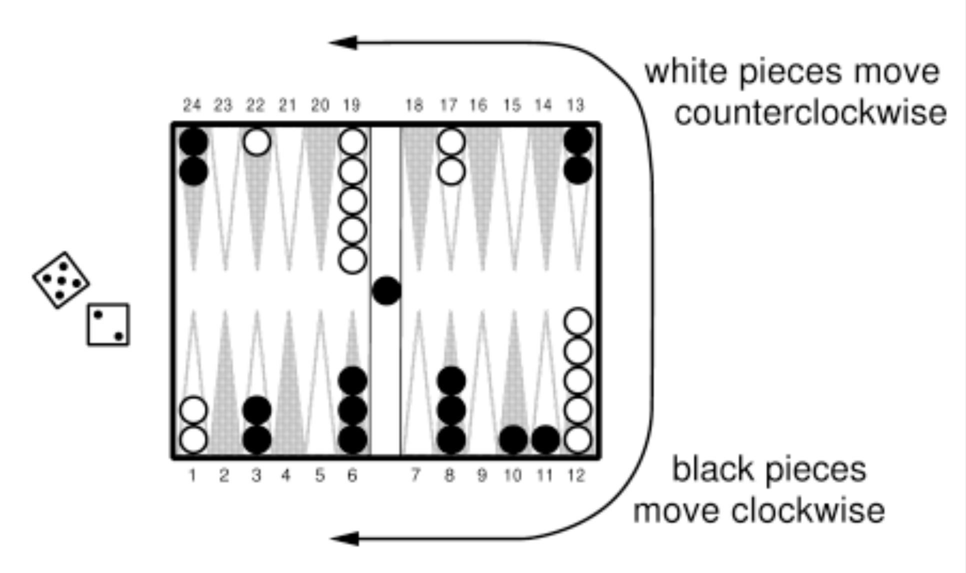
Task of an agent embedded in an environment

repeat forever
1) sense world
2) reason
3) choose an action to perform
4) get feedback (usually reward = 0)
5) learn

the environment may be the physical world or an artificial one



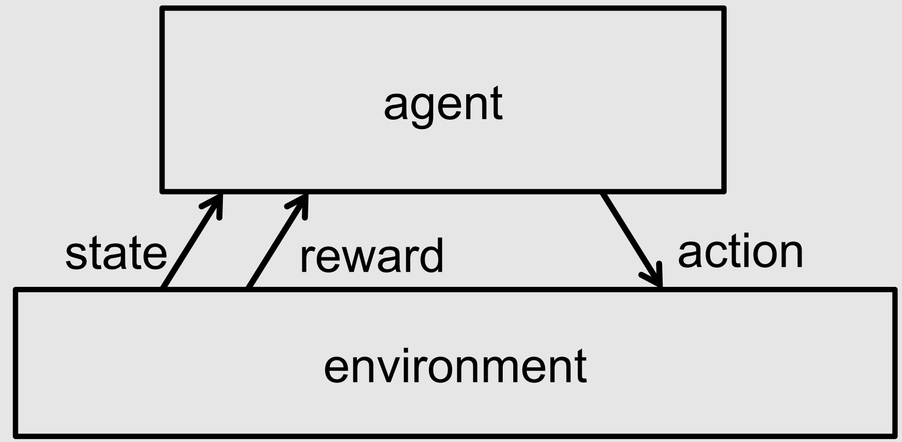
Example: RL Backgammon Player
[Tesauro, *CACM* 1995]



- world
  - 30 pieces, 24 locations
- actions
  - roll dice, e.g. 2, 5
  - move one piece 2
  - move one piece 5
- rewards
  - win, lose
- TD-Gammon 0.0
  - trained against itself (300,000 games)
  - as good as best previous BG computer program (also by Tesauro)
- TD-Gammon 2
  - beat human champion

Reinforcement learning

- set of states S
- set of actions A
- at each time t, agent observes state $s_t \in S$ then chooses action $a_t \in A$
- then receives reward r_t and changes to state s_{t+1}



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2}$$

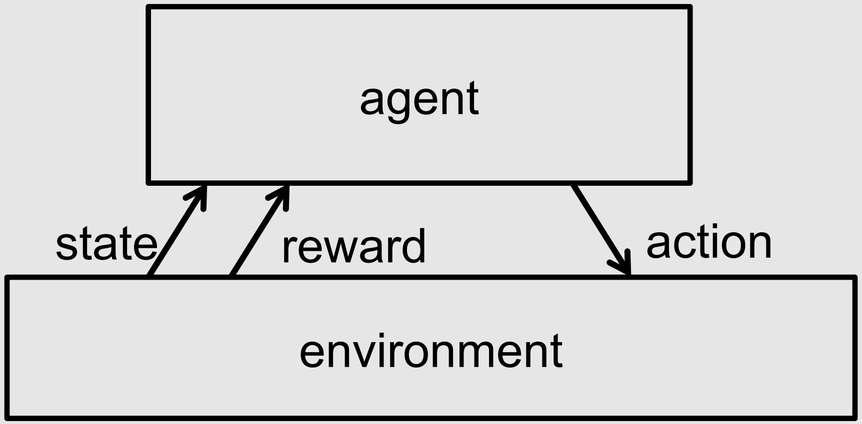
RL as Markov decision process (MDP)

- Markov assumption

$$P(s_{t+1} \mid s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(s_{t+1} \mid s_t, a_t)$$

- also assume reward is Markovian

$$P(r_{t+1} \mid s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(r_{t+1} \mid s_t, a_t)$$



