Computational and Numerical Methods Lab - 5


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Abhimanyu Karia: 202201435
Devarshi Patel : 202201447
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Jacobi Method

- The Jacobi method is an iterative algorithm for determining the solutions of a strictly diagonally dominant system of linear equations. Each diagonal element is solved for, and an approximate value is plugged in. The process is then iterated until it converges.

- Equation to be followed:
  - $$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1, j \neq i}^{n} a_{ij} x_j^{(k)} \right)$$

Gauss-Siedel Method

- Improvisation of Jacobi method
- In Jacobi method the value of the variables is not modified until next iteration, whereas in Gauss-Seidel method the value of the variables are modified as soon as new value is evaluated
- Equation used:
  - $$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

Comparision between both methods

- Gauss-Siedel method converges more rapidly than Jacobi method as it uses modified values for evaluation whenever available.

For both Jacobi and Seidal Method our algorithm runs as follows:

1. Create a object of class LinearSolve and pass required matrix A and vector b.
2. Call the get_roots function specifying method and initial guess.
3. When a call enters jacobi or seidal function, first thing function does is check if the matrix is diagonally dominant.
   - If the matrix is dominant, no change in the matrix.
   - If the matrix isn't diagonally dominant but row exchanges can make it diagonally dominant return new matrix alongside permutation performed to make it dominant (same permutation is done in the vector b as done in the matrix A).
   - If the matrix isn't diagonally dominant and can't be made dominant through row exhanges return NULL and function ends here.
4. Now perform Jacobi/Seidal method to get required answer.
5. Store iteration number and calculate distance between two iteration results (i.e new answer and previous iteration answer) (Note : Here we have used euclidean norm to calculate distance).
6. Stopping Condition : Norm < Maximum_specified_error.
7. Return Answer, iteration list and norms in consecutive iterations.
8. Compare results of Gauss-Seidal, Jacobi and Inbuilt numpy function.
9. Plot Iteration vs Norm graph for Seidal and Jacobi to compare the results.

Q1)

In [22]:
```
import numpy as np
import matplotlib.pyplot as plt
import copy
```

In [23]:
```
class LinearSolve:
    def __init__(self,matrix,vector) -> None:
        self.matrix = matrix
        self.vector = vector
        self.res = np.array([])
        self.err = 1e-7
    def get_upper_permute(self,matrix):
        n = len(matrix)
        upper = copy.deepcopy(matrix)
        permute = np.eye(n)
        for i in range(n):
            if(upper[i][i] == 0):
                j = i+1
                while(j < n and upper[j][i] == 0):
                    j += 1
                if(j == n):
                    continue
                else:
                    permute[[i,j]] = permute[[j,i]]
                    upper = np.dot(permute,upper)
            for j in range(i+1,n):
                m = upper[j][i]/upper[i][i]
                upper[j][i] = 0
```

```
            for k in range(i+1,n+1):
                upper[j][k] = upper[j][k] - m*upper[i][k]
        return upper,permute

    def get_inv(self,matrix):
        matinv = np.linalg.inv(matrix)
        return matinv

    def get_lower(self,matrix,upper,permute):
        pa = np.dot(permute,matrix)
        upp_inv = self.get_inv(upper)
        return np.dot(pa,upp_inv)

    def gauss(self,A_n,b):
        n = len(A_n)
        A = copy.deepcopy(A_n)
        #st = deque()
        for i in range(n):
            A[i].append(b[i])

        A,_ = self.get_upper_permute(A)
        x = [0]*n
        for i in range(n-1,-1,-1):
            x[i] = A[i][n]
            k = 0
            for j in range(n-1,i,-1):
                x_j = x[j]*A[i][j]
                if(abs(x_j) < self.err):
                    continue
                x[i] -= x[j]*A[i][j]
            x[i] /= A[i][i]
            if(abs(x[i]) < self.err):
                x[i] = 0
        self.res = x
        return np.array(x)

    def vector_norm(self,x1,x2):
        return np.linalg.norm(np.array(x1) - np.array(x2))

    def jacobi(self,A,b,guess):
        C,permute = self.diagonal_dominance(A)
        if C == False:
            return None,None,None
        n = len(A)
        new_b = copy.deepcopy(b)
        for i in range(n):
            new_b[permute[i]] = b[i]
        iter = []
        ite = 1
        norm = []
        x = copy.deepcopy(guess)
        mat = [0]*n
        norm_val = 0
        new_x = copy.deepcopy(x)
        while True:
            for i in range(n):
                sum_val = 0
                for j in range(n):
                    if i != j :
                        sum_val -= C[i][j]*x[j]
```

```
                new_x[i] = (new_b[i] + sum_val)/C[i][i]

            norm_val = self.vector_norm(new_x,x)
            # print(new_x)
            # print(x)
            if(norm_val < self.err):
                break
            iter.append(ite)
            ite += 1
            norm.append(norm_val)
            x = copy.deepcopy(new_x)
        return np.array(x),iter,norm

    def diagonal_dominance(self,A):
        flag = True
        n = len(A)
        column_idx = [0]*n
        uset = set()
        for i in range(n):
            column_idx[i] = A[i].index(max(A[i],key=abs))
            #print(column_idx[i])
            if column_idx[i] in uset:
                print('Matrix is not Diagonally Dominant and hence result cannot
                return False,column_idx
            val = 0
            for j in range(n):
                if j != column_idx[i]:
                    val += abs(A[i][j])
            if val >= abs(A[i][column_idx[i]]):
                print('Matrix is not Diagonally Dominant and hence result cannot
                return False,column_idx
            uset.add(column_idx[i])
        C = copy.deepcopy(A)
        for i in range(n):
            if i != column_idx[i]:
                flag = False
            C[column_idx[i]] = A[i]
        if not flag:
            print('Matrix is not Diagonally Dominant but permutation of rows can
        else:
            print('Matrix is Diagonally Dominant')
        return C,column_idx

    def seidal(self,A,b,guess):
        C,permute = self.diagonal_dominance(A)
        if C == False:
            return None,None,None
        n = len(A)
        new_b = copy.deepcopy(b)
        for i in range(n):
            new_b[permute[i]] = b[i]
        x = copy.deepcopy(guess)
        iter = []
        norm = []
        ite = 1
        mat = [0]*n
        norm_val = 0
        old_x = copy.deepcopy(x)
        while True:
            for i in range(n):
```

