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Thapar Institute of Engineering and Technology, Patiala
Department of Mathematics
Auxilary Examination

Date/Duration: 20-02-2025/3 Hours	Course Code: UMA401/UCS410
Max. Marks: 100	Course Name: Probability and Statistics
Faculty (Dr.): Mamta Gulati.	

- **There are a total of 5 Questions each having two parts. Attempt all questions.**
- **Non programmable calculators are allowed.**
- **Appropriate tables are given at the end of the paper.**

1. (a) State and prove the total probability theorem. And hence show that for the partition E_i for $i = 1, \ldots, k$ for the sample space and an event A such that $P(A) > 0$,

$$P(E_r|A) = \frac{P(E_r)P(A|E_r)}{\sum_{i=1}^{k} P(E_i)P(A|E_i)}$$

(b) For the probability density function for continuous random variable X given by

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2, \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the expected value of X and X^2, i.e. $E(X)$ and $E(X^2)$.
(ii) Using the above find the value of $E((2X+1)^2)$.

[10 + 10 marks]

2. (a) 4 coins are tossed. Let X be the number of heads and Y be the number of heads minus the number of tails.

(i) Find the probability mass function of X and Y.
(i) Find $P[-2 \leq Y \leq 4]$.

(b) If X is a normal random variable with mean $\mu = 40$ and standard deviation $\sigma = 6$, find the value of x that has (a) 45% of the area to the left and (b) 14% of the area to the right.

[10 + 10 marks]

3. (a) For $i = 1, 2, \ldots, n$, if each X_i is a random variable having normal distribution with mean μ and variance σ^2 and $\bar{X}$ is random variable representing the sampling mean distribution, then show that

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

is an biased estimator for the population variance.

(b) If X is a Poisson's random variable such that $P(X = 1) = P(X = 2)$, then find the value of $P(X = 4)$.

[12 + 8 marks]

4. (a) A population has a density function given by

$$f(x) = \begin{cases} (k+1)x^k; & 0 \le x \le 1, \\ 0, & otherwise. \end{cases}$$

For n-observations $x_1, x_2, \ldots, x_n$ made from this population, find the maximum likelihood estimate of k.

(b) Let the random variable X has a probability density function

$$f(x) = \begin{cases} 3(1-x)^2; & 0 < x < 1, \\ 0, & otherwise. \end{cases}$$

then show that $Y = (1-X)^3$ is a uniformly distributed random variable.

[12 + 8 marks]

5. (a) In a random sample of 400 students in TIET, 120 are female. For the collected sample can it be said that males and females are in the ratio of 5 : 3 in the population at 1% level of significance?

(b) For two samples of paints (A and B) with sizes 40 and 50, the mean drying time is 30 hours and 42 hours respectively. The corresponding values for $\sigma_A = 6$ and $\sigma_B = 8$. Find the 96% confidence interval for $\mu_A - \mu_B$.

[10 + 10 marks]

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976

Note: Evaluated answer sheets will be shown on 28-02-2025 at 5:00 PM in room number G256.

-End of question paper-