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2}$$

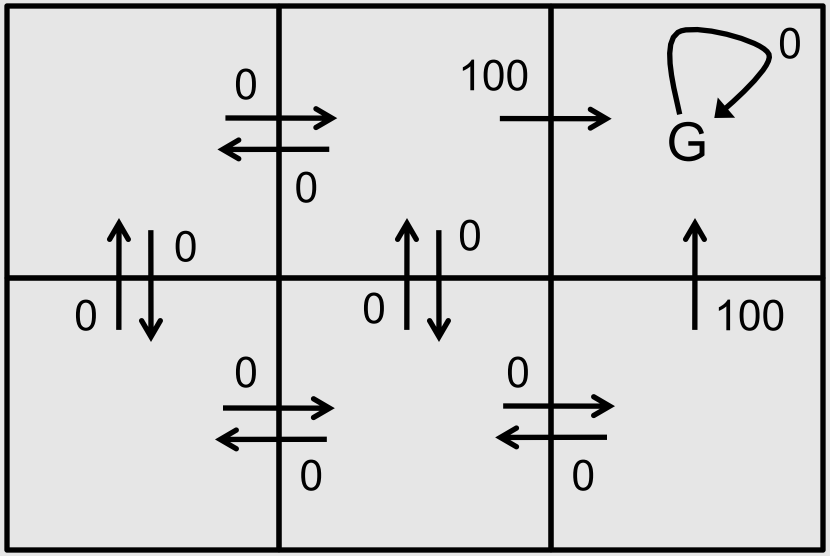
Goal: learn a policy $\pi : S \rightarrow A$ for choosing actions that maximizes

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \quad \text{where } 0 \le \gamma < 1$$

for every possible starting state s_0

Reinforcement learning task

- Suppose we want to learn a control policy $\pi : S \rightarrow A$ that maximizes $\sum_{t=0}^{\infty} \gamma^t E[r_t]$ from every state $s \in S$



each arrow represents an action a and the associated number represents deterministic reward $r(s, a)$

Value function for a policy

- given a policy $\pi : S \rightarrow A$ define

$$V^{\pi}(s) = \sum_{t=0}^{\infty} \gamma^t E[r_t]$$

assuming action sequence chosen according to π starting at state s

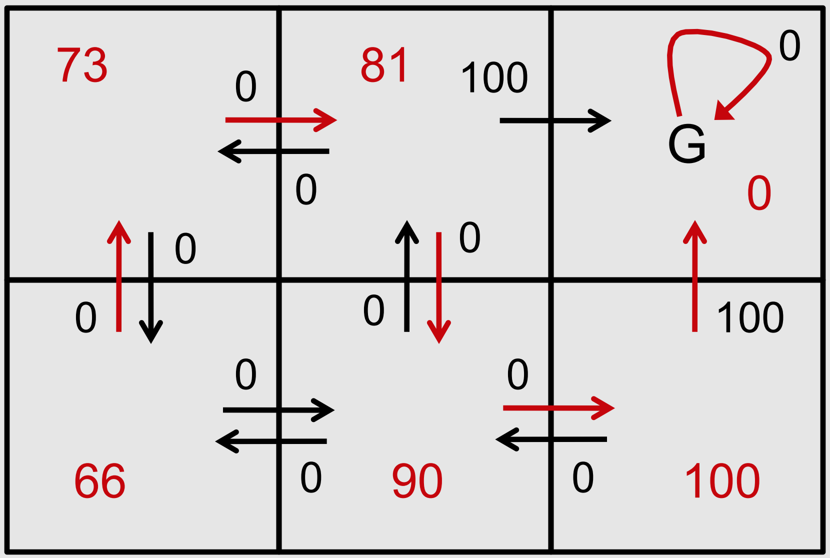
- we want the optimal policy π^* where

$$\pi^* = \arg\max_{\pi} V^{\pi}(s) \quad \text{for all } s$$

we'll denote the value function for this optimal policy as $V^*(s)$

Value function for a policy π

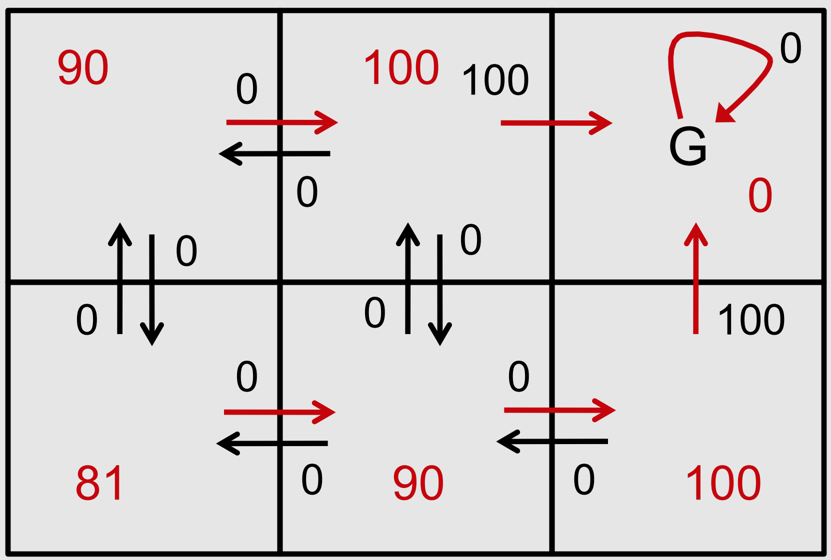
- Suppose π is shown by red arrows, $\gamma = 0.9$



$V^{\pi}(s)$ values are shown in red

Value function for an optimal policy π^*

- Suppose π^* is shown by red arrows, $\gamma = 0.9$



$V^*(s)$ values are shown in red

Using a value function

If we know $V^*(s)$, $r(s_t, a)$, and $P(s_t \mid s_{t-1}, a_{t-1})$ we can compute $\pi^*(s)$

$$\pi^*(s_t) = \arg\max_{a \in A} \left[r(s_t, a) + \gamma \sum_{s \in S} P(s_{t+1} = s \mid s_t, a) V^*(s) \right]$$

Value iteration for learning $V^*(s)$

initialize $V(s)$ arbitrarily

loop until policy good enough

{

  loop for $s \in S$

  {

    loop for $a \in A$

    {

$$Q(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in S} P(s' | s,a) V(s')$$

    }

$$V(s) \leftarrow \max_a Q(s,a)$$

  }

}

Value iteration for learning $V^*(s)$

- $V(s)$ converges to $V^*(s)$
- works even if we randomly traverse environment instead of looping through each state and action methodically
  - but we must visit each state infinitely often
- implication: we can do online learning as an agent roams around its environment

- assumes we have a model of the world: i.e. know $P(s_t \mid s_{t-1}, a_{t-1})$
- What if we don't?

