Design Optimization of Drive Screw Linear Actuator

(ME782 – Design Optimization)

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Abstract

It is often required to reduce cost or mass or any physical quantity of a product in industrial applications. Hence optimization techniques are required to carry out that analysis. Present project is done to reduce the cost of linear actuator subjected to constraints imposed by mechanical strength, space restrictions, machining limitations, and functional requirements. A mathematical model is developed to convert requirements in terms of equations. Monotonicity analysis, SQP with branch and bound method is used to optimize the cost. The genetic algorithm is also used to verify results. The results obtained in all three cases are in agreement with each other. Each of the solution methods gives various insightful information about the problem. Monotonicity analysis reveals that the d3 diameter is actually unbounded and needs to be well defined. The geometry of standard unified threads provides the required relation between d2 and d3 diameters and lower bound of d3 governed by thrust required to avoid wearing. The value of objective function greatly depends upon the choice of addition constraint, by relaxing the constraint added by bearing thrust requirement ($d3>=0.1875$) objective value can be further reduced from 0.36 to 0.32. The SQP analysis provides the Lagrange multiplier values which are critical to sensitivity analysis. For different initial guesses, SQP provides objective function value almost the same as optimum value, but with a non-integer number of teeth. Four iterations of the branch and bound technique are required to converge the solution to integer values. Sensitivity analysis shows that the objective function value is sensitive for the extra constraints added, which is expected since we have added them to bound d3. The sensitivity of the objective function is also more for L2 and L3. Any optimization problem can be solved using various methods and one should use discretion to choose the best method for the application depending on the formulation of the problem and which aspect of the problem is to be studied.

Introduction

The present design optimization problem statement is intended to optimize the design of the drive screw linear actuator to reduce its total cost of production. The drive screw has a wide range of application where rotary motion is to be converted into linear motion. Some of the common applications are Screw jack, lead screw of lathe machine, linear motors. The application ranges from light to heavy-duty applications. Considering the wide variety of application of drive screw, it is important to select the most suitable material and dimensions so as to minimize the cost while ensuring that drive screw meets constraints imposed by mechanical strength, space restrictions, machining limitations, and functional requirements. For the problem under consideration, the stainless steel is selected as material, specifying the material reduces the complications of the problem.

The linear actuator assembly consists of a prime mover, drive gear, pinion gear, and load-carrying nut and chassis which supports the whole assembly in bearing.

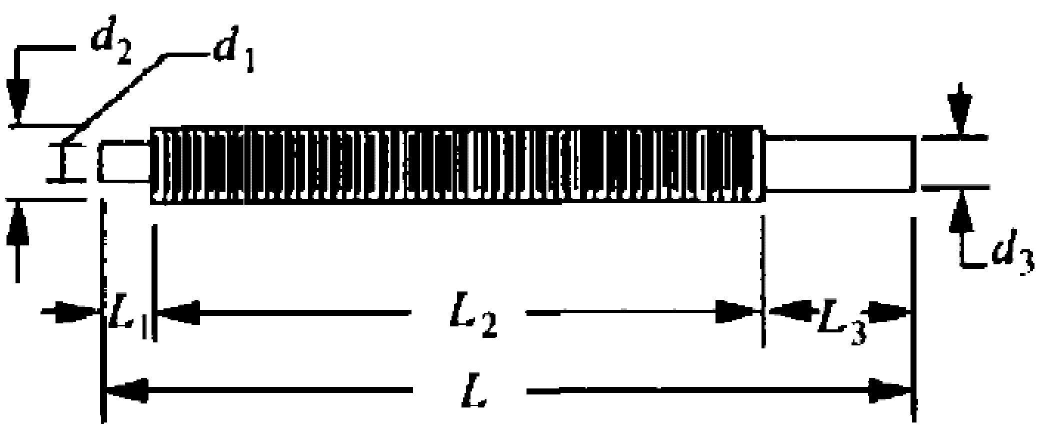

Figure 1: Schematic of design of drive screw

Objective of the Report:

The objective of the report is to minimize the material cost of drive screw linear actuator made out of stainless steel and subjected due to mechanical strength, space restrictions, machining limitations, and functional requirements

Report outline:

The report starts with the introduction of the problem and objective function. The mathematical model describes the physical significance of the problem and various constraints imposed by strength, space restriction, and functional requirements. The assumptions are enlisted thereafter. The mathematical model is followed by the optimization model. This optimization model is later on solved using monotonicity analysis, SQP, generic algorithm and sensitivity are checked. The findings of the project are then included in results and conclusion

Mathematical model:

1. Objective function

The objective of the optimization problem is to reduce the total manufacturing cost of the drive screw. The total cost of production involves the cost of machining and the cost of the material. The cost of machining is a function of the shape of drive screw, the drive screw is assumed to have definite shape indicated by screw and the only variation in geometric parameters is allowed. Variation of dimensional parameter will not affect the manufacturing process to be used and since the production rate is assumed high, the cost of machining can be assumed to be constant as long as the above two assumptions are valid. The problem statement simplifies to a reduction of cost of drive screw, which is the only function of material cost, effectively we can also model the problem as minimize weight of drive screw.

$$\mathcal{F} = C_m * \frac{\pi}{4} * \left({d_1}^2 * L1 + {d_2}^2 * L2 + {d_3}^2 * L3\right)$$

Where,

C_m = Material cost ($/inch^3)

d_1, d_2, d_3= Diameters of drive screw indicated by figure 1.

L_1, L_2, L_3 = Lengths of a screw having diameters as d_1, d_2, d_3 respectively.

2. Assumptions

1. The material of the drive screw is stainless steel.
2. Production rate is high.
3. The shape of the drive screw is pre-defined.
4. Threads are standard unified threads.

5. Assembly force is concentrated at the midpoint of a lead screw.
6. Frictional forces are only assumed to exist between threads and load nut, negligible everywhere else.
7. Slenderness ratio $L/d_i \geq 10$, in order to model drive screw as beam.

3. Design variables

d_1 = Diameter of shaft/gear interface.
d_2= Diameter of threaded portion.
d_3= Diameters of the lower bearing surface.
N_S= Number of teeth of the pinion gear.
N_m= Number of teeth on the drive gear.
N_T= Number of threads per inch.
L_2, L_3 = Lengths of a screw having diameters as d_2, d_3 respectively.
Refer to Figure 1: Schematic of design of drive screw.

4. Design parameters

T_m = Motor torque = 2 inch-ounce

S_m = Motor speed = 300 rpm

τ_{all} = Maximum allowable shear stress = 22,000 psi

σ_{all}=Maximum allowable bending stress = 20,000 psi

F_a = Force required to assembly drive screw into the assembly = 6lb

K = Stress concentration factor = 3

W =Drive screw load = 3lb

μ = friction coefficient = 0.35

L_1 =Length of gear/drive screw interface = 0.405 inch

α_n =Thread angle = 60˚

S = Linear cycle rate = 0.0583 inch/sec

5. Constraints

Design for Bending stress due to assembly force Fa

$$\frac{M}{I}\frac{d_1}{2} \leq \sigma_{all}$$

Where,

M = bending moment due to assembly force = Fa*L/2.

L = Total length of screw = $L_1 + L_2 + L_3$

Shear stress due to torsion:

Design against fatigue failure in shear. Drive screw is required to sustain a given number of stress cycle under shear stress.

$$\frac{KT({}^{d_1}/_2)}{J} \leq \tau_{all}$$

Where;

$T = Applied\ torque = C_2 * T_m * N_s/N_m$

$C_2 = \frac{1}{16}(lb/ounce)$ i.e(Conversion factor)

K = Stress concentration factor.

J = Polar moment of inertia (${}^{\pi d_1{}^4}/_{32}$).

