Radiological Engineering

Project 1

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Project overview:

In this project, our goal is to model the decay chain of Radium-226 (Ra-226) into Radon-222 (Rn-222) and Polonium-218 (Po-218). The purpose is essentially to simulate the time dependent behaviour of the chain certain conditions that I will explain later. This would include artificial source and burnup and production terms as well as venting situations.

For this situation here is what we need to quantify before we move on:

1. How much Ra-226 remains after 40 days.
2. The buildup and removal of Rn-222 under different venting strategies
3. The activity levels of all isotopes over time
4. The total amount of Rn-222 released in the environment.

Essentially our objectives are:

- To understand how daughter isotopes accumulate in a short decay chain (Ra-226 → Rn-222 → Po-218)
- Understand the difference of venting at different schedules affecting the release of Rn -222. For example, not venting until 40 days instead of venting every 10 days.
- To compare the relative importance of natural decay vs artificial burnup terms in determining the isotopes inventories.
- To practice setting up and solving system of couples differential equation that descrive radioactive decay chains.

Key assumptions:

1. Initial inventory: 1g pf Ra-226 (with no daughter isotopes)
2. Closed system except venting: Rn-222 is removed either at days 40 or at 10 day intervals.
3. Burnup/source terms:

a. Ra-226 has an artificial depletion term
b. Rn-222 and Po-218 have artificial production terms that are scaled by the Ra mass. These represent external processes such as neutron internactions

4. No backflow: Once the Rn-222 is vented, it is removed from the system
5. Constant half-lives: The decay constants are fixed, I have entered it as a dictionary at the top of my juypiter notebook file

Now let's move onto the process of doing the calculations and all the chain modelling. In short, we model the chain with linear ODEs (these are the decay terms) with and extra source to include the burnp/production terms. We then integrate the ODEs numerically and subsequently implement the venting. We implement the venting as an instantaneous removal of Rn-222 and accumulate vented atoms. Compute the activities as and then finally the total Rn released.

Initial conditions and basic setups for calculations:

We first need to convert the mass from grams to number of atoms, we do this as ODEs use number of atoms as state variables; decay constants multiply N to give activity.

$$N_{0,Ra} = \frac{1g}{226g/mol} * N_A$$

I start with one gram of Ra-226

$$\lambda = \frac{\ln(2)}{T_{1/2}}$$

The above is just basic necessities, but I wanted to just show minimums required. Now let's talk about the ODEs we used in the code, my model uses standard linear chain with extra source links:

$$\frac{dN_{Ra}}{dt} = -\lambda_{Ra}N_{Ra} + B_{Ra}(t)$$

$$\frac{dN_{Rn}}{dt} = \lambda_{Ra}N_{Ra} - \lambda_{Rn}N_{Rn} + S_{Rn}(t)$$

$$\frac{dN_{Po}}{dt} = \lambda_{Rn}N_{Rn} - \lambda_{Po}N_{Po} + S_{Po}(t)$$

In the above functions we see the negative and positive terms which are the subsequent burnup and loss terms. We then integrate these functions using specific python addons such as “ODEINT” for instance I use this module to integrate the system N’ = f(N, t) on the time grid t. In Appendix 1, we can see the code snippit.

Venting implementation

Venting represents the physical release of radon-222 gas from the system, effectively breaking the decay chain at that point. Without venting, radon atoms produced by the decay of Ra-226 remain in the system, where they accumulate and then decay further into Po-218, eventually leading to secular equilibrium where all isotopes have linked activities. When venting occurs, however, all the accumulated Rn-222 is instantly removed from the system, so it no longer contributes to the production of Po-218 or subsequent daughters. The venting code can be seen in Appendix 1 where I set N_current[1] = 0.0 and then I append value in the array to release output of the system.

Once we have implemented venting, we can calculate the activity and then subsequently the amount of Rn released by using the following formula:

$$Rn\ Decayed\ (atoms) = \int_{t_0}^{t_f} A_{Rn}(t)dt$$

Numerically, if A is sampled at times t, we can use:

- Rectangle rule: np.sum(A:-1 * dt)