Q learning

define a new function, closely related to V^*

$$V^*(s) \leftarrow E\left[r(s,\pi^*(s))\right] + \gamma E_{s'|s,\pi^*(s)}\left[V^*(s')\right]$$

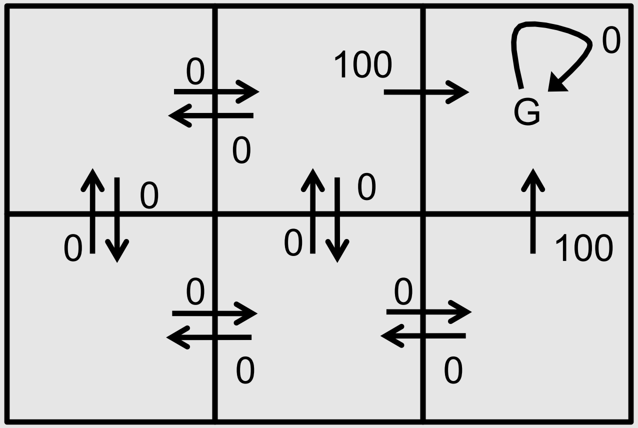
$$Q(s,a) \leftarrow E\left[r(s,a)\right] + \gamma E_{s'|s,a}\left[V^*(s')\right]$$

if agent knows $Q(s, a)$, it can choose optimal action without knowing $P(s' \mid s, a)$

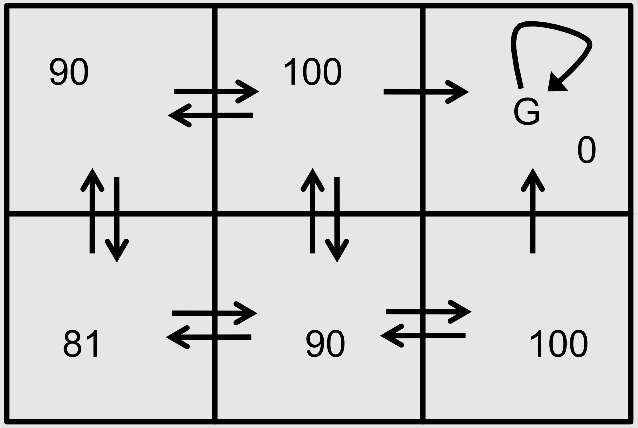
$$\pi^*(s) \leftarrow \arg\max_a Q(s,a) \qquad V^*(s) \leftarrow \max_a Q(s,a)$$

and it can learn $Q(s, a)$ without knowing $P(s' \mid s, a)$

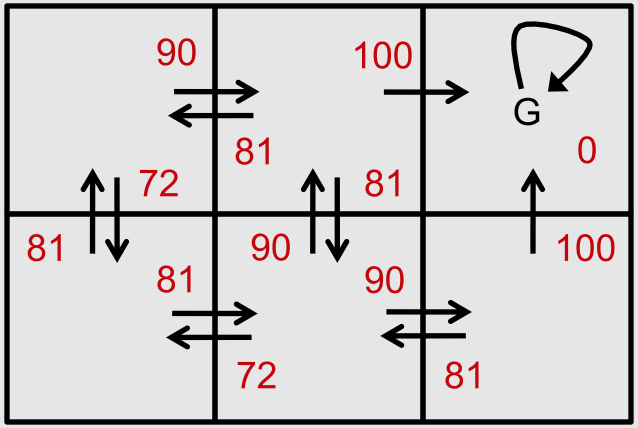
Q values



$r(s, a)$ (immediate reward) values



$V^*(s)$ values



$Q(s, a)$ values

Q learning for deterministic worlds

for each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

observe current state s

do forever

select an action a and execute it

receive immediate reward r

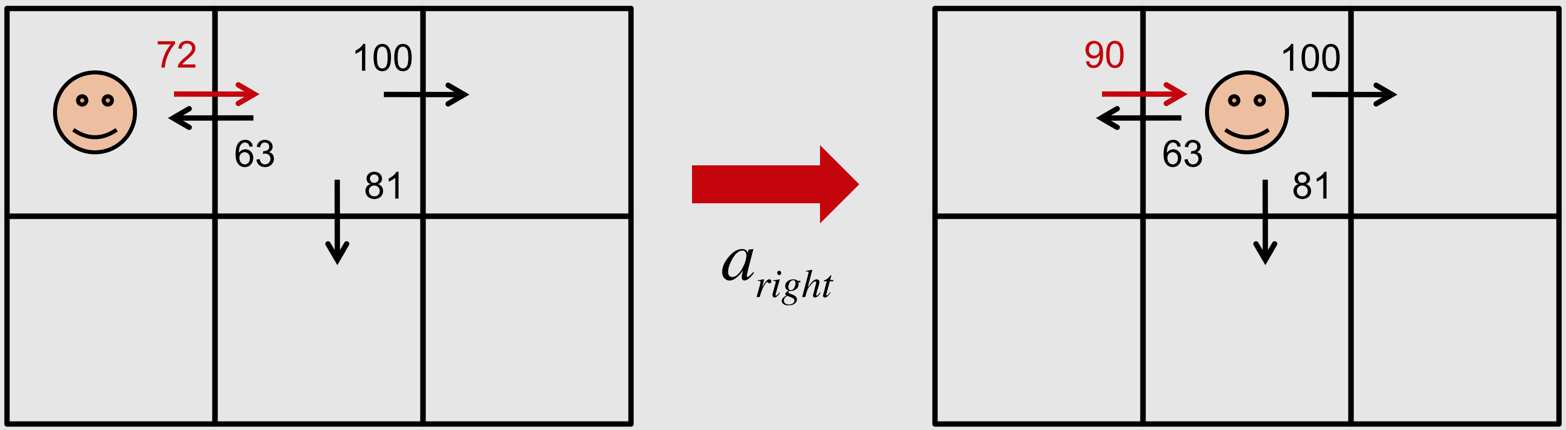
observe the new state s'