The specified linear rate of oscillation:

The drive is required to move the load at a specific linear rate.

$$C_4 * N_m S_m/(N_s N_T) \leq S$$

$C_4 = 60^{-1}$ (no. of threads/revolution)(min/S)

No slip condition:

$(W*\frac{d_2}{2})[\frac{\pi * \mu * d_2 + N_T{}^{-1} * Cos(\alpha_n)}{\pi * d_2 Cos(\alpha_n) - N_T{}^{-1} * \mu}] \leq T$

Upper bound for threads per inch due to mass production

$N_T \leq 24$

Limit on the number of teeth on gears to avoid interference

$N_m \geq 8 \; and \; N_s \leq 52$

Space constraints: packaging consideration

$8.75 \leq L_1 + L_2 + L_3 \leq 10.0$

$7.023 \leq L_2 \leq 7.523$

$1.1525 \leq L_3 \leq 1.6525$

$d_2 \leq 0.625$

To facilitate assembly and from a manufacturing standpoint, some constraints are necessary and are not taken as constraints explicitly

$d_1 < d_2 \; and \; d_3 < d_2$

Optimization problem

The problem can be represented in negative null form by substituting the values of design parameters.

Objective function:

Minimize $\mathcal{F} = C_m * \frac{\pi}{4} * \left({d_1}^2 * L1 + {d_2}^2 * L2 + {d_3}^2 * L3\right)$

Subject to:

$$g_1 : 38.88 + 96L_2 + 96L_3 - \pi\sigma_{all}{d_1}^3 \leq 0$$

$$g_2 \ : \ 6(N_s/N_m) - \pi\tau_{all}{d_1}^3)$$

$$g_3 \ : \ 8.345 - L_2 - L_3 \leq 0$$

$$g_4 \ : \ -9.595 + L_2 + L_3 \leq 0$$

$$g_5 \ : \ L_2 - 7.523 \leq 0$$

$$g_6 \ : \ 7.023 - L_2 \leq 0$$

$$g_7 \ : \ L_3 - 1.6525 \ \leq 0$$

$$g_8 \ : \ 1.1525 - L_3 \leq 0$$

$$g_9 \ : \ d_2 - 0.625 \ \leq 0$$

$$g_{10} : \ 5(N_m/N_s) - 0.0583N_T \ \leq \ 0$$

$$g_{11} : (1.5d_2)\left[\frac{\pi\mu d_2 + \ 0.5{N_T}^{-1}}{0.5\pi d_2 - 0.35{N_T}^{-1}}\right] - 0.125(N_s/N_m) \leq 0$$

$$g_{12} : \ N_T - 24 \leq 0$$

$$g_{13} : 8 - N_m \leq 0$$

$$g_{14} : \ N_s - 52 \leq 0$$

The d_1,d_2,d_3 are continuous variables and N_M,N_T,N_S are integer variables.

Threads per inch for UN threads is integer.

Model Validity:

In Checking the solid actuator model validity, the Initial set of values infeasible set is provided to check it is giving some values.

For initial condition, x0 = [0.2,0.2,0.2,7.3,1.4,10,30,22]

fvalue= 0.36006 * C_m Model is thus validated.

Optimization Methods:

- ## Monotonicity Analysis:

Table 1: Monotonicity table

	d_1	d_2	d_3	L_2	L_3	N_m	N_s	N_T
F	+	+	+	+	+	0	0	0
g1	0	-	0	+	+	0	0	0
g2	-	0	0	0	0	-	+	0
g3	0	0	0	-	-	0	0	0
g4	0	0	0	+	+	0	0	0
g5	0	0	0	+	0	0	0	0
g6	0	0	0	-	0	0	0	0
g7	0	0	0	0	+	0	0	0
g8	0	0	0	0	-	0	0	0
g9	0	+	0	0	0	0	0	0
g10	0	0	0	0	0	+	-	-
g11	0	NA	0	0	0	+	-	-
g12	0	0	0	0	0	0	0	+
g13	0	0	0	0	0	-	0	0
g14	0	0	0	0	0	0	+	0
g15	+	-	0	0	0	0	0	0
g16	0	-	+	0	0	0	0	0

g_2 is active with respect to **d_1.**

d_3 is unbounded. We can put either lower bound to d_3 or express lower bound in terms of some other diameter or any other variables.