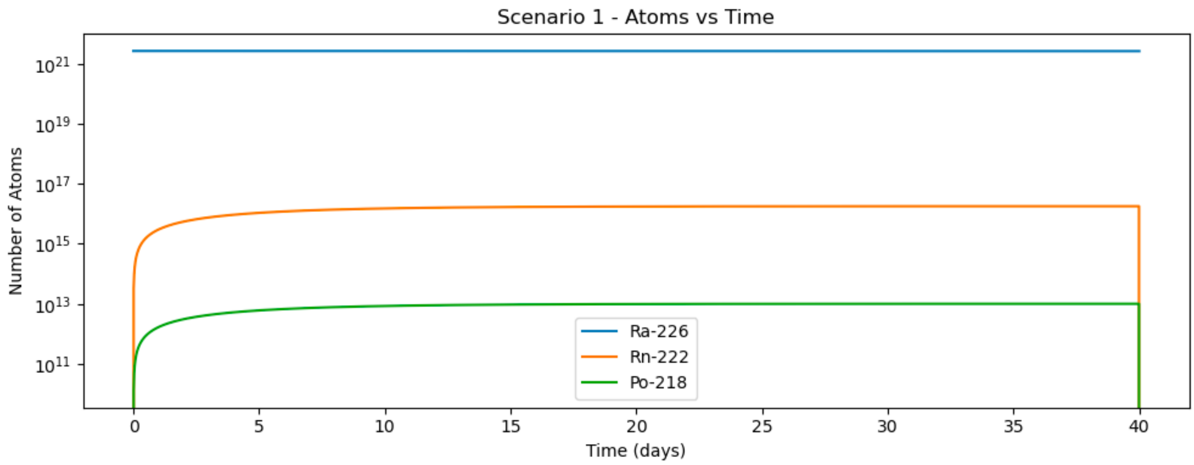
Rn vented:

- $Rn\ vented = \sum_{Vent\ times} N_{Rn}(t^{-}vent)$

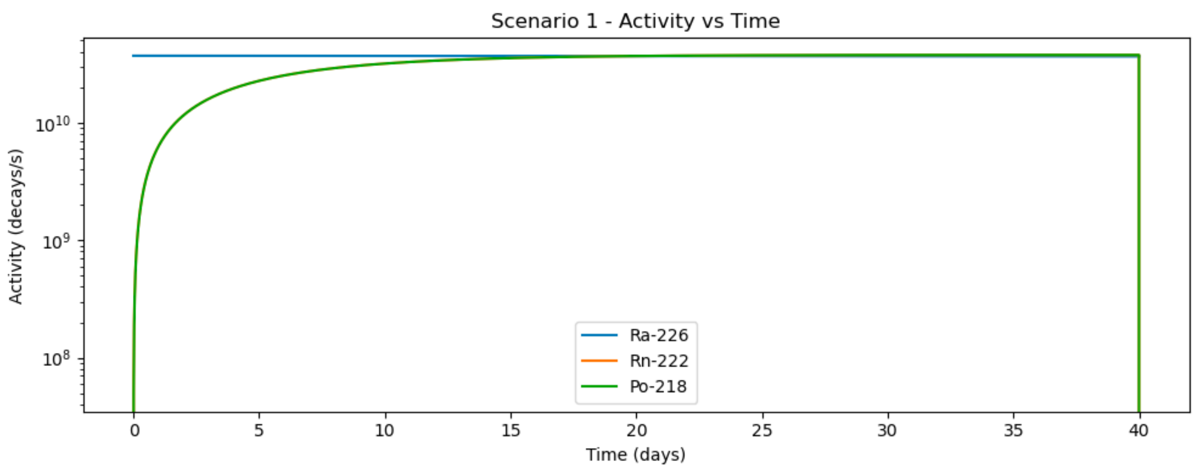
Here $N_{Rn}(t^{-}vent)$ denotes the inventory *just before* the vent; in my code I use `sol[-1,1]` or `N_current[1]` which is the post-integration, pre-vent inventory. We can see the full code in appendix 2.

Scenario 1:

In the first scenario, the system is modeled as completely closed for the entire 40-day period, with no removal of radon until the very end. This allows Rn-222 produced from Ra-226 decay to accumulate continuously, and it also leads to the production of Po-218 through the subsequent decay of radon. At the 40-day mark, we implement a single venting step, where the entire inventory of Rn-222 present at that instant is released from the system. Po-218 is also reset to zero, since it is assumed to be vented along with its parent. This scenario therefore represents a case where radon is allowed to build up to its maximum levels before being removed, making it useful for comparing against more frequent venting schedules to highlight the impact of delayed release. Here are the graphs that are outputted from scenario 1:

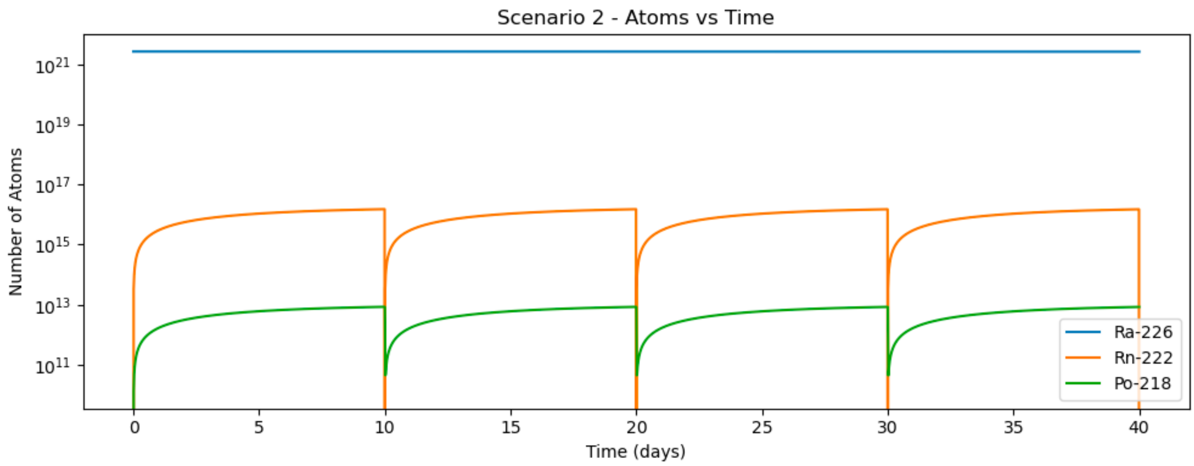


On the logarithmic scale, the Ra-226 inventory appears essentially as a flat line around 10^{21}–10^{22} atoms, which reflects its very long half-life and the fact that only a negligible fraction decays over 40 days. In contrast, Rn-222 rises rapidly at first as it is produced from Ra-226, before reaching a plateau at around 10^{15} atoms. This plateau indicates that the production rate of Rn-222 from Ra-226 has balanced its own radioactive decay, creating a dynamic equilibrium. Po-218 shows a similar pattern but at a lower magnitude, rising to about 10^{12} atoms before leveling off. This lag and lower plateau occur because Po-218 depends on the prior buildup of Rn-222 and also has a much shorter half-life. Overall, the plot illustrates how the parent remains virtually unchanged, while the daughters build up and then stabilize as equilibrium conditions are approached in the closed system.

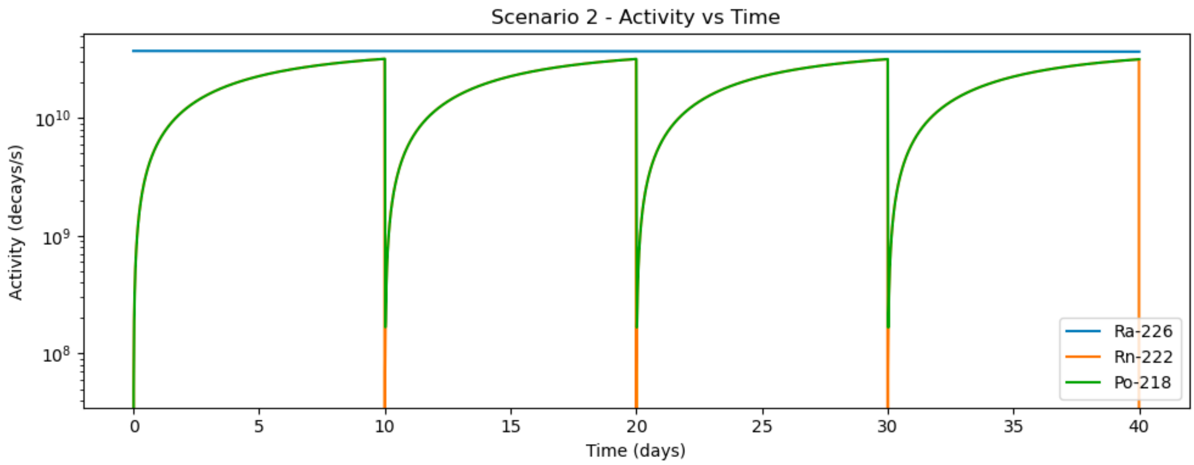


In the activity plot, Ra-226 remains nearly constant at its initial value, again reflecting its very long half-life. The activities of Rn-222 and Po-218 rise from zero and then overlap almost perfectly, forming a single curve on the logarithmic scale. This behavior occurs because in equilibrium, every decay of a Rn-222 atom produces a Po-218 atom, so their decay rates become identical. As a result, the activities of these two isotopes converge and remain locked together over time.

Scenario 2:



In the second scenario, the Ra-226 inventory again remains essentially flat around 10^{21}–10^{22} atoms, showing little change over 40 days. However, the behavior of the daughter isotopes is very different from Scenario 1. Because venting is applied every 10 days, the inventories of Rn-222 and Po-218 repeatedly drop to zero at each venting event. After each release, both isotopes begin to accumulate again as they are regenerated from the decay of Ra-226, rising back toward the same equilibrium levels observed in Scenario 1. This produces a characteristic sawtooth pattern: sharp drops to zero followed by exponential regrowth. The graph highlights how regular venting prevents long-term accumulation of radon and its progeny, in contrast to the continuous buildup seen when venting is delayed.