update table entry

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

$s \leftarrow s'$

Updating Q



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow 0 + 0.9 \max\{63, 81, 100\}$$
$$\leftarrow 90$$

Q learning for *nondeterministic* worlds

for each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

observe current state s

do forever

    select an action a and execute it

    receive immediate reward r

    observe the new state s'

    update table entry

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n\left[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')\right]$$

    $s \leftarrow s'$

where α_n is a parameter dependent on the number of visits to the given (s, a) pair

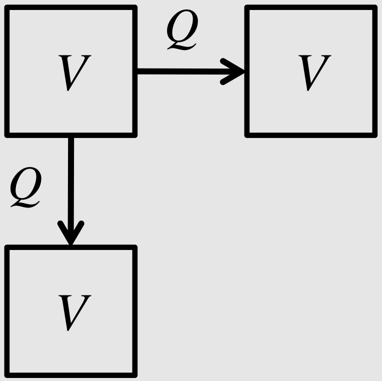
$$\alpha_n = \frac{1}{1 + \text{visits}_n(s,a)}$$

Convergence of Q learning

- Q learning will converge to the correct Q function
  - in the deterministic case
  - in the nondeterministic case (using the update rule just presented)

- in practice it is likely to take many, many iterations

Q's vs. V's



- Which action do we choose when we're in a given state?
- V's (model-based)
  - need to have a 'next state' function to generate all possible states
  - choose next state with highest V value.
- Q's (model-free)
  - need only know which actions are legal
  - generally choose next state with highest Q value.

Exploration vs. Exploitation

- in order to learn about better alternatives, we shouldn't always follow the current policy (exploitation)
- sometimes, we should select random actions (exploration)
- one way to do this: select actions probabilistically according to:

$$P(a_i \mid s) = \frac{c^{\hat{Q}(s, a_i)}}{\sum_j c^{\hat{Q}(s, a_j)}}$$

where $c > 0$ is a constant that determines how strongly selection favors actions with higher Q values

Q learning with a table

As described so far, Q learning entails filling in a huge table

states

actions

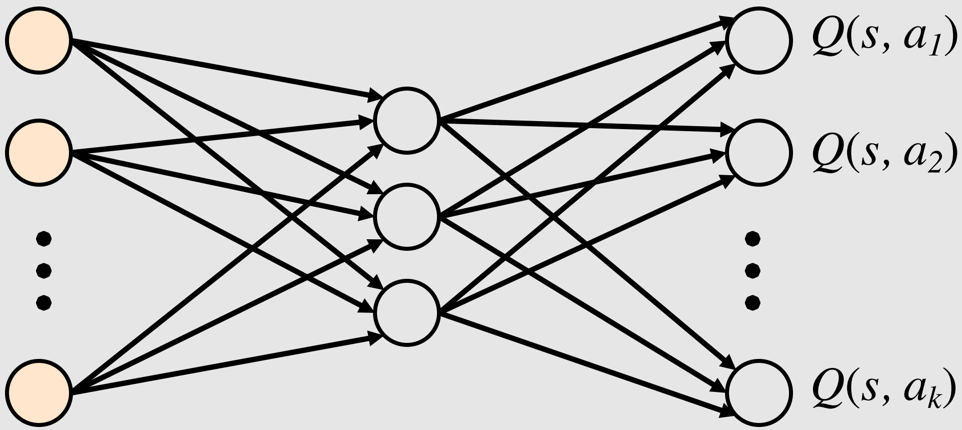
	s_0	s_1	s_2	. . .	s_n
a_1			.		
a_2			.		
			.		
a_3	. . .		$Q(s_2, a_3)$		
.					
.					
.					
a_k					

A table is a very <u>verbose way to</u> represent a function

Representing Q functions more compactly

We can use some other function representation (e.g. a neural net) to compactly encode a substitute for the big table

encoding of the state (s)



each input unit encodes a property of the state (e.g., a sensor value)

or could have one net for each possible action

Why use a compact Q function?

1. Full Q table may not fit in memory for realistic problems
2. Can generalize across states, thereby speeding up convergence

   i.e. one instance 'fills' many cells in the Q table

Notes

1. When generalizing across states, cannot use $\alpha=1$
2. Convergence proofs only apply to Q tables
3. Some work on bounding errors caused by using compact representations (e.g. Singh & Yee, *Machine Learning* 1994)

Q tables vs. Q nets

Given: 100 Boolean-valued features

10 possible actions

Size of Q table

10×2^{100} entries

Size of Q net (assume 100 hidden units)

$$\underbrace{100 \times 100}_{\text{weights between inputs and HU's}} + \underbrace{100 \times 10}_{\text{weights between HU's and outputs}} = 11{,}000 \text{ weights}$$

Representing Q functions more compactly

- we can use other regression methods to represent Q functions

    k-NN

    regression trees

    support vector regression

    etc.