$$g_{17}: 0.1875 - d_3 \ \leq 0 \ \boldsymbol{or} \ (\ k * d_2) - d_3 \ \leq 0$$

$$h_{10}: \ d_3 = d_2 - \frac{1.299}{N_T}$$

Constraint g17 describes the minimum diameter for a lower bearing surface. This is required to provide adequate thrust to keep the shaft from wearing into lower bearing support.

h_{10} is redefined as strict equality such that, $d_3$must be equal to the root diameter of nominal thread diameter d_2.

g_{17} constraints become active with respect to **d_3**

$$d_3 = 0.1875$$

$$0.1875 = d_2 - \frac{1.299}{N_T}$$

$$d_1 = \left(\frac{6(\frac{N_s}{N_m})}{\pi\tau_{all}}\right)^{1/3}$$

Analyzing the monotonicity for L_2 and L_3.

From the Monotonicity table, It can be observed that objective function and constraints g_3 , g_6 have opposite Monotonicity with respect to variable L_2 . Similarly, objective function and constraints g_3, g_8 have opposite Monotonicity with respect to variable L_3.

From the monotonicity table, either g_3 is active with respect to both L_2 and L_3 or g_6 is active with respect to L_2 and g_8 is active with respect to L_3. We can evaluate both the options one by one.

Let's assume that g_3 is active with respect to L_2.

$$L_2 = -L_3 + 8.345 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

	L_2	L_3
F	+	+
g3	-	-
g4	+	+
g5	+	0
g6	-	0
g7	0	+
g8	0	-
h10	0	0

The new formulated problem becomes

$$f = C_m * \left(\frac{\pi}{4}\right) * (0.405 * d_1^2 + L_3(d_3^2 - d_2^2) + 8.345 * d_2^2)$$

$$g4:\ -1.25 \le 0$$

$$g5:\ -L_3 + 0.822 \le 0$$

$$g6: L_3 - 1.322 \le 0$$

$$g7: L_3 - 1.6525 \le 0$$

$$g8: -L_3 + 1.1525 \le 0$$

$$h_{10}: -d_3 + d_2 - \frac{1.299}{N_T} = 0$$

As d_3 is always less than d_2. The function has negative monotonicity with respect to variable L_3.

	L_3
F	-
g4	0
g5	-
g6	+
g7	+
g8	-

From the above table, g6 and g7 have opposite monotonicity with respect to L_3.

$$g6: L_3 \leq 1.322$$

$$g7: L_3 \leq 1.6525$$

g6 is dominating constraint over g7. Hence g6 has to be active with respect to L_3

$$L_3 = 1.322$$

$$L_2 = 7.023 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots From(1)$$

$d_3 = d_2 - \frac{1.299}{N_T}$=0.1875

Substituting all above values of the variable to formulate new objective function

$$N_T = \left[\frac{1.299}{d_2 - 0.1875}\right]$$

$$d_1 = \left(\frac{6(\frac{N_s}{N_m})}{\pi \tau_{all}}\right)^{1/3} = 0.04428 * \left(\frac{N_s}{N_m}\right)^{1/3}$$

$$L_3 = 1.322$$

$$L_2 = 7.023$$

Substituting above values and reformulating problem,

Model 1:

$$f = C_m * \left(\frac{\pi}{4}\right) * \left[0.0179327 \left(\frac{N_s}{N_m}\right)^{1/3} + 7.023\, d_2^2 + 0.0464765\right]$$

$$g1: 0.013369 - d_2^3 \leq 0$$

$$g9: d_2 - 0.625 \leq 0$$

$$g10:\ 5\left(\frac{N_m}{N_s}\right) - 0.0583\ \left[\frac{1.299}{d_2 - 0.1875}\right] \leq 0$$

$$g11: 1.5\ d_2\left[\frac{\pi f d_2 + \frac{0.5}{\left[\frac{1.299}{d_2-0.1875}\right]}}{0.5\pi d_2 - \frac{f}{\left[\frac{1.299}{d_2-0.1875}\right]}}\right] - 0.125\left(\frac{N_s}{N_m}\right) \leq 0$$