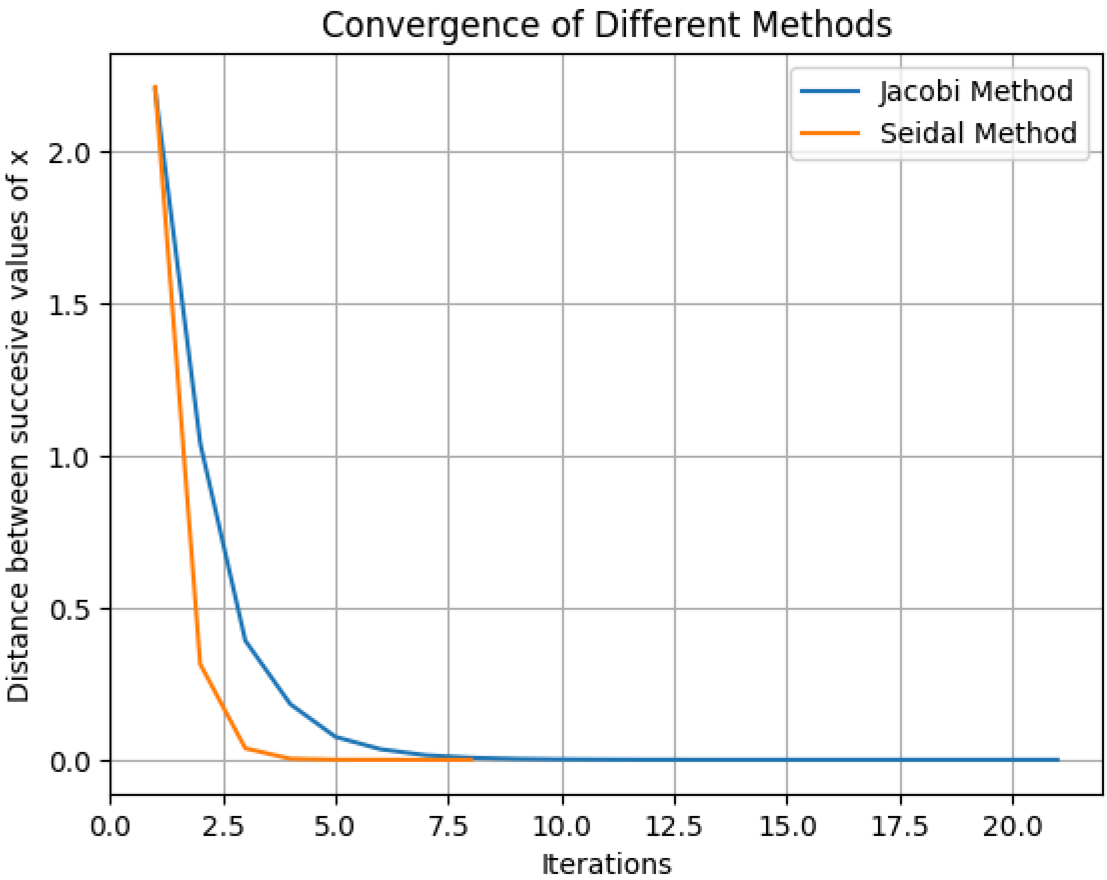
```python
                sum_val = 0
                for j in range(n):
                    if i != j :
                        sum_val -= C[i][j]*x[j]
                x[i] = (new_b[i] + sum_val)/C[i][i]
            #print(x)
            norm_val = self.vector_norm(x,old_x)
            if(norm_val < self.err):
                break
            iter.append(ite)
            ite += 1
            norm.append(norm_val)
            old_x = copy.deepcopy(x)
        return np.array(x),iter,norm

    def inbuilt(self,A_n,b):
        return np.linalg.solve(A_n,b)

    def get_roots(self,method = 'None',guess = np.array([])):
        match method:
            case 'gauss':
                return self.gauss(self.matrix,self.vector)
            case 'numpy':
                return self.inbuilt(self.matrix,self.vector)
            case 'seidal':
                return self.seidal(self.matrix,self.vector,guess)
            case 'jacobi':
                return self.jacobi(self.matrix,self.vector,guess)
            case 'None':
                return self.inbuilt(self.matrix,self.vector)
```