Q learning with function approximation

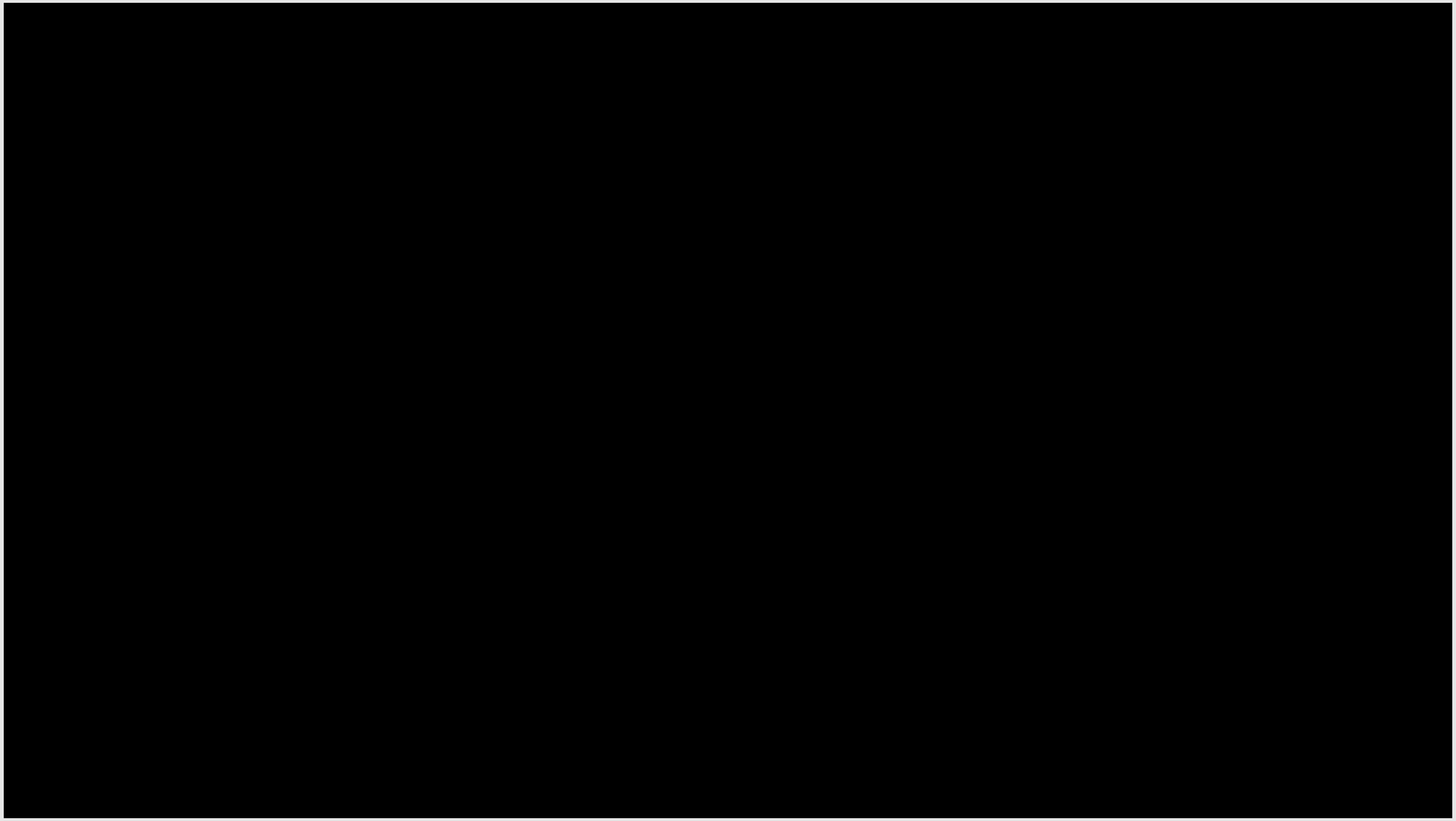
1. measure sensors, sense state s_0
2. predict $\hat{Q}_n(s_0, a)$ for each action a
3. select action a to take (with randomization to ensure exploration)
4. apply action a in the real world
5. sense new state s_1 and immediate reward r
6. calculate action a' that maximizes $\hat{Q}_n(s_1, a')$
7. train with new instance

$$\boldsymbol{x} = s_0$$

$$y \leftarrow (1-\alpha)\hat{Q}(s_0, a) + \alpha\left[r + \gamma \max_{a'} \hat{Q}(s_1, a')\right]$$

Calculate Q-value you would have put into Q-table, and use it as the training label

ML example: reinforcement learning to control an autonomous helicopter



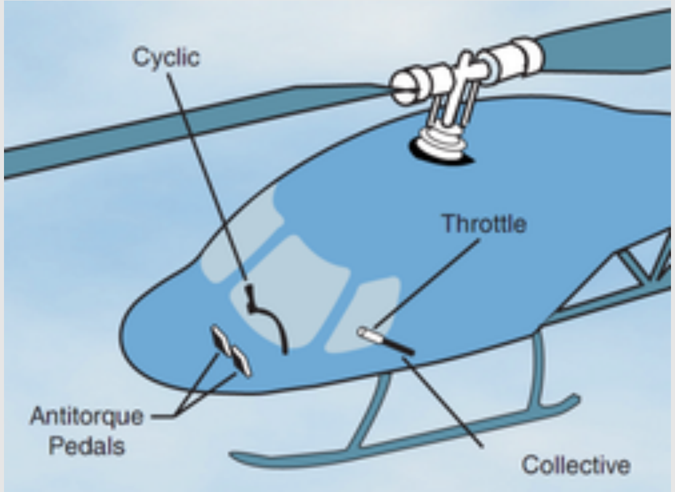
video of Stanford University autonomous helicopter from http://heli.stanford.edu/