$$g12:\ \left[\frac{1.299}{d_2 - 0.1875}\right] - 24 \leq 0$$

$$g13: -N_m + 8 \leq 0$$

$$g14: N_s - 52 \leq 0$$

$$g15: -d_2 + 0.04428 * \left(\frac{N_s}{N_m}\right)^{1/3} \leq 0$$

Monotonicity table for d_2

g_{11} have positive monotonicity with respect to d_2 in the range we are operating

	d_2
F	+
g1	-
g9	+
g10	+
g11	+
g12	-
g15	-

As g_{15} cannot be active with respect d_2, as these constraints are added additionally to ensure $d_2 > d_3$ and $d_2 > d_1$ (strict inequality)

Constraint that can be active is g1 and g12

$$g1: d_2 \geq 0.237337$$

$$g12: d_2 \geq 0.2416$$

g_{12} is dominating constraint over g1 and hence g1 is active.

$$d_2 = 0.2416$$

From equality constraint, we found that

$$N_T = 24$$

Monotonicity table for N_m

	N_m
F	-
g10	+
g11	+
g13	-

g13 is active with respect to N_m.

$N_m = 8$

Now,g_{12} is active

$N_T = 24$

$g10\colon N_s \geq 28.58$

$g11\colon N_s \geq 17.81$

Hence g_{10} is dominating constraint over g_{11}.

Hence constraint g10 is active.

As the given program is of integer type we can reformulate the problem with dominating constraints.

Model 2:

$$f = C_m * \left(\frac{\pi}{4}\right) * \left[0.0179327\left(\frac{N_s}{N_m}\right)^{1/3} + 0.44835\right]$$

$$g10\colon\ 5\left(\frac{N_m}{N_s}\right) - 1.3992 \leq 0$$

$$g13\colon - N_m + 8 \leq 0$$

$$g14\colon N_s - 52 \leq 0$$

Monotonicity for N_m and N_s

	N_m	N_s
f	-	+
g10	+	-
g12	0	0
g13	-	0
g14	0	+

From monotonicity table constraint g10 is active.

$$\left(\frac{N_m}{N_s}\right) = 0.01166\, N_T = 0.27984$$

$$\left(\frac{N_s}{N_m}\right) = 3.57347$$

$$d_1 = \left(\frac{6(\frac{N_s}{N_m})}{\pi\tau_{all}}\right)^{1/3} = 0.04428 * \left(\frac{N_s}{N_m}\right)^{1/3} = 0.067697$$

Because function form in objective function and constraints (active) is the same, problem possesses multiple optimal solutions.

One of the solutions can be

$N_s = 29$

$N_m = 8$

$N_T = 24$

Total solution can be

$\boldsymbol{d_1 = .067697}$

$\boldsymbol{d_2 = 0.2416}$

$\boldsymbol{d_3 = 0.1875}$

$\boldsymbol{L_2 = 7.023}$

$\boldsymbol{L_3 = 1.322}$

$\boldsymbol{N_T = 24}$

$\boldsymbol{\left(\frac{N_s}{N_m}\right) = 3.57347}$; $\boldsymbol{N_m = 8}$; $\boldsymbol{N_s = 29}$

Value of Objective function = 0.3600*C_m

- SQP with the help of Branch and Bound method:

Octave is used to solve the problem.

Code is sent in zip file.(.m File)

Initial solution by SQP:

For initial condition, x0 = [0.2,0.2,0.2,7.3,1.4,10,30,22],

X = [0.069325, 0.241625, 0.187500,7.023000, 1.322000, 8.008071,30.733496, 24.000000]

Fmin = 0.36006

```
>> screw
warning: function 'screw' defined within script file 'C:\Users\lenovo\Desktop\Octave\Practice\Nonlinear\screw.m'
Symbolic pkg v2.8.0: Python communication link active, SymPy v1.4.
warning: function 'screw' defined within script file 'C:\Users\lenovo\Desktop\Octave\Practice\Nonlinear\screw.m'
warning: called from
    screw at line 25 column 20
x =

    0.069325
    0.241625
    0.187500
    7.023000
    1.322000
    8.008071
   30.733496
   24.000000

fmin =  0.36006
info =  103
iter =  100
```

