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Assignment - r
Parameter Estimation

1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2, --- x_n$

$L(x_1, x_2, x_3 -- x_n) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_1-\mu)^2}{2\sigma^2}} \right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_2-\mu)^2}{2\sigma^2}} \right)$

Taking ln both sides.

$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$ — ①

derivative with respect to ①

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1} \frac{2(x_i-\mu)^2}{2\sigma^2}$

$= \sum_{i=1}^{n} (x_i - \mu) = 0$

$n x_i - n\mu = 0$

$x_i = \mu$

$\theta_1 = \mu$ is the sample mean.

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Taking derivative w.r.t. σ^2 of eq①

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i-\mu_i)^2}{2(\sigma^2)^2} = 0$$

$$-n + \sum_{i=1}^{n} \frac{(x_i-\mu_i)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu_i)^2$$

here

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu_i)^2$$

2) Binomial

$$^nC_{x_i}\ \sigma^{x_i}(1-\sigma)^{n-x_i}$$

$$L = \prod_{i=1}^{n}\ ^nC_{x_i}\ \sigma^{x_i}(1-\sigma)^{n-x_i}$$

log both side

$$\log L = \sum_{i=1}^{n} \log(^nC_{x_i}) + \log \sigma^{x_i} + \log(1-\sigma)^{n-x_i}$$

differention with respect to σ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\sigma}\sum x_i = \frac{n^2}{1-\sigma} + \frac{\sum x_i}{1-\sigma}$$

$$\frac{1}{\theta(1-\theta)} \sum x^i - \frac{n}{1-\theta}$$

$$\frac{\sum x^i}{\theta} = n^2$$

$$\theta = \frac{\sum x^i}{n^2}$$