Stanford autonomous helicopter

sensing the helicopter's state

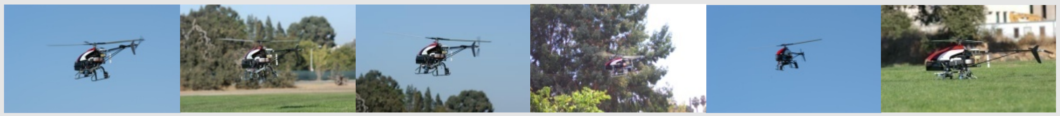
- orientation sensor
  - accelerometer
  - rate gyro
  - magnetometer
- GPS receiver ("2cm accuracy as long as its antenna is pointing towards the sky")
- ground-based cameras

actions to control the helicopter

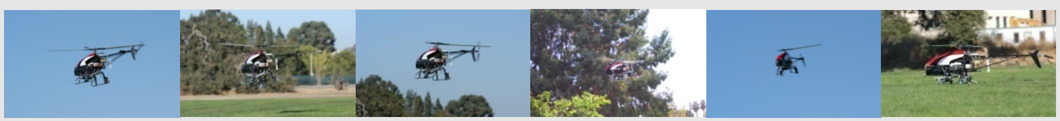


Experimental setup for helicopter

1. Expert pilot demonstrates the airshow several times



2. Learn a reward function based on desired trajectory
3. Learn a dynamics model
4. Find the optimal control policy for learned reward and dynamics model
5. Autonomously fly the airshow



6. Learn an improved dynamics model. Go back to step 4

Learning dynamics model $P(s_{t+1} \mid s_t,\ a)$

- state represented by helicopter's

position	(x, y, z)
velocity	$(\dot{x}, \dot{y}, \dot{z})$
angular velocity	$(\omega_x, \omega_y, \omega_z)$

- action represented by manipulations of 4 controls

$$\left(u_1, u_2, u_3, u_4\right)$$

- dynamics model predicts accelerations as a function of current state and actions
- accelerations are integrated to compute the predicted next state

Learning dynamics model $P(s_{t+1} \mid s_t,\ a)$

dynamics model

$$\ddot{x}^b = A_x \dot{x}^b + g_x^b + w_x,$$
$$\ddot{y}^b = A_y \dot{y}^b + g_y^b + D_0 + w_y,$$
$$\ddot{z}^b = A_z \dot{z}^b + g_z^b + C_4 u_4 + D_4 + w_z,$$
$$\dot{\omega}_x^b = B_x \omega_x^b + C_1 u_1 + D_1 + w_{\omega_x},$$
$$\dot{\omega}_y^b = B_y \omega_y^b + C_2 u_2 + D_2 + w_{\omega_y},$$
$$\dot{\omega}_z^b = B_z \omega_z^b + C_3 u_3 + D_3 + w_{\omega_z}.$$

- A, B, C, D represent model parameters
- g represents gravity vector
- w's are random variables representing noise and unmodeled effects

- linear regression task!

Learning a desired trajectory

- repeated expert demonstrations are often suboptimal in different ways
- given a set of M demonstrated trajectories

$$y_j^k = \begin{bmatrix} s_j^k \\ u_j^k \end{bmatrix} \quad \text{for } j = 0, \ldots, N-1, k = 0, \ldots, M-1$$

action on j^{th} step of trajectory k

state on j^{th} step of trajectory k

- try to infer the implicit desired trajectory

$$z_t = \begin{bmatrix} s_t^* \\ u_t^* \end{bmatrix} \quad \text{for } t = 0, \ldots, H$$