Lower bound (8.008071,30.733496, 24.000000) = 0.36006

Upper bound (8,30,24) = 0.36006

First iteration

Branching x(7)

Program name - Screw31	Screw32
X(7)>=31	X(7)<=30
(8.01528,31.2556,24)	(8.011592,29.9665,24)
0.36007	0.36004

Going ahead with **g_{18}:** **X(7)<=30**

Second iteration Branching x(7)

Screw21	Screw22

X(7)>=30	X(7)<=29
(8.0097,30,24)	(8.007,28.958,24)
0.36004	0.36000

Going ahead with g_{19} : **X(7)<=29**

Third iteration

Branching x(7)

Screw41	Screw42
X(7)>=29	X(7)<=28
(8.0078,29,24)	Infeasible
0.36000	No solution

Going ahead with g_{20} : **X(7)>=29**

Fourth iteration

Branching x(6)

Screw51	Screw52
X(6)>=9	X(6)<=8
Infeasible	(8.0000,29.0000,24.0000)
No solution	0.36000

Adding this constraint g_{21} : **X(6)<=8**

The final solution is

X = [0.068019, 0.241625, 0.187500, 7.023000, 1.322000, 8.000000, 29.000000, 24.000000]

Fmin = 0.36000

Output Image:

```
>> screw52
Symbolic pkg v2.8.0: Python communication link active, SymPy v1.4.
warning: function 'screw31' defined within script file 'C:\Users\lenovo\Desktop\Octave\Practice\Nonlinear\screw31.m'
warning: called from
    screw52 at line 29 column 20
x =

    0.068019
    0.241625
    0.187500
    7.023000
    1.322000
    8.000000
   29.000000
   24.000000

fmin =  0.36000
info =  104
iter =  10
```

- Using Evolutionary Algorithm (Genetic Algorithm):

Genetic Algorithm evolutionary method rather than gradient-based methods. It creates points (embryo with better characteristic) with better results. We used this code to verify the results.

The code is sent separately.

As the problem has multiple solutions because the ratio $\left(\frac{N_s}{N_m}\right) = \mathbf{3.57347}$

Hence $N_s$and N_m can take any values that have ration nearly equal to 3.57347

but values of $\boldsymbol{d_1 = 0.0711, d_2 = 0.2416, d_3 = 0.1875, L_1 = 7.0231, L_2 = 1.3210}$

are in close agreement with SQP results($d_1 = 0.068, d_2 = 0.2416, d_3 = 0.1875, L_1 = 7.023, L_2 = 1.322$) and monotonicity results.

Fval=0.3601*C$_m$

```
Command Window
x =

    0.0711    0.2416    0.1875    7.0231    1.3210    8.0000   31.0000   23.0000


fval =

    0.3601

>>
```

Results:

Problem is solved used 3 technique

1) Monotonicity analysis

2) SQP with branch and bound algorithm

3) Genetic Algorithm

Method/Variables	d_1	d_2	d_3	L_1	L_2	N_s	N_m	N_T	**Function Value**
Monotonicity Analysis	0.06769	0.2416	0.1875	7.023	1.322	8	29	24	**0.3600*C_m**
SQP with Branch and Bound Method	0.068	0.2416	0.1875	7.023	1.322	8	29	24	**0.3600*C_m**
Genetic Algorithm	0.0711	0.2416	0.1875	7.0231	1.3210	8	29	24	**0.3601*C_m**

These values of Objective function depend on the assumption made for well boundedness of variable d_3. For our case, we have taken $d_3 \geq 0.1875(referance\ from\ literature)$ and used a geometric constraint written below which needs to be satisfied

$$d_3 = d_2 - \frac{1.299}{N_T}$$

Objective value can be further reduced by changing the lower bound on d_3 **depending upon an assumption made**. (0.3200*C_m). Sensitivity analysis mentioned below also shows the maximum value of Lagrange multiplier with respect to constraint added for d_3.