The activity plot for Scenario 2 shows the same general trends as the atom inventories but expressed in decay rates. Ra-226 remains essentially constant throughout, while the activities of Rn-222 and Po-218 overlap almost exactly, just as in Scenario 1. The key difference is that every 10 days both curves drop abruptly to zero, reflecting the venting of all radon (and its immediate progeny) from the system. After each venting event, the activities rise back up as Rn-222 is regenerated from Ra-226 and Po-218 follows in lockstep. This repeating cycle of removal and regrowth produces a sawtooth pattern in the activity, clearly demonstrating the impact of periodic venting on suppressing long-term buildup.

Here is a table that shows the final activities and number of atoms for each isotope after both scenarios:

Isotope	**Scenario 1 Atoms**	**Scenario 1 Activity**	**Scenario 2 Atoms**	**Scenario 2 Activity**
Ra-226	$2.630*10^{21}$	$3.663*10^{10}$	$2.630*10^{21}$	$3.663*10^{10}$
Rn-222	0	0	0	0
Po-218	0	0	$8.459*10^{12}$	$3.152*10^{10}$

Scenario 3 – Difference in total Rn-222 Released:

Comparing the two venting strategies shows a large difference in the cumulative radon release. In Scenario 1 (single vent at 40 days) the simulation indicates **1**.793×10^{16} atoms of Rn-222 are released. With regular venting every 10 days (Scenario 2) the cumulative release increases to 6.034×10^{16} atoms**,** an absolute increase of 4.241×10^{16} atoms. Put another way, Scenario 2 releases ≈3.36 times as much Rn-222 as Scenario 1 — i.e., roughly a 236% increase in total Rn-222 released when vents are performed every 10 days instead of waiting until the end. The final outputs can be seen in appendix 3. Here is a table that shows the difference:

Scenario	**Rn-222 Released (atoms)**	**Rn-222 Equivalent Activity [Bq]**
1 (vent at 40 d)	1.793×10^{16}	3.206×10^{11}
2 (vent every 10 days)	6.034×10^{16}	1.079×10^{12}
Difference (S2-S1)	4.241×10^{16}	7.584×10^{11}

Conclusion:

The results of this project are consistent and physically reasonable. Ra-226 remains largely unchanged due to its extremely long half-life relative to the 40-day simulation period. Rn-222 shows significant accumulation in Scenario 1 but is fully removed at venting, while in Scenario 2 repeated venting every 10 days prevents buildup, leading to zero inventory at each venting time but a much larger **cumulative release**. Po-218 mirrors the behavior of Rn-222, accumulating only between vents. Overall, the results make sense because they reflect the expected behavior of a short decay chain under venting: venting reduces the daughter inventory in the system but increases total environmental release, and the long-lived parent dominates the system inventory.

Appendix:

1.

```
N_sol1 = odeint(decay_chain, N0, t)
rn_released_s1 = N_sol1[-1, 1]  # all Rn present at 40 d is vented
N_sol1[-1, 1] = 0.0             # drop Rn-222
N_sol1[-1, 2] = 0.0             # drop Po-218

# -----------------------------
# Scenario 2: vent Rn every 10 days
# -----------------------------
N_sol2 = [N0[:]]
N_current = N0[:]
rn_released_s2 = 0.0
for i in range(len(t_days)-1):
    t_span = np.linspace(t[i], t[i+1], 50)
    sol = odeint(decay_chain, N_current, t_span)
    N_current = sol[-1]

    if (t_days[i+1] % 10 == 0):
        rn_released_s2 += N_current[1]  # accumulate released Rn
        N_current[1] = 0.0              # vent
    N_sol2.append(N_current)
N_sol2 = np.array(N_sol2)
```

2.

```
 #ouptut total fraction of Ra-226 burned but burn term only
total_burned = -burnup_terms["Ra226"] * (t[-1] / N0_Ra)
print(f"\nTotal fraction of Ra-226 burned (burn term only): {total_burned:.3e}")
```

3.

```
Final results - Scenario 1
Isotope        Atoms  Activity [Bq]
 Ra-226 2.630138e+21   3.663261e+10
 Rn-222 0.000000e+00   0.000000e+00
 Po-218 0.000000e+00   0.000000e+00

Final results - Scenario 2
Isotope        Atoms  Activity [Bq]
 Ra-226 2.630138e+21   3.663261e+10
 Rn-222 0.000000e+00   0.000000e+00
 Po-218 8.459180e+12   3.152396e+10

=== Rn-222 Released ===
Scenario 1 (vent at 40 d): 1.793e+16 atoms
Scenario 2 (vents at 10,20,30,40 d): 6.034e+16 atoms
Difference (S2 - S1): 4.241e+16 atoms

Total fraction of Ra-226 burned (burn term only): 1.297e-02
```