Learning a desired trajectory

colored lines: demonstrations of two loops
black line: inferred trajectory

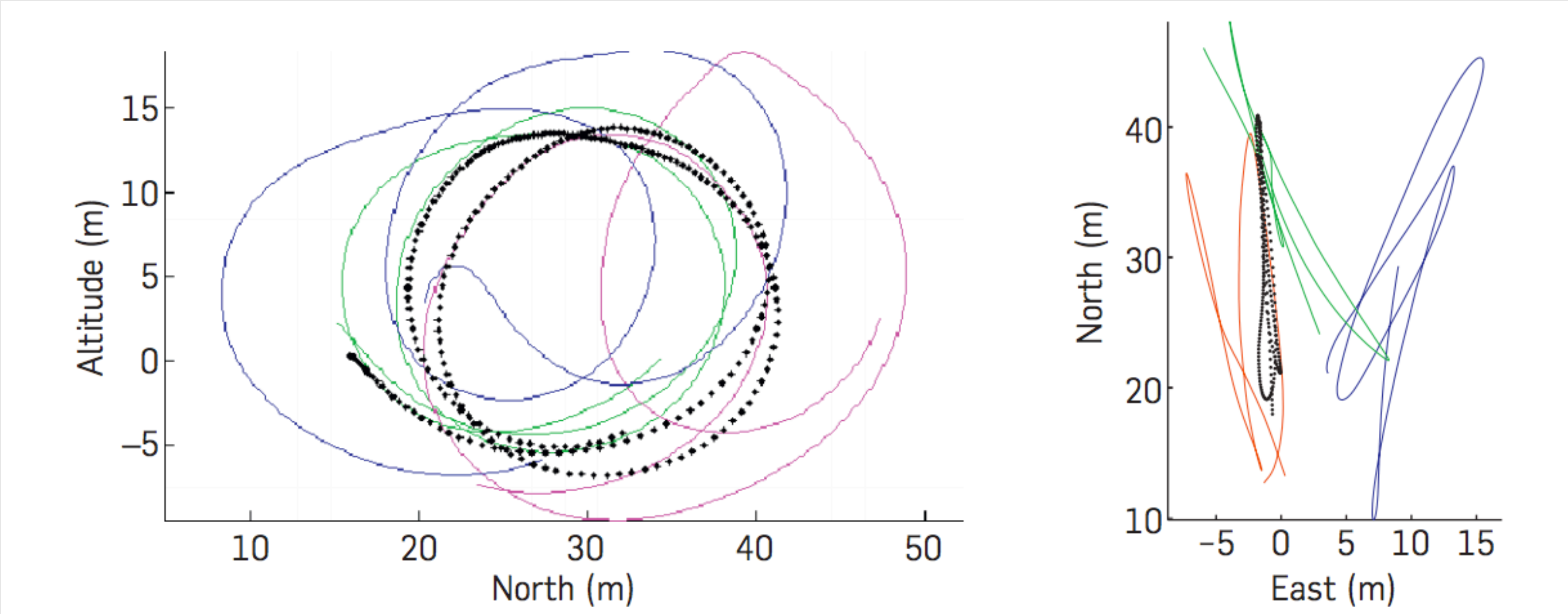


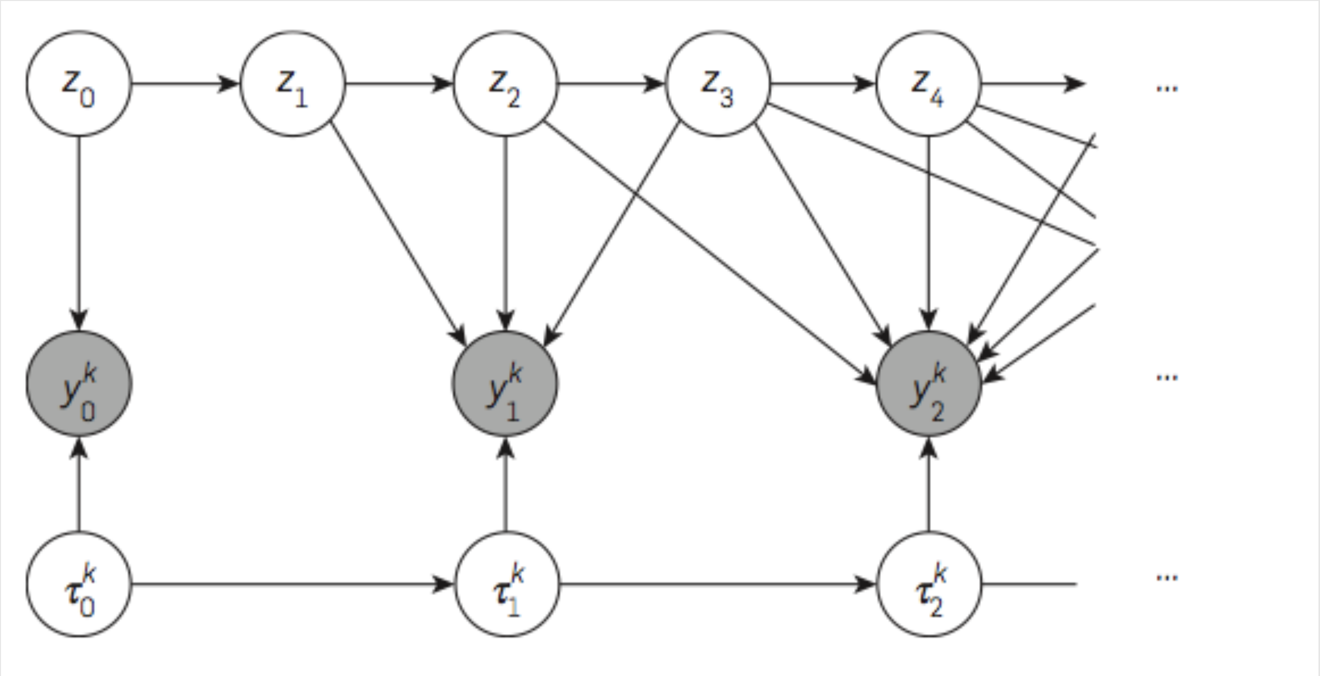
Figure from Coates et al.,*CACM* 2009

Generative model for desired trajectories

desired trajectory

demonstrated trajectory

time indices



$$z_{t+1} = f\left(z_t\right) + w_t^{(z)}, \quad w_t^{(z)} \sim \mathcal{N}\left(0, \Sigma^{(z)}\right)$$