Discussions:

Problem solution is converged and results are in good agreement.

By changing the initial guess Algorithm give different values of two variables but the same values of all other variables with the same cost function value. Also from monotonicity analysis, it can be concluded that the problem has **multiple optimum** points.

This is because referring to Model 2 in the monotonicity section, Function form of cost function is similar to one of the active constraints (g_{10}).

Model 2:

$$f = C_m * \left(\frac{\pi}{4}\right) * \left[0.0179327\left(\frac{N_s}{N_m}\right)^{1/3} + 0.44835\right]$$

$$g10:\ 5\left(\frac{N_m}{N_s}\right) - 1.3992 \leq 0$$

$$g13: -N_m + 8 \leq 0$$

$$g14: N_s - 52 \leq 0$$

$\frac{N_s}{N_m}$ Takes a specific value (3.57347).

Hence any Integer values of N_s and N_m having a ratio of approximately equal to 3.57347 is the solution.

Another solution is

$$d_1 = 0.068, d_2 = 0.2416, d_3 = 0.1875, L_1 = 7.023, L_2 = 1.322, N_m = 12, N_s = 43, N_T = 24$$

From the SQP algorithm with the brand and bound method, the values of Lagrange Multipliers are obtained.

All the **Lagrange multipliers** are either **zero or positive**. Hence satisfies constraints imposed by **KKT conditions.**

Sensitivity Analysis:

Lagrange values are obtained with the help of SQP with branch and bound method. Results obtained are tabulated as follows:

Table 2: Sensitivity Analysis

Active Constraints	**Lagrange Multiplier**
g_2	**0.0000451**
g_3	**0.02711**
g_6	**0.0182419**
g_{10}	**0**
g_{12}	**0.006011**

g_{17}	**3.05**
g_{20}	**0.00012263**
g_{21}	**0.00003383**
H	**2.6655**

- All the **Lagrange multipliers** are either **zero or positive**. Hence satisfies constraints imposed by **KKT conditions.**
- Lower bound on d_3 has the largest Lagrange multiplier value. Hence cost function is most sensitive with respect to d_3. This is because initially, d_3 was unbounded (evident from monotonicity table), to make the problem well-bounded the constraint is added which is active.
- Lengths L_2 and L_3 also have a dominating effect on cost function than other variables. These lengths are constrained by space consideration.
- Other variables do not have significant changes in objective function like d_1.
- Remaining all constraints which are not active have Lagrange multipliers zero.

Output Image

```
fmin =    3.600042147057215e-01
info =  104
iter =  10
nf =  26
inlambda =

   2.665535223256100e+00
   0.000000000000000e+00
   4.510785280119698e-05
   2.761165424198330e-02
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   3.383089329002606e-05
   1.226369643720522e-04
   0.000000000000000e+00
   0.000000000000000e+00
   3.054896377404667e+00
   1.824196058328324e-02
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   0.000000000000000e+00
   6.011337225055708e-03

>> |
```

Conclusion:

- Based on Solution Obtained the resulting diameters are in positive fraction up to three digits. In actual case making such Diameters is not possible. Those have to be rounded off at the cost of some increase in the objective function. The decrease in cost function will determine whether to go with such precision diameters or not. More constraints required to be added to make more realistic values of the design variable.

- Any optimization problem can be solved using various methods and one should use discretion to choose the best method for the application depending on the formulation of the problem and which aspect of the problem is to be studied. This project is solved by monotonicity analysis, SQP with branch and bound, genetic algorithms. Problems where sensitivity is prime importance along with the solution, SQP holds the advantage.
- SQP with branch and bound method is better for the current project where sensitivity study can be easily done. Monotonicity analysis gave us a complete solution in this case but not necessarily always. The Genetic algorithm is also used to solve this problem. Genetic algorithm is a very powerful tool which can solve complex problems using the evolutionary method

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