Examples

In [24]:

```python
A = [[9,1,1],[2,10,3],[3,4,11]]
b = [10,19,0]
guess = [0,0,0]
ls = LinearSolve(A,b)
val_j,j_iter,j_norm = ls.get_roots(method = 'jacobi',guess = guess)
print('Roots are (jacobi method): ',val_j)
val_s,s_iter,s_norm = ls.get_roots(method = 'seidal',guess = guess)
print('Roots are (seidal method): ',val_s)
val_x = ls.get_roots(method='numpy')
print('Roots using numpy: ',val_x)
if val_j is not None:
    plt.plot(j_iter,j_norm,label = 'Jacobi Method')
    plt.plot(s_iter,s_norm,label = 'Seidal Method')
    plt.legend()
    plt.grid(True)
    plt.xlabel('Iterations')
    plt.ylabel('Distance between succesive values of x')
    plt.title('Convergence of Different Methods')
    plt.show()
```

```
Matrix is Diagonally Dominant
Roots are (jacobi method):  [ 1.00000002  2.00000004 -0.99999996]
Matrix is Diagonally Dominant
Roots are (seidal method):  [ 1.  2. -1.]
Roots using numpy:  [ 1.  2. -1.]
```

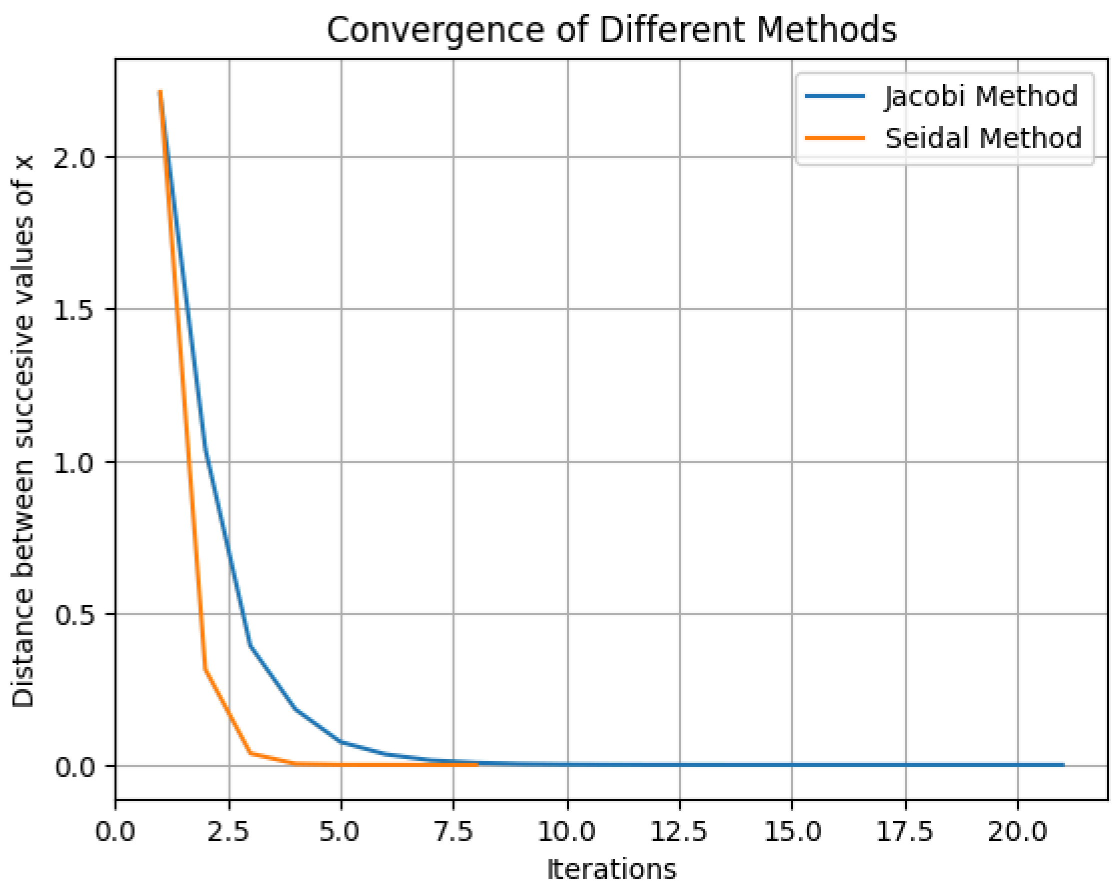


Result:

1. Matrix is diagonally dominant.
2. Roots generated using both methods are approximately same.
3. Gauss siedal method converges faster than Jacobi method

In [25]:
```
A = [[2,10,3],[9,1,1],[3,4,11]]
b = [19,10,0]
guess = [0,0,0]
ls = LinearSolve(A,b)
val_j,j_iter,j_norm = ls.get_roots(method = 'jacobi',guess = guess)
print('Roots are (jacobi method): ',val_j)
val_s,s_iter,s_norm = ls.get_roots(method = 'seidal',guess = guess)
print('Roots are (seidal method): ',val_s)
val_x = ls.get_roots(method='numpy')
print('Roots using numpy: ',val_x)
if val_j is not None:
    plt.plot(j_iter,j_norm,label = 'Jacobi Method')
    plt.plot(s_iter,s_norm,label = 'Seidal Method')
    plt.legend()
    plt.grid(True)
    plt.xlabel('Iterations')
    plt.ylabel('Distance between succesive values of x')
    plt.title('Convergence of Different Methods')
    plt.show()
```

```
Matrix is not Diagonally Dominant but permutation of rows can make it dominant
Roots are (jacobi method):  [ 1.00000002  2.00000004 -0.99999996]
Matrix is not Diagonally Dominant but permutation of rows can make it dominant
Roots are (seidal method):  [ 1.  2. -1.]
Roots using numpy:  [ 1.  2. -1.]
```



Result:

1. Matrix is not diagonally dominant but can be made diagonally dominant.
2. Roots generated using both methods are approximately same.
3. Gauss siedal method converges faster than Jacobi method

In [26]:
```
A = [[2,10,3],[9,9,1],[3,4,11]]
b = [10,19,0]
guess = [0,0,0]
ls = LinearSolve(A,b)
val_j,j_iter,j_norm = ls.get_roots(method = 'jacobi',guess = guess)
print('Roots are (jacobi method): ',val_j)
val_s,s_iter,s_norm = ls.get_roots(method = 'seidal',guess = guess)
print('Roots are (seidal method): ',val_s)
val_x = ls.get_roots(method='numpy')
print('Roots using numpy: ',val_x)
if val_j is not None:
    plt.plot(j_iter,j_norm,label = 'Jacobi Method')
    plt.plot(s_iter,s_norm,label = 'Seidal Method')
    plt.legend()
    plt.grid(True)
    plt.xlabel('Iterations')
    plt.ylabel('Distance between succesive values of x')
    plt.title('Convergence of Different Methods')
    plt.show()
```

```
Matrix is not Diagonally Dominant and hence result cannot be computed
Roots are (jacobi method):  None
Matrix is not Diagonally Dominant and hence result cannot be computed
Roots are (seidal method):  None
Roots using numpy:  [ 1.22745626  0.95962315 -0.68371467]
```

Result:

1. Roots not generated for both methods because matrix is not diagonally dominant and can't be made diagonally dominant by exchanging rows.