dynamics model

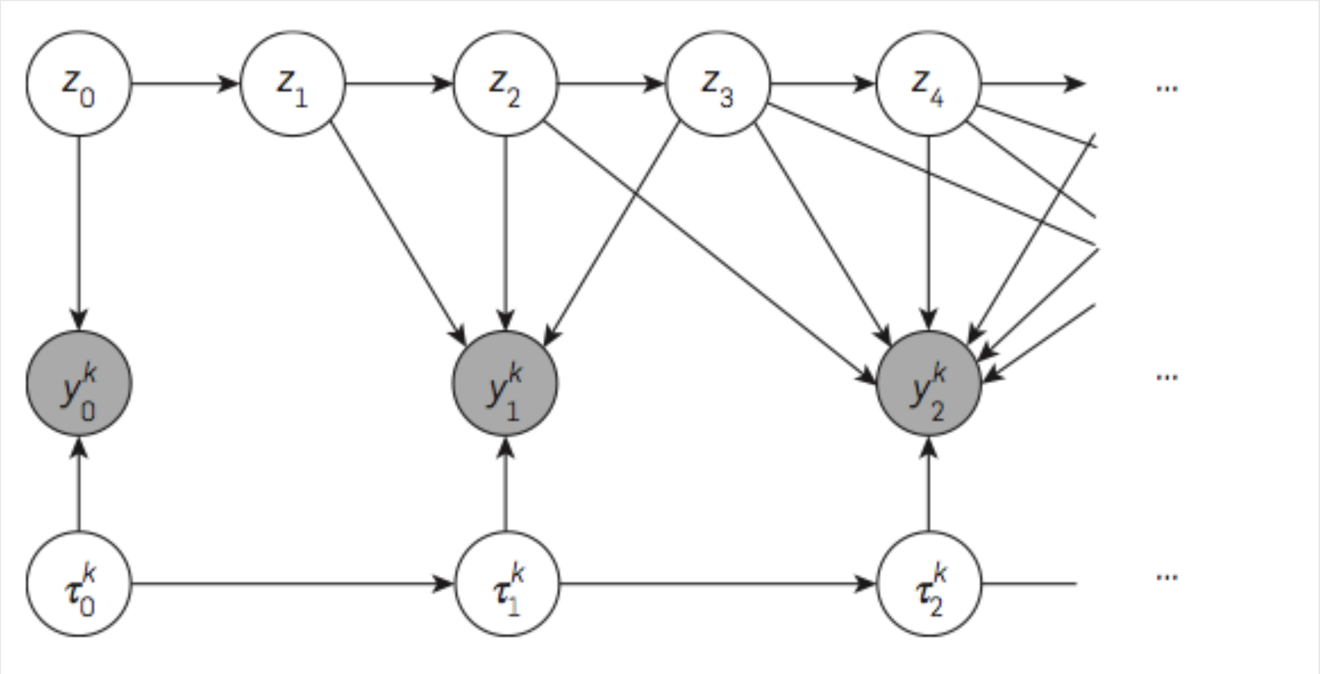
noise

Generative model for desired trajectories

desired trajectory

demonstrated trajectory

time indices

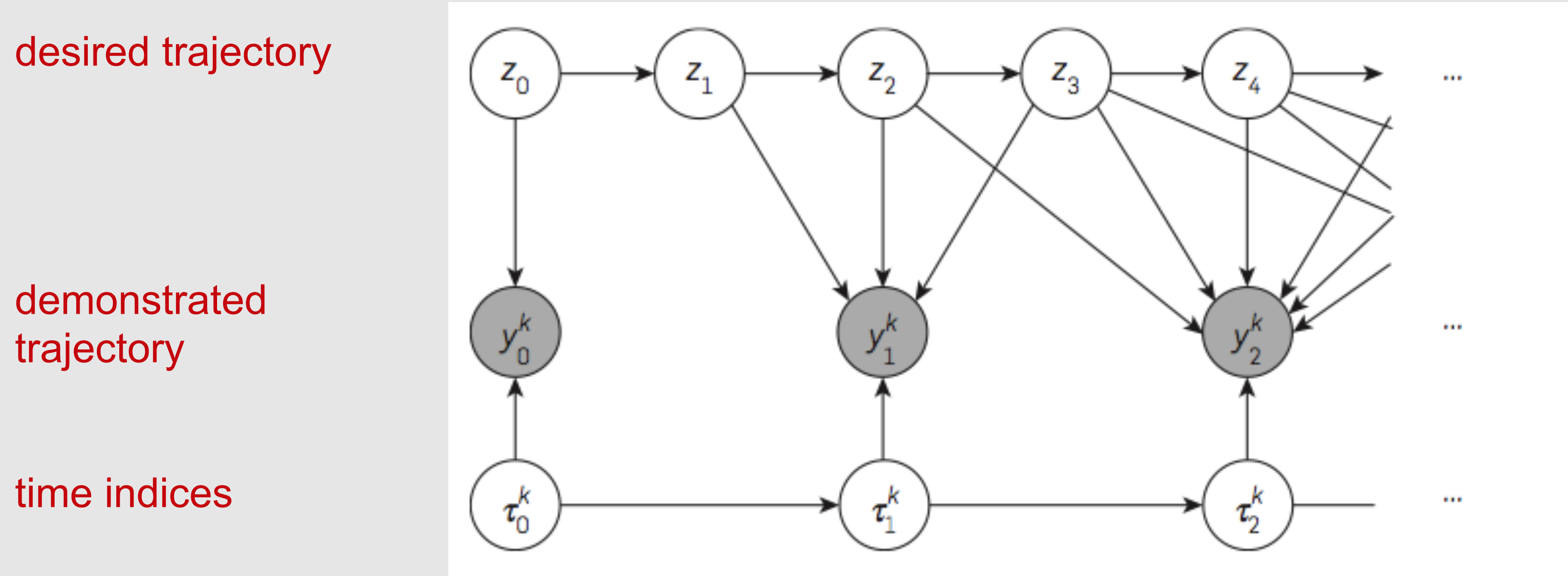


$$y_j^k = z_{\tau_j^k} + w_j, \quad w_t^{(y)} \sim \mathcal{N}\left(0, \Sigma^{(y)}\right)$$

desired trajectory at time step indexed by $\tau^k{}_j$

noise

Generative model for desired trajectories



$$P\left(\tau_{j+1}^k \mid \tau_j^k\right) = \begin{cases} d_1^k & \text{if } \tau_{j+1}^k - \tau_j^k = 1 \\ d_2^k & \text{if } \tau_{j+1}^k - \tau_j^k = 2 \\ d_3^k & \text{if } \tau_{j+1}^k - \tau_j^k = 3 \\ 0 & \text{otherwise} \end{cases}$$

parameters specifying probability of the subsampling interval

Learning reward function

- EM is used to infer desired trajectory from set of demonstrated trajectories
- The reward function is based on deviations from the desired trajectory

Finding the optimal control policy

- finding the control policy is a reinforcement learning task

$$\pi^* \leftarrow \arg\max_\pi E\left[\sum_t r(s_t, a) \mid \pi\right]$$

- RL learning methods described earlier don't quite apply because state and action spaces are both continuous
- A special type of Markov decision process in which the optimal policy can be found efficiently
  - reward is represented as a linear function of state and action vectors
  - next state is represented as a linear function of current state and action vectors
- They use an iterative approach that finds an approximate solution because the reward function used is quadratic

THANK